**Report of the Year Work 2017-2018**

Zhongtian Cai

898708

**Q. 1**

Calculating the maximum length for a single element:

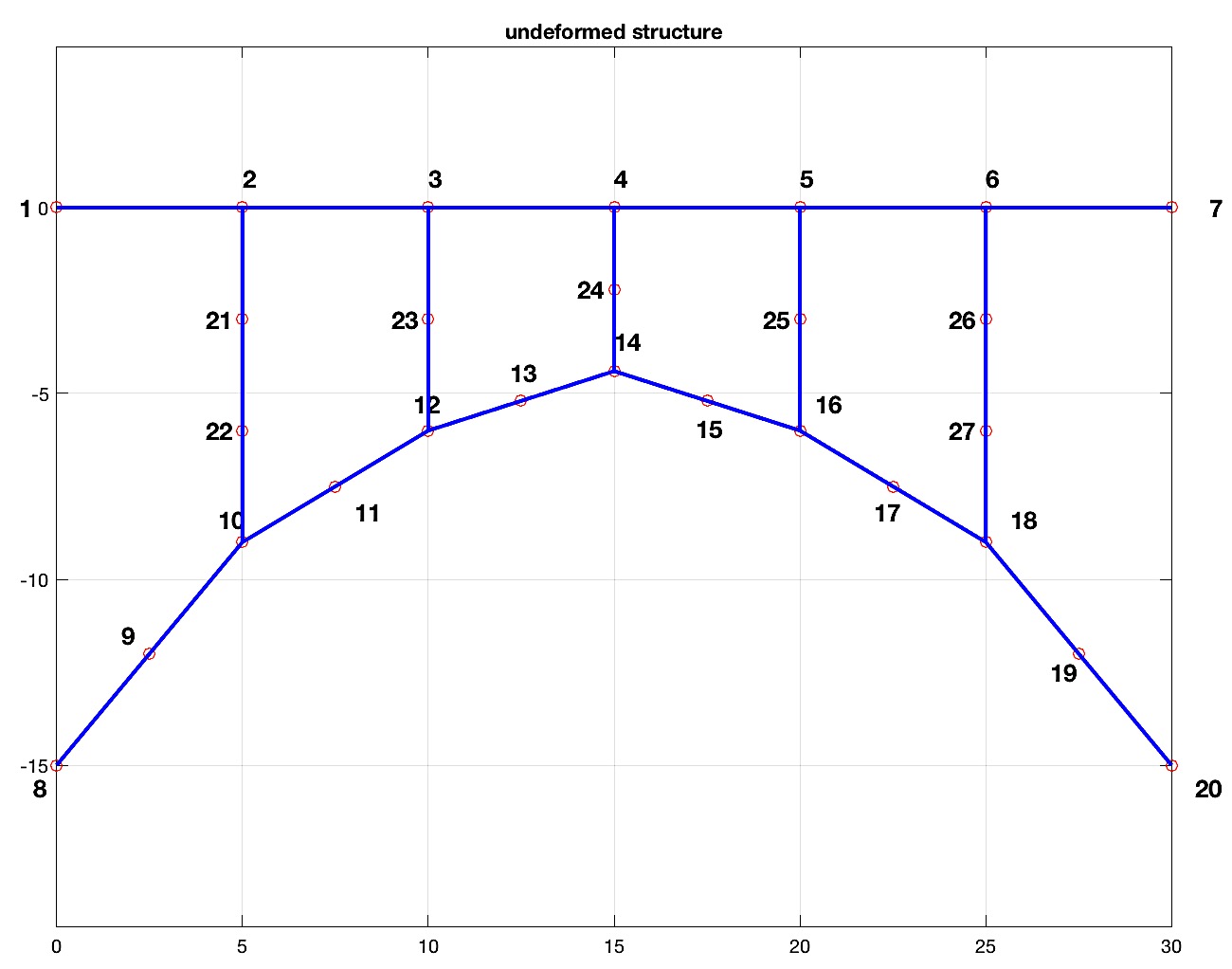
(L10-p.8)

Considering a safety coefficient ,

Thus for red beams,

and for blue beams,

According to the maximum length of each element, subdivide the structure into 27 nodes and 30 beam elements:



**Q. 2**

The structure’s natural frequencies and modes of vibration could be calculated simply using the function in the given software. Also it can be calculated using MATLAB after exporting the structural matrices and some procedures of partitioning. The first few natural frequencies (ascendingly sorted, considering a maximum of 25 Hz) are:

[Hz]

4.1273

8.7453

14.8520

16.0560

16.9685

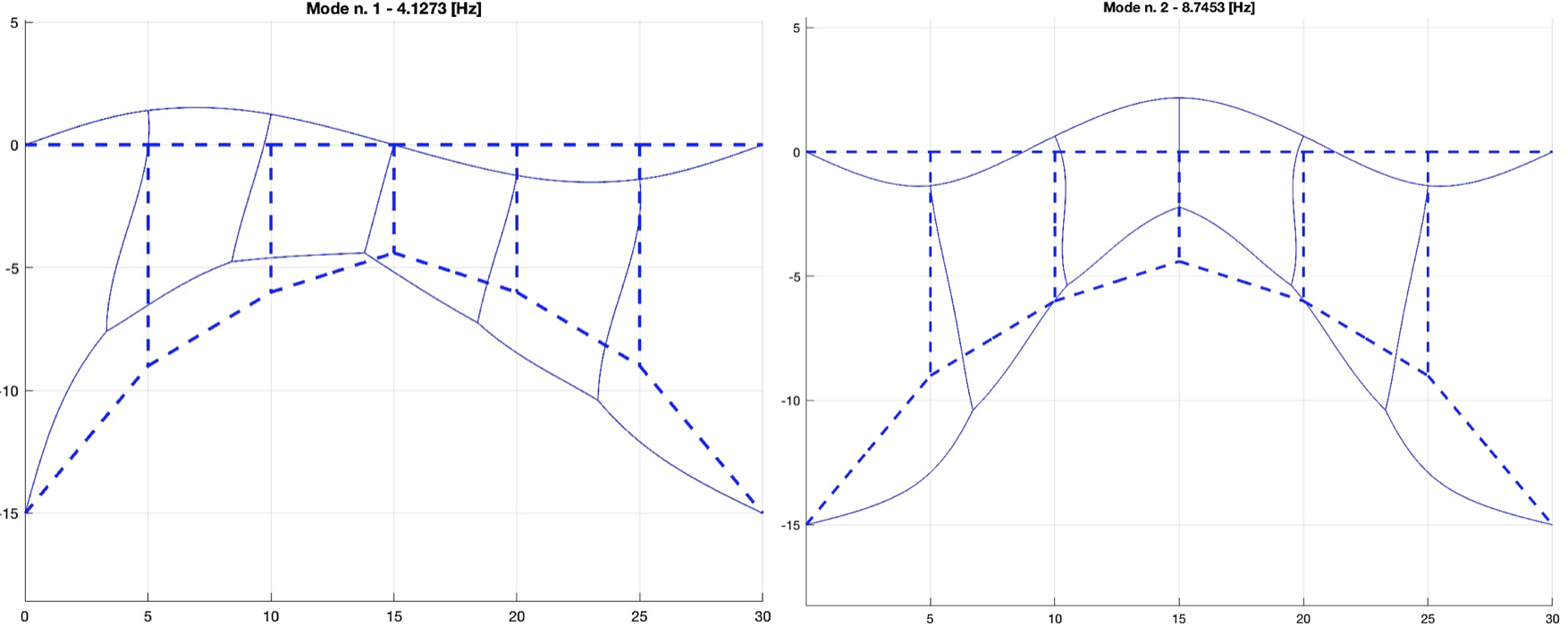
18.8969

22.0651

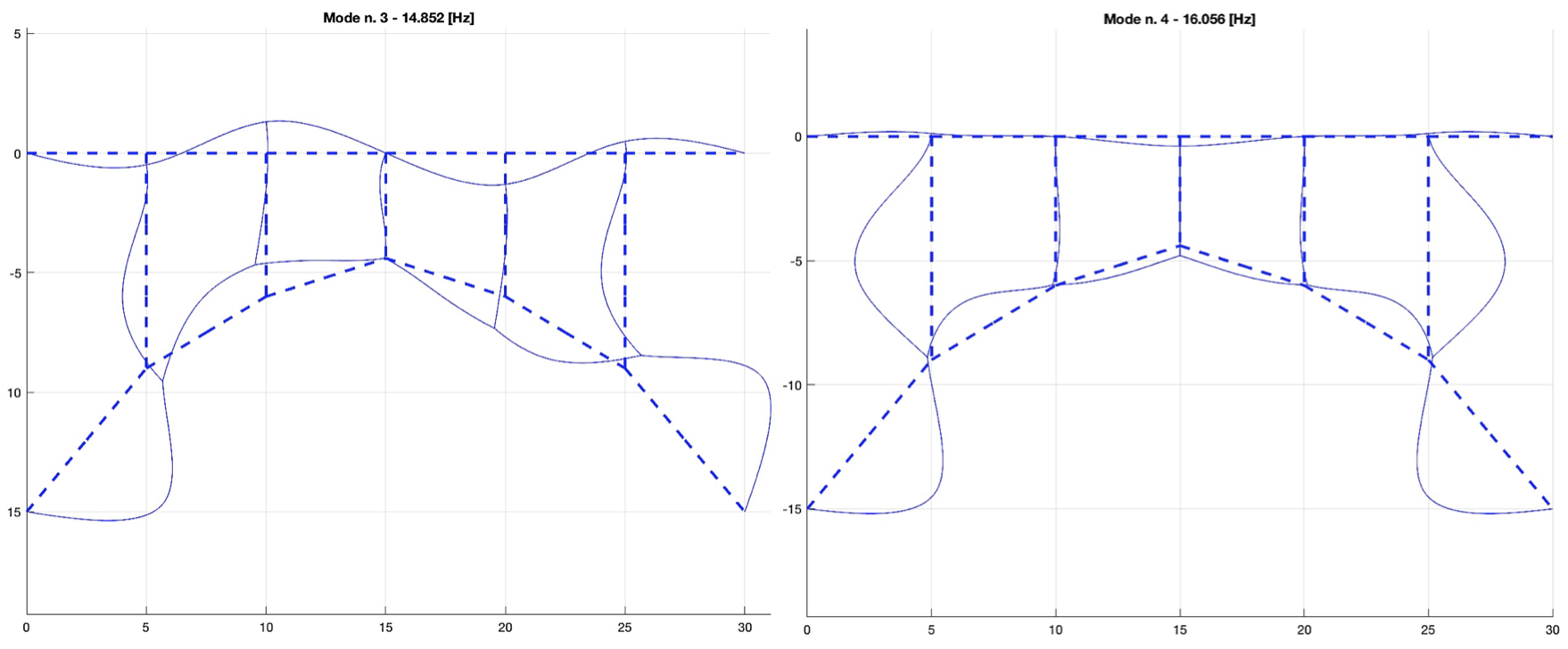
24.8797

…

The plot of Mode No. 1&2 with the scale factor equals to 8,



The plot of Mode No. 3&4 with the scale factor equals to 8,



**Q. 3**

For calculating the natural frequencies of the damped structure, we can formulate

then substitute the formulas with

one can get

where

Assuming

By the definition of the eigenvalues and eigenvectors, one can know that is exactly the eigenvalues of the matrix where is the number of d.o.f. of the system.

To be noticed that the , and here in our case should actually be defined as , and all with dimension , which for sure need further partitioning after being exported from the software.

are in form of complex conjugate pairs like with . Here the are the natural frequencies of the damped structure we are looking for. Using MATLAB it is easy to find that the first few natural frequencies (ascendingly sorted, considering a maximum of 25 Hz) are:

[Hz]

4.1268

8.7450

14.8518

16.0557

16.9683

18.8966

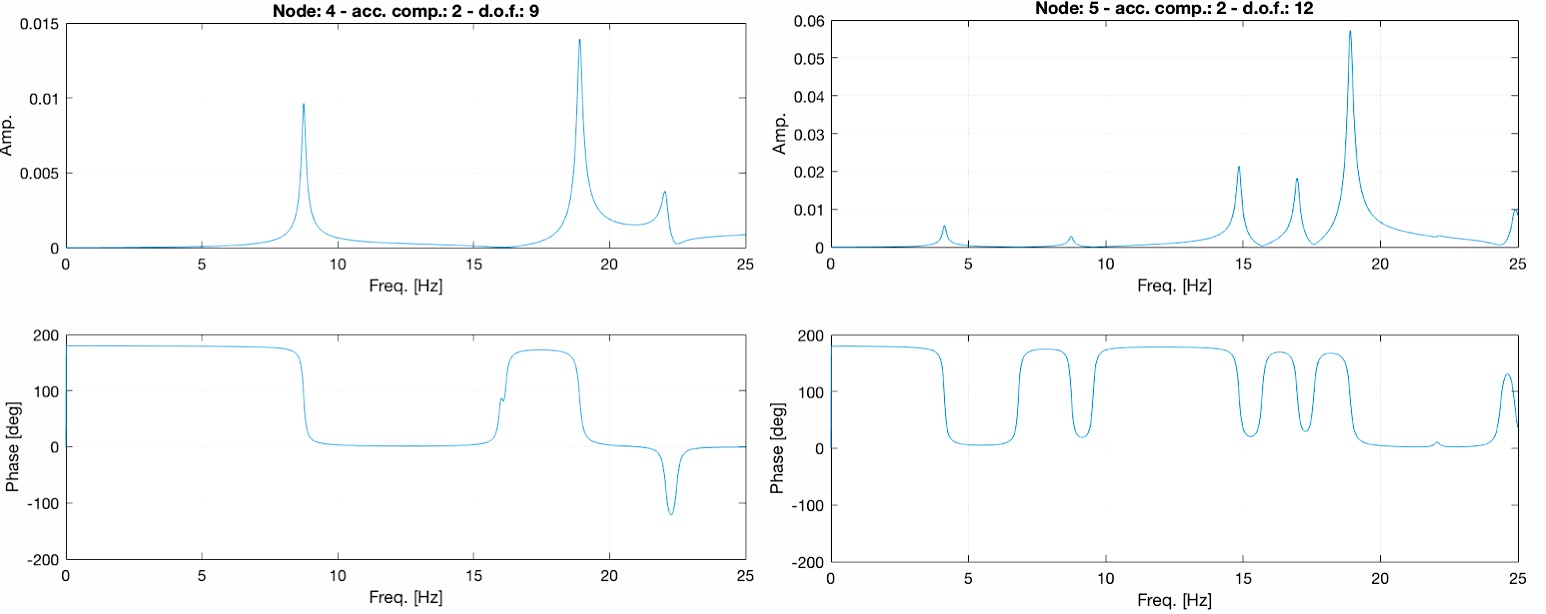
22.0649

24.8794

…

**Q. 4**

Simply applying the built-in function of the software, one can get plots as follows:



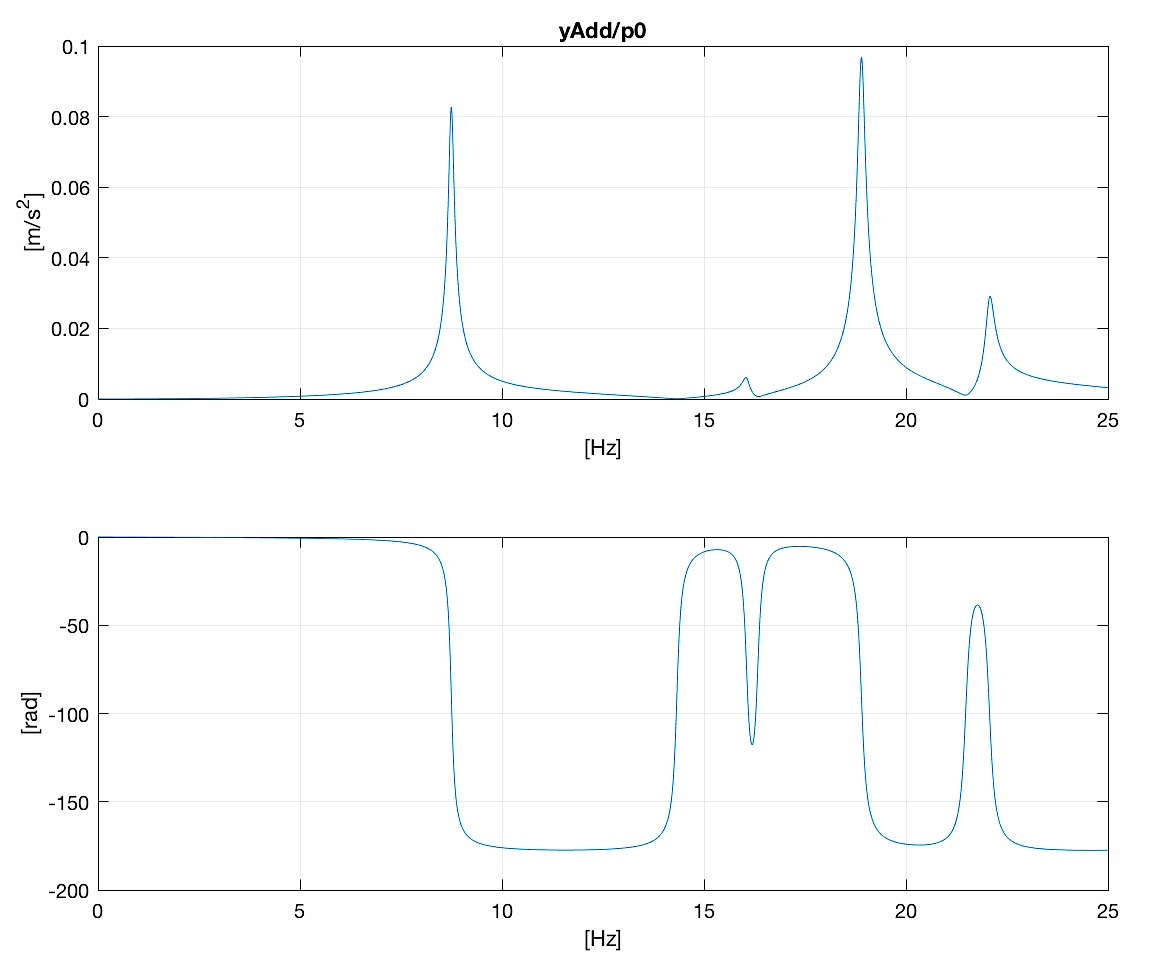
One can also derive the same result from MATLAB, calculating the frequency response respectively at the 9th and 12th degree of freedom (as is shown in the above plots).

**Q. 5**

For an input of a distributed shear force, the expression of energetic quantities Virtual work of time dependent forces is

where

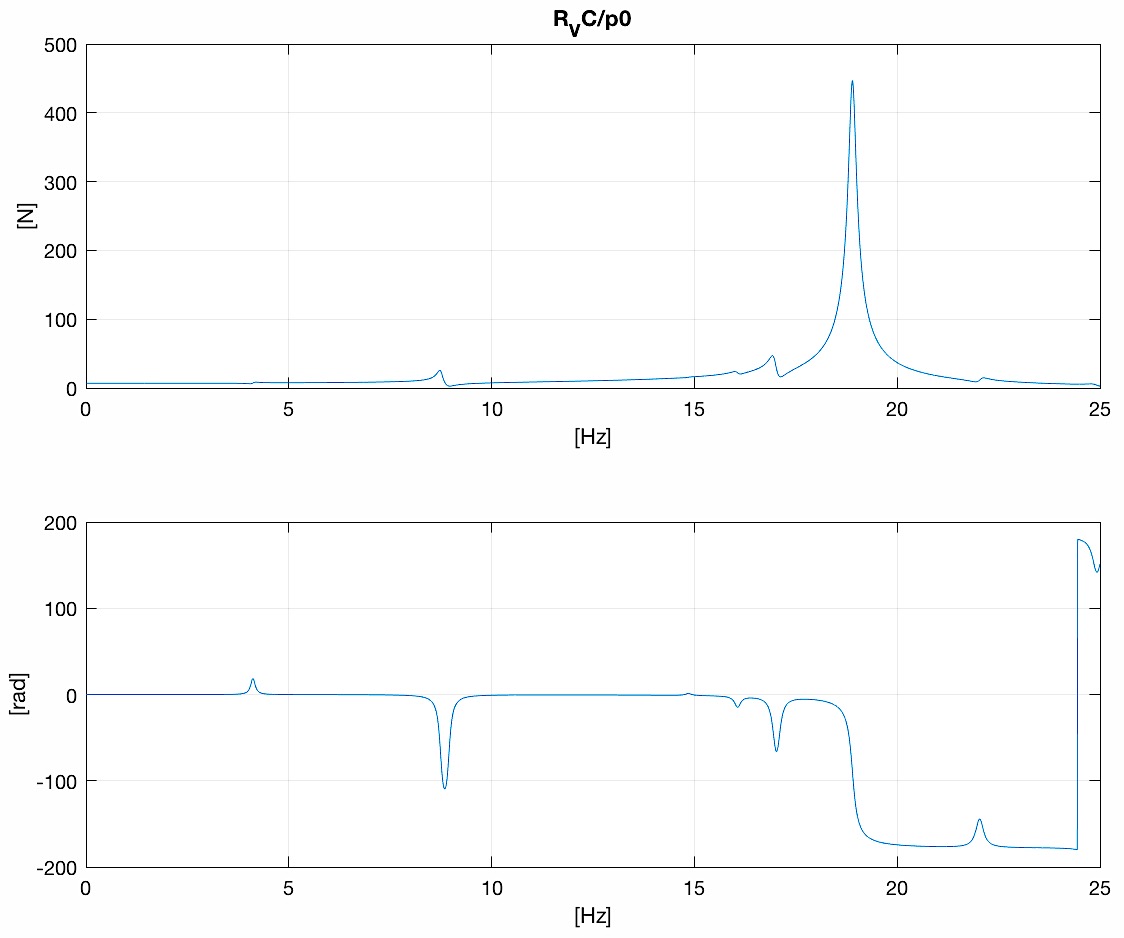
In our case, the distributed force is loaded on the element No.2 (between node No. 2&3) and the element No.3 (between node No. 3&4), where the local coordinates happen to be identical with the global coordinate, with data , . Thus one can derive the vector , taking into account the loads overlaid on node No.3 (d.o.f. No. 6&7). It is shown as following the Bode diagram of the FRF of the vertical acceleration at Point A:



As for the constraint force, using the equation

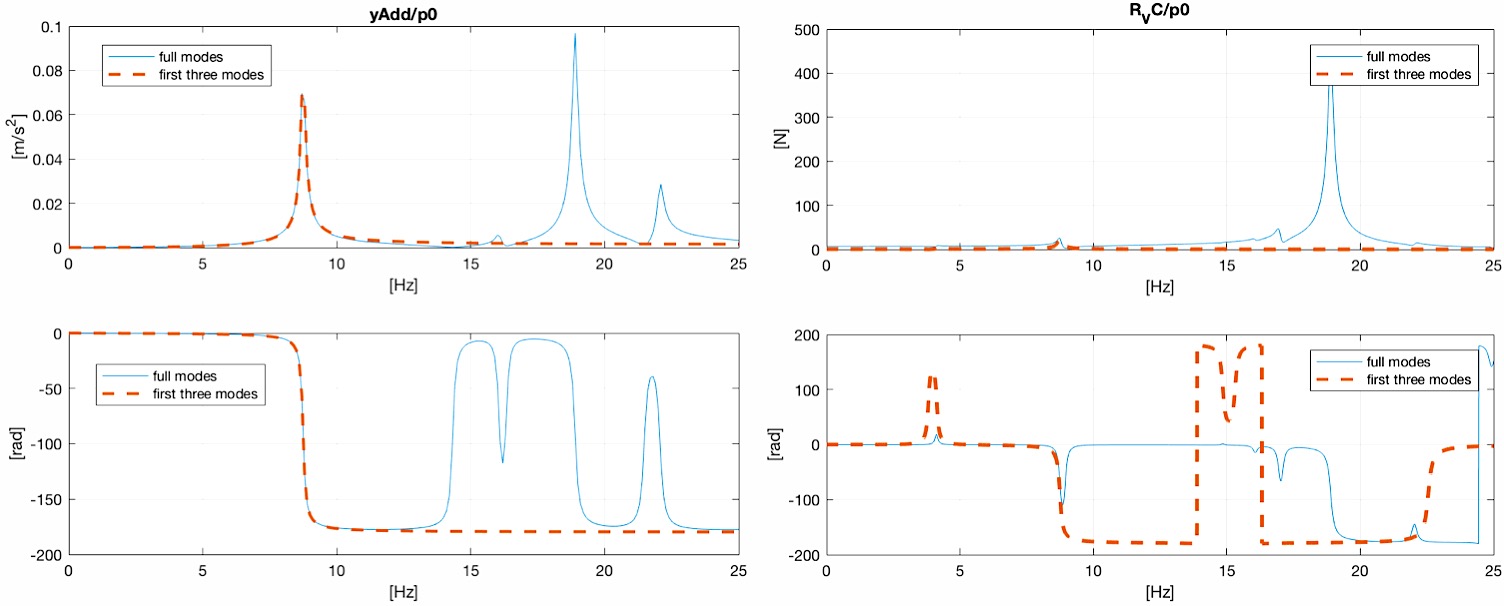
where is the unknown constraint force, is the known forces applied to the constraints which equals to , is the displacements of the constraints which also equals to . Thus,

The corresponding Bode diagrams are as follows:



**Q. 6**

By applying coordinate transformation indicated in L03, and then truncating all the modes other than the three with lowest frequencies which are the first three main and important modes, one can get the plots:

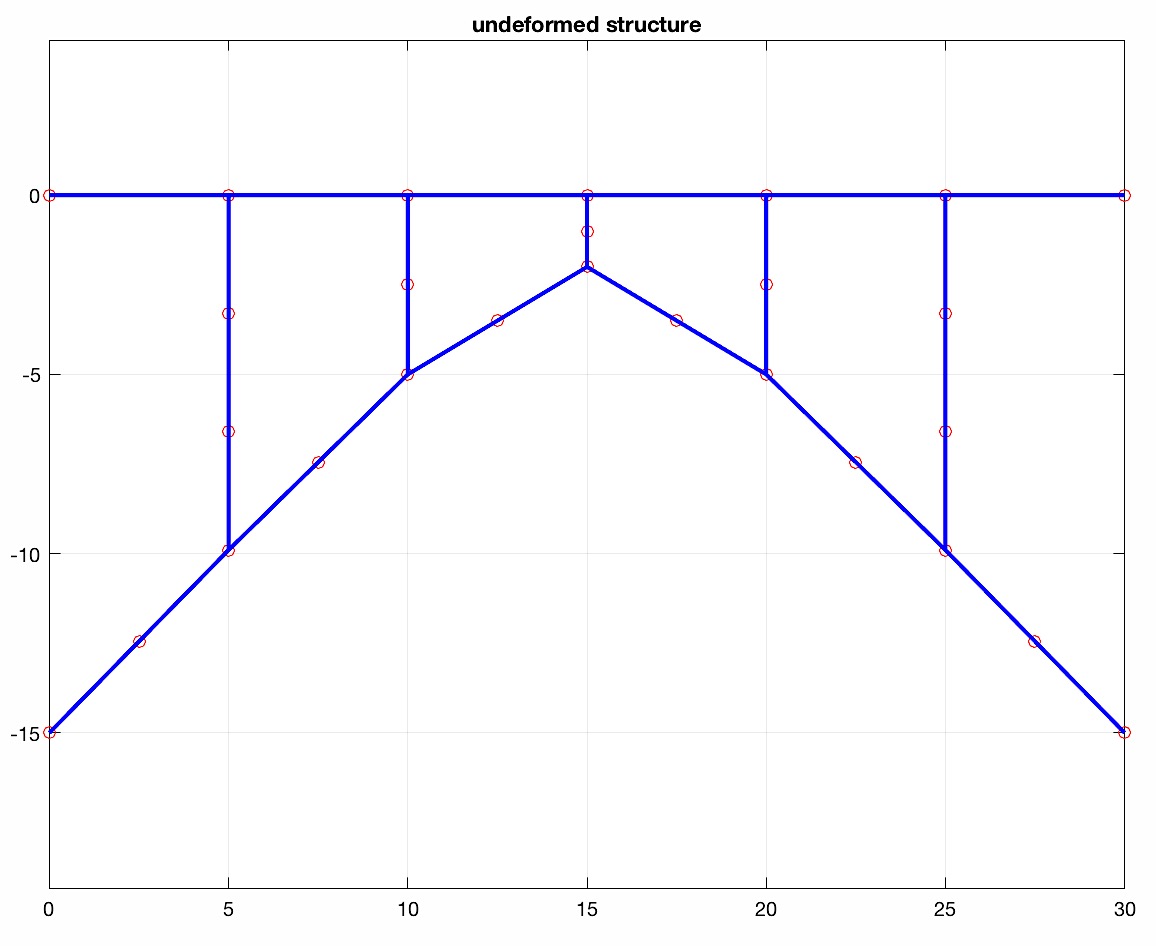


**Q. 7**

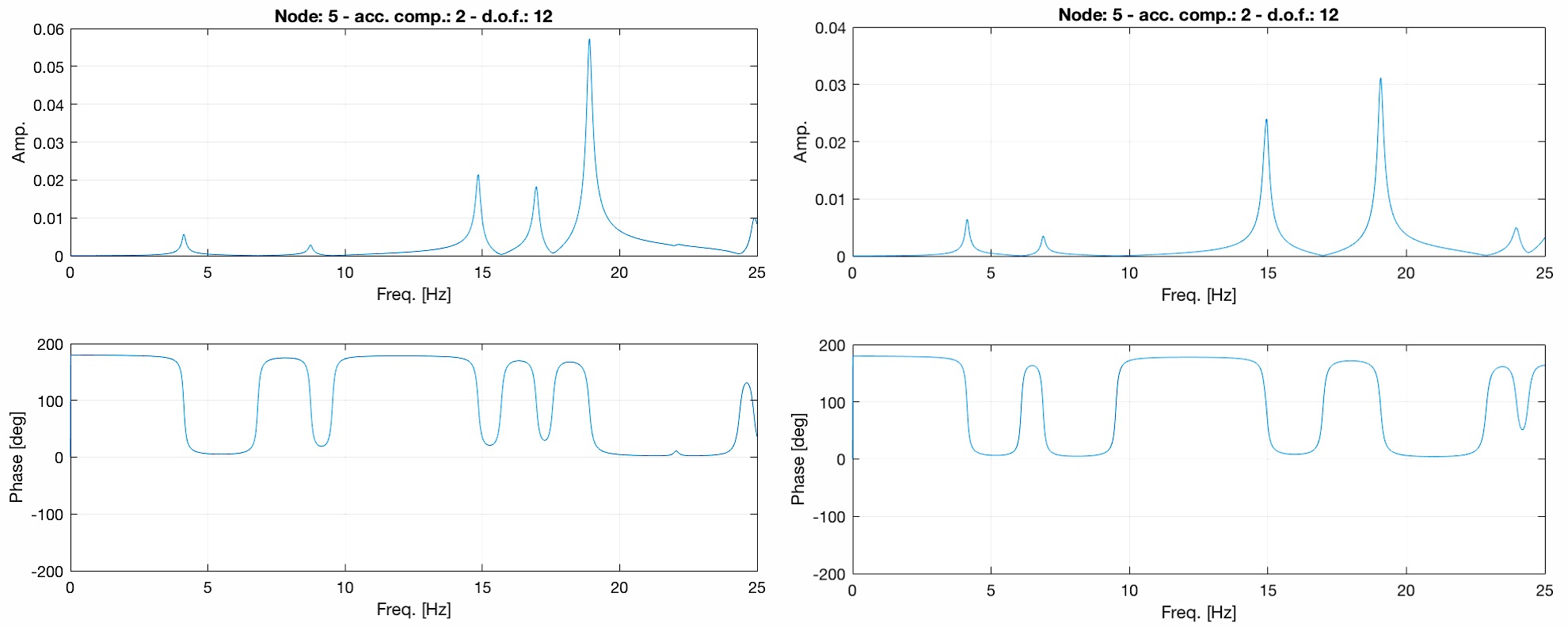
By simply changing a little bit the structure, without enhancing the section shape, the desired performance is obtained with even decrease of the total mass.

In detail, by changing the lengths of the vertical beams in the middle respectively from to , from to , and from to . The total mass is slightly decreased by approximately , from to .

The new structure is shown as follow:



In order for an easy comparison, the old FRF Bode Diagram and the new one are presented side by side as follows (on left: before; on right: after):



The maximum amplitude of the FRF of the vertical acceleration of point B is decreased significantly from 0.0574 to 0.0311, by more than 45%.

**Appendix**

%% Q2

clear all;

load('CAI\_898708\_mkr.mat')

MFF=M(1:73,1:73);

MFC=M(1:73,74:81);

MCF=M(74:81,1:73);

MCC=M(74:81,74:81);

RFF=R(1:73,1:73);

RFC=R(1:73,74:81);

RCF=R(74:81,1:73);

RCC=R(74:81,74:81);

KFF=K(1:73,1:73);

KFC=K(1:73,74:81);

KCF=K(74:81,1:73);

KCC=K(74:81,74:81);

n=length(MFF);

A=[MFF zeros(n);zeros(n) MFF];

B=[RFF KFF;-MFF zeros(n)];

[modes, eigenvalues]=eig(MFF\KFF);

freq=sqrt(diag(eigenvalues))/2/pi;

freq\_sorted=sort(freq)

%% Q3

% [modes, eigenvalues]=eig(-(A)\B);

% freq=imag(diag(eigenvalues))/2/pi;

% freq\_posi=zeros(length(freq)/2,1);

% for i=1:length(freq)/2

% freq\_posi(i)=freq(i\*2-1);

% end

% freq\_posi=sort(freq\_posi)

%% Q4

i=sqrt(-1);

C0=1;

Q0=zeros(n,1);

Q0(12)=C0;

vett\_f=0:0.01:25;

for k=1:length(vett\_f)

ome=2\*pi\*vett\_f(k);

A=-ome^2\*MFF+i\*ome\*RFF+KFF;

x0=A\Q0;

yA=x0(9);

yB=x0(12);

yAdd=-ome^2\*yA;

yBdd=-ome^2\*yB;

mod1(k)=abs(yAdd);

fas1(k)=angle(yAdd);

mod2(k)=abs(yBdd);

fas2(k)=angle(yBdd);

end

figure

subplot 211;plot(vett\_f,mod1);grid;title('yAdd/C');xlabel('[Hz]');ylabel('[m/s^2]');

subplot 212;plot(vett\_f,fas1\*180/pi);grid;xlabel('[Hz]');ylabel('[rad]');

figure

subplot 211;plot(vett\_f,mod2);grid;title('yBdd/C');xlabel('[Hz]');ylabel('[m/s^2]');

subplot 212;plot(vett\_f,fas2\*180/pi);grid;xlabel('[Hz]');ylabel('[rad]');

%% Q5

i=sqrt(-1);

Q0=zeros(n,1);

Q0(3)=-5/2;Q0(4)=-25/12;Q0(6)=-5;Q0(9)=-5/2;Q0(10)=25/12;

vett\_f=0:0.01:25;

for k=1:length(vett\_f)

ome=2\*pi\*vett\_f(k);

A=-ome^2\*MFF+i\*ome\*RFF+KFF;

x0=A\Q0;

yA=x0(9);

yAdd=-ome^2\*yA;

x0d=i\*ome\*x0;

x0dd=-ome^2\*x0;

R0=MCF\*x0dd+RCF\*x0d+KCF\*x0;

R\_VC=R0(6);

mod1(k)=abs(yAdd);

fas1(k)=angle(yAdd);

mod2(k)=abs(R\_VC);

fas2(k)=angle(R\_VC);

end

figure

subplot 211;plot(vett\_f,mod1);grid;title('yAdd/p0');xlabel('[Hz]');ylabel('[m/s^2]');

subplot 212;plot(vett\_f,fas1\*180/pi);grid;xlabel('[Hz]');ylabel('[rad]');

figure

subplot 211;plot(vett\_f,mod2);grid;title('R\_VC/p0');xlabel('[Hz]');ylabel('[N]');

subplot 212;plot(vett\_f,fas2\*180/pi);grid;xlabel('[Hz]');ylabel('[rad]');

%% Q6

i=sqrt(-1);

Q0=zeros(n,1);

Q0(3)=-5/2;Q0(4)=-25/12;Q0(6)=-5;Q0(9)=-5/2;Q0(10)=25/12;

vett\_f=0:0.01:25;

for k=1:length(vett\_f)

ome=2\*pi\*vett\_f(k);

A=-ome^2\*MFF+i\*ome\*RFF+KFF;

x0=A\Q0;

yA=x0(9);

yAdd=-ome^2\*yA;

x0d=i\*ome\*x0;

x0dd=-ome^2\*x0;

R0=MCF\*x0dd+RCF\*x0d+KCF\*x0;

R\_VC=R0(6);

mod1(k)=abs(yAdd);

fas1(k)=angle(yAdd);

mod2(k)=abs(R\_VC);

fas2(k)=angle(R\_VC);

end

MFF\_new=modes'\*MFF\*modes;

RFF\_new=modes'\*RFF\*modes;

KFF\_new=modes'\*KFF\*modes;

Q0\_new=modes'\*Q0;

a=find(freq==freq\_sorted(1));

b=find(freq==freq\_sorted(2));

c=find(freq==freq\_sorted(3));

q0=zeros(n,1);

for k=1:length(vett\_f)

ome=2\*pi\*vett\_f(k);

A\_new=-ome^2\*MFF\_new+i\*ome\*RFF\_new+KFF\_new;

q0\_full=A\_new\Q0\_new;

q0(a)=q0\_full(a);q0(b)=q0\_full(b);q0(c)=q0\_full(c);

x0\_new=modes\*q0;

yA\_new=x0\_new(9);

yAdd\_new=-ome^2\*yA\_new;

%

x0d\_new=i\*ome\*x0\_new;

x0dd\_new=-ome^2\*x0\_new;

R0\_new=MCF\*x0dd\_new+RCF\*x0d\_new+KCF\*x0\_new;

R\_VC\_new=R0\_new(6);

mod3(k)=abs(yAdd\_new);

fas3(k)=angle(yAdd\_new);

mod4(k)=abs(R\_VC\_new);

fas4(k)=angle(R\_VC\_new);

end

figure

subplot 211;plot(vett\_f,mod1,'linewidth',0.5);hold on;...

plot(vett\_f,mod3,'--','linewidth',2);...

grid;title('yAdd/p0');xlabel('[Hz]');ylabel('[m/s^2]');...

legend('full modes','first three modes');

subplot 212;plot(vett\_f,fas1\*180/pi,'linewidth',0.5);hold on;...

plot(vett\_f,fas3\*180/pi,'--','linewidth',2);...

grid;xlabel('[Hz]');ylabel('[rad]');...

legend('full modes','first three modes');

figure

subplot 211;plot(vett\_f,mod2,'linewidth',0.5);hold on;...

plot(vett\_f,mod4,'--','linewidth',2);...

grid;title('R\_VC/p0');xlabel('[Hz]');ylabel('[N]');...

legend('full modes','first three modes');

subplot 212;plot(vett\_f,fas2\*180/pi,'linewidth',0.5);hold on;...

plot(vett\_f,fas4\*180/pi,'--','linewidth',2);...

grid;xlabel('[Hz]');ylabel('[rad]');...

legend('full modes','first three modes');