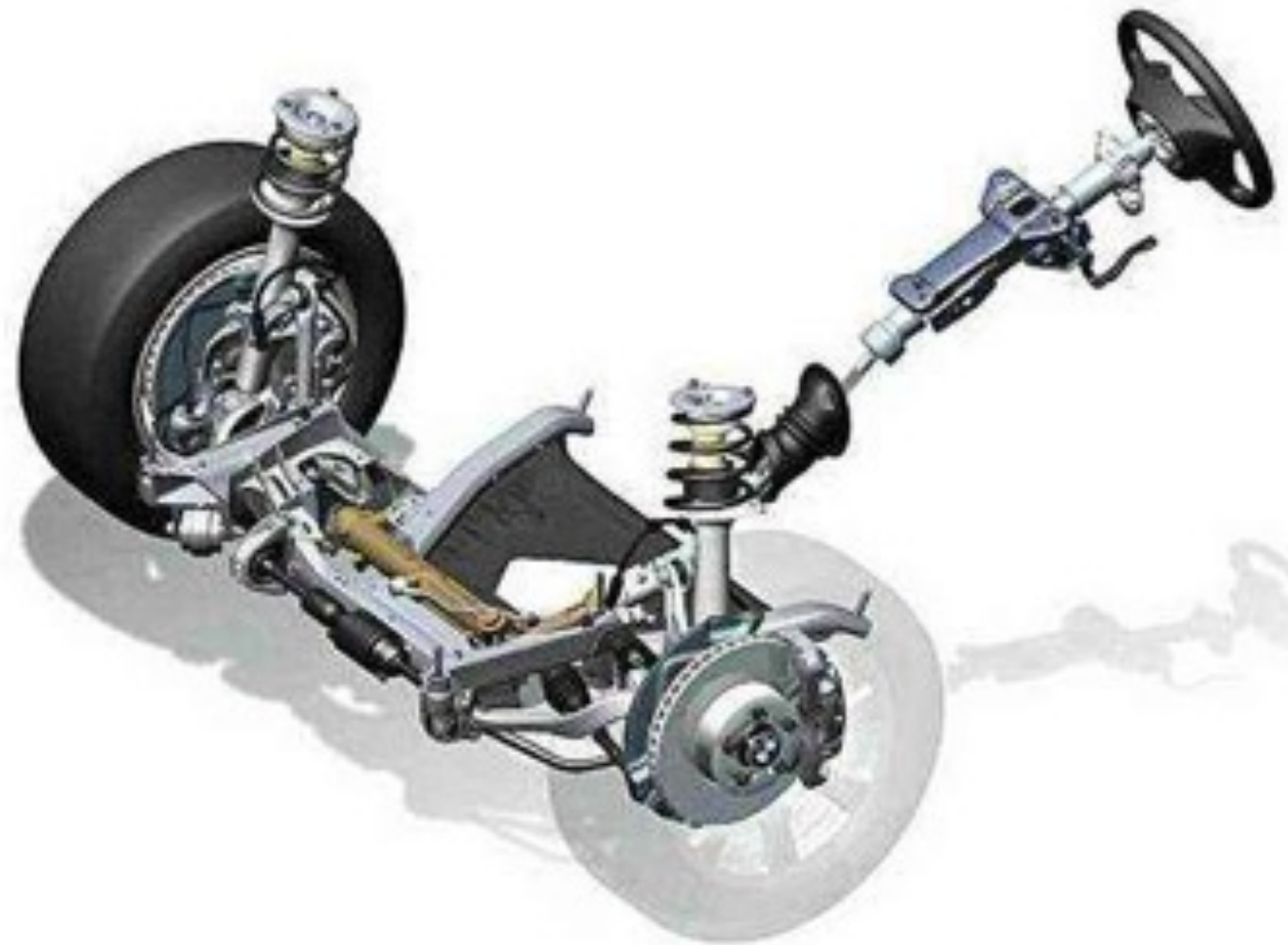
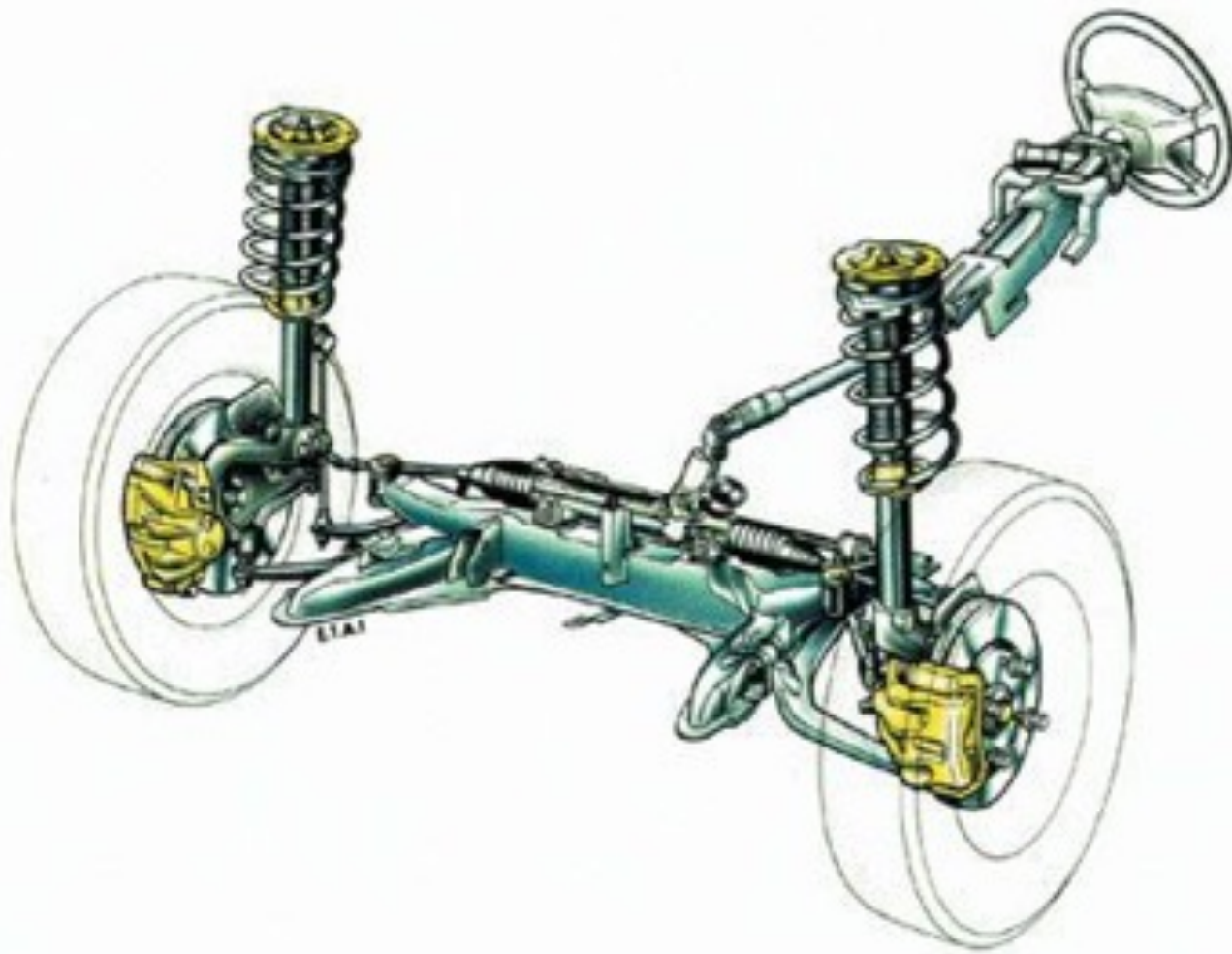


SIMULATION OF VEHICLE STEERING SYSTEM

Zhongtian. Cai
898708

STEPS:

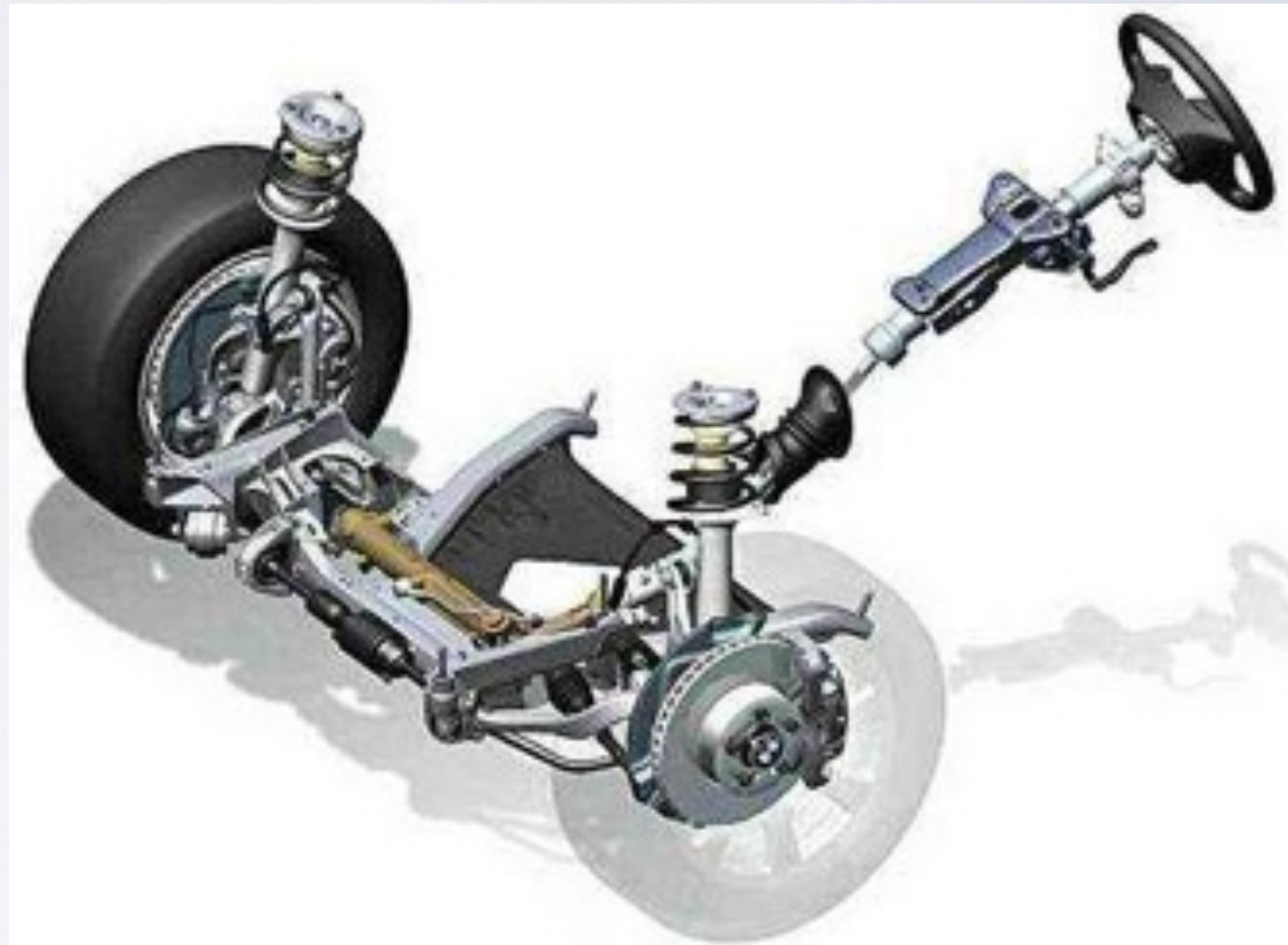
- Study different components of a common steering system
- Mathematically modeling the Steering System
- Include the Steering System within the entire vehicle model
- Apply adequate experiments (simulation) to verify the model



STEERING SYSTEM OF A COMMON
COMMERCIAL 4-WHEEL VEHICLE

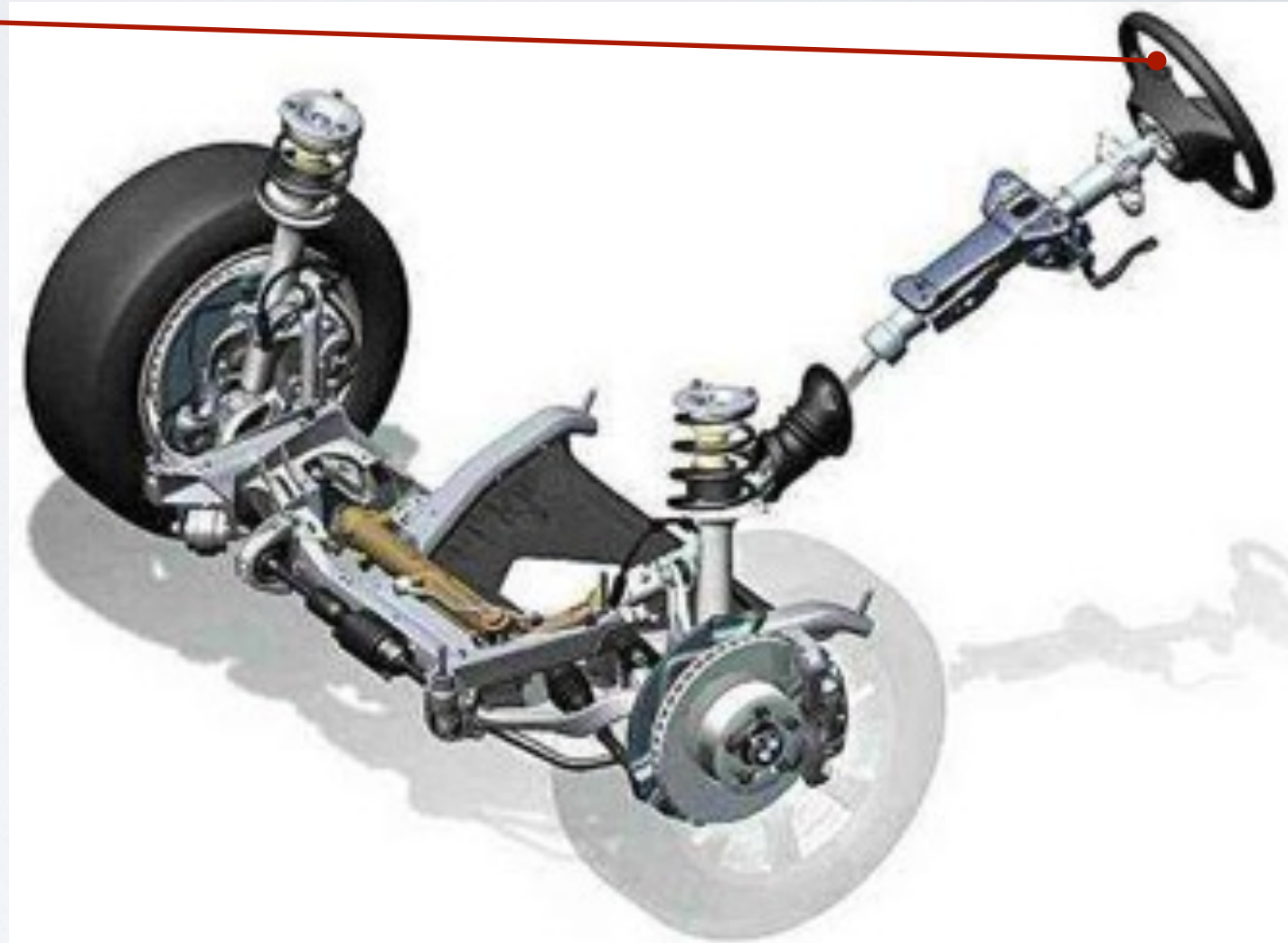
SIMPLIFIED COMPONENTS:

- Steering Wheel (SW)
- Steering Column (SC)
- Torque Bar
- Diverter
- Hydraulic Power Steering (HPS)
- Ackermann Steering Linkages
- Front Axle
- Kingpins
- Wheels



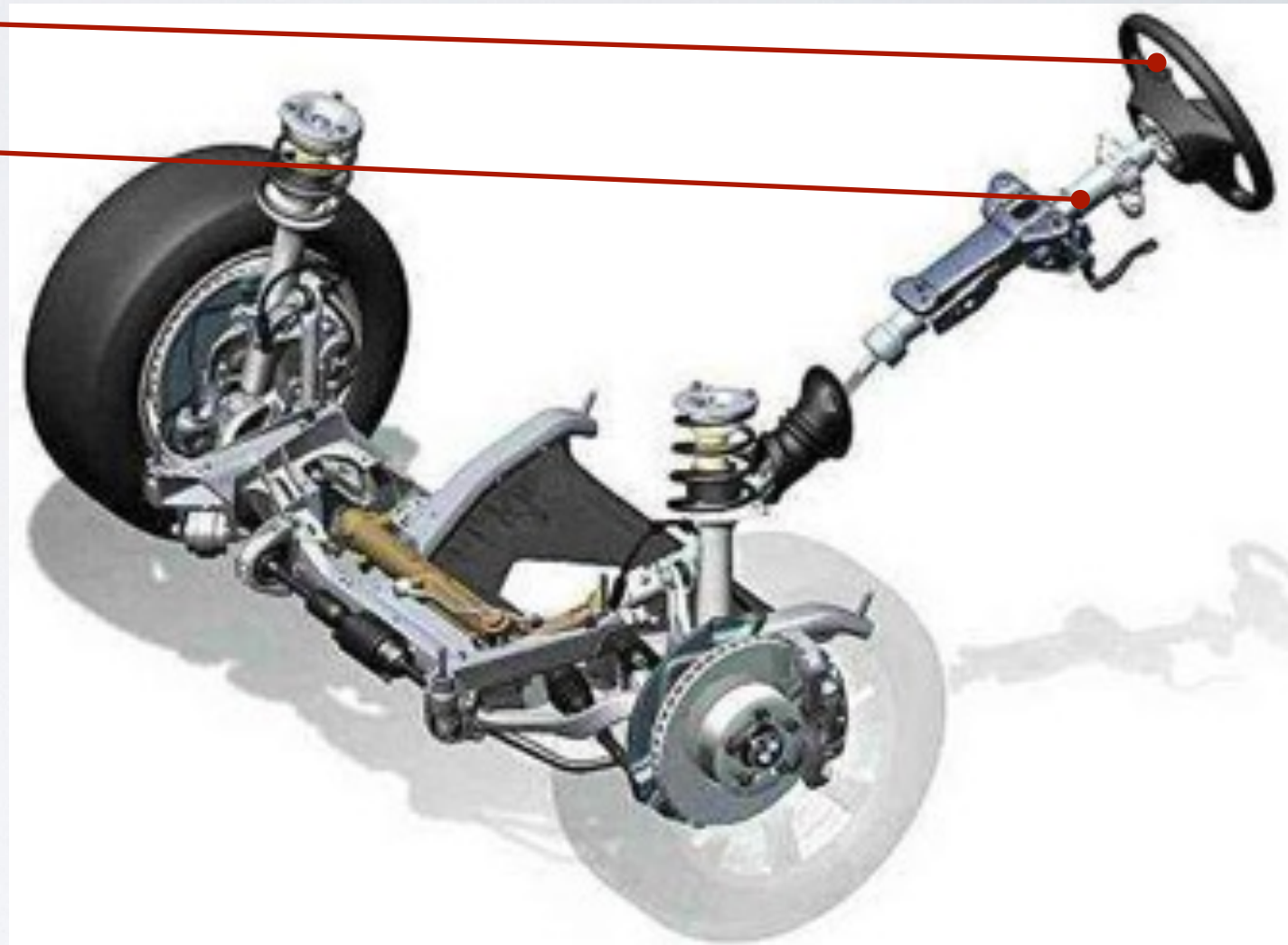
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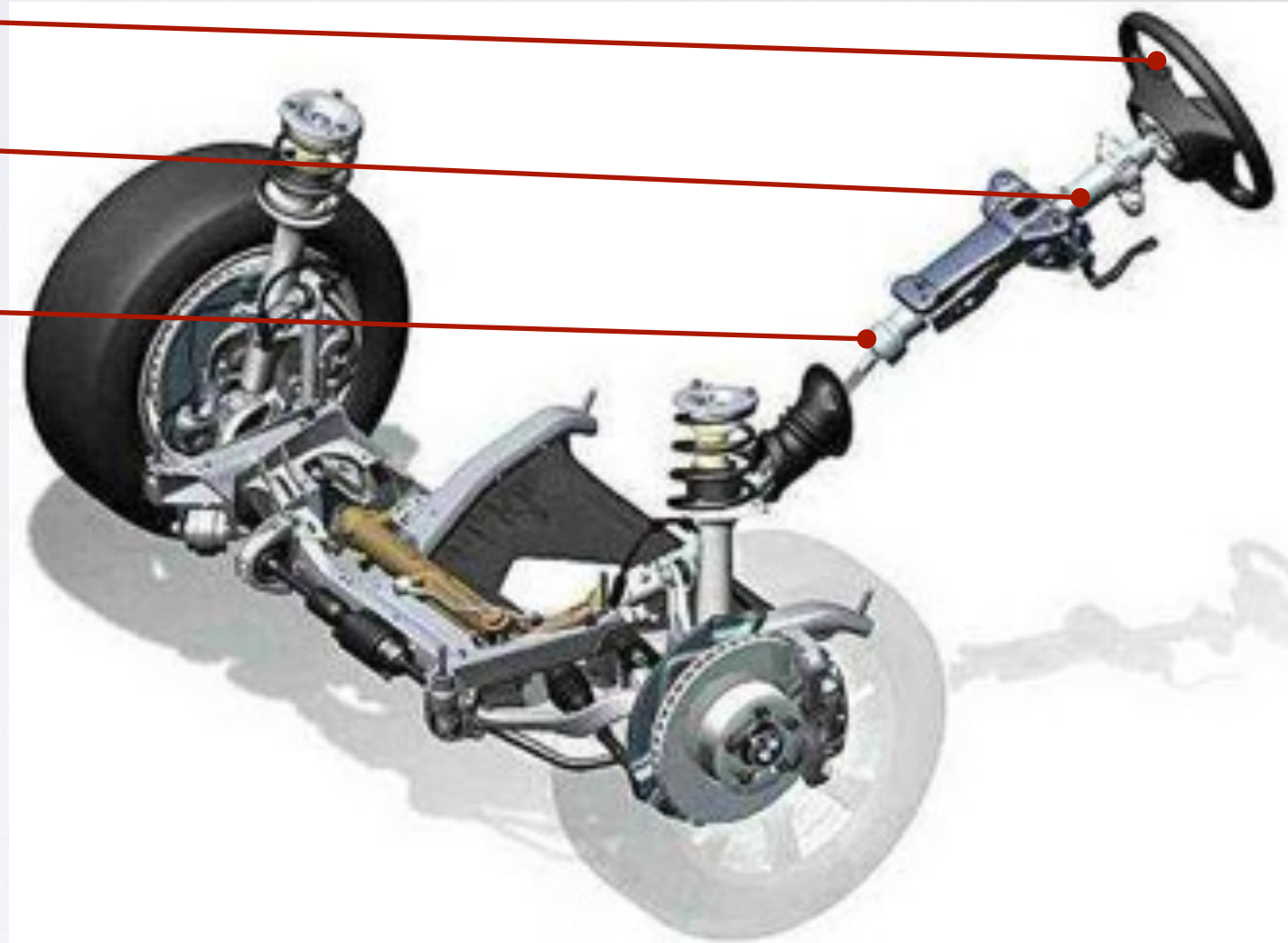
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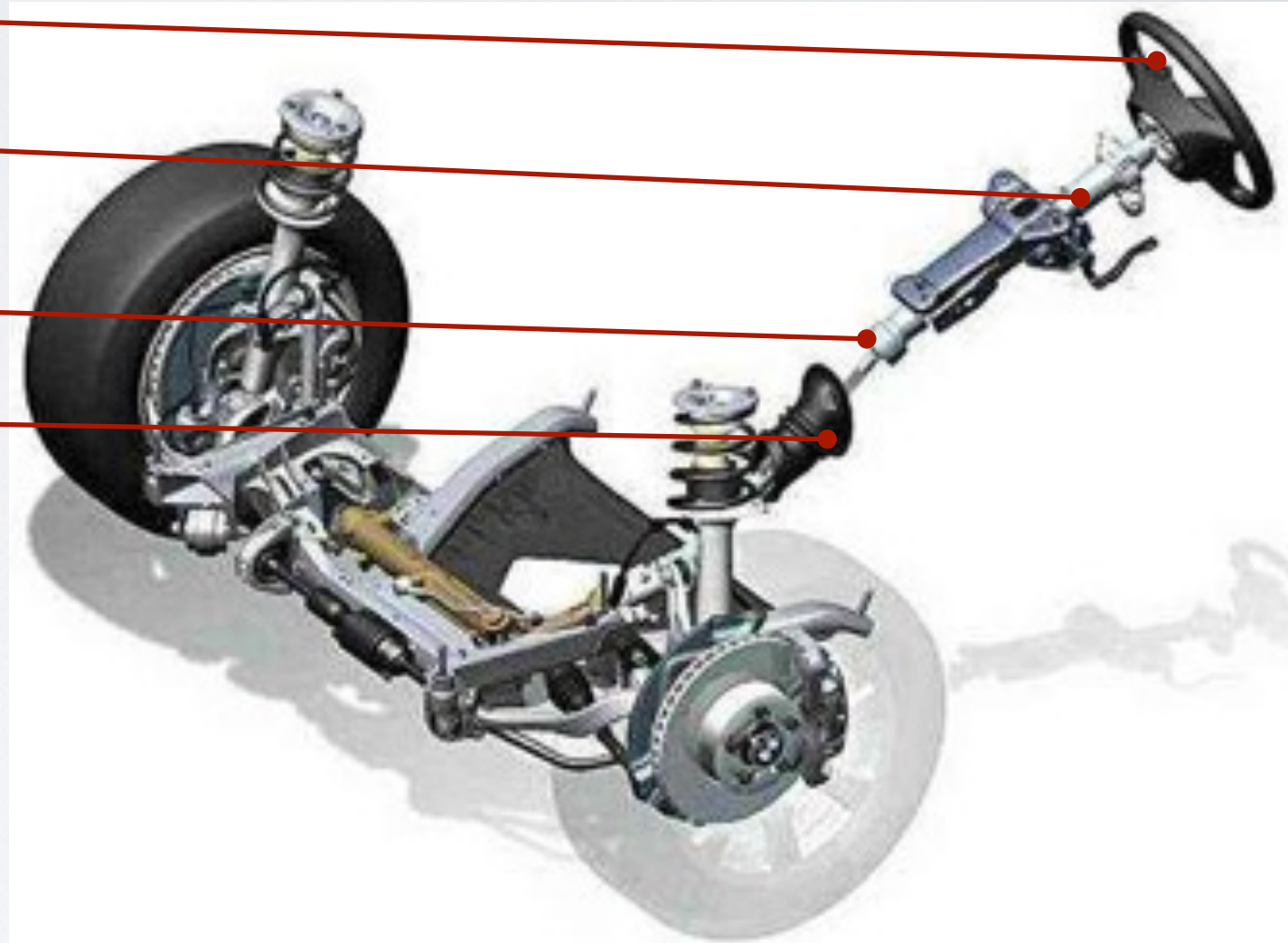
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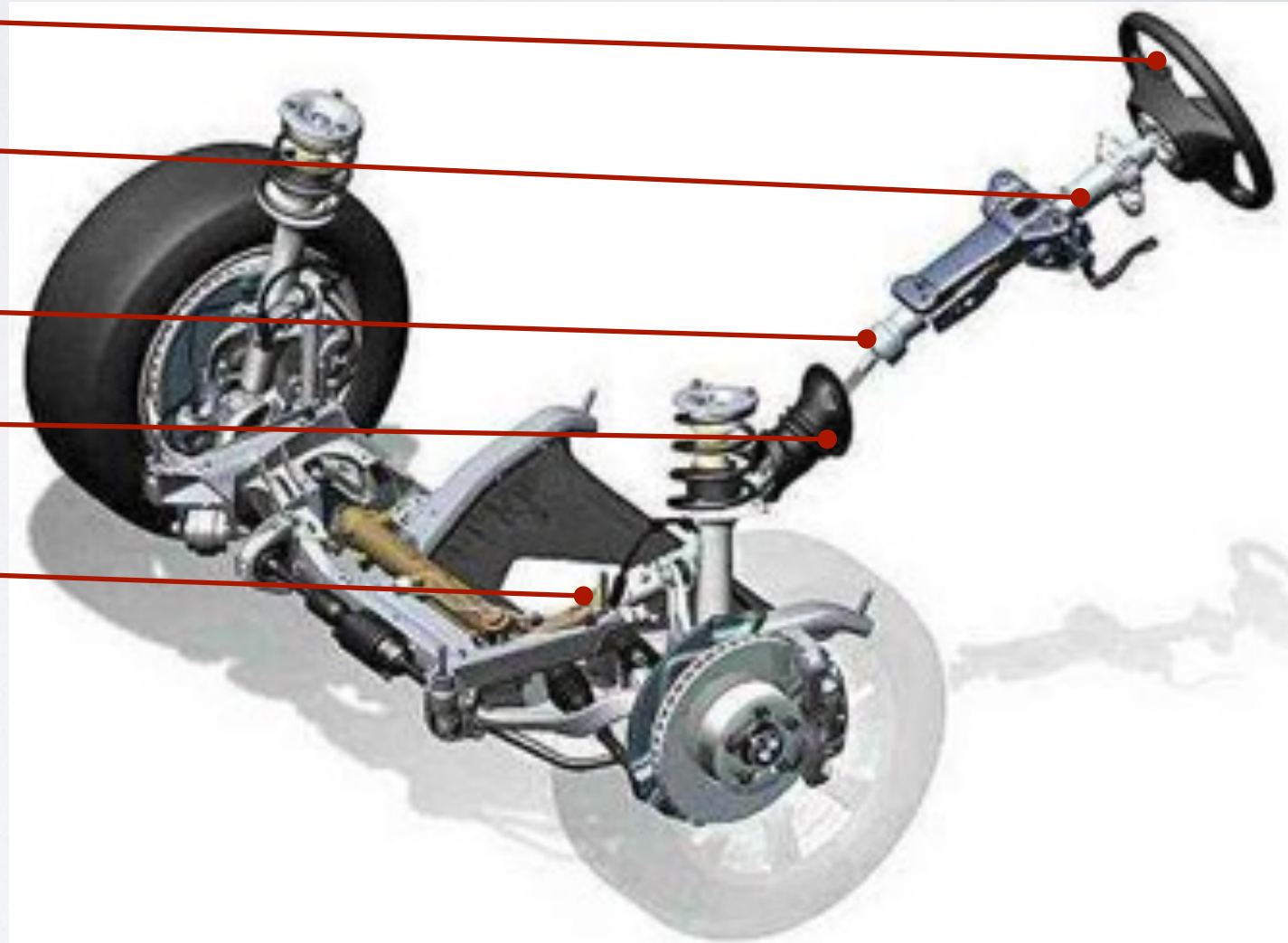
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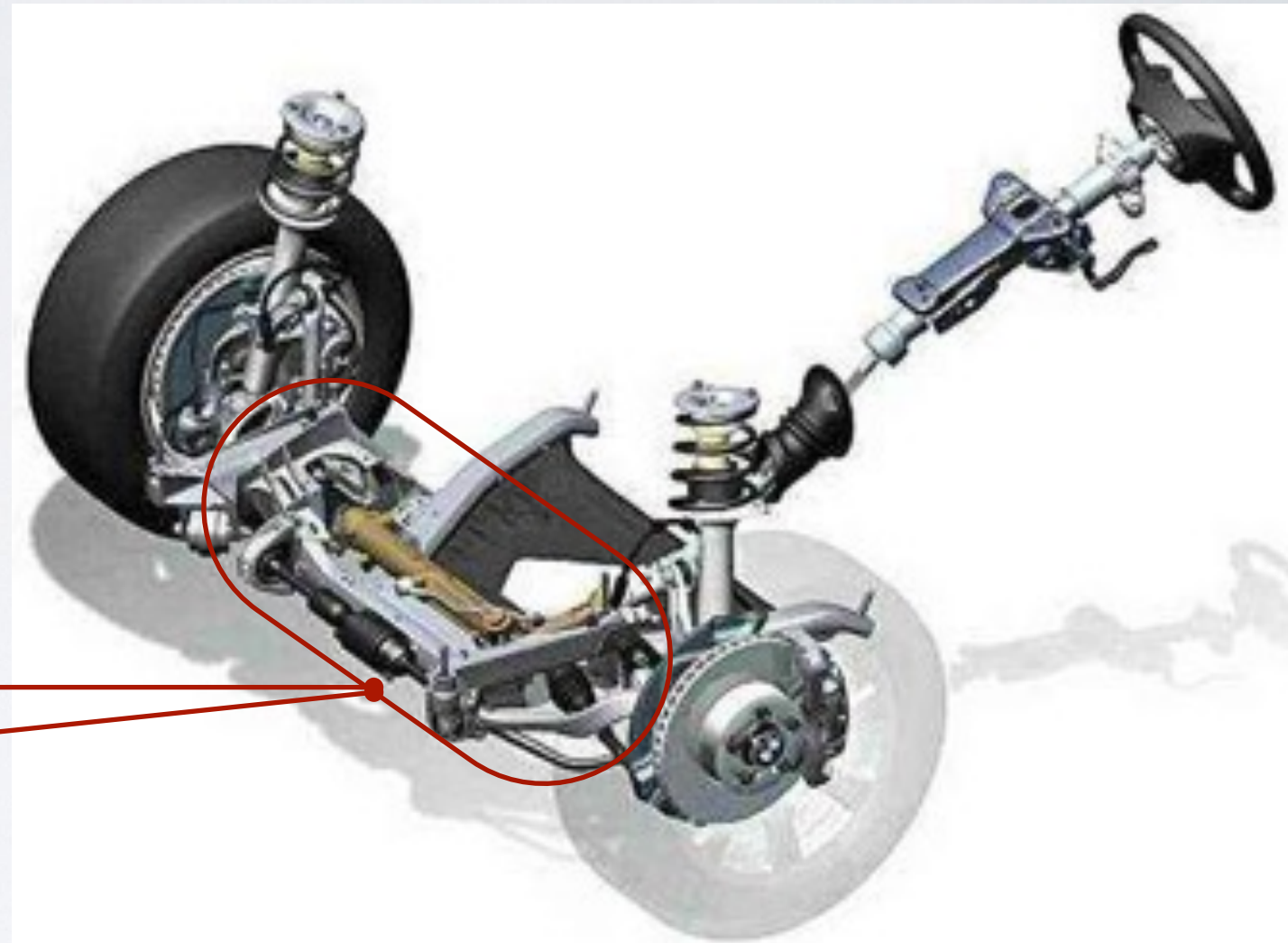
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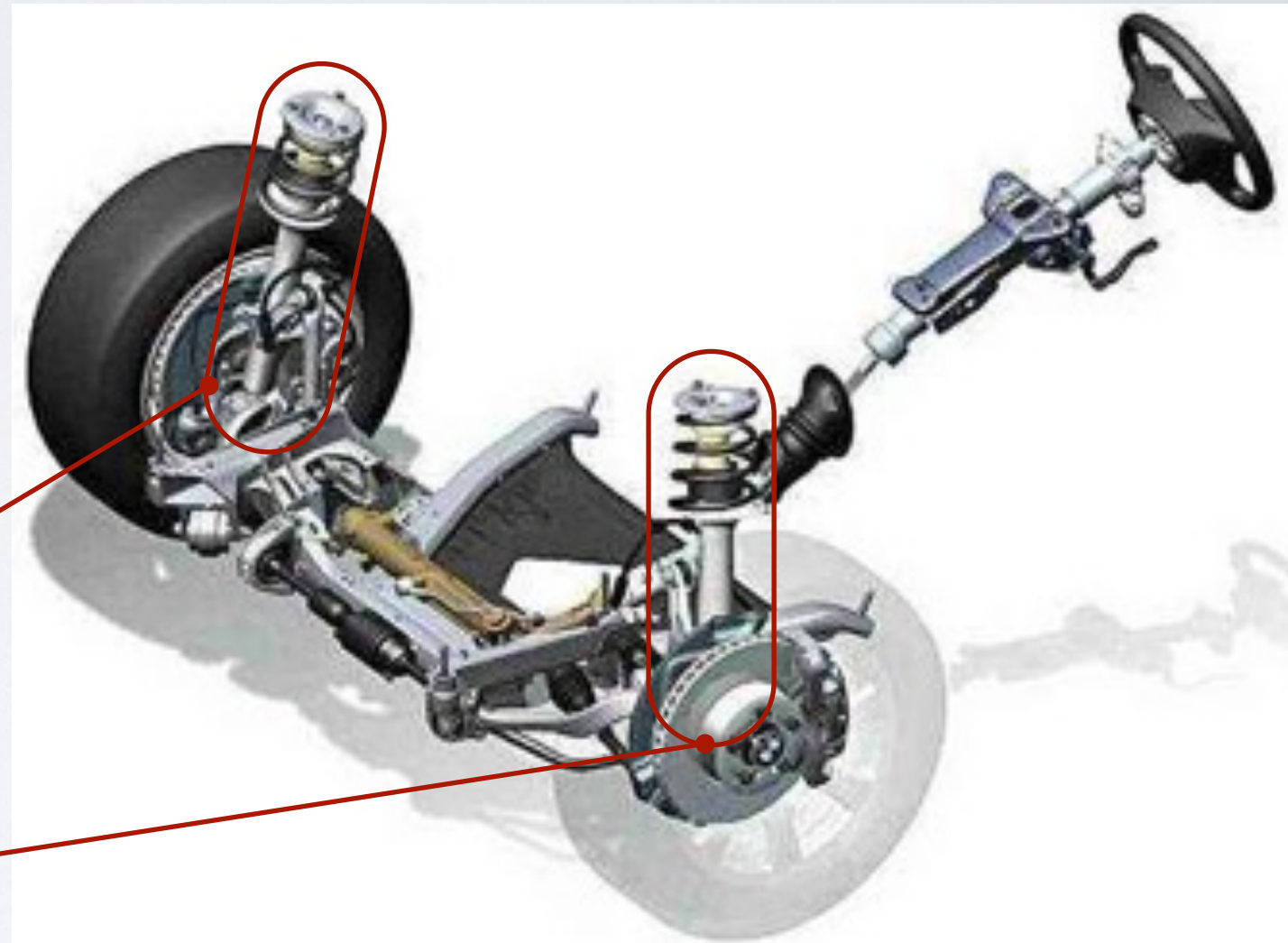
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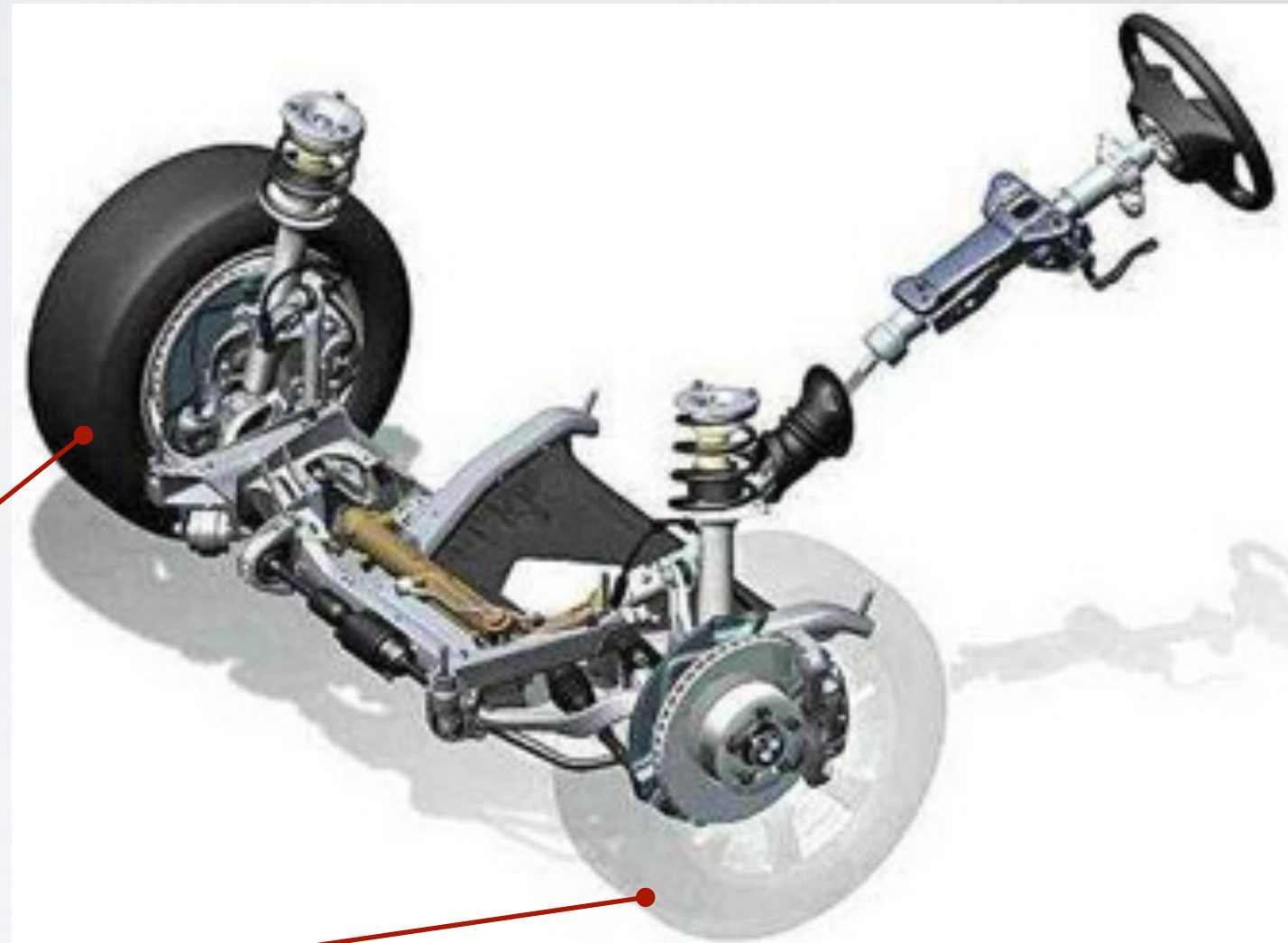
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- Ackermann Steering Geometry

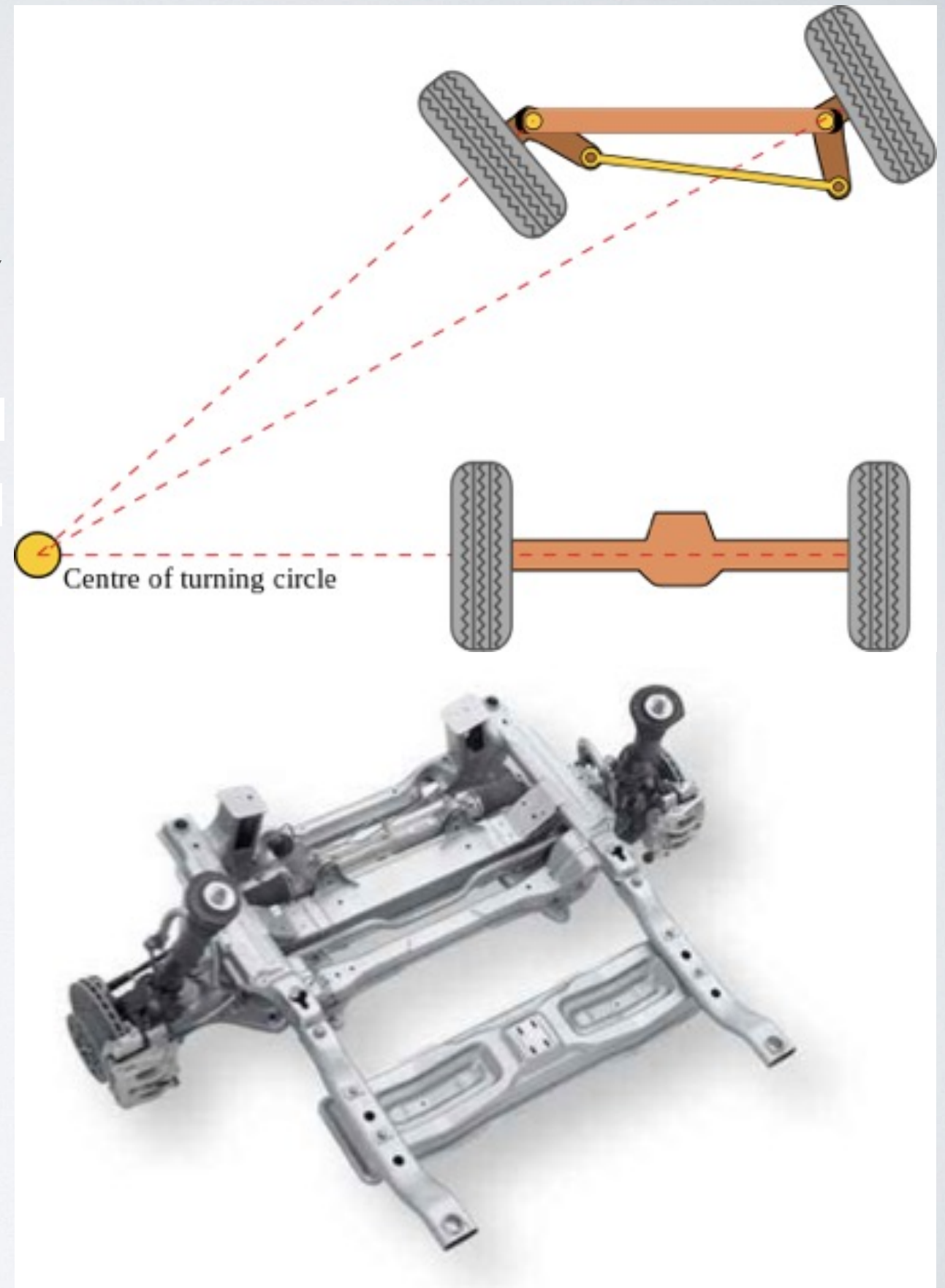
From Wikipedia, the free encyclopedia

Ackermann steering geometry is a geometric arrangement of linkages in the [steering](#) of a [car](#) or other [vehicle](#) designed to solve the problem of wheels on the inside and outside of a turn needing to trace out [circles](#) of different [radii](#).

It was invented by the German carriage builder [Georg Lankensperger](#) in Munich in 1817, then patented by his agent in England, [Rudolph Ackermann](#) (1764–1834) in 1818 for horse-drawn carriages. [Erasmus Darwin](#) may have a prior claim as the inventor dating from 1758.

- Front Axle

Connects the masses of wheels with the chassis.



- Ackermann Steering Geometry

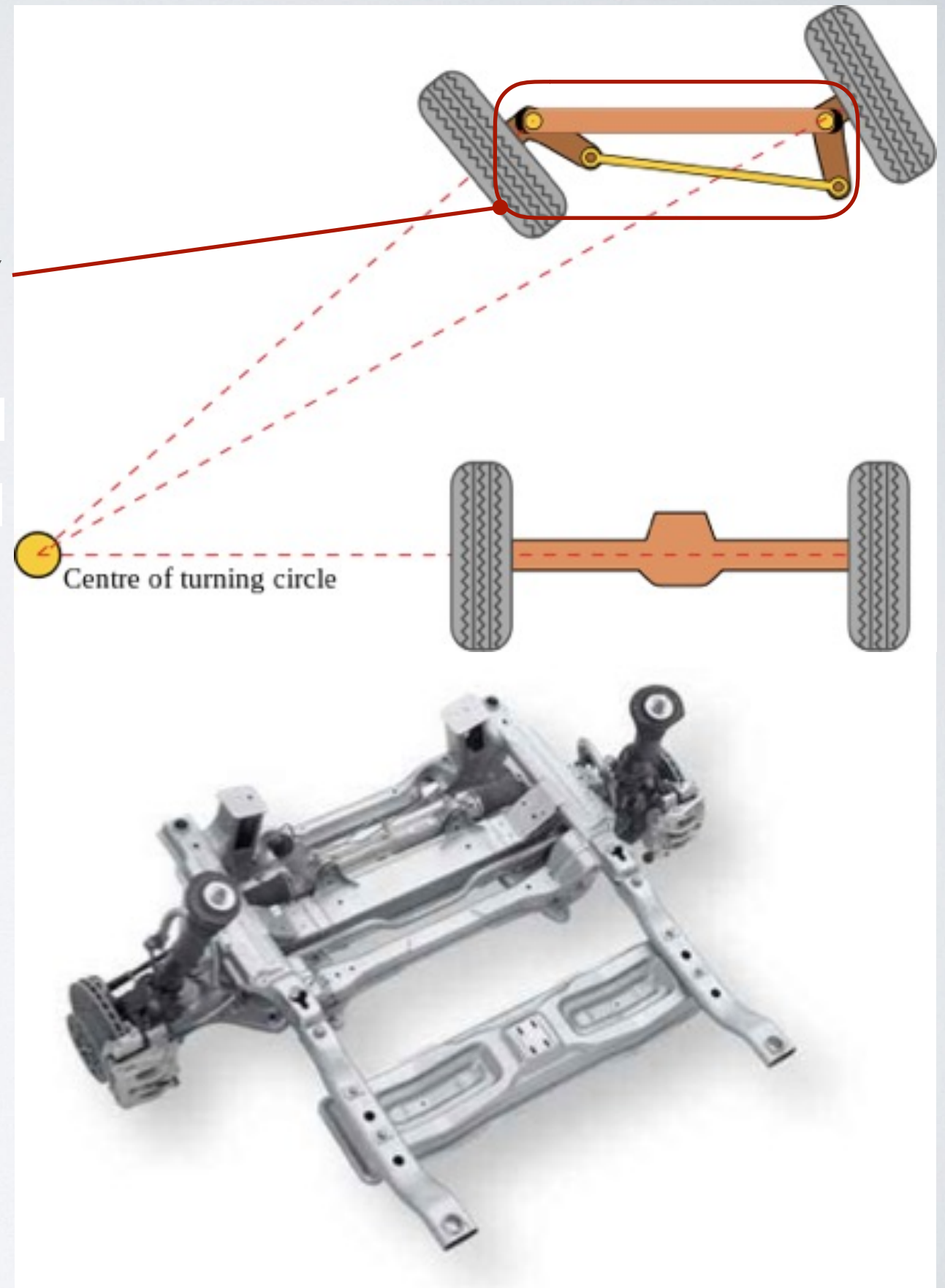
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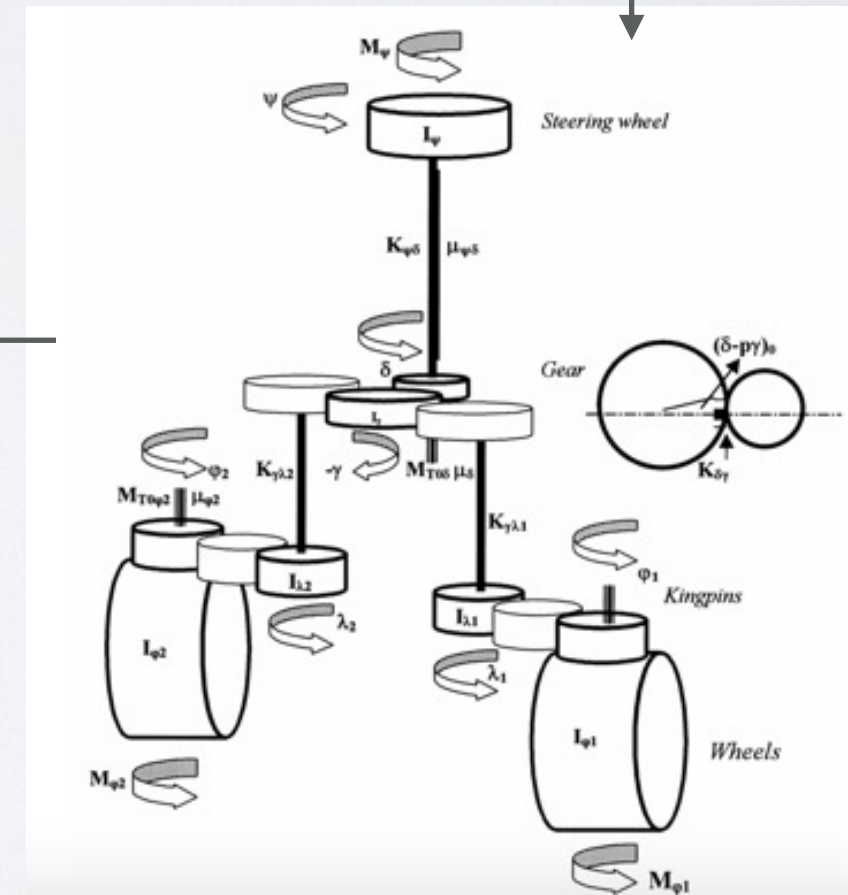
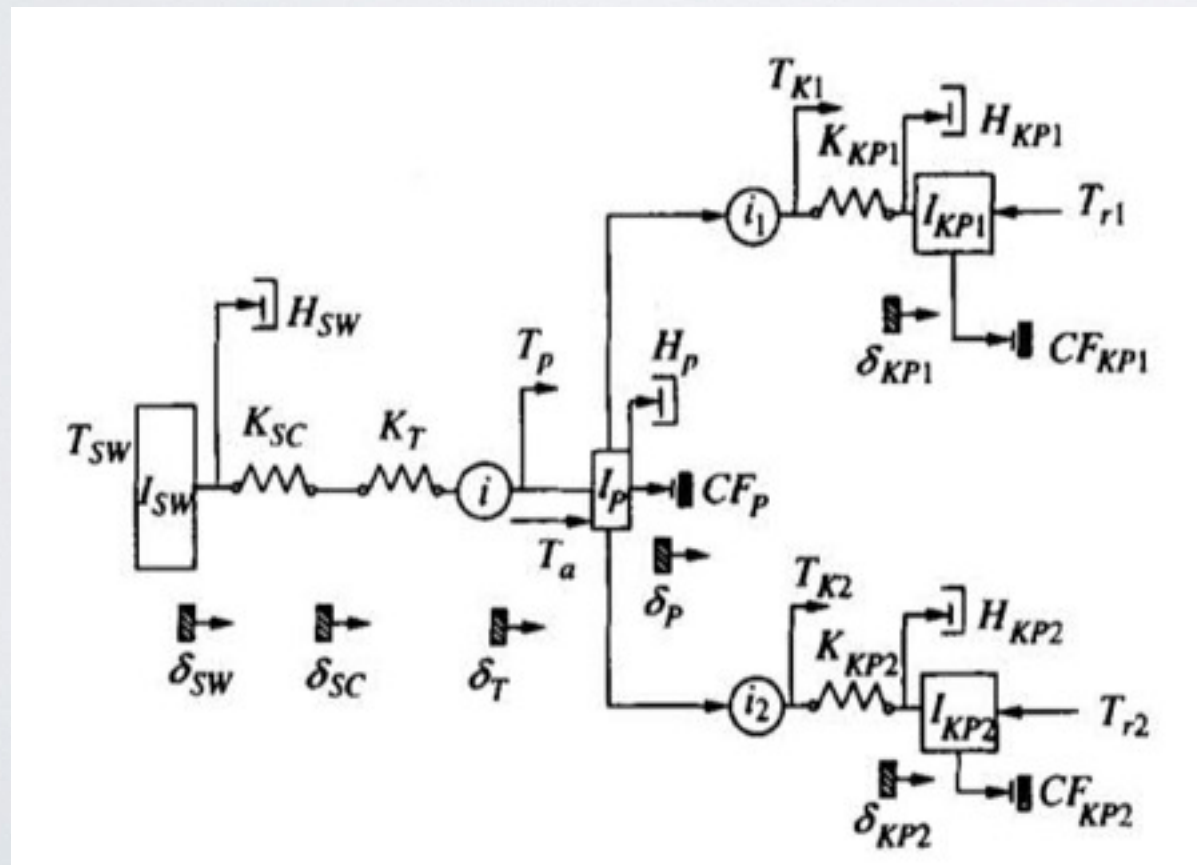
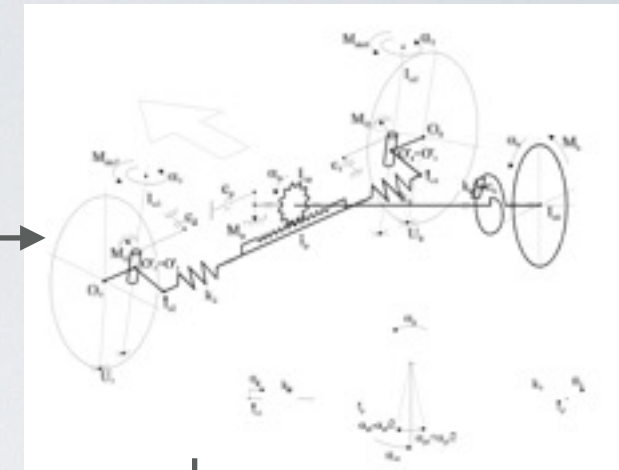
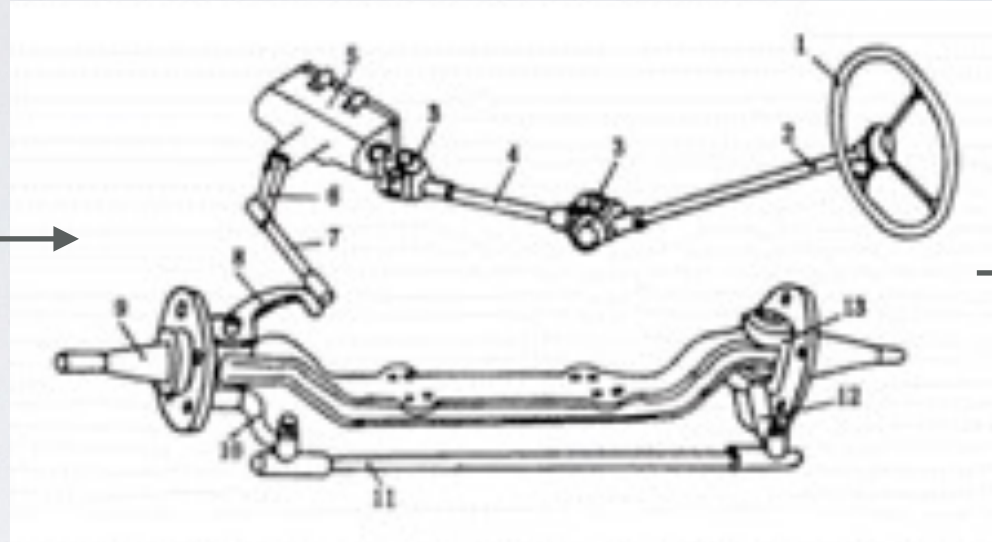
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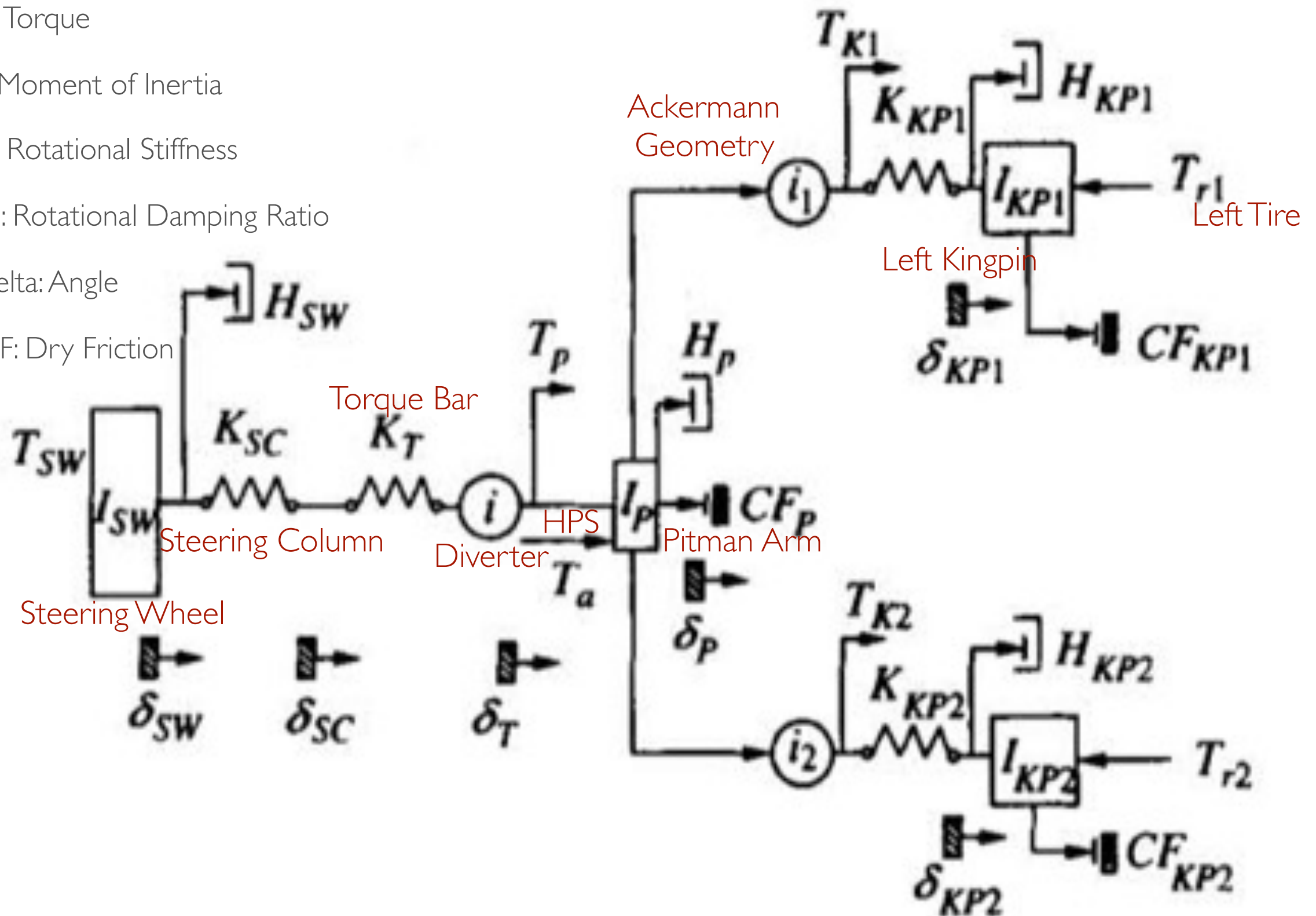


SIMPLIFY —> MODELING

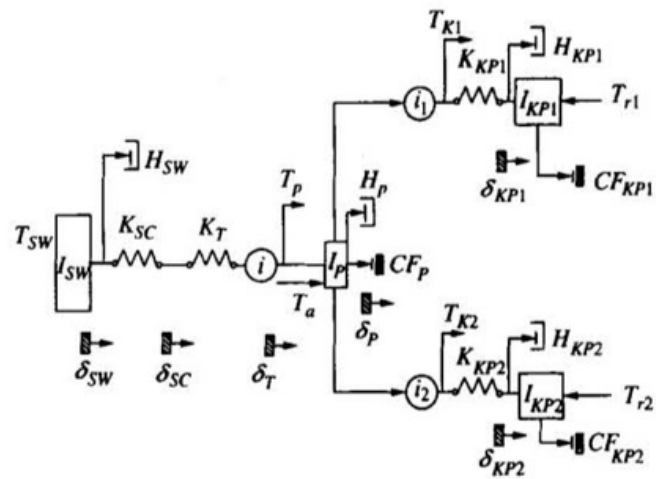


MODELING OF THE STEERING SYSTEM

- T: Torque
- I: Moment of Inertia
- K: Rotational Stiffness
- H: Rotational Damping Ratio
- delta: Angle
- CF: Dry Friction



MODELING OF THE STEERING SYSTEM



$$\begin{cases} I_{SW} \ddot{\delta}_{SW} = T_{SW} - H_{SW} \dot{\delta}_{SW} - K_{SC} (\delta_{SW} - \delta_{SC}) \\ I_P \ddot{\delta}_P = T_P \eta_F + T_a \eta_{PS} - H_P \dot{\delta}_P - \frac{T_{K1}}{i_1} \eta_B - \frac{T_{K2}}{i_2} \eta_B - \text{sgn}(\dot{\delta}_P) CF_P \\ I_{KP1} \ddot{\delta}_{KP1} = T_{K1} - T_{r1} - H_{KP1} \dot{\delta}_{KP1} - \text{sgn}(\dot{\delta}_{KP1}) CF_{KP1} \\ I_{KP2} \ddot{\delta}_{KP2} = T_{K2} - T_{r2} - H_{KP2} \dot{\delta}_{KP2} - \text{sgn}(\dot{\delta}_{KP2}) CF_{KP2} \end{cases}$$

where

I_{SW} : Moment of inertia of steering wheel

I_P : Moment of inertia of pitman arm

I_{KP1} : Moment of inertia of left tire around its kingpin

I_{KP2} : Moment of inertia of right tire around its kingpin

δ_{SW} : Rotation of steering wheel

δ_P : Rotation of pitman arm

δ_{KP1} & δ_{KP2} : Rotation of left & right tire around its kingpin

δ_{SC} : Rotation of steering column, $\delta_{SC} = \frac{K_T \cdot i \cdot \delta_P + K_{SC} \cdot \delta_{SW}}{K_T + K_{SC}}$

T_{SW} : Acting torque on steering wheel by driver

T_P : Acting torque on pitman arm caused by driver, $T_P = i \cdot K_{SC} \cdot (\delta_{SW} - \delta_{SC})$

T_a : Assisting torque on pitman arm by HPS, $T_a = \frac{i \cdot p \cdot A \cdot t}{2\pi}$ where t is the screw lead, A is the effective area of the oil pressure, p is the pressure difference b/t the two ends of the nut

T_{K1} : Acting torque of left kingpin on the tire, $T_{K1} = K_{KP1} (\frac{\delta_P}{i_1} - \delta_{KP1})$

T_{K2} : Acting torque of right kingpin on the tire, $T_{K2} = K_{KP2} (\frac{\delta_P}{i_2} - \delta_{KP2})$

T_{r1} : Resistant torque of left tire when steering

T_{r2} : Resistant torque of right tire when steering

K_{SC} : Torsional stiffness of the steering column

K_T : Torsional stiffness of the torque bar

K_{KP1} & K_{KP2} : Torsional stiffness of the left & right kingpin

H_{SW} : Damping ratio of the steering wheel module

H_P : Damping ratio of the pitman bar

H_{KP1} & H_{KP2} : Damping ratio of the left & right kingpin

i : Transmission ration of the diverter

i_1 & i_2 : Transmission ration of the left & right steering structure

CF_P : Dry friction coefficient of the pitman bar

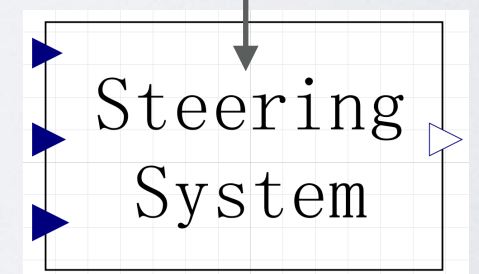
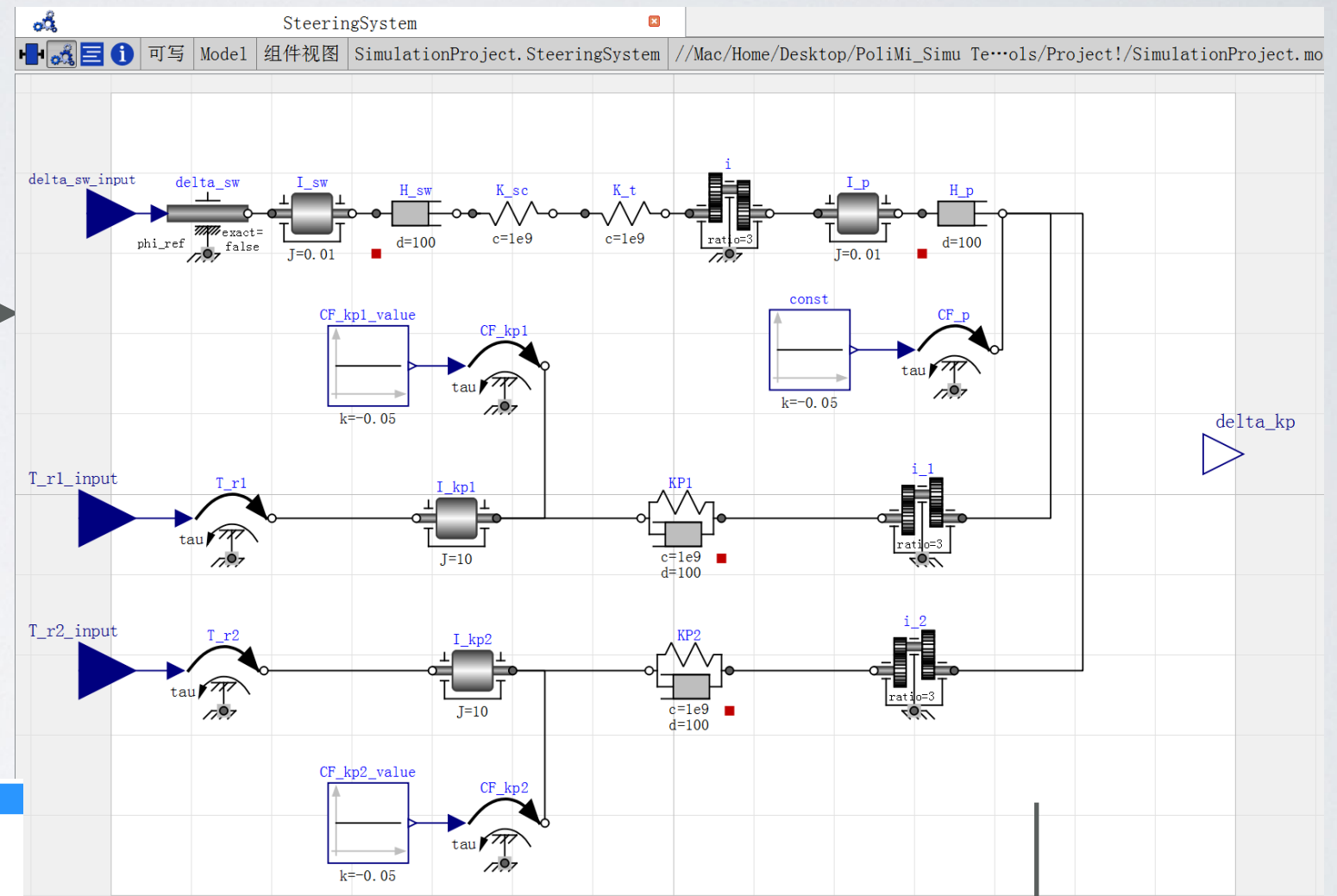
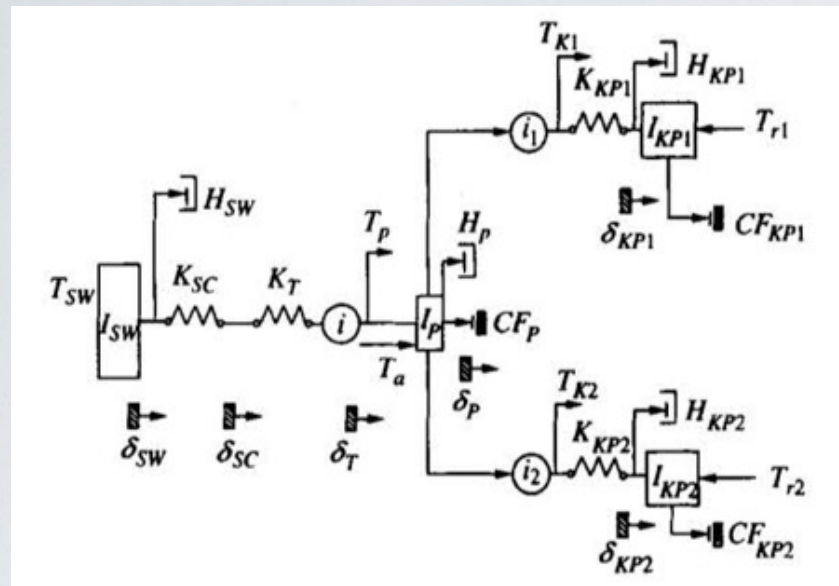
CF_{KP1} & CF_{KP2} : Dry friction coefficient of the left & right kingpin

η_F : Forward transmission efficiency of the diverter

η_B : Backward transmission efficiency of the diverter

η_{PS} : Efficiency of the power steering system

MODELING OF THE STEERING SYSTEM IN MODELICA:



An important library:
- Modelica/ Mechanics/ **Rotational**

- Modelica
- UsersGuide
- Blocks
- ComplexBlocks
- StateGraph
- Electrical
- Magnetic
- Mechanics
- MultiBody
- Rotational**
- UsersGuide
- Examples
- Components
- Sensors
- Sources
- Interfaces
- Icons
- Translational
- Fluid
- Media
- Thermal
- Math

- Rotational**
- UsersGuide
- Examples
- Components
- Fixed
- Inertia
- Disc
- Spring
- Damper
- SpringDamper
- ElastoBacklash
- ElastoBacklash2
- BearingFriction
- Brake
- Clutch
- OneWayClutch
- IdealGear
- LossyGear
- IdealPlanetary
- Gearbox
- IdealGearR2T
- IdealRollingWheel
- InitializeFlange
- RelativeStates
- TorqueT...Adaptor
- AngleTo...Adaptor

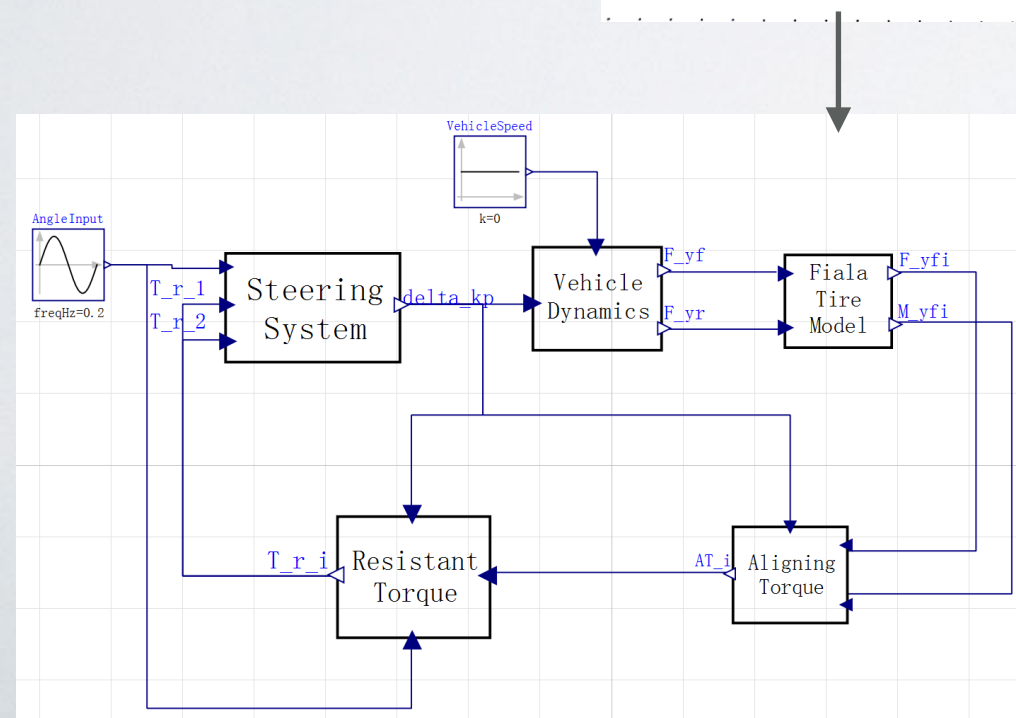
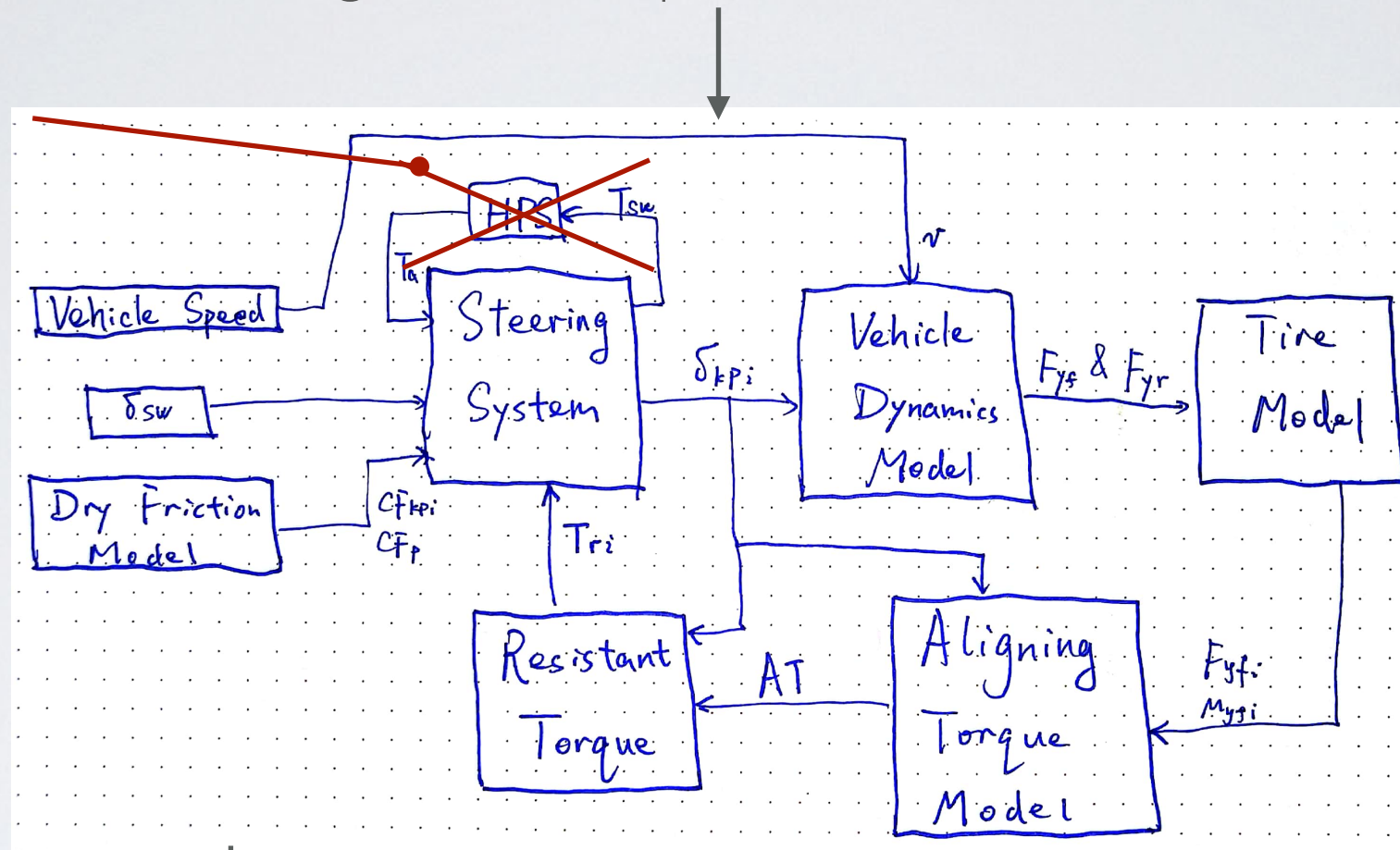
- Rotational**
- UsersGuide
- Examples
- Components
- Sensors
- Sources
- Position
- Speed
- Accelerate
- Move
- Torque
- Torque2
- LinearS...tTorque
- Quadrat...tTorque
- ConstantTorque
- SignTorque
- ConstantSpeed
- TorqueStep

- Interfaces**
- Flange_a
- Flange_b
- Support
- InternalSupport

SIMULATION PLATFORM: ENTIRE VEHICLE MODEL

- To verify the steering system, an experiment platform which integrates other parts of vehicle was built.

For verifying the model, assume null torque is provided by HPS.



- An **experiment** is designed: Steering back and forth (SW angle input: sine waveform) while the vehicle remains static (vehicle speed $v=0$ m/s)

SIMULATION PLATFORM: ENTIRE VEHICLE MODEL

- **Vehicle Dynamics Model:** Single-track Model (Bicycle Model)
- Assumption: the steering angle of two front wheels are the same.

2.1 Equations of motion of the single-track model

The equations of motion of the single-track model (Figure 1) may be expressed in a body-fixed frame with the origin at the vehicle's Centre of Gravity (CG) as follows:

$$m(\dot{V}_x - V_y \dot{\psi}) = f_{Fx} \cos \delta - f_{Fy} \sin \delta + f_{Rx} \quad (1)$$

$$m(\dot{V}_y + V_x \dot{\psi}) = f_{Fx} \sin \delta + f_{Fy} \cos \delta + f_{Ry} \quad (2)$$

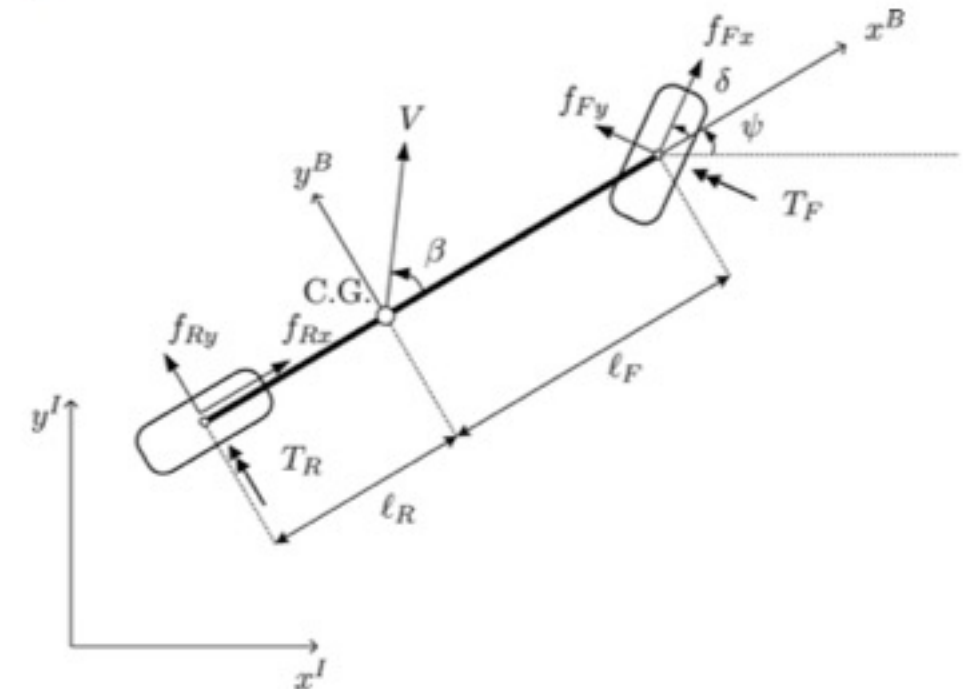
$$I_z \ddot{\psi} = (f_{Fy} \cos \delta + f_{Fx} \sin \delta) \ell_F - f_{Ry} \ell_R, \quad (3)$$

where

$$V_x = V \cos \beta, \quad V_y = V \sin \beta.$$

In the above equations m is the vehicle's mass, I_z is the moment of inertia of the vehicle about the vertical axis, V_x and V_y are the body-frame components of the vehicle velocity V , ψ is the yaw angle of the vehicle, and δ is the steering angle of the front wheel. By f_{ij} ($i = F, R$ and $j = x, y$) we denote the longitudinal and lateral friction forces at the front and rear wheels, respectively.

Figure 1 Single-track vehicle model



The vehicle slip angle is given by

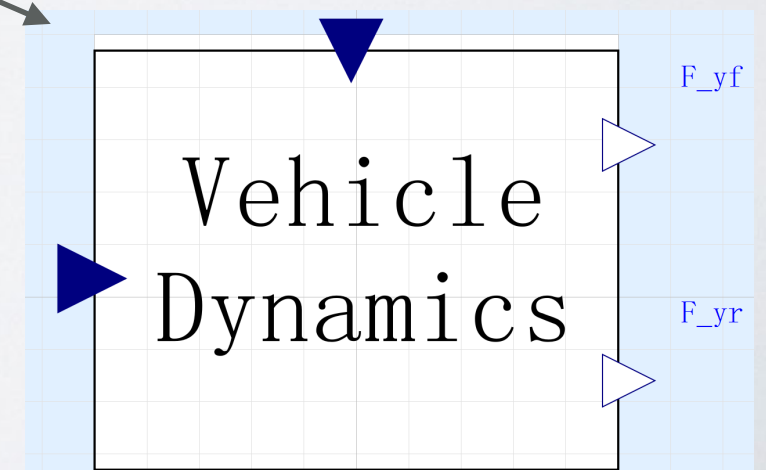
$$\beta = \text{atan}\left(\frac{\dot{y}}{\dot{x}}\right) - \psi = \text{atan}\left(\frac{V_y}{V_x}\right)$$

where \dot{x} and \dot{y} are the inertial frame components of the vehicle speed.

SIMULATION PLATFORM: ENTIRE VEHICLE MODEL

- **Vehicle Dynamics Model:** Single-track Model (Bicycle Model)
- Assumption: the steering angle of two front wheels are the same.
- Simplification: in the **experiment**, $v=0$ m/s, thus according to the bicycle model, all the lateral tire forces are null.

```
112 model VehicleDynamics
113   Modelica.Blocks.Interfaces.RealInput delta_kp annotation( ...);
115   Modelica.Blocks.Interfaces.RealInput v annotation( ...);
117   Modelica.Blocks.Interfaces.RealOutput F_yf annotation( ...);
119   Modelica.Blocks.Interfaces.RealOutput F_yr annotation( ...);
121 equation
122   F_yf = 0;
123   F_yr = 0;
124   annotation( ...);
127 end VehicleDynamics;
128
```



SIMULATION PLATFORM: ENTIRE VEHICLE MODEL

- Tire Model: **Fiala Tire Model**
- Pros: - Analytical Model (Lower computational load)
 - Consider the tire as a rigid circular plate:
 - Thus good for the study focused on aligning torques and analysis on manipulation performance etc.

$$\bar{M}_z = \bar{F}_y \cdot \frac{D_x}{a} - \bar{F}_x \left(\frac{D_y}{a} + \frac{y_b}{a} \right)$$

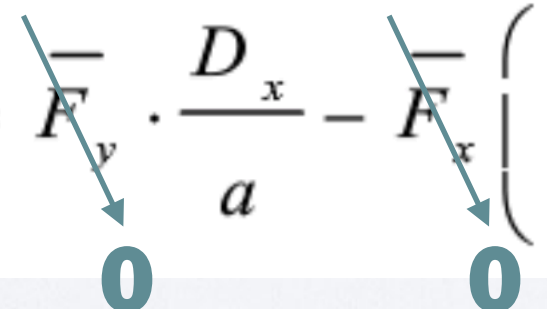
Diagram illustrating the Fiala Tire Model equation, showing the relationship between the self-aligning torque (\bar{M}_z) and the dimensionless lateral/longitudinal force (\bar{F}_y), dimensionless longitudinal/lateral trail ($\frac{D_x}{a}$), dimensionless longitudinal/lateral trail ($\frac{D_y}{a}$), and dimensionless translational deformation ($\frac{y_b}{a}$).

Labels and arrows pointing to the equation terms:

- Self-aligning torque (\bar{M}_z)
- Dimensionless lateral/longitudinal force (\bar{F}_y)
- Dimensionless longitudinal/lateral trail ($\frac{D_x}{a}$)
- Dimensionless longitudinal/lateral trail ($\frac{D_y}{a}$)
- Dimensionless translational deformation ($\frac{y_b}{a}$)

SIMULATION PLATFORM: ENTIRE VEHICLE MODEL

- Tire Model: **Fiala Tire Model**
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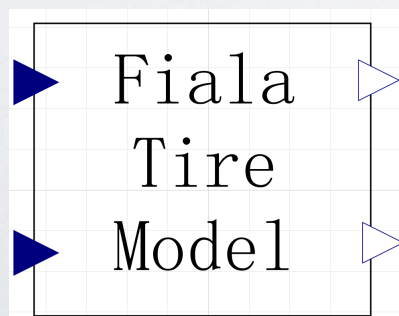
$$\bar{M}_z = \bar{F}_y \cdot \frac{D_x}{a} - \bar{F}_x \left(\frac{D_y}{a} + \frac{y_b}{a} \right)$$


- In the **experiment**, the vehicle remains static

SIMULATION PLATFORM: ENTIRE VEHICLE MODEL

- Tire Model: **Fiala Tire Model**
- Pros: - Analytical Model (Lower computational load)
 - Consider the tire as a rigid circular plate:
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```
129 model TireModel
130   Modelica.Blocks.Interfaces.RealInput F_yf annotation( ...);
132   Modelica.Blocks.Interfaces.RealInput F_yr annotation( ...);
134   Modelica.Blocks.Interfaces.RealOutput F_yfi annotation( ...);
136   Modelica.Blocks.Interfaces.RealOutput M_yfi annotation( ...);
138   equation
139     F_yfi=0;
140     M_yfi=0;
141   annotation( ...);
145 end TireModel;
```


SIMULATION PLATFORM: ENTIRE VEHICLE MODEL

- **Aligning Torque Model**

- Consists of three components:

- Gravitational aligning torque caused by vertical load **F_z**

$$M_z = F_z E_z \sin \delta_i \cos \delta_c \sin \delta_{KP}$$
$$E_z = E_{KP} \cos \delta_i$$

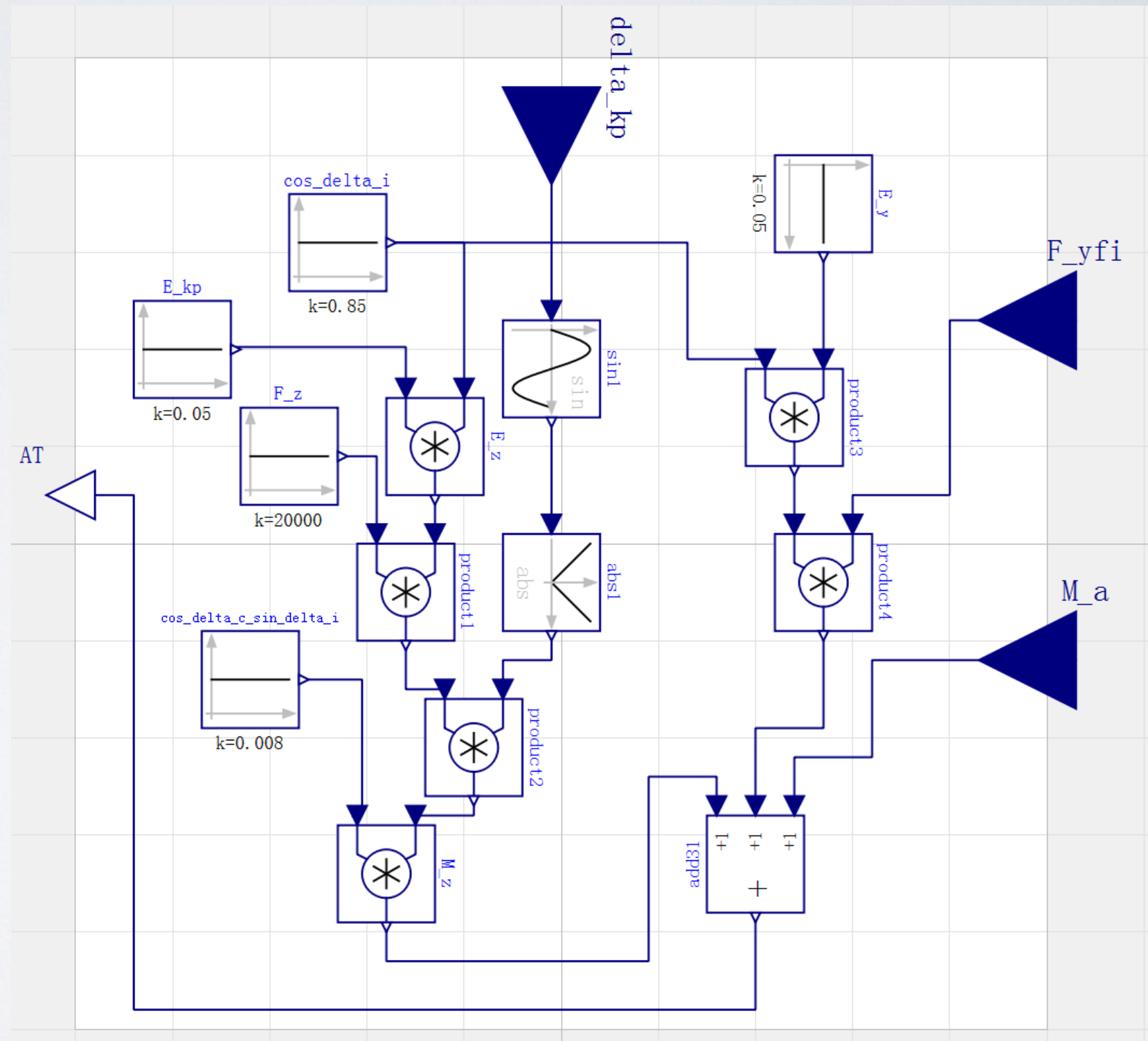
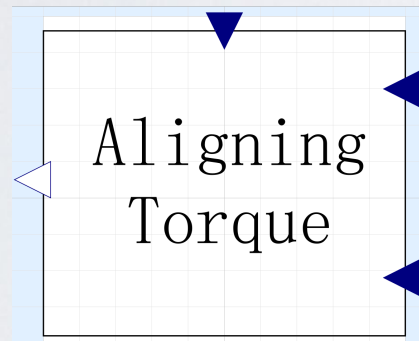
- Torque caused by lateral force **F_y**

$$M_y = F_y \cdot E_y \cdot \cos \delta_i$$
$$E_y = R_j \cdot \sin \delta_c$$

- Self-aligning torque (provided by the tire model)

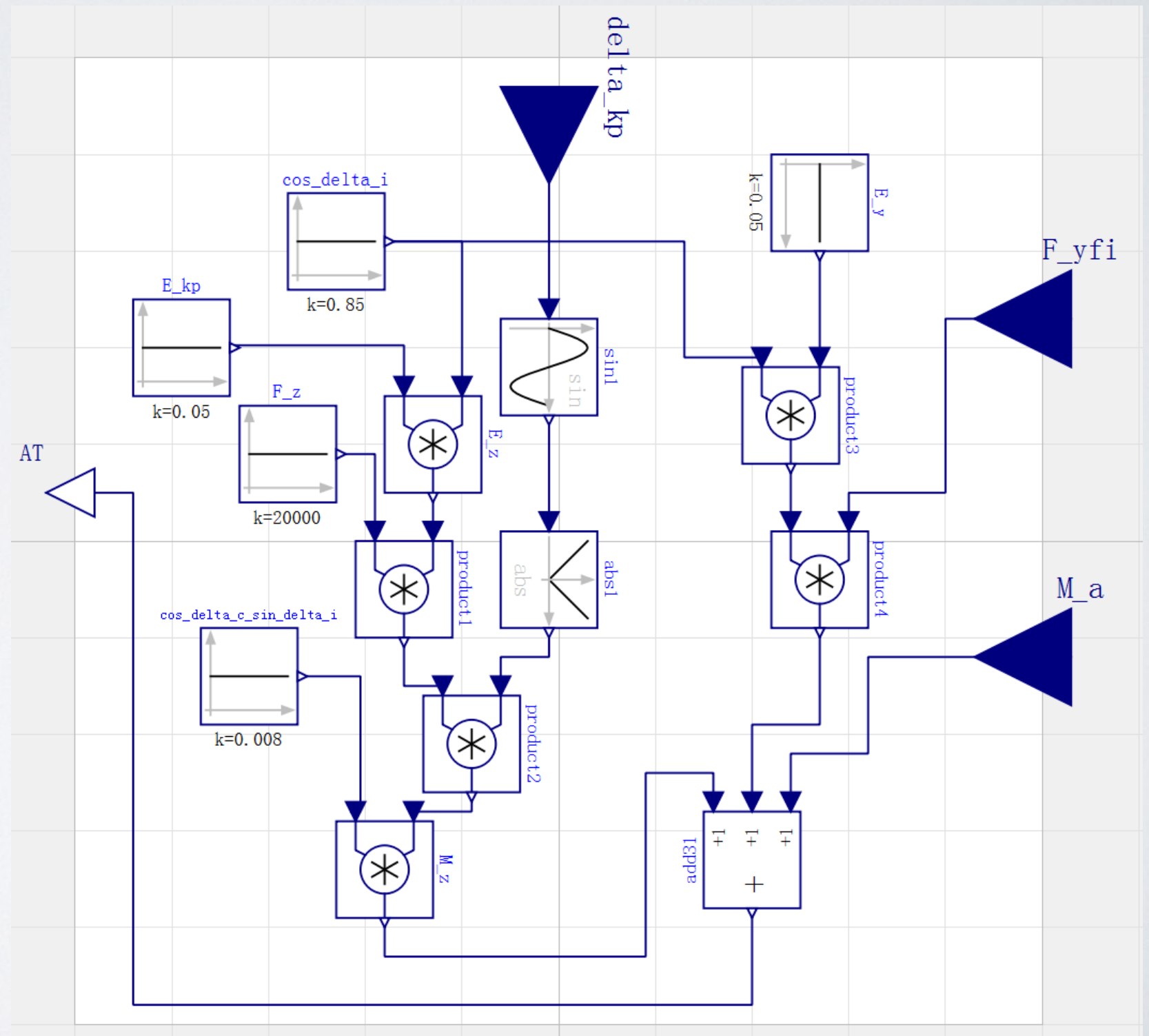
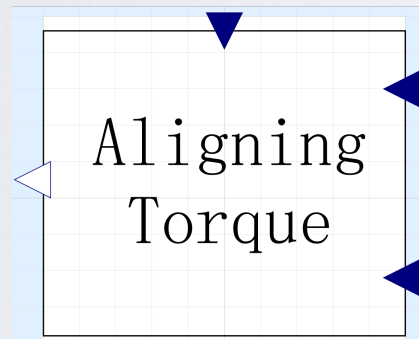
SIMULATION PLATFORM: ENTIRE VEHICLE MODEL

- **Aligning Torque Model**



SIMULATION PLATFORM: ENTIRE VEHICLE MODEL

- **Aligning Torque Model**



SIMULATION PLATFORM: ENTIRE VEHICLE MODEL

- **Resistant Torque Model**

- Frictional torque **T_m** :

- when $v \leq 5$ km/h,

$$T_m = \begin{cases} K_f \delta_{KP} & K_f \delta_{KP} \leq T_{mmax} \\ T_{mmax} & K_f \delta_{KP} > T_{mmax} \end{cases}$$

where

$$T_{mmax} = \frac{f}{3N} \sqrt{\frac{G^3}{P}} \quad (= \text{constant})$$

- when $v > 5$ km/h,

~~$$T_m = \frac{0.001 G K R_i}{N}$$~~

- Resistant torque **T_r** :

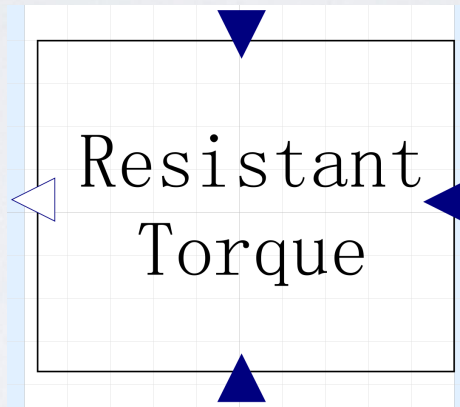
$$T_r = T_m + \text{sgn}(\delta_{sw} \dot{\delta}_{sw}) \cdot AT$$

AT is:

- resistive when steering
- active when aligning

SIMULATION PLATFORM: ENTIRE VEHICLE MODEL

- **Resistant Torque Model**

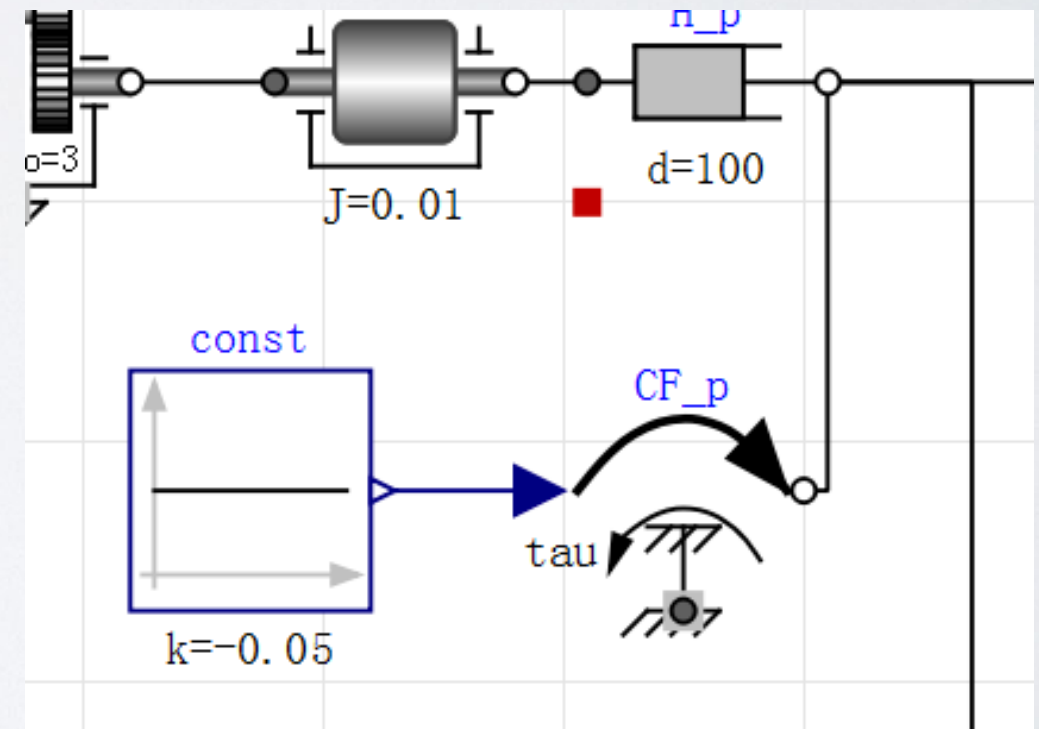
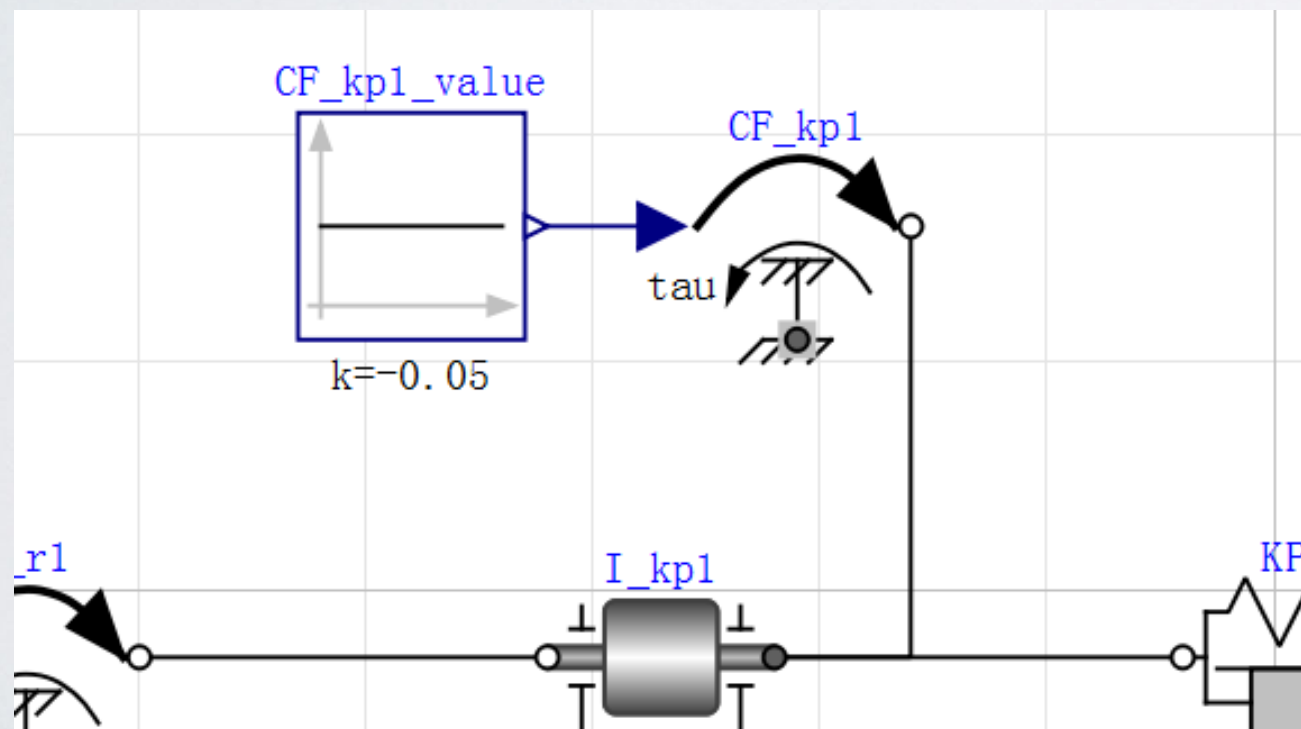


```
227 model ResistantTorque
228 + Modelica.Blocks.Interfaces.RealInput AT annotation( ...);
230 + Modelica.Blocks.Interfaces.RealInput delta_kp annotation( ...);
232 + Modelica.Blocks.Interfaces.RealInput delta_sw annotation( ...);
234 + Modelica.Blocks.Interfaces.RealOutput T_r annotation( ...);
236 parameter Real T_m_max = 5 "Maximum Tire-Road Friction Torque";
237 parameter Real K_f = 2 "Equivalent Stiffness of Tires";
238 Real T_m "Tire-Road Friction Torque";
239 Real delta_kp_abs "Absolute Value of delta_kp";
240 equation
241   delta_kp_abs = if delta_kp >= 0 then delta_kp else -delta_kp;
242   T_m = if K_f * delta_kp_abs <= T_m_max then K_f * delta_kp_abs else T_m_max;
243   T_r = if delta_kp * der(delta_kp) >= 0 then T_m + AT else T_m - AT;
244 + annotation( ...);
247 end ResistantTorque;
```

SIMULATION PLATFORM: ENTIRE VEHICLE MODEL

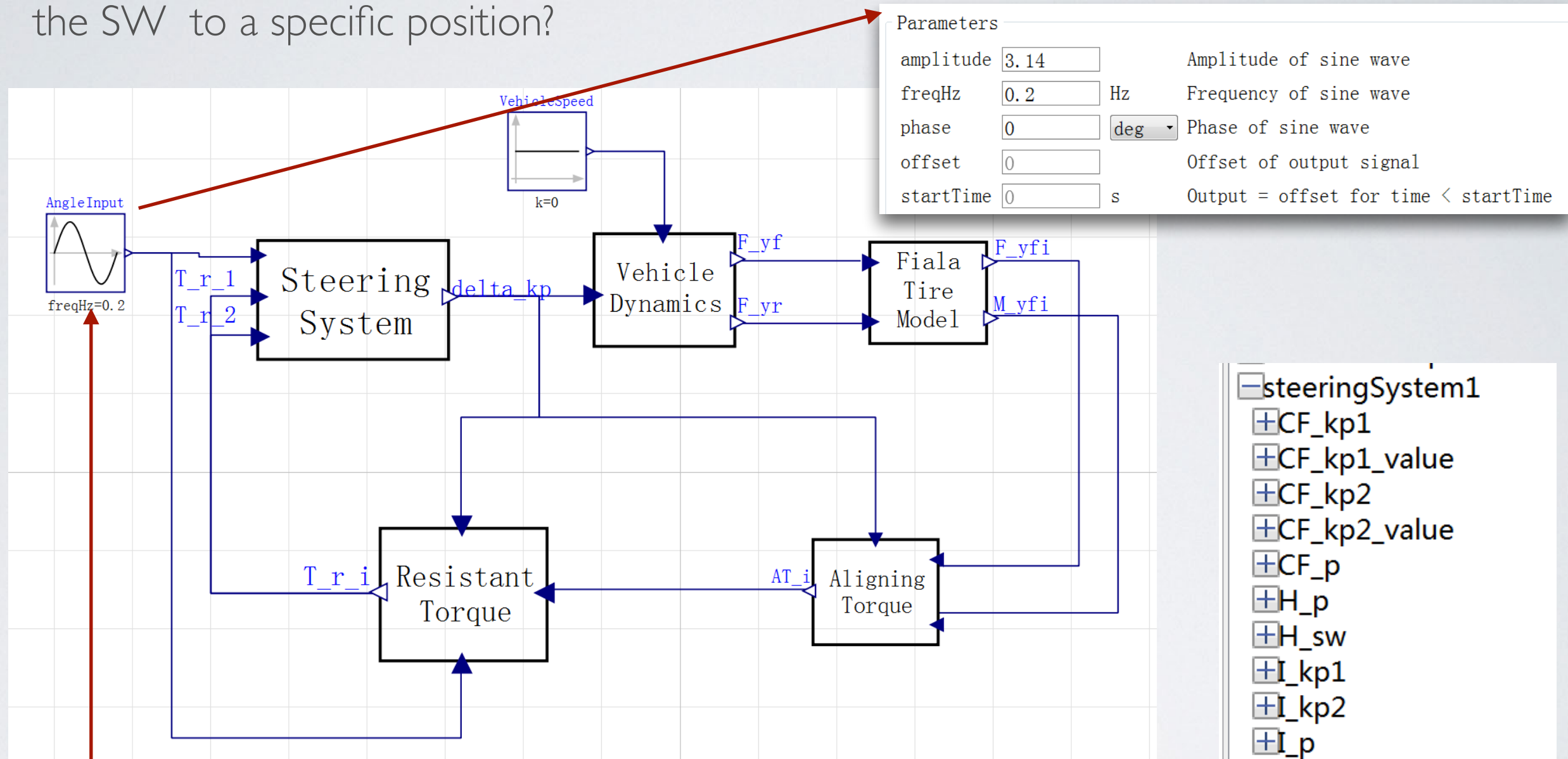
- **Dry Friction Model**

$$CF_{KPi} = \frac{GK}{M} \left(f_1 r_1 + \frac{2E_{KP} f_2 r_2}{L_{AB}} \right) \quad (= \text{constant})$$

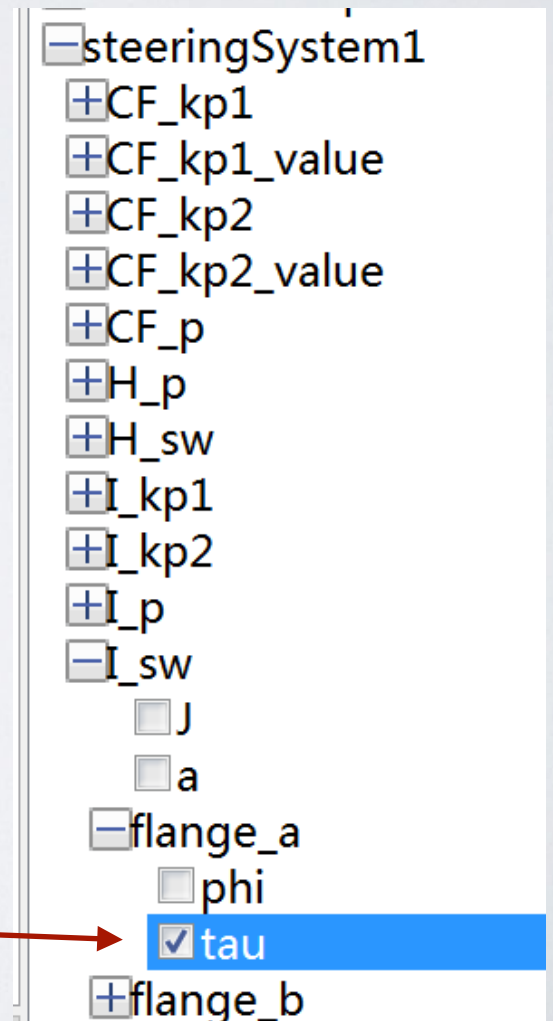


SIMULATION RESULTS

- Experiment: How much force (torque) is required from the driver to steer the SW to a specific position?



- Input:** SW angle sine waveform
- Output:** Torque on SW



SIMULATION RESULTS

- Question: is the model correct?

仿真配置 - Simulation...rolledSimulation

常规 输出 仿真选项 仿真存档

仿真间隔

Start Time 0 secs

End Time 50 secs → Ten cycles

Number of Intervals 2500

Interval: 0.02 secs

积分

方法: dassl

误差: 0.0001

雅可比:

DASSL/IDA Options

☒ 根查找

☒ 结束后重启

初始化步长:

最大步长:

最大积分秩: 5

☒ Save experiment annotation inside model

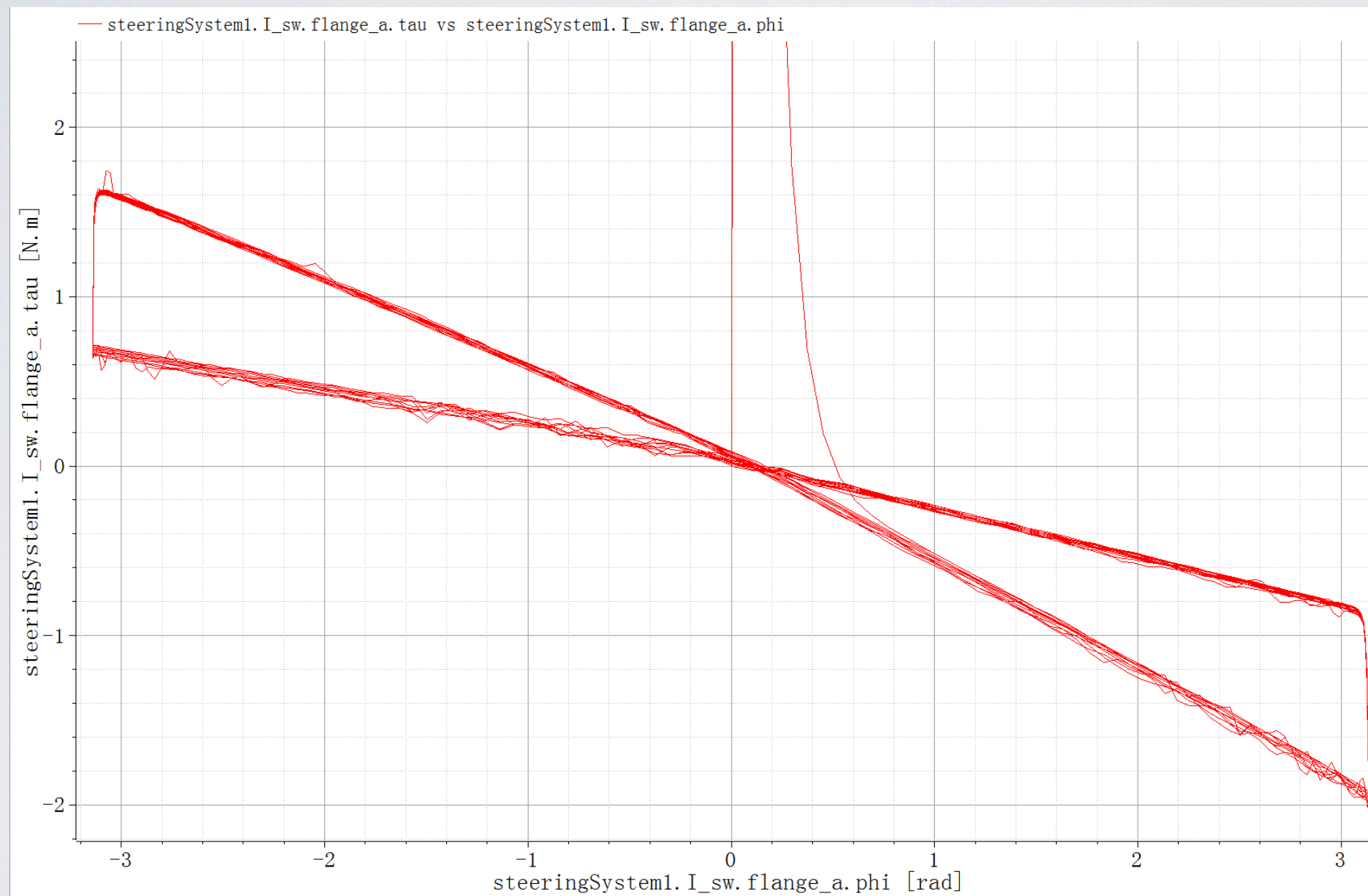
☐ Save __OpenModelica_simulationFlags annotation inside model

☒ 仿真

确定 取消

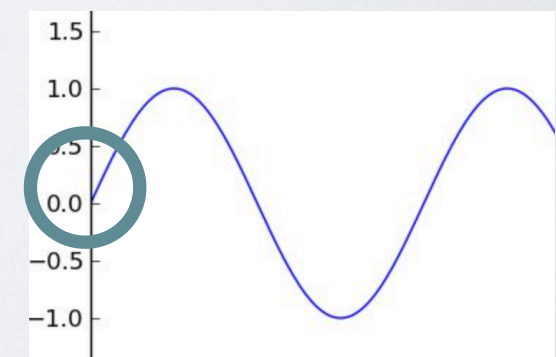
SIMULATION RESULTS

- Question: is the model correct?



Steering angle on SW

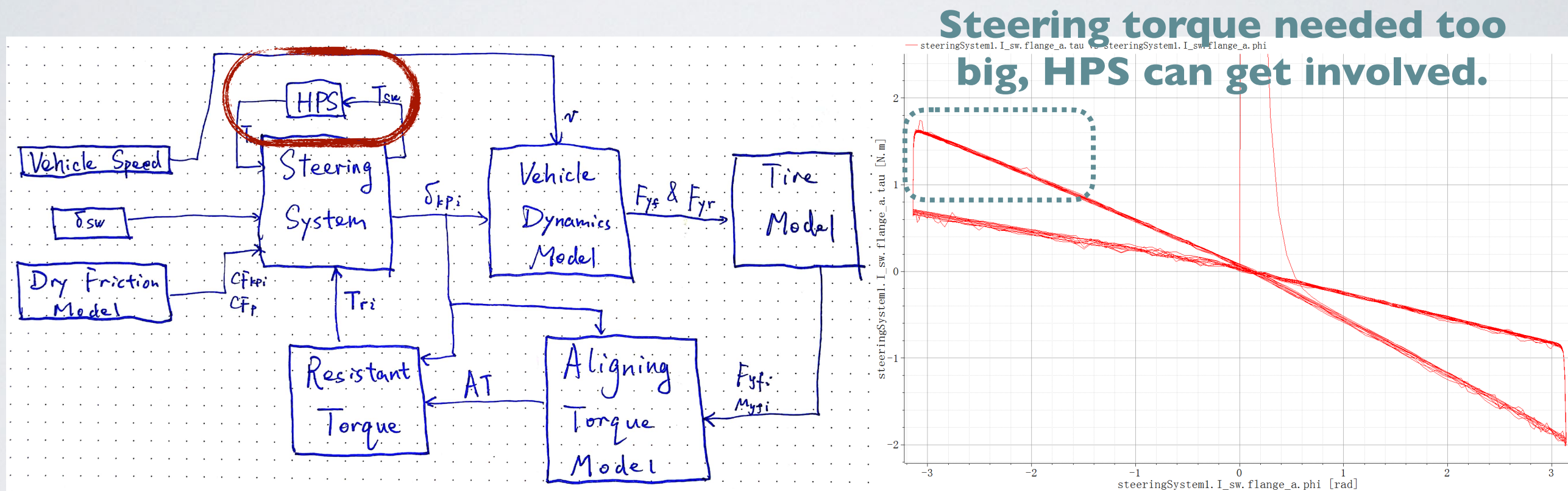
- Intuitively (Qualitatively) speaking correct
- An obvious overshoot because the starting steering acceleration too big.



Driving torque needed on SW

SIMULATION RESULTS

- Question: what can this platform do?
 - As a test platform for further design of HPS



- Develop a standard test platform for manipulating performance analysis
- etc...