# Section 4: Arrow’s Impossibility Theorem and the Median-Voter Theorem

As mentioned in the previous section, the market can ensure that everyone’s interests are expressed and protected, which is Pareto-optimal, but democracy cannot and can only seek to ensure that the interests of the majority are expressed and protected at the expense of the minority. Attention: the interests of the minority may not be lower than those of the majority. If the minority can buy the votes of the majority, in this case they will pay to the majority and still have surplus, which is precisely the implication of the third version of Coase theorem.

However, does democracy really at least ensure that the interests of the majority are expressed and protected? Arrow’s Impossibility Theorem or the Cycling Paradox has broken the myth of democracy.

Arrow’s impossibility theorem points out that a fundamental defect of the majority rule is that there will be cycling in decision-making. Suppose the voting preferences of three voters A, B, and C over three proposals, ranked from highest to lowest, are as follows: A prefers X is over Y, and Y over Z; B prefers Y over Z, and Z over X; C prefers Z over X, and X over Y.

Notice that a majority (A and C) prefer X over Y, a majority (A and B) prefer Y over Z, but a majority (B and C) prefer Z over X! This is the paradox. The majority-rule choices can be intransitive.

Where cycling is possible, the voting sequence (the “agenda”) can determine the final outcome. Suppose A can determine the order of voting. He could call first for a vote between Y and Z, the winner to be matched against X. Policy Y would win the first vote. But in the second vote, his favored policy X would win over Y. Given the same power over the agenda, B or C could also assure outcomes in their own favor.

Is it possible to design a voting without cycling? Arrow’s conclusion is that there is no voting that can guarantee efficiency, protect everyone’s interests without relying on the majority rule. In other words, when the ranking of the interests of all in a society is known, it is impossible to rank the social interests by ranking the individual’s interests, and it is impossible to accurately express everyone’s interests by a certain procedure, so it is impossible to make a desirable public choice.

The above is the case where there are multiple alternatives and there is cycling. What if an issue is to be decided in dichotomous (yes/no) voting? Here it is the Median-voter Theorem that applies. Under reasonable assumptions the outcome will coincide with the preferences of the median voter. To see why, imagine lining up the voters in order of their support for the issue. The voter in the middle (the median voter) must necessarily be among the majority. If there is a strong preponderance on one side, by definition the median voter will be in that group. Or, at the extreme, if the other voters are equally split, then the middle voter’s own choice makes the majority.

This median-voter theorem seems to make little sense in politics of reality because it is generally impossible for the other voters to be equally split, but it is actually significant. The economic implication (or political implication) is that the offsetting forces in the voting are not important, but it is the “residual value” that determines the outcome. For example, suppose there are 1 million eligible voters in a country, of which 300,000 of them vote to the left and 400,000 vote to the right, so only 100,000 votes actually matter. According to the postulate of self-interest, in democracy, politicians pursue vote maximization. The median-voter theorem implies that what a politician should do is to get the votes that can overwhelm 100,000 (instead of 300,000 or 400,000) with the least cost. Thus, it is the interests of the minority instead of the majority) that are important.

By contrast, in the market, any interests will be cared about, whether of the majority or the minority. Just as mentioned in Lecture 9, big chickens eat big rice, small chickens eat small rice, and even broken rice is eaten by worms. As long as there is demand, there will be always supply and there will be always a market to take care of the demand.