Readme

The present scripts are made to help with the numerical analysis of data from radio nuclei transport experiments as demonstrated in *simulation_ example.m*. The relevant functions are documented at the beginning of their file.
The used underlying physical model describes the concentration profile $c(t,x)$ of unabsorbed radioactive nuclei in the ground where t denotes time and x the depth below the surface. The temporal evolution of c is expected to obey the transport equation

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} + v\frac{\partial c}{\partial x} - \lambda c \tag{1}$$

where D and v are diffusion and drift velocity coefficient respectively and λ is the decay constant of the nuclei. In the case of constant D and v, eq. 1 has an analytical solution. For the more complex case of spatially varying diffusion and drift velocity, a numerical scheme for solving the transport equation, as used in *transport_ radio_ nuclei.m* , is written down in the following.
We may discretize the spacial coordiante into segments $i = 1, ..., N$ of thickness dx. We then discretize eq. 1 at each segment as

$$\frac{\partial c_i}{\partial t} = \frac{J_{in,i} - J_{out,i}}{dx} - \lambda c_i \tag{2}$$

where $J_{in,i}$ and $J_{out,i}$ are the conserved incoming and outgoing fluxes of each segment which we discretize as

$$J_{in,i} = \frac{D_{i-1} + D_i}{2}\,\frac{c_{i-1} - c_i}{dx} + \frac{v_{i-1} + v_i}{2}\,\frac{c_{i-1} + c_i}{2}, \qquad J_{out,i} = J_{in,i+1} \tag{3}$$

thus leading to

$$\begin{aligned}\frac{\partial c_i}{\partial t} = & \frac{1}{2\,dx^2}\left[(D_{i-1} + D_i)\; c_{i-1} - (D_{i-1} + 2D_i + D_{i+1})\; c_i + (D_i + D_{i+1})\; c_{i+1}\right] + \\ & + \frac{1}{4\,dx}\left[(v_{i-1} + v_i)\; c_{i-1} + (v_{i-1} + v_{i+1})\; c_i + (v_i + v_{i+1})\; c_{i+1}\right] - \lambda\, c_i.\end{aligned} \tag{4}$$

This scheme then can be time iterated with conventional solvers. The boundary conditions read

$$J_{in,1} = v_1\, c_0(t), \qquad J_{out,N} = \left(\frac{v_{N-1} - v_N}{2}\right)(1.5c_N - 0.5c_{N-1})\,. \tag{5}$$

Here, the time course of the concentration at the surface $c_0(t)$ can be chosen freely. For the case of constant diffusion and drift velocity coefficientt this scheme simplifies to form in [Frissel et al. (1974)].