In [1]:
```
import sys
sys.path.insert(0, '/home/dominic/Software/tudat-bundle/build-tudat-bundl
```

Re-entry trajectory

Context

This examples focuses on the application of aerodynamic guidance in the context of a re-entry trajectory of the Space Transportation System (STS).

The aerodynamic guidance updates the angle of attack and the bank angle of the vehicle based on its flight conditions. The angle of attack is set to 40deg for a Mach number above 12, to 10deg for a Mach number below 6, and varies linearly between them. The bank angle is computed such that the derivative of the flight path angle over time equals 0, and the flight path angle is then constant. To do so, this example also showcases how to extract and use the flight condition and body properties during the simulation.

The initial state of the STS is most notably its initial altitude of 120km, velocity of 7.5km/s, and its flight path angle of -0.6deg.

A high number of dependent variable are also propagated in this example. All of them are then plotted at the end of this script.

Import statements

The required import statements are made here, at the very beginning.

Some standard modules are first loaded. These are `numpy` and `matplotlib.pyplot`.

Then, the different modules of `tudatpy` that will be used are imported.

In [2]:
```
# Load standard modules
import math
import numpy as np
from matplotlib import pyplot as plt
```

In [3]:
```
# Load tudatpy modules
from tudatpy.kernel.interface import spice
from tudatpy.kernel import numerical_simulation
from tudatpy.kernel.numerical_simulation import environment_setup, enviro
from tudatpy.kernel.numerical_simulation import propagation_setup, propag
from tudatpy.kernel.astro import element_conversion
from tudatpy.kernel import constants
from tudatpy.util import result2array
```

Aerodynamic guidance class

First of all, let's create a class that contains the aerodynamic guidance. This class needs to be inherited from `propagation.AerodynamicGuidance`.

During the initialisation of this class, the current system of simulated bodies will have to be input.

Then, the class must contain an `updateGuidance()` function that will be called at each simulation time step, taking the time as an input. Most importantly, this function updates both the angle of attack (`self.angle_of_attack`) and bank angle (`self.bank_angle`) of the vehicle.

The angle of attack α should be updated as a function of the Mach number M as follows:

- $\alpha = 40$ deg if $M > 12$.
- $\alpha = 10$ deg if $M < 6$.
- α varies linearly between the two boundaries for other M.

In practice, the following Logistic function is used so that the transition in α between $M=12$ and $M=6$ is smoother: $$ \alpha = \frac{30}{1 + e^{-2 (M-9) }} + 10 $$

The bank angle σ is computed so that, ideally, the flight path angle γ should stay constant over time ($\dot{\gamma} = 0$). The change in flight path angle over time $\dot{\gamma}$ is related to the bank angle σ trough the equation below. We thus compute σ so that this equation equals 0. $$ V \dot{\gamma} = \frac{L}{m} \cos \sigma - \left(g_d - \frac{V^2}{r} \right) \cos \gamma + 2 \omega_E V \cos \delta \sin \chi + \omega^2_E r \cos \delta (\cos \delta \cos \gamma + \sin \gamma \sin \delta \cos \chi) $$ In this equation, χ and δ are the heading and latitude angles. L, m, V, and ω_E are the lift force, vehicle mass, airspeed, and Earth rotation rate. g_d is the downwards (towards the centre of Earth) component of the gravitational acceleration. All of these values are either directly obtained or computed from the flight conditions, the aerodynamic coefficient interface, or the aerodynamic coefficient interface.

In [4]:
```
# Create a class for the aerodynamic guidance of the STS, inheriting from
```

```python
class STSAerodynamicGuidance:

    def __init__(self, bodies: environment.SystemOfBodies):

        # Extract the STS and Earth bodies
        self.vehicle = bodies.get_body("STS")
        self.earth = bodies.get_body("Earth")

        # Extract the STS flight conditions, angle calculator, and aerody
        environment_setup.add_flight_conditions( bodies, 'STS', 'Earth' )
        self.vehicle_flight_conditions = bodies.get_body("STS").flight_co
        self.aerodynamic_angle_calculator = self.vehicle_flight_condition
        self.aerodynamic_coefficient_interface = self.vehicle_flight_cond

        self.current_time = float("NaN")

    def getAerodynamicAngles(self, current_time: float):
        self.updateGuidance( current_time )
        return np.array([self.angle_of_attack, 0.0, self.bank_angle])

    # Function that is called at each simulation time step to update the
    def updateGuidance(self, current_time: float):

        if( math.isnan( current_time ) ):
            self.current_time = float("NaN")
        elif( current_time != self.current_time ):
            # Get the (constant) angular velocity of the Earth body
            earth_angular_velocity = np.linalg.norm(self.earth.body_fixed
            # Get the distance between the vehicle and the Earth bodies
            earth_distance = np.linalg.norm(self.vehicle.position)
            # Get the (constant) mass of the vehicle body
            body_mass = self.vehicle.mass

            # Extract the current Mach number, airspeed, and air density
            mach_number = self.vehicle_flight_conditions.mach_number
            airspeed = self.vehicle_flight_conditions.airspeed
            density = self.vehicle_flight_conditions.density

            # Set the current Angle of Attack (AoA). The following line e
            # * the AoA is constant at 40deg when the Mach number is abov
            # * the AoA is constant at 10deg when the Mach number is belo
            # * the AoA varies close to linearly when the Mach number is
            # * a Logistic relation is used so that the transition in AoA
            self.angle_of_attack = np.deg2rad(30 / (1 + np.exp(-2*(mach_n

            # Update the variables on which the aerodynamic coefficients
            current_aerodynamics_independent_variables = [self.angle_of_a
            # Update the aerodynamic coefficients
            self.aerodynamic_coefficient_interface.update_coefficients(
                current_aerodynamics_independent_variables, current_time)

            # Extract the current force coefficients (in order: C_D, C_S,
            current_force_coefficients = self.aerodynamic_coefficient_int
            # Extract the (constant) reference area of the vehicle
            aerodynamic_reference_area = self.aerodynamic_coefficient_int

            # Get the heading, flight path, and latitude angles from the
```

```
            heading = self.aerodynamic_angle_calculator.get_angle(environ
            flight_path_angle = self.aerodynamic_angle_calculator.get_ang
            latitude = self.aerodynamic_angle_calculator.get_angle(enviro

            # Compute the acceleration caused by Lift
            lift_acceleration = 0.5 * density * airspeed ** 2 * aerodynam
            # Compute the gravitational acceleration
            downward_gravitational_acceleration = self.earth.gravitationa
            # Compute the centrifugal acceleration
            spacecraft_centrifugal_acceleration = airspeed ** 2 / earth_d
            # Compute the Coriolis acceleration
            coriolis_acceleration = 2 * earth_angular_velocity * airspeed
            # Compute the centrifugal acceleration from the Earth
            earth_centrifugal_acceleration = earth_angular_velocity ** 2

            # Compute the cosine of the ideal bank angle
            cosine_of_bank_angle = ((downward_gravitational_acceleration
            # If the cosine lead to a value out of the [-1, 1] range, set
            if (cosine_of_bank_angle < -1):
                self.bank_angle = np.pi
            elif (cosine_of_bank_angle > 1):
                self.bank_angle = 0.0
            else:
                # If the cos is in the correct range, return the computed
                self.bank_angle = np.arccos(cosine_of_bank_angle)
            self.current_time = current_time
```

Configuration

NAIF's `SPICE` kernels are first loaded, so that the position of various bodies such as the Earth can be make known to `tudatpy`.

Then, the start and end simulation epochs are setups. In this case, the start epoch is set to `0`, corresponding to the 1st of January 2000. The times should be specified in seconds since J2000. Please refer to the API documentation of the `time_conversion module` here for more information on this.

In [5]:
```
# Load spice kernels
spice.load_standard_kernels()

# Set simulation start epoch
simulation_start_epoch = 0.0

# Set the maximum simulation time (avoid very long skipping re-entry)
max_simulation_time = 3*constants.JULIAN_DAY


# ## Environment setup
#
# Let’s create the environment for our simulation. This setup covers the
#
```

Create the bodies

Bodies can be created by making a list of strings with the bodies that is to be included in the simulation.

The default body settings (such as atmosphere, body shape, rotation model) are taken from `SPICE`.

These settings can be adjusted. Please refere to the Available Environment Models in the user guide for more details.

Finally, the system of bodies is created using the settings. This system of bodies is stored into the variable `bodies`.

In [6]:
```
# Create default body settings for "Earth"
bodies_to_create = ["Earth"]

# Create default body settings for bodies_to_create, with "Earth"/"J2000"
global_frame_origin = "Earth"
global_frame_orientation = "J2000"
body_settings = environment_setup.get_default_body_settings(
    bodies_to_create, global_frame_origin, global_frame_orientation)

# Create system of bodies (in this case only Earth)
bodies = environment_setup.create_system_of_bodies(body_settings)
```

Create the vehicle

Let's now create the 5000kg vehicle for which Earth re-entry trajectory will be simulated.

In [7]:
```
# Create vehicle object and set its constant mass
bodies.create_empty_body("STS")
bodies.get_body( "STS" ).set_constant_mass(5.0e3)
```

Add an aerodynamic coefficient interface

An aerodynamic coefficient interface is now added to the STS vehicle. These coefficients are interpolated from files that tabulate them as a function of angle of attack and Mach number.

In [8]:
```
# Define the aerodynamic coefficient files (leave C_S empty)
aero_coefficients_files = {0: "input/STS_CD.dat", 2:"input/STS_CL.dat"}

# Setup the aerodynamic coefficients settings tabulated from the files
coefficient_settings = environment_setup.aerodynamic_coefficients.tabulat
    force_coefficient_files=aero_coefficients_files,
    reference_area=2690.0*0.3048*0.3048,
    independent_variable_names=[environment.angle_of_attack_dependent, en
    are_coefficients_in_aerodynamic_frame=True,
    are_coefficients_in_negative_axis_direction=True
)

# Add predefined aerodynamic coefficients database to the body
environment_setup.add_aerodynamic_coefficient_interface(bodies, "STS", co
```

In [9]:
```
# ### Add rotation model based on aerodynamic guidance

# Create the aerodynamic guidance object
aerodynamic_guidance_object = STSAerodynamicGuidance(bodies)
rotation_model_settings = environment_setup.rotation_model.aerodynamic_an
    'Earth', '', 'STS_Fixed', aerodynamic_guidance_object.getAerodynamicA
environment_setup.add_rotation_model( bodies, 'STS', rotation_model_setti
```

Propagation setup

Now that the environment is created, the propagation setup is defined.

First, the bodies to be propagated and the central bodies will be defined. Central bodies are the bodies with respect to which the state of the respective propagated bodies is defined.

In [10]:
```
# Define bodies that are propagated
bodies_to_propagate = ["STS"]

# Define central bodies of propagation
central_bodies = ["Earth"]
```

Create the acceleration model

The acceleration settings that act on the `STS` vehicle are now defined. In this case, these simply consist in the Earth gravitational effect modelled as a point mass and of the aerodynamic acceleration of the Earth atmosphere.

The acceleration settings defined are then applied to `STS` vehicle in a dictionary.

This dictionary is finally input to the propagation setup to create the acceleration models.

In [11]:
```
# Define the accelerations acting on the STS (Earth as a Point Mass, and
accelerations_settings_STS = dict(
    Earth=[
        propagation_setup.acceleration.point_mass_gravity(),
        propagation_setup.acceleration.aerodynamic(),
    ]
)

acceleration_settings = {"STS": accelerations_settings_STS}

# Create the acceleration models
acceleration_models = propagation_setup.create_acceleration_models(
    bodies, acceleration_settings, bodies_to_propagate, central_bodies
)
```

Define the initial state

The initial state of the vehicle that will be propagated is now defined. Most importantly, the `STS` vehicle starts 120km above Earth, at a velocity og 7500m/s, and a flight path angle of -0.6 deg (from the horizon).

This initial state always has to be provided as a cartesian state, in the form of a list with the first three elements reprensenting the initial position, and the three remaining elements representing the initial velocity.

In this case, let's make use of the `spherical_to_cartesian_elementwise()` function that is included in the `element_conversion` module, so that the initial state can be input as Spherical elements, and then converted in Cartesian elements.

Finally, the initial state has to be converted from the Earth-fixed frame in which it is defined with the Spherical elements to the inertial frame.

In [12]:
```
# Set the initial state of the STS as spherical elements, and convert the
initial_radial_distance = bodies.get_body("Earth").shape_model.average_ra

# Convert the initial state
initial_earth_fixed_state = element_conversion.spherical_to_cartesian_ele
    radial_distance=initial_radial_distance,
    latitude=np.deg2rad(20),
    longitude=np.deg2rad(140),
    speed=7.5e3,
    flight_path_angle=np.deg2rad(-0.6),
    heading_angle=np.deg2rad(15),
)

# Convert the state from the Earth-fixed frame to the inertial frame
earth_rotation_model = bodies.get_body("Earth").rotation_model
initial_state = environment.transform_to_inertial_orientation(
    initial_earth_fixed_state, simulation_start_epoch, earth_rotation_mod
)
```

Define the dependent variables to save

In this example, we are interested in saving not only the propagated state of the vehicle over time, but also a set of so-called dependent variables, that are to be computed (or extracted and saved) at each integration step.

This page of the tudatpy API website provides a detailled explanation of all the dependent variables that are available.

In [13]:
```
# Define the list of dependent variables to save during the propagation
dependent_variables_to_save = [
    propagation_setup.dependent_variable.flight_path_angle("STS", "Earth"
    propagation_setup.dependent_variable.altitude("STS", "Earth"),
    propagation_setup.dependent_variable.bank_angle("STS", "Earth"),
    propagation_setup.dependent_variable.angle_of_attack("STS", "Earth"),
    propagation_setup.dependent_variable.aerodynamic_force_coefficients("
    propagation_setup.dependent_variable.airspeed("STS", "Earth"),
    propagation_setup.dependent_variable.total_acceleration_norm("STS"),
    propagation_setup.dependent_variable.mach_number("STS", "Earth")
]
```

Create the propagator settings

The propagator is finally setup.

First, a termination condition is defined so that the propagation as soon as one of these conditions is fulfilled:

- The altitude gets below 25km.
- The simulation time gets above 3 days.

Combinated termination settings are then needed, which can be done using the `propagation_setup.propagator.hybrid_termination()` function.

Then, the translational propagator settings are defined. These are used to simulate the orbit of `Delfi-C3` around Earth.

In [14]:
```
# Define a termination conditions to stop once altitude goes below 25 km
termination_altitude_settings = propagation_setup.propagator.dependent_va
    dependent_variable_settings=propagation_setup.dependent_variable.alti
    limit_value=25.0e3,
    use_as_lower_limit=True)
# Define a termination condition to stop after a given time (to avoid an
termination_time_settings = propagation_setup.propagator.time_termination
# Combine the termination settings to stop when one of them is fulfilled
combined_termination_settings = propagation_setup.propagator.hybrid_termi
    [termination_altitude_settings, termination_time_settings], fulfill_s

# Create the propagation settings
propagator_settings = propagation_setup.propagator.translational(
    central_bodies,
    acceleration_models,
    bodies_to_propagate,
    initial_state,
    combined_termination_settings,
    output_variables=dependent_variables_to_save
)
```

Create the integrator settings

The last step before starting the simulation is to setup the integrator that will be used.

In this case, a RK4 integrator is used with a step fixed at 0.5 seconds.

In [15]:
```
# Create numerical integrator settings
fixed_step_size = 0.5
integrator_settings = propagation_setup.integrator.runge_kutta_4(
    simulation_start_epoch, fixed_step_size
)
```

Propagate the trajectory

The re-entry trajectory is now ready to be propagated.

This is done by calling the `SingleArcSimulator()` function of the `numerical_simulation module`. This function requires the `bodies`, `integrator_settings`, and `propagator_settings` that have all been defined earlier.

After this, the dependent variable history is extracted. The column indexes corresponding to a given dependent variable in the `dep_vars` variable are printed when the simulation is run, when `SingleArcSimulator()` is called. Do mind that converting to an ndarray using the `result2array()` utility will shift these indexes, since the first column (index 0) will then be the times.

In this example, we are not interested in analysing the state history. This can however be accessed in the `dynamics_simulator.state_history` variable.

In [16]:
```
# Create the simulation objects and propagate the dynamics
dynamics_simulator = numerical_simulation.SingleArcSimulator(
    bodies, integrator_settings, propagator_settings
)

# Extract the resulting simulation dependent variables
dependent_variables = dynamics_simulator.dependent_variable_history
# Convert the dependent variables from a dictionary to a numpy array
dependent_variables_array = result2array(dependent_variables)
```

```
State vector contains:
Vector entries, Vector contents
[0:6], translational state of body STS
Dependent variables being saved, output vector contains:
Vector entry, Vector contents
0, Velocity orientation angle  flight path angle of STS w.r.t. Earth
1, Altitude of STS w.r.t. Earth
2, Aerodynamic body orientation angle bank angle of STS w.r.t. Earth
3, Aerodynamic body orientation angle angle of attack of STS w.r.t. Eart
h
4, Aerodynamic force coefficients of STS
7, Airspeed of STS w.r.t. Earth
8, Total acceleration norm of STS
9, Mach number of STS w.r.t. Earth
```

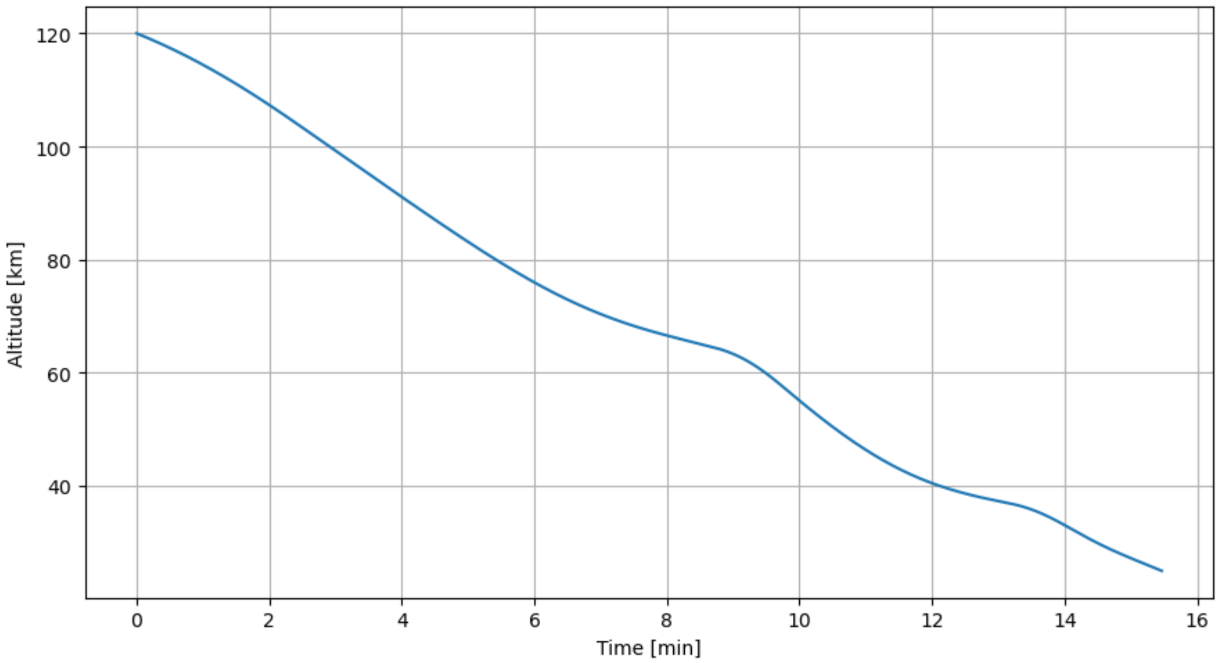
Post-process the propagation results

The results of the propagation are then processed to a more user-friendly form.

Altitude over time

First, let's plot the altitude of the `STS` vehicle over time.

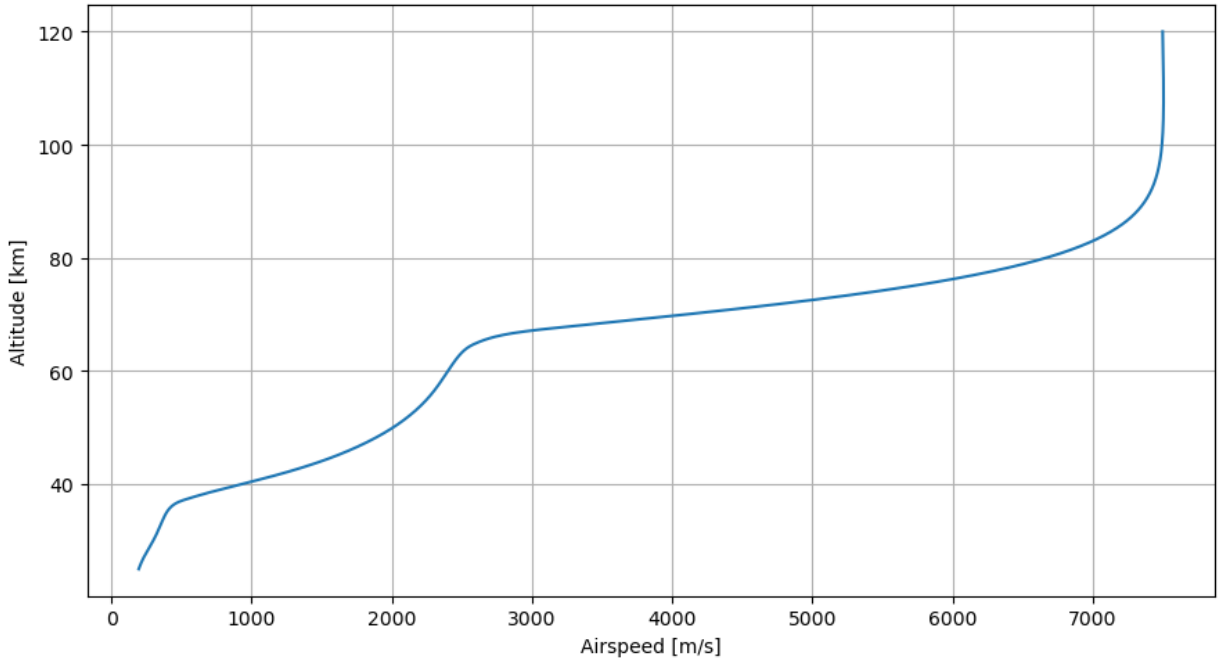
In [17]:
```
# Extract the time from the dependent variables array (and convert from s
time_min = dependent_variables_array[:,0] / 60

# Define a matplotlib.pyplot figure
plt.figure(figsize=(9, 5))
# Plot the altitude over time
plt.plot(time_min, dependent_variables_array[:,2]/1e3)
# Add label to the axis
plt.xlabel("Time [min]"), plt.ylabel("Altitude [km]")
# Add a grid
plt.grid()
# Use a tight layout to save space
plt.tight_layout()
plt.show()
```



Airspeed vs altitude

Let's now plot the altitude of the vehicle as a function of its airspeed. This gives insights into how the vehicle decelerates.

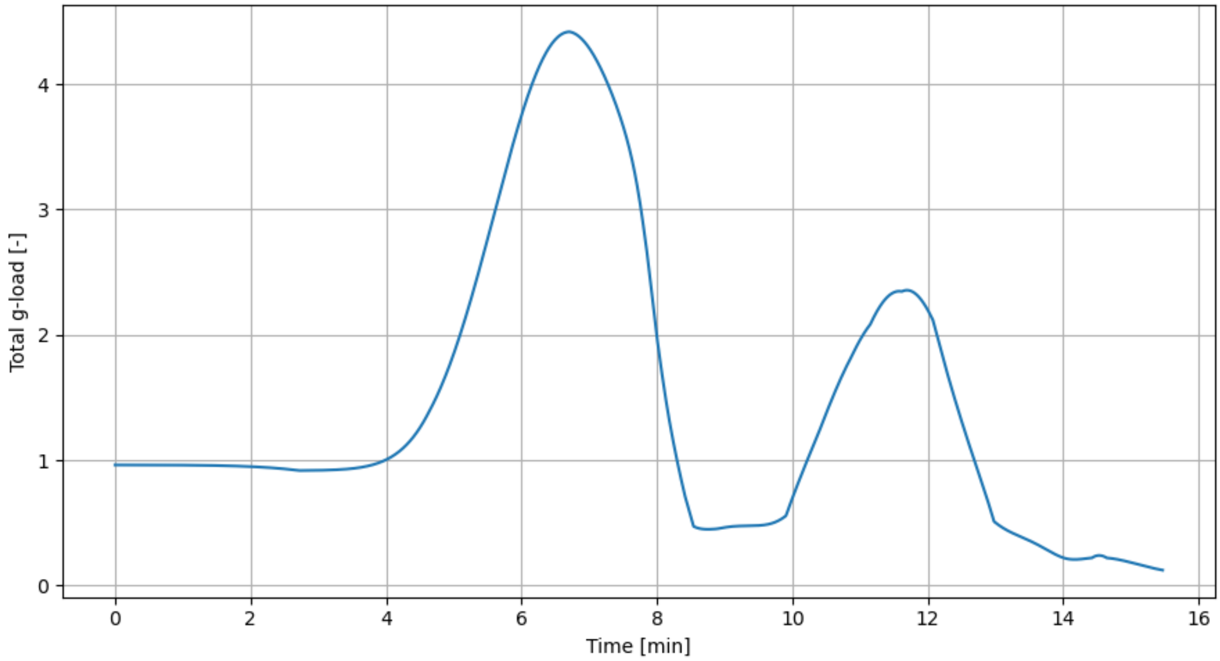
In [18]:
```
# Plot the airspeed vs altitude
plt.figure(figsize=(9, 5))
plt.plot(dependent_variables_array[:,8], dependent_variables_array[:,2]/1
plt.xlabel("Airspeed [m/s]"), plt.ylabel("Altitude [km]")
plt.grid()
plt.tight_layout()
plt.show()
```



g-load over time

The following plot then shows the total acceleration on the vehicle in `g` (Earth's gravitational acceleration at sea level).

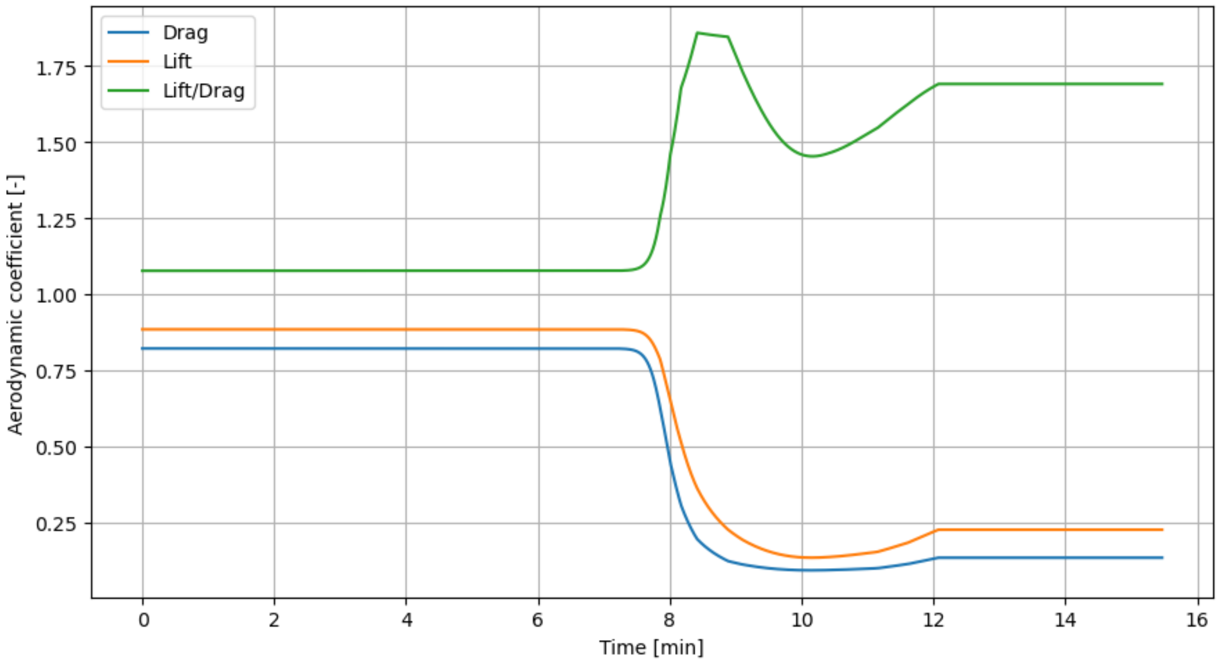
In [19]:
```
# Plot the g-load over time
plt.figure(figsize=(9, 5))
plt.plot(time_min, dependent_variables_array[:,9]/9.81)
plt.xlabel("Time [min]"), plt.ylabel("Total g-load [-]")
plt.grid()
plt.tight_layout()
plt.show()
```



Aerodynamic coefficient over time

Plotting the aerodynamic coefficients over time can also give a good set of insights into what happens during re-entry.

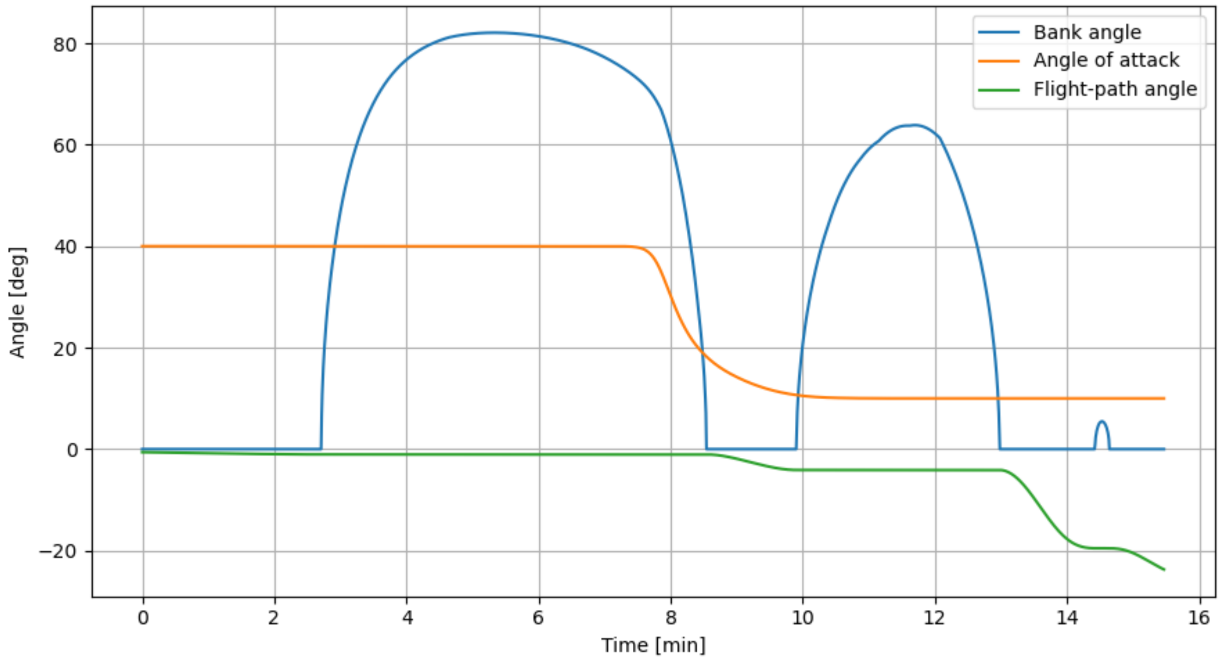
In [20]:
```
# Plot C_D, C_L, and L/D over time
plt.figure(figsize=(9, 5))
plt.plot(time_min, dependent_variables_array[:,5], label="Drag")
plt.plot(time_min, dependent_variables_array[:,7], label="Lift")
plt.plot(time_min, dependent_variables_array[:,7]/dependent_variables_arr
plt.xlabel("Time [min]"), plt.ylabel("Aerodynamic coefficient [-]")
# Also add a legend
plt.legend()
plt.grid()
plt.tight_layout()
plt.show()
```



Angles over time

Plotting the angle of attack and bank angle over time allows to check if the aerodynamic guidance behaves as expected. Moreover, plotting the flight path angle over time allows to check how efficient the guidance was at keeping it constant.

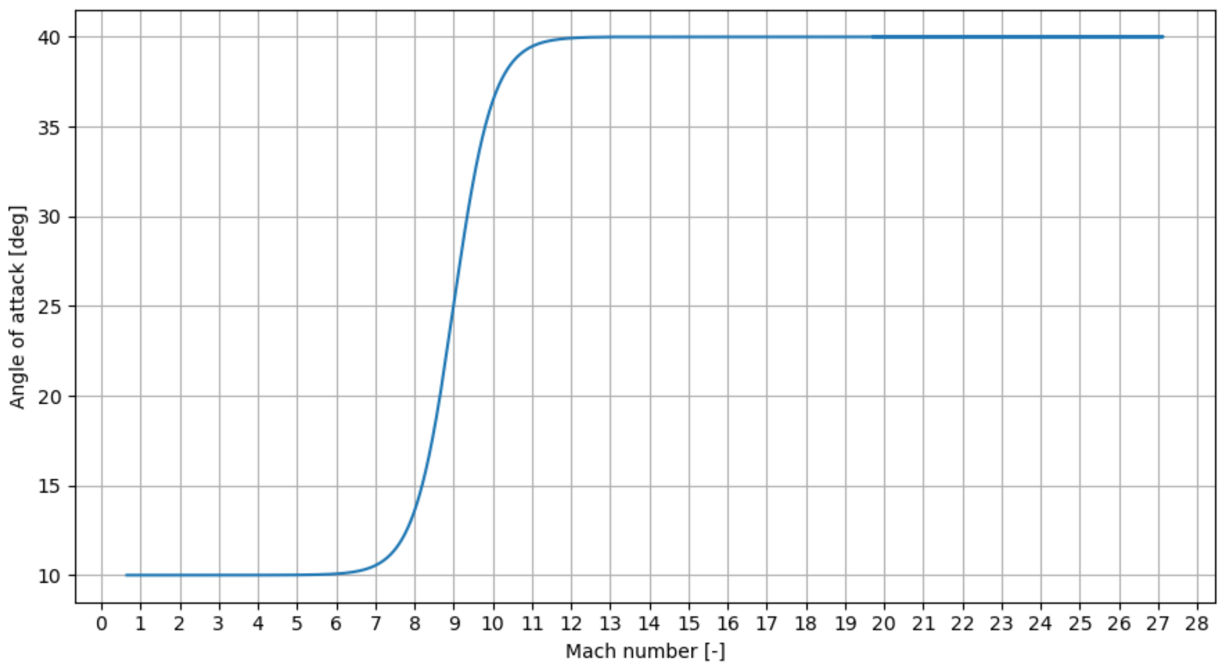
In [21]:
```
# Plot various angles over time (bank angle, angle of attack, and flight-
plt.figure(figsize=(9, 5))
plt.plot(time_min, np.rad2deg(dependent_variables_array[:,3]), label="Ban
plt.plot(time_min, np.rad2deg(dependent_variables_array[:,4]), label="Ang
plt.plot(time_min, np.rad2deg(dependent_variables_array[:,1]), label="Fli
plt.xlabel("Time [min]"), plt.ylabel("Angle [deg]")
plt.legend()
plt.grid()
plt.tight_layout()
plt.show()
```



Angle of attack vs Mach number

Plotting the angle of attack as a function of the Mach number allows to check that it indeed is of 10deg below Mach 6, of 40deg above Mach 12, and that it varies more or less linearly (and smoothly) in-between.

In [22]:
```
# Plot the AoA over Mach number
plt.figure(figsize=(9, 5))
plt.plot(dependent_variables_array[:,10], np.rad2deg(dependent_variables_
plt.xlabel("Mach number [-]"), plt.ylabel("Angle of attack [deg]")
# Set the x-axis ticks spacing to 1
plt.xticks(np.arange(0, 28.1, 1))
plt.grid()
plt.tight_layout()
plt.show()
```



Derivative of flight path angle over time

Plotting the derivative of the flight path angle over time finally allows to analyse how constant the flight path angle really was.

In [23]:

```
flight_path_angle = dependent_variables_array[:,1]
# Compute the derivative of the flight path angle over time (dot(gamma) =
flight_path_angle_derivative = np.fabs(( flight_path_angle[1:flight_path_
# Plot the derivative of the flight path angle over time
plt.figure(figsize=(9, 5))
plt.plot(time_min[0:-1], np.rad2deg(flight_path_angle_derivative))
plt.xlabel("Time [min]"), plt.ylabel("Absolute flight-path angle rate [de
# Make the y-axis logarithmic
plt.yscale("log")
# Have a tick on the y-axis every power of 10
plt.yticks(10**np.arange(-12, 0.1, 1))
plt.grid()
plt.tight_layout()
plt.show()
```