

Dominic Grant

1510819

6/28/20

<https://leetcode.com/dominicgrant/>

PA 2:

Theorem 1 Proof:

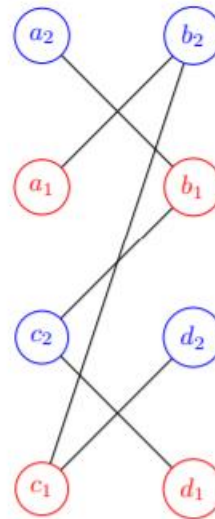
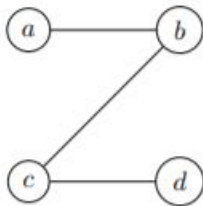


Figure 1: Example 1

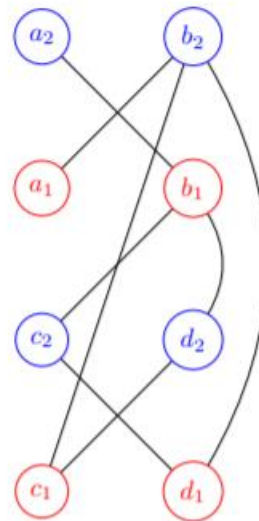
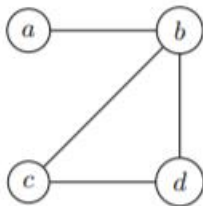


Figure 2: Example 2

Example 1 is a bipartite. If the graph, in this case figure 1, has no odd cycles, it is considered a bipartite. Figure 2 isn't a bipartite as it has an odd cycle of 3. The notable thing is that Figure 1 connects to all the connected nodes without repeating any of them unlike Figure 2. This is what labels it as a bipartite.

However, this also affects the number of connected components. Both figure 1 and 2 have a number of 1. However, once going into their  $G'$ , their examples, they change. More precisely, Example 1 changes in there is now 2 connected components of  $a_2 b_1 c_2 d_1$  and  $a_1 b_2 c_1 d_2$ . All of the nodes are connected but they take two lines without reuse of the nodes. This is different from the odd cycle example which connects back to the other side of the nodes so to speak, which gives it a number of 1 for its connected components. These examples show and prove the theorem "Let  $C$  be the number of connected components in  $G$ . Then the number of connected components in  $G_0$  is  $2C$  if and only if the graph  $G$  is bipartite."

b. DFS will be a good efficient algorithm so the code will have somewhere around a  $O(n+m)$  runtime.

Code was accepted by Leetcode.