

Compile time adjoint in C++

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Overview

- ① **Preliminaries:** resources, basic ideas, etc
- ② **Mapping indices:** separate tree structure and its data
- ③ **Language extensions:** functionality that would be very helpful

Preliminaries

Zero or more

```
// no state, but instantiable  
template<class... T> struct list {};
```

```
// a more useful form...  
template<class T, T...> struct seq {};
```

```
// _the_ operation  
template<class L> struct front;  
  
template<template<class...> class L, class T1, class... T>  
struct front<L<T1, T...> >  
{  
    using type = T1;  
};
```



Simple C++11 metaprogramming

With variadic templates, parameter packs and template aliases

Peter Dimov, 26.05.2015

I was motivated to write this after I read Eric Niebler's thought-provoking [Tiny Metaprogramming Library](#) article. Thanks Eric.

C++11 changes the playing field

The wide acceptance of [Boost.MPL](#) made C++ metaprogramming seem a solved problem. Perhaps MPL wasn't ideal, but it was good enough to the point that there wasn't really a need to seek or produce alternatives.

C++11 changed the playing field. The addition of variadic templates with their associated parameter packs added a compile-time list of types structure directly into the language. Whereas before every metaprogramming library defined its own type list, and MPL defined several, in C++11, type lists are as easy as

```
// C++11
template<class... T> struct type_list {};
```

and there is hardly a reason to use anything else.

Template aliases are another game changer. Previously, "metafunctions", that is, templates that took one type and produced another, looked like

```
// C++03
template<class T> struct add_pointer { typedef T* type; };
```

and were used in the following manner:



Compressible N-S adjoint: auto. diff. at field loop iteration level

Differentiate a function *losslessly*

Parity preserving transform

write something like this ...

```
fn(A const &a, B const &b,  
    R &r)  
{  
    auto c0 = 7;  
    auto c1 = 9;  
  
    auto t0 = a * b;    // 1  
    auto t1 = c0 + t0;  // 2  
    auto t2 = c1 + t0;  // 3  
  
    r = t1 / t2;        // 4  
}
```

to implement something like this

```
fn(A const &a, B const &b,  
    R &r)  
{  
    auto c0 = 7;  
    auto c1 = 9;  
  
    auto t0 = a * b;    // 1  
    auto t1 = c0 + t0;  // 2  
    auto t2 = c1 + t0;  // 3  
  
    // somehow add this...  
    t1.d += (1/t2)      * r.d; // 4'  
    t2.d -= (t1/t2^2)   * r.d; // 4'  
    t0.d += t2.d;       // 3'  
    t0.d += t1.d;       // 2'  
    a.d  += b * t0.d;   // 1'  
    b.d  += a * t0.d;   // 1'  
}
```


Observations

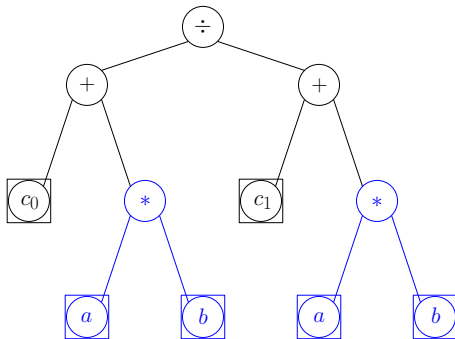
- ① Given a pure-functional algorithm, differentiate it
- ② Implement the transpose of the chain of derivatives (the adjoint)
- ③ The required 'extra' code is in the reversed sequence of the original and the data flow is reversed
- ④ Eager and lazy evaluation: ctor-dtor pairs?

Two hurdles

1. Dealing with duplicate nodes

Eager evaluation and capture by reference?

```
fn(A const &a, B const &b,  
    R &r)  
{  
    auto c0 = 7;  
    auto c1 = 9;  
  
    auto t0 = a * b;  
    auto t1 = c0 + t0;  
    auto t2 = c1 + t0;  
  
    r = t1 / t2;  
}
```



2. Dealing with nested scoping

The *complete* tree, including **cm**, is needed for the transform

```
mul_dbl(A const &a, B const &b)
{
    auto cm = 2; // locally scoped
    return cm * a * b;
}
```

```
fn(A const &a, B const &b, R &r)
{
    auto c0 = 7;
    auto c1 = 9;

    auto t0 = mul_dbl(a, b);
    auto t1 = c0 + t0;
    auto t2 = c1 + t0;

    r = t1 / t2;
}
```

2. Dealing with nested scoping

The *complete* tree, including **cm**, is needed for the transform

```
fn(A const &a, B const &b,  
    R &r)  
{  
    auto c0 = 7;  
    auto c1 = 9;  
    auto cm = 2;  
    auto t0 = cm * a * b;  
    auto t1 = c0 + t0;  
    auto t2 = c1 + t0;  
  
    r = t1 / t2;  
}
```

```
fn(A const &a, B const &b,  
    R &r)  
{  
    auto c0 = 7;  
    auto c1 = 9;  
    auto cm = 2;  
    auto t0 = cm * a * b;  
    auto t1 = c0 + t0;  
    auto t2 = c1 + t0;  
  
    // the transform...  
    t1.d += (1/t2) * r.d;  
    t2.d -= (t1/t2^2) * r.d;  
    t0.d += t2.d;  
    t0.d += t1.d;  
    a.d += cm * b * t0.d;  
    b.d += cm * a * t0.d;  
}
```

State of affairs

- ① Eager evaluation avoids duplicate branch evaluation - but lazy evaluation will also be needed
- ② 'Capture by reference' to keep the tree small - but cannot work with nested scoping
- ③ 'Capture by value' is too inefficient - the tree will get very large very quickly
- ④ A monolithic tree, supporting eager and lazy evaluation, of minimal size, and impartial to scoping is required

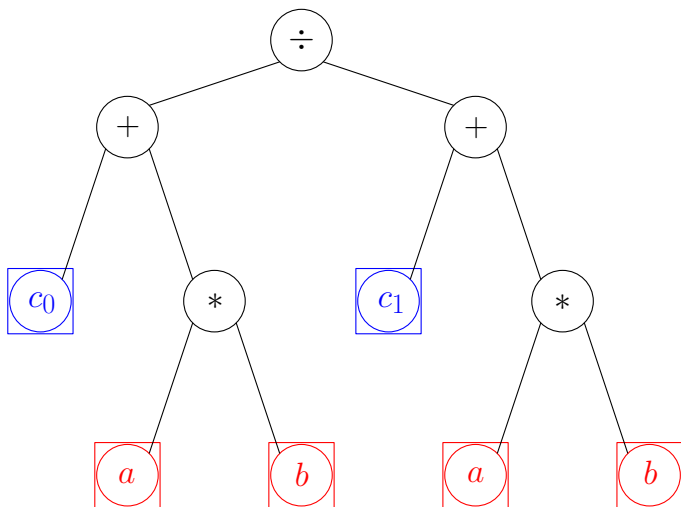
Expression tree to type list

Two kinds of data

```
auto fn(A const &a, B const &b)
{
    auto c0 = UQ(7);
    auto c1 = UQ(9);

    auto t0 = a * b;
    auto t1 = c0 + t0;
    auto t2 = c1 + t0;
    return t1 / t2;
}
```


Two kinds of data



```
template<std::size_t ID, typename T>
struct Unique
{
    T value;
};
```

```
// UQ
#define UQ(v) Unique<__COUNTER__, decltype(v)>{v}
```

Tree type to list of types

Generate tree with operator overloading

```
Binary<Div,  
  Binary<Add, C0,  
    Binary<Mul, A, B> >  
  Binary<Add, C1,  
    Binary<Mul, A, B> > >
```

Group hierarchically and prune duplicates (at each binary node),
then flatten (at the root node)

```
{A,  
 B,  
 Binary<Mul, A, B>,  
 Binary<Add, C0, [...]>,  
 Binary<Add, C1, [...]>,  
 Binary<Div, [...], [...]>}
```

Mapping nodes to data

Map **C0** and **C1** to values

```
L = {A, // 0
      B, // 1
      Binary<Mul, A, B>, // 2
      Binary<Add, C0, [...]>, // 3 <--
      Binary<Add, C1, [...]>, // 4 <--
      Binary<Div, [...], [...]> } // 5
```

```
tuple<float, float> vars = {7.0, 9.0};
```

Indices of constants in 'L'

```
IC = {3, 4}
```

Map elements in 'L' to storage offsets (2 = *null marker*)

```
DC = {2, 2, 2, 0, 1, 2}
```

dual

```
// input                                output
//-----                             -----
DC_SIZE = 6      \
                  --- dual --->    DC = {2, 2, 2, 0, 1, 2}
IC = {3, 4}      /                   0  1  2  3  4  5
```

github.com/DominicJones/snippets/blob/master/Cxx/mp_functions.cpp

Mapping nodes to data

Map **A** and **B** to addresses

```
L = {A, // 0 <--  
      B, // 1 <--  
      Binary<Mul, A, B>, // 2  
      Binary<Add, C0, [...]>, // 3  
      Binary<Add, C1, [...]>, // 4  
      Binary<Div, [...], [...]> } // 5
```

```
tuple<float*, float*> args = {&a, &b};
```

Map offsets of left child nodes (*6 = null marker*)

```
IL_L = {6, 6, 0, 6, 6, 3}
```

Map offsets of right child nodes

```
IL_R = {6, 6, 1, 2, 2, 4}
```

Evaluate operator list

```
tuple<float, float>   vars = {7.0, 9.0};
```

```
tuple<float*, float*> args = {&a, &b};
```

Iterate over lists to compute primal and adjoint

```
DC    = {2, 2, 2, 0, 1, 2}
```

```
IL_L  = {6, 6, 0, 6, 6, 3}
```

```
IL_R  = {6, 6, 1, 2, 2, 4}
```

```
L = {A,                                // 0
      B,                                // 1
      Binary<Mul, A, B>,                 // 2
      Binary<Add, C0, [...]>,           // 3
      Binary<Add, C1, [...]>,           // 4
      Binary<Div, [...], [...]>         // 5}
```

Results

Case 1: nodes: 82, depth: 37, inputs: 2, constants: 12

Version	compilation	original	auto diff	manual diff
alt::tuple	2.2s	1x	1.25x	1.48x
std::tuple	4.5s	1x	1.25x	1.48x

Case 2: nodes: 331, depth: 25, inputs: 5, constants: 103

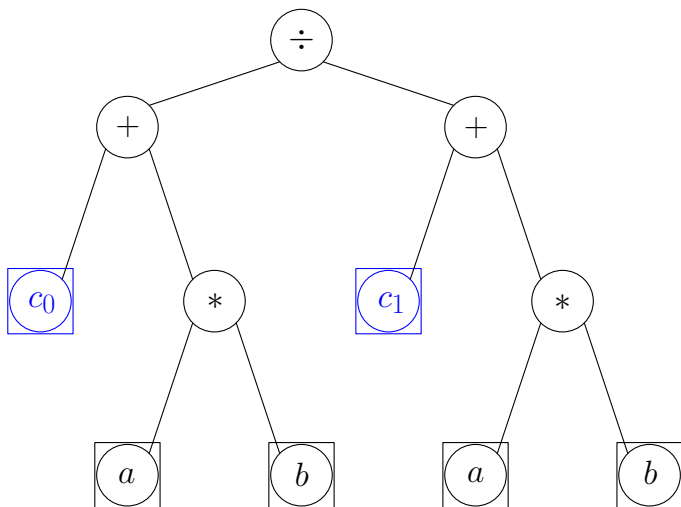
Version	compilation	original	auto diff	manual diff
alt::tuple	59s	1x	5.7x	1.9x
std::tuple	27m	1x	4.7x	1.9x

Conclusion

- ① Manipulation the tree is obtained at immense effort
- ② Works in the range of acceptably well (better than most alternatives) to exceptionally well (better than hand coded)
- ③ Prunes global tree (computation optimisation)
- ④ Orders data by access (cache optimisation)
- ⑤ Compile-time features of the language are too limited to use this approach neatly (cf. preprocessor macros)
- ⑥ Inlining gives up too readily; `__attribute__((always_inline))` used ubiquitously
- ⑦ Only supports having one result
- ⑧ Not obvious what is going on with `std::tuple`

Float template parameter

Distinguishing different constants



⚠ *Values as types* ⚠

```
// C++ does not permit 'auto' to resolve as 'float'
template<auto V>
struct _float
{
    constexpr operator auto() const { return V; }
};
```

```
// type distinguished by value
auto constexpr c0 = -4.2_f;
static_assert(c0 == _float<-4.2>{});
```

Values as types in D

```
// exactly what is wanted
struct _float(float v)
{
    static immutable auto value = v;
}
```

“_float” workaround

```
// exponent ignored...
template<auto H, auto L, auto E>
struct _float
{
    auto constexpr static value =
        (H + float(L) / multiplier<10, E, 1>::value);

    constexpr operator auto() const { return value; }
};
```

```
// and for operator+
template<auto H, auto L, auto E>
auto constexpr operator-(_float<H, L, E>)
{
    return _float<(-H), (-L), E>{};
}
```

...made palatable

```
// makes life easier
template<char...> struct mp_chars {};
```

```
// user-defined literal
template<char... Cs>
auto constexpr operator""_f()
{
    return make_float_t<0, 0, 0, 0,
                        sizeof...(Cs), mp_chars<Cs...> >{};
}
```

```
// seamless conversion to literals
auto constexpr c0 = -4.2_f;
float t0 = 2 * c0;
```

Reflect variable location

The trusty preprocessor

```
auto fn(A const &a, B const &b)
{
    auto c0 = UQ(7); // auto = Unique<724, float>{7}
    auto c1 = UQ(9); // auto = Unique<725, float>{9}

    auto t0 = a * b;
    auto t1 = c0 + t0;
    auto t2 = c1 + t0;
    return t1 / t2;
}
```

A fundamental problem

Write a transform function to yield:

```
auto c0 = 7;  
auto c1 = 9;  
  
transform(c0 + c0) --> 2 * c0  
transform(c1 + c0) --> c1 + c0
```

Impossible! ... despite it appearing so trivial

Three kinds of reflection?

	123456789	123456789	123456789
	10	20	30
1			
2	<code>// main.cpp</code>		
3			
4	<code>decltype(x)*</code>	<code>y</code>	<code>= &x;</code>
5			
	<code>reflect</code>	<code>reflect</code>	
	<code>type</code>	<code>address</code>	

Reflect *location* of `x`, at `[18, 4, main.cpp]`?

Reflect location in D

```
struct Terminal(T, string file = __FILE__,
               size_t line = __LINE__)
{
    T value;

    ...
}
```

```
Terminal!float c0 = 7; // Terminal!(float, "main.d", 724)
Terminal!float c1 = 9; // Terminal!(float, "main.d", 725)
```

One definition per line could be run-time checked with a debug build which requires all Terminals be singletons.

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