Special Problem F Solutions

Exactly one of the five matrices below represents an orthogonal projection onto a line and exactly one represents a reflection across a line. Identify both and justify your choices. It will be necessary to utilize a fundamental idea concerning a linear transformation and its matrix and to take into account the geometry associated with the two particular linear transformations here.

$$A = \tfrac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$B = \tfrac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$C = \tfrac{1}{3}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$D = -\tfrac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$E = \tfrac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

The column vectors of the standard matrix for a linear transformation are the images of the standard basis vectors under that transformation.

A projection onto a line will map any vector onto the set of all vectors parallel to that line. The only matrix above whose column vectors are collinear is $B$. Too, $B$ has the property that $B^2 = B$, a necessity for a projection.

A reflection onto a line preserves the length of vectors and the angles between them. Since the lengths of the standard basis vectors are 1 and they are orthogonal, the same must be true of the column vectors of the standard matrix for a reflection onto a line. Only $E$ has these properties.