Special Problem 1b(i)

1b. Let $n = 2$, $\vec{a} = \begin{bmatrix}1\\1\end{bmatrix}$ and $\vec{b} = \begin{bmatrix}2\\3\end{bmatrix}$. Sketch $\ell(\vec{a},\vec{b})$ and $f(\ell(\vec{a},\vec{b}))$ if

i. $f\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}x\\y^2\end{bmatrix}$

Here, we compute the image of the line segment in $\mathbf{R}^2$ with endpoints at $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}2\\3\end{bmatrix}$ under the transformation f described above. The line segment is $\ell\left(\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}2\\3\end{bmatrix}\right) = \left\{t\begin{bmatrix}1\\1\end{bmatrix} + (1-t)\begin{bmatrix}2\\3\end{bmatrix} \,|\, 0 \le t \le 1\right\} = \left\{\begin{bmatrix}2-t\\3-2t\end{bmatrix} \,|\, 0 \le t \le 1\right\}$ and its image under f is a segment of a curve described as follows.

$$f\left(\left\{\begin{bmatrix}2-t\\3-2t\end{bmatrix} \,|\, 0 \le t \le 1\right\}\right) = \left\{f\left(\begin{bmatrix}2-t\\3-2t\end{bmatrix}\right) \,|\, 0 \le t \le 1\right\} = \left\{\begin{bmatrix}2-t\\(3-2t)^2\end{bmatrix} \,|\, 0 \le t \le 1\right\}.$$

To identify this curve segment, simply let $x = 2 - t$ and $y = (3 - 2t)^2$. Solving for t from the first equation and substituting into the second, we have $y = (3 - 2[2 - x])^2 = (2x - 1)^2$. This is the segment of a parabola with vertex at (1/2, 0) opening upward from $f\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\1\end{bmatrix}$ to $f\left(\begin{bmatrix}2\\3\end{bmatrix}\right) = \begin{bmatrix}2\\9\end{bmatrix}$.