LaTeX-ing

Linear Algebra

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This document contains instructions for compiling the source to Jim Hefferon's undergraduate textbook *Linear Algebra*.

1   Summary of *Linear Algebra*.

Linear Algebra covers the material that is usually done in a three or four hour a week, one semester US course: solving linear systems, vector spaces, linear maps, determinants, and eigenvalues and eigenvectors. The pedagogical approach of the book is to help a student make the transition from calculus to upper division mathematics. Therefore the presentation stresses careful motivation, many examples, clarity in the proofs, and developmental exercise sets. The Preface contains a more detailed description.

1.1   Best When Viewed By ...

To have a look go to `http://joshua.smcvt.edu/linearalgebra`. The files there can be viewed with *Acrobat Reader* (or, that I know of, any PDF reader).

Get both the book and the answers and put them in the same directory, so that clicking on a question number takes you to its answer and clicking on the answer takes you back to the question.

1.2   It is Free

These materials are freely available, in both readable and source form.

First, you are free to download the book. If you are an instructor then you are also free to have students in your class use paper or electronic copies as a course text, either as the main text or as an auxilliary text. This includes the freedom to print out copies and sell them from your bookstore. But note that hard copies are for sale from major distributers such as Amazon and the college bookstore distributer; the price is less than it used to cost to get our print shop to make comb-bound versions.

Besides being free to use the output form of the text, you are also free to use the source. Some examples of how this can be helpful are: an instructor can add a few favorite exercises here and there, perhaps include a new Topic, adjust the development in a subsection, or even rewrite a portion entirely. Another example is: someone who finds an error, say in the answer to an exercise, can send me the correction (email me a section and subsection number and a description, ideally and also a correction).

I welcome contribution of work back to the project. I of course reserve the ability to not use some things but all the contributions that I do use are acknowledged in the file `Acknowledgements`.

2   Use the Source

If you want to modify the materials then you need to know that the files here use the LaTeX macro system for the text formatter TeX. It is the standard system for writing mathematics.

So you need a distribution of TeX. I recommend TeXLive, `http://www.tug.org/texlive`.

If you are unfamiliar with TeX then this might not be the right job on which to learn. It is not especially tough but you might want to start with a two page, or a ten page document. Just saying.

To get this document to come out, these work for me under Linux: '`pdflatex 1st_readme`'. If that works — if you can view it under *Acrobat Reader* — then your setup is at least partially correct. However, I must say that each TeX distribution has a slightly different approach and, I'm sorry, I can't help with that. You'll have to find that information in the distribution documentation.

What good is this 'How to Run It' file if it must be run? It builds character. Anyway, if you are having trouble even getting started, or only want a quick look, I've included the output material in the file `1st_readme.pdf`.

If you are just starting working with LaTeX then I recommend finding the *Short Guide to LaTeX2e* at `http://www.ctan.org/`, which might be enough for you to get through making small changes in the materials.

The graphics here are in MetaPost. This should come with a TeX distribution, but you need to search for the manual and you may also want *The MetaFont Book* by Knuth (ISBN 0-201-13444-6).

2.1 Compiling from source

Unzip the contents of `linear_algebra.zip`. This makes a new directory *book*.

The book's source comes in a number of separate files. In addition to separate files for each section, such as `gr1.tex` for the first section of the first chapter, here are some other key files.

file name	what it is
`book.tex`	Main file
`bookjhconcrete.sty`	Formatting instructions
`linalgjh.sty`	Macros for Linear Algebra
`jhanswer.tex`	Main file for the answers
`cover.tex`	Make the front, back, and spine

After you get the files, you will have in your directory many `.tex` files, a few `.mp` files, and a few other types such as LaTeX style files `.sty`. The `.tex` files are for LaTeX and the `.mp` files are for MetaPost for the graphics (a few graphics are in forms such as `.jpg`). There are also some graphic files for the Asymptote system, in the subdirectory `/asy`.

2.1.1 The easy way

If you are on a Unix-y machine, such as Linux, try running `./make_book_for_web.sh` from the command line. That's how I produce a version of the book.

If you are not on a Unix machine then you have to read `make_book_for_web.sh` and mimic the steps. (really, it would be easier to learn Unix.) These steps will give a printable version.

`mpost dotprod.mp`	Make the dot product symbol.
`mpost ch1.mp` ...`mpost ch5.mp`, `mpost appen.mp`, `mpost voting.mp`	Make the line art.
`pdflatex book`	Produce the book and the future references
`makeindex -s book.isty -p odd book.idx`	Alphabatize index.
`pdflatex book`	Produce the final version of the book
`pdflatex jhanswer`	Produce the answers
`pdflatex jhanswer`	Resolve references from the first run

If something fails, make sure you have the latest TeX installation. Try it again.

If you still have trouble, in case it helps I can tell you what I have that works for me. I have the TeXLive that comes with Ubuntu's package system. For line-by-line details, see the file `1ST_README`.

2.1.2 When Good Systems Do Things That Are Bad

What can go wrong? Almost anything. As Tom Wolfe quotes the early astronauts, "It can blow at any seam." Discouraging, isn't it?

The MetaPost-ing can fail in a number of ways. First, you need the *3D* material from CTAN. Second, you need all of the MetaPost material that I've generated (e.g., *venn.mp* for Venn diagrams), although you'll have gotten that from me unless I've made a mistake. Third, typesetting the labels, etc., may result in MetaPost giving you a LaTeX error. The thing about this failure is that the error message will say that the first label is bad. But it isn't the first label; it is likely somewhere much further down the file. To figure it out you must run '`pdflatex mpxerr.tex`' (this oddly-named file is where MetaPost writes the labeling lines for them to be automatically LaTeX-ed). Then you'll get an error message that at least points to the correct line, and presumably a little staring at it will give you a clue. (I've wasted any number of hours looking at line one when the problem was somewhere else entirely.)

LaTeX-ing the book can then give you a wide variety of troubles.

It is a big book, and I use a number of styles. If you are having trouble that seems to come from, say, *color.sty*, your first try is to go to CTAN and download the latest. In particular, I find that *hyperref* has been a frequent changer over the course of time that I have been working on the text (understandably, obviously).

If you are having trouble with a part, see what commenting it out does. Go into *book.tex*, for instance and comment out that part of the `includeonly` material (watch your commas! if you say '`,vs3%,`' instead of '`,vs3,%`' then you may find LaTeX complaining about not finding *vs3map1.tex* since *map1* starts the next line).

One area that can be annoying is that errors in the answer file do not tell you the line number in the original source file. Instead, they tell you the line number in the source file for the answers (probably *bookans.tex*). That file was written when you LaTeX-ed the book. So don't edit that file since your changes will disappear the next time you do the book again. Instead, you can look up that line in the answer's source, then look for that same line in the original source. Edit that one. Then LaTeX the book again. Thank goodness for fast computers. (Getting the line numbers from the original source into *bookans.tex* somehow would be a fine project for someone.)

One trouble that I've had comes when switching between a *hyperref*-ed and non-*hyperref*-ed versions. LaTeX complains about not liking the cross-reference file (something about "fifthoffive"). My solution is to delete all the cross-reference files (under Linux, `rm -f *.aux *.toc`), and then rerun the LaTeX command line twice. Not very elegant, I know, and once I accidentally left out the `.` between the `*` and the `aux`, but I am over that now.

2.1.3 Slides

I have some slides for a classroom projector. These are keyed to the text in a couple of ways.

First, they reference numbers for theorems, lemmas, etc. These references are generated automatically, meaning that I did not input them by hand, so in theory they are accurate.

Second, I have changed almost all the examples from the text. This way students see twice as many.

Third, the system is set so that the source of the statements of definitions, theorems, etc., is the text. Thus there is guaranteed, in theory, to be no discrepency with the text.

You can generate slides by changing into the `\slides` directory and running `./make_slides.sh`. Some of the graphics take a few minutes to run so you need to be patient.

3 Guide to the Macros

You can use the same macros to write your materials as are used in the text. For example, when you make up an exam and you want to refer to the vector space of polynomials of degree three, use `$\polyspace_3$`, avoiding students' confusion and also simplifying your day. This section describes how to use those macros.

First, the traditional disclaimer. In developing the book, I got to know a little bit about how to write TeX and LaTeX macros — but I didn't get to know a lot! If you can improve what is here, I'd welcome that contribution.

3.1 Linear algebra macros

The macros that seemed to me to be specific to typesetting the linear algebra material are in the file `linalgjh.sty`. For instance, here are macros for displaying systems of linear equations, matrices, row and column vectors, special vector spaces, linear maps, etc.

Note that the book uses the *amsmath* package, including the commutative diagram extension *amscd*, so all of those wonderful macros are available. In particular, displayed equations are shown with `equation*` (the `*` makes them not-numbered) and all of the alignment structures like `align` and matrix structures are available.

Another style that gets loaded is *mathrsfs*, for the Ralph Smith Formal Script fonts to make, for instance, the script R for the range space.

linsys Make a linear system, in such a way that the columns line up. Here is an example of a three-unknowns system; you want to do this only as a displayed equation.

```
\begin{equation*}
  \begin{linsys}{3}
     x &+ &3y &+ &a &= &7 \\
     x &- &3y &+ &a &= &7
  \end{linsys}
\end{equation*}
```

If a row is without some of the variables, be sure to nonetheless add the appropriate &'s.

```
\begin{equation*}
  \begin{linsys}{3}
      x &+ &3y &+ &a &= &7 \\
        & &3y & & & &= &7
  \end{linsys}
\end{equation*}
```

In the special case that a row starts with a negative sign, do not use $\&-$. That is, do not start the second line below with `&- &3y`.

```
\begin{equation*}
  \begin{linsys}{3}
      x &+ &3y &+ &a &= &7 \\
        & &-3y&+ &a &= &7
  \end{linsys}
\end{equation*}
```

Remarks.

1. In the exercises I might have three or four linear systems on a horizontal line and to get them to line up (to be **t**-aligned) I included an optional argument governing that vertical alignment.

2. *LATEX note.* The variables are put in the columns right-justified, while the additions or subtractions are centered. Between the columns is put 4/18-ths of an em (TEXbook, p. 167–170; that's a medmuskip).

3. *LATEX note.* The argument saying how many variables (which in the examples above is the 3) is there instead of some quite large number because it isn't as simple as $arg + 1$ times **rc**. But I wasn't sure, and might be convinced to change this by someone who knows what they are doing here.

grstep Denote a step of Gauss's Method.

```
\begin{equation*}
  \begin{linsys}{2}
    x &+ &y &= &0  \\
    x &- &y &= &1
  \end{linsys}
  \grstep{-\rho_1+\rho_2}
  \begin{linsys}{2}
    x &+ &y &= &0  \\
      & &-2y&= &1
  \end{linsys}
\end{equation*}
```

Show more than one row operation at a time with `\grstep[2\rho_5]{\rho_1+\rho_3}` for two row operations, or `\grstep[2\rho_5 \\ 3\rho_6]{\rho_1+\rho_3}` for three. Swap two rows with `\rho_1\swap\rho_2`.

matrix structures Generic matrices are made with `mat`.

```
\begin{equation*}
  \begin{mat}
    1 &2  &3  \\
    4 &5  &6
  \end{mat}
\end{equation*}
```

Note that there is no need to specify the number of columns; see the *amsmath* documentation.

There is an optional argument to make the columns right-aligned.

```
\begin{equation*}
  \begin{mat}[r]
    1  &2  &13  \\
    4  &5  &6
  \end{mat}
\end{equation*}
```

I restrict its use to those matrices (and column vectors) that contain only numbers, no variables at all.

There are places where I needed something a little different. In particular, I needed augmented matrices:

```
\begin{equation*}
  \begin{amat}{2}
    1  &2  &3  \\
    4  &5  &6
  \end{amat}
\end{equation*}
```

produces a matrix that is two-by-three, with a vertical bar between the final and next-to-final columns. The argument 2 means that there are two columns before the vertical bar. (This also has an optional argument to right-align.) I also sometime need matrices partitioned into columns:

```
\begin{equation*}
  \begin{pmat}{c|c|c}
    1  &2  &3  \\
    4  &5  &6
  \end{pmat}
\end{equation*}
```

produces a two-by-three matrix that has vertical bars separating the columns.

I make displayed determinant arrays with `vmat`

```
\begin{equation*}
  \begin{vmat}
    a  &c  \\
    b  &d
  \end{vmat}
  =ad-bc
\end{equation*}
```

and in-line determinants with `\deter{A}`. (Again, there is an optional argument to right-align.)

vectors Make column vectors with `\colvec{1 \\ 2 \\ 3}`. (There is an optonal argument to right-align that I use for number-only vectors `\colvec[r]{1 \\ 2 \\ 3}`) Make row vectors with `\rowvec{1  &2  &3}`.

decimal point-aligned columns I use the *dcolumn* package:

```
\begin{equation*}
  \begin{aligncolondecimal}{3}
    15.12  &0.345
  \end{aligncolondecimal}
\end{equation*}
```

makes a column vector aligned on the decimal with room for at most three decimal places on the right.

strings A digit 3 in a square is `\digitinsq{3}`. Not surprisingly, a digit 3 in a circle is `\digitincirc{3}`. A sequence of strings is shown this way.

```
\begin{equation*}
  \begin{strings}{ccccc}
        \vec{e}_1 &\mapsto &\vec{e}_2 &\mapsto &\zero \\
        \vec{e}_3 &\mapsto &\zero
  \end{strings}
\end{equation*}
```

There are five `c`'s because the longest line is of length five.

names for things Note that there is a page (inside the book's cover) covering the notation conventions.

I tried to remember to make up macros to name things, rather than retype the thing each time I ran across it. Here is a list.

1. The reals `\Re`, the rationals `\Q`, the complex numbers `\C`, the integers `\Z`, and the natural numbers `\N` come out in the traditional "blackboard bold".

2. A vector is `\vec{v}_j`. The zero vector is `\zero`. The length of a vector is `\norm{\vec{v}}`, and the absolute value of a number is `\absval{r}`. An angle can be expressed in degrees as `$53\degs$`. The distance between two vectors is `\dist (\vec{v}_1,\vec{v}_2)`.

3. The dot product of two vectors `\vec{v}\dotprod\vec{w}`. Please note that (as it is set up coming from me) this is different than a `\cdot`.

4. The vector space of degree n polynomials `\polyspace_n`, and the vector space of n-by-m matrices `\matspace_{\nbym{n}{m}}`. The vector space of linear maps from V to W `\linmaps{V}{W}`.

5. The span of a set S of vectors `\spanof{S}`.

6. The row space of a matrix is `\rowspace{M}` and the columnspace is `\colspace{M}`.

7. A set is `\set{\colvec{x \\ y}\suchthat 2x+y=0}`. The union and intersection of the sets S and T is `S\union T` and `S\intersection T`. The complement of a set is `\complement{S}`.

8. The empty set is `\emptyset`.

9. A sequence, such as a string, is `\sequence{s_0,s_1,\dots,s_n}`. The concatenation of two sequences is `\cat{B_1}{B_2}`.

10. A basis is `\basis{\vec{\beta}_1,\dots,\vec{\beta}_n}`. The standard basis for real n-space is `\stdbasis_n`.

11. Isomorphic spaces is `V\isomorphicto W`.

12. The matrix representing a linear map h with respect to the bases B and D is `\rep{h}{B,D}`.

13. The size of a general matrix is `\nbym{n}{m}` while the special case of a square matrix is `\nbyn{n}`.

14. A map h's range space `\rangespace{h}` and null space `\nullspace{h}`, and generalized range space `\genrangespace{h}` and generalized null space `\gennullspace{h}`

15. The direct sum of two subspaces `V\directsum W`.

16. The function, as specified by its domain and codomain, is described by `\map{f}{D}{C}`. Its action on a single element x is `x\mapsunder{f} f(x)`. Two maps are composed with `\composed{g}{f}`. The identity map is `\identity`.

17. The projection of a vector into a subspace `\proj{\vec{v}}{S}`.

18. The restriction of a map to some subdomain is `\restrictionmap{f}{S}`.

19. The rank of a matrix `\rank (A)` and the nullity of a matrix `\nullity (A)`. The transpose of a matrix `\trans{A}`. The trace of a matrix `\trace (A)`. The adjoint of a matrix is `\adj (A)`.

20. The size of a box is `\size (B)`.

21. The signum of a permutation ϕ is `\sgn (\phi)`.

22. For the Topic on voting, voter preferences are indicated by `\votepreflist{7}{1}{5}` for the column vector, and `\voteprefloop{1}{2}{3}` for the circle (for this one, the first argument appears at the ten o'clock position, the second argument at six o'clock, and the third argument at two o'clock).

23. A generic field is `\F`.

aligned vdots To make `\vdots` come out inside a bunch of aligned equations, I use `\vdotswithin{}` from the *mathtools* package.

```
\begin{align*}
  a_{1,1}x_1+\cdots+a_{1,n}x_n &= d_1           \\
                                   &\vdotswithin{=} \\
  a_{m,1}x_1+\cdots+a_{m,n}x_n &= d_m
\end{align*}
```

Otherwise the three vertical dots are not centered on the equals sign.

3.2   Book layout macros

The macros that do chapter and section headings, or cross references, or how the exercises are laid out, are in the file `bookjh.sty`.

theorem-like structures I have already declared the `theorem`, `lemma`, `definition`, and `corollary` environments. These will come out shaded. To change this, or the color of the shading or its border, see also *bookjh.sty*

```
\begin{definition}
  A \definend{big} vector space is one where you can't  see the end when
  you are standing at the zero vector, even with your glasses on.
\end{definition}
```

I have also declared (not shaded): `example` and `remark`.

By the way, note the `\definend`; I use this for the term being defined (I though I would automate indexing, but I've not got around to that).

proofs Use this.

```
\begin{proof}
   Because I said so; that's why.
\end{proof}
```

The end-of-proof symbol can be changed as `\renewcommand{\qedsymbol}{$\box$}`. I avoid ever having a proof end with a displayed equation so the question of where to put the end-of-proof symbol in that case never arises.

footnotes Footnotes come out on the same page as the reference. I only use footnotes to suggest that a reader can look for more in the Appendix `\appendrefs{codomains}`.

graphics I am using MetaPost for all of the graphics and all of the output drivers that I am interested in handle *.mp*'s straight off. Instead I say this.

```
\begin{center}
   \includegraphics{gr2.7}
\end{center}
```

Putting two graphics side-by-side, or putting parallel text, are all a bit messy. I use `vcenteredhbox{}`. Just mimic what I did somewhere.

exercises and answers The exercise portions of a subsection looks like you might expect.

```
\begin{exercises}
  \item
    First Exercise
    \begin{answer}
      Answer to the first exercise.
    \end{answer}
  \item
    Second Exercise
    \begin{answer}
      Answer to the second.
    \end{answer}
  \recommended \item
    Third Exercise, recommended.
    \begin{answer}
        Answer to the third.
    \end{answer}
\end{exercises}
```

Exercises are, of course, numbered automatically, in the same sequence as the other numbered parts of the text.

My goal is to have all exercises have answers, even the proof exercises. Of course, the answers are not written directly to the text; they are written to one or two separate files. Use *jhanswer.tex* to print them.

Remarks.

1. Some of the exercises have parts. I use the environment `exparts`. The items in such an environment are `\partsitem`'s.

   ```
   \recommended\item Do this one right away.
     \begin{exparts}
       \partsitem Do this first.
       \partsitem Do this second.
     \end{exparts}
     \begin{answer}
       \begin{exparts}
         \partsitem Answer to the first.
         \partsitem Answer to the second.
       \end{exparts}
     \end{answer}
   ```

   I sometimes use `exparts*`, which leaves the items in a horizontal list, but it doesn't work very well (it sometimes causes TeX's `Underfull hbox` errors; this is not as easy to fix as using a paralist because I don't want the beginning of a part on one line and the end on another).

2. Any desired changes to the spacing, etc., of exercises is in `exerciselist`. See also the style for the answers in *bookans.sty*.

3. You can redefine `\recommendationmark` to change the mark used to denote recommended exercises. You can also mark puzzles with `\puzzle\item`. This can be combined with `\recommended`.

4. If you are putting in an exercise without an answer, your best bet is to put an answer environment that is blank. But really, you shouldn't do that. I have found that doing the answers to all of

the exercises improved both the questions and the presentation greatly, since I often found that I needed a detail here, or some tweak there. (If what you want is that students can't read the answer in their answer list, that is a different matter. See the next two remarks.)

5. These answers are typically quite detailed, giving the work and not just the answer; I approached the answers expecting that the reader wants to learn and did not take the approach that the reader is trying to cheat. (*Remark.* As a practicing teacher, I am very aware that it is not a simple issue. Nonetheless, in my opinion, a reader is entitled to enough exercise and answer pairs that, having tried a good selection and not just read the questions and peeked at the answers, they can move on well-prepared to the next subsection. I figure that a dozen such pairs in each subsection is about right. Then, to provide a selection and also to allow someone a second crack at a problem type that gave them trouble the first time, I aimed for two dozen questions in each subsection. If this question/answer situation bothers you, I would very much welcome additional submissions of exercise sets for which students do not have access to the answers.)

6. I used to have it set up so that you can have the answers written to either a single file, or separate files for the answers to recommended exercises and the answers to the others. Although I no longer do that, I haven't deleted the code so maybe you could get it to work. (In this Internet age, trying to control access to such files quickly proved to be folly. For a while I only released the entire answers to people who emailed me with a claim to being an instructor, but soon the silliness of that became too much to even hold a pretense. I'll just say that every twenty year old on the planet who can come up in minutes with a download address for the answers to exercises for any popular math text. It is just a fact.)

7. *LaTeX note.* You can use `\ref`'s inside answers, even though they are written to other files. The style for the answers reads in the reference files.

8. An answer that is taken from a cited source may not blend stylistically with the others (insofar as I have a style). I use the disclaimer `\answerasgiven`.

tfae A small environment for The Following are Equivalent-type lists.

clearemptydoublepage Taken straight from the *Companion*. It does what it says.

cross references I use `\nearbydefinition{def:BigThm}`-like references. This has the disadvantage that the referrer has to know what type of textual element it is referring to (in this case a definition), but LaTeX doesn't give you a way to know in this case (that I can see). Note that I try to make the `\label{def:BigThm}` moderately descriptive.

I have set up nearby cross references (meaning there is no need to reference the chapter number and the section number) for: `\nearbydefinition`, `\nearbyfigure`, `\nearbylemma`, `\nearbyexample`, `\nearbycounterexample`, `\nearbytheorem`, `\nearbycorollary`, `\nearbyexercise`, `\nearbyremark`, `\nearbynotice`, and `\nearbynote`. (I don't use most of these, but they are there anyway.)

topics Begin a new topic with `\topic{My Favorite Topic}`. The exercises come out looking a little differently automatically, because I couldn't think of a common treatment that was both convenient and consistent.

optional subsections A subsection is marked as optional with `\subsectionoptional` instead of LaTeX's standard `\subsection`. This asterisk's name in the Table of Contents. I have also followed the practice of beginning each optional subsection with a italicised comment that it is optional, but this is not part of the macro.

page headings This work is all done through the wonderful *fancyhdr* style. See the documentation for that package.

hyperlink style I have set the links to come out in blue. Change that by altering the

```
\usepackage{
          ..,
```

```
                    linkcolor=blue,
                    ..}{hyperref}
```

entry.

4   Making Your Exams

I have included an exam style that does what you'd think it would do, number the exercises automatically, automatically put headers and page numbers, and optionally allow exercises to have attached answers that are printed to a separate file.

Here is a sample exam, without answers. See the web page for more extensive examples.

```
\begin{exam}
  \item \pts{15}
    How much wood could a woodchuck chuck?
    \begin{enumerate}
      \item If he would chuck wood?
      \item If not?
    \end{enumerate}
  \item \pts{85}
    Justify the Axiom of Choice, since we all know it is true anyway.
\end{exam}
```

5   Bugs

See the `TODO.org` file in the source.