Answer Key

FALL 2013 FINAL EXAM FOR SM261
755 DECEMBER 18, 2013

SHOW ALL WORK

PART 1. On this part (problems 1-5) you may **NOT** use a calculator. Show all your work.

1. Complete the following definitions.

a. A function T from $\mathbb{R}^m$ to $\mathbb{R}^n$ is called a linear transformation if...

for all $\vec{v}, \vec{w} \in \mathbb{R}^m$ and all scalars $a, b \in \mathbb{R}$
$T(a\vec{v} + b\vec{w}) = aT(\vec{v}) + bT(\vec{w})$

b. A vector $\vec{b}$ in $\mathbb{R}^n$ is called a linear combination of the vectors $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m\}$ in $\mathbb{R}^n$ if... there are scalars $c_1, \ldots, c_m \in \mathbb{R}$ such that $\vec{b} = c_1\vec{v}_1 + \ldots + c_m\vec{v}_m$.

c. A set of vectors $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m\}$ in $\mathbb{R}^n$ forms a linearly independent set for $\vec{V}$ if... the only scalars $c_1, \ldots, c_m \in \mathbb{R}$ for which $c_1\vec{v}_1 + \ldots + c_m\vec{v}_m = \vec{0}$ holds is with $c_1 = c_2 = \ldots = c_m = 0$.

typo
$V \to \mathbb{R}^n$

d. If A is an $m \times n$ matrix, then the rank of A is...

the number of non-zero rows in the row-reduced echelon form.

Note: My definitions may be different than yours, so use your best judgement in grading here.

2. Consider the linear system of equations

$$x + 2y - z = 2$$
$$2x + y + z = 0$$
$$x - y + 2z = -2$$

a. If we write the system in the matrix form $A\vec{v} = \vec{b}$, what are A, $\vec{v}$ and $\vec{b}$?

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}$$

b. Find the reduced row echelon form of the augmented matrix $[A, \vec{b}]$ for this system. Show all steps.

$$\operatorname{rref}(A, \vec{b}) = \begin{pmatrix} 1 & 0 & 1 & -2/3 \\ 0 & 1 & -1 & 4/3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

c. Describe all solutions to the system.

$$\{\vec{v} \mid A\vec{v} = \vec{b}\} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| \begin{array}{l} x = -z - 2/3 \\ y = z + 4/3 \end{array} \right\} = \left\{ \begin{pmatrix} -2/3 \\ 4/3 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} t \right\}$$

d. Is the set of all solutions a subspace of $\mathbb{R}^3$? Explain.

No, it does not contain $\vec{0}$.

3. Let $A = \left[\begin{smallmatrix} 2 & 1 & -1 \\ 4 & 3 & 0 \end{smallmatrix}\right]$ and $B = \left[\begin{smallmatrix} 3 & 2 \\ 1 & 2 \\ 0 & 1 \end{smallmatrix}\right]$. Find $(AB)^{-1}$.

$$AB = \begin{pmatrix} 7 & 5 \\ 15 & 14 \end{pmatrix}$$

$$(AB)^{-1} = \begin{pmatrix} 14/23 & -5/23 \\ -15/23 & 7/23 \end{pmatrix}$$

Note: $\det AB = 98 - 75 = 23 \neq 0$

4. Circle all the answers which correctly complete the following sentence. If $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$ are non-zero vectors in $\mathbb{R}^3$ and S is the subspace spanned by them, $S = lin\{\vec{v}_1, \vec{v}_2, \vec{v}_3\} = Span(\vec{v}_1, \vec{v}_2, \vec{v}_3)$, then $\dim(S)$ ...

(a) Always equals 3.

(b) Equals 3 only if $\vec{v}_1$ is a linear combination of $\vec{v}_2$ and $\vec{v}_3$.

(c) Is less than 3 if $\vec{v}_1$ is a linear combination of $\vec{v}_2$ and $\vec{v}_3$.

(d) Equals 2 if $\vec{v}_1$ is a linear combination of $\vec{v}_2$ and $\vec{v}_3$.

(e) Could possibly equal 0.

5. Find the determinant of the matrix

$$A = \begin{pmatrix} 0^{+} & 2 & 1 & 6 \\ 2^{-} & 6 & 0 & 1 \\ 6^{+} & 0 & 1 & 2 \\ 0^{-} & 1^{+} & 0 & 0 \end{pmatrix}$$

Expand across the bottom row:

$$\det A = +1 \cdot \det \begin{pmatrix} 0 & 1 & 6 \\ 2 & 0 & 1 \\ 6 & 1 & 2 \end{pmatrix}$$

Now, expand down the last column:

$$\det A = +6 \cdot \det \begin{pmatrix} 2 & 0 \\ 6 & 1 \end{pmatrix} - 1 \cdot \det \begin{pmatrix} 0 & 1 \\ 6 & 1 \end{pmatrix} + 2 \cdot \det \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$$

$$= 12 + 6 - 4 = \underline{14}$$

This is the end of Part 1. Turn in your answers and proceed to Part 2.

PART 2. You may use a calculator on this part. Show all your work.

6. Define the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^2$ by

$$T\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} x & y \\ z & w \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}.$$

Find the matrix of T with respect to the standard bases for $\mathbb{R}^4$ and $\mathbb{R}^2$.

$$T\vec{e}_1 = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \quad T\vec{e}_2 = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \quad T\vec{e}_3 = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \quad T\vec{e}_4 = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$$

$$[T] = \begin{pmatrix} 3 & 5 & 0 & 0 \\ 0 & 0 & 3 & 5 \end{pmatrix}.$$

7. Consider the following matrix:

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{pmatrix}.$$

Find and label eigenvalues and associated eigenvectors of A. Use them to find an *integer* matrix P such that $P^{-1}AP$ is a diagonal matrix.

$$P = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 0 & -2 & -1 \end{pmatrix}$$

$$P^{-1}AP = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The characteristic polynomial is $-(x^3 - 4x^2 + 3x)$

$= -x\cdot(x-1)(x-3)$

$\lambda_1 = 0$ $\qquad \lambda_2 = 1$ $\qquad \lambda_3 = 3$

$\vec{v}_1 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ $\qquad \vec{v}_2 = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ $\qquad \vec{v}_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

(see #2c above)

Solve:

$$\begin{cases} 2y - z = 0 \\ 2x + z = 0 \\ x - y + z = 0 \end{cases}$$

Solve

$$\begin{cases} -2x + 2y - z = 0 \\ 2x - 2y + z = 0 \\ x - y - z = 0 \end{cases}$$

8. Let A be the matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ -1 & 1 & 1 & -1 & 1 \\ 1 & 2 & 3 & 4 & 1 \\ 2 & 1 & 2 & 5 & 4 \end{pmatrix}.$$

We have

$$\text{rref}(A) = \begin{pmatrix} 1 & 0 & 1/3 & 2 & 0 \\ 0 & 1 & 4/3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

a. Find a basis for the null space (or kernel) of A, $Null(A) = ker(A)$. Find the nullity, $\dim(ker(A)) = \dim(Null(A))$.

$$x_1 + \tfrac{1}{3}x_3 + 2x_4 = 0$$
$$x_2 + \tfrac{4}{3}x_3 + x_4 = 0$$
$$x_5 = 0$$

$$\{\vec{x} \mid A\vec{x} = \vec{0}\} = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -1/3 \\ -4/3 \\ 1 \\ 0 \\ 0 \end{pmatrix} x_3 + \begin{pmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} x_4 \right\}$$

b. Find a basis for the range of A, $Range(A) = Image(A)$. Find the rank of A.

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \\ 1 \\ 4 \end{pmatrix} \right\}$$

(pivots are in columns 1, 2, 5)

Rank$(A) = 3$

9. Let V denote the subspace of $\mathbb{R}^4$ given by the span $Span(\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4)$, where $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 5 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 6 \end{bmatrix}$, and $\vec{v}_4 = \begin{bmatrix} 0 \\ 2 \\ -1 \\ -2 \end{bmatrix}$.

a. Find a basis B for V.

$$B = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$$

pivots in columns 1, 2, 3

since $\operatorname{rref}\begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & -2 & 0 & 2 \\ 2 & 3 & 1 & -1 \\ 3 & 5 & 6 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

b. Is the vector $\vec{v} = \begin{bmatrix} 1 \\ 6 \\ -1 \\ 1 \end{bmatrix}$ in V? If so, find $[\vec{v}]_B$. If not, explain why.

No, the system

$$\begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & -2 & 0 & 2 \\ 2 & 3 & 1 & -1 \\ 3 & 5 & 6 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ -1 \\ 1 \end{pmatrix}$$

is inconsistent:

$$\operatorname{rref}\begin{pmatrix} 1 & 1 & 2 & 0 & 1 \\ 0 & -2 & 0 & 2 & 6 \\ 2 & 3 & 1 & -1 & -1 \\ 3 & 5 & 6 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

10. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation which satisfies $T\left[\begin{smallmatrix}2\\4\end{smallmatrix}\right] = \left[\begin{smallmatrix}2\\1\end{smallmatrix}\right]$ and $T\left[\begin{smallmatrix}1\\3\end{smallmatrix}\right] = \left[\begin{smallmatrix}1\\5\end{smallmatrix}\right]$. Find the matrix of T relative to the standard basis $\{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]\}$.

Since T is linear,

$$T\begin{bmatrix}0\\2\end{bmatrix} = T\begin{bmatrix}2\\6\end{bmatrix} - T\begin{bmatrix}2\\4\end{bmatrix} = 2\cdot T\begin{bmatrix}1\\3\end{bmatrix} - T\begin{bmatrix}2\\4\end{bmatrix}$$
$$= 2\cdot\begin{pmatrix}1\\5\end{pmatrix} - \begin{pmatrix}2\\1\end{pmatrix}$$
$$= \begin{pmatrix}2\\10\end{pmatrix} - \begin{pmatrix}2\\1\end{pmatrix}$$
$$= \begin{pmatrix}0\\9\end{pmatrix}$$

Divide both sides by 2 $\Rightarrow$ $T\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\9/2\end{pmatrix}$

$$\begin{pmatrix}2\\1\end{pmatrix} = T\begin{bmatrix}2\\4\end{bmatrix} = T\begin{bmatrix}2\\0\end{bmatrix} + T\begin{bmatrix}0\\4\end{bmatrix}$$
$$= 2\cdot T\begin{pmatrix}1\\0\end{pmatrix} + 4\cdot T\begin{pmatrix}0\\1\end{pmatrix}$$
$$= 2\cdot T\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\18\end{pmatrix}$$
$$\Rightarrow 2\cdot T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}2\\-17\end{pmatrix}$$
$$\Rightarrow T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\-17/2\end{pmatrix}$$
$$\Rightarrow [T] = [T(\vec{e}_1), T(\vec{e}_2)] = \underline{\begin{pmatrix}1 & 0\\ -17/2 & 9/2\end{pmatrix}}$$

11. Let V be the subspace of $\mathbb{R}^4$ with basis $\left\{ \begin{bmatrix} 0\\3\\4\\0 \end{bmatrix}, \begin{bmatrix} -4\\0\\0\\3 \end{bmatrix}, \begin{bmatrix} 1\\1\\1\\1 \end{bmatrix} \right\}$.

a. Find an orthonormal basis for V.

$$\vec{u}_1 = \begin{pmatrix} 0\\3/5\\4/5\\0 \end{pmatrix}, \quad \vec{u}_2 = \begin{pmatrix} -4/5\\0\\0\\3/5 \end{pmatrix}$$

$$\vec{w}_3 = \begin{pmatrix} 1\\1\\1\\1 \end{pmatrix} - a_1\vec{u}_1 - a_2\vec{u}_2 \text{. Want } \vec{w}_3 \perp \vec{u}_1,\ \vec{w}_3 \perp \vec{u}_2$$

$$\vec{u}_1 \cdot \vec{w}_3 = 0 \rightarrow 0 = \frac{7}{5} - a_1 \Rightarrow a_1 = 7/5$$

$$\vec{u}_2 \cdot \vec{w}_3 = 0 \Rightarrow 0 = -1/5 - a_2 \Rightarrow a_2 = -1/5$$

$$\vec{w}_3 = \begin{pmatrix} 1\\1\\1\\1 \end{pmatrix} - \left(\frac{7}{5}\right)\begin{pmatrix} 0\\3/5\\4/5\\0 \end{pmatrix} - \left(-\frac{1}{5}\right)\begin{pmatrix} -4/5\\0\\0\\3/5 \end{pmatrix} = \left(\frac{21}{25}, \frac{4}{25}, -\frac{3}{25}, \frac{28}{25}\right)$$

$$\|\vec{w}_3\| = \sqrt{2} \Rightarrow \vec{u}_3 = \vec{w}_3/\sqrt{2}.$$

b. Find a non-zero vector in $\mathbb{R}^4$, if it exists, perpendicular to all the elements of V. If it does not exist, explain why.

$$\text{Let } A = \begin{pmatrix} 0 & 3 & 4 & 0\\ -4 & 0 & 0 & 3\\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\vec{v} \perp V \Rightarrow A\vec{v} = \vec{0} \Rightarrow \vec{v} \in Null(A)$$

$$rref(A) = \begin{pmatrix} 1 & 0 & 0 & -3/4\\ 0 & 1 & 0 & 7\\ 0 & 0 & 1 & -21/4 \end{pmatrix} \Longrightarrow \vec{v} = \begin{pmatrix} 3\\-28\\21\\4 \end{pmatrix} t, \ t \in \mathbb{R}$$

$$x_1 = \frac{3}{4}x_4$$
$$x_2 = -7x_4$$
$$x_3 = \frac{21}{4}x_4$$
$$x_4 = x_4$$

Yes such a $\vec{v}$ exists,

eg $\begin{pmatrix} 3\\-28\\21\\4 \end{pmatrix}$

12. True or False. If true, give a reason. If false, give a reason or a counterexample.

1. If A is a square matrix and $A^{2016} = I$, then A is invertible.

True $\det(A)^{2016} = 1 \Rightarrow \det A \neq 0$
$\Rightarrow A$ is invertible.

2. Any set of vectors which spans $\mathbb{R}^n$ is a subset of some basis of $\mathbb{R}^n$.

False. The vectors $(1,0), (0,1), (1,1)$ span $\mathbb{R}^2$ but are not a subset of a basis since they are linearly dependent.

3. Every square matrix has at least one real eigenvalue.

False $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ has char poly x^2+1, whose roots are $\pm i$.

4. If V is the set of all vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ in $\mathbb{R}^2$ with $|x| = |y|$, then V is a subspace of $\mathbb{R}^2$.

False. $\begin{pmatrix} 1 \\ 1 \end{pmatrix} \in V$, $\begin{pmatrix} 1 \\ -1 \end{pmatrix} \in V$ but $\begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} \notin V$.
Not closed under vector addition.

5. If $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+1 \\ y-1 \end{bmatrix}$, then T is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$.

False $T\begin{pmatrix} 0 \\ 0 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

13. Let R denote counterclockwise rotation about the origin in $\mathbb{R}^2$ by $\pi/4$.

a. Determine the matrix representation of T with respect to the basis $B = \{(1,1),(1,-1)\}$.

$R\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}0\\ \sqrt{2}\end{pmatrix} = c_1\begin{pmatrix}1\\1\end{pmatrix} + c_2\begin{pmatrix}1\\-1\end{pmatrix} \Rightarrow \begin{matrix} c_1 = \frac{\sqrt{2}}{2} \\ c_2 = -\frac{\sqrt{2}}{2}\end{matrix}$

$R\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}\sqrt{2}\\0\end{pmatrix} = c_1\begin{pmatrix}1\\1\end{pmatrix} + c_2\begin{pmatrix}1\\-1\end{pmatrix} \Rightarrow \begin{matrix} c_1 = \sqrt{2}/2 \\ c_2 = \sqrt{2}/2\end{matrix}$

$\longrightarrow [R]_B = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{pmatrix}$

b. Determine the matrix representation of T with respect to the the standard basis.

$T(e_1) = \begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$.

$T(e_2) = \begin{pmatrix}-1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$

$\Rightarrow [T] = \begin{pmatrix}1/\sqrt{2} & -1/\sqrt{2}\\ 1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}$

14. True or False. If true, give a reason. If false, give a reason or a counterexample.

a. If S and T are linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^n$, then the set of all vectors $\vec{v}$ in $\mathbb{R}^n$ such that $S(\vec{v}) = T(\vec{v})$ is a subspace of $\mathbb{R}^n$.

True $\{\vec{v} \mid S(\vec{v}) = T(\vec{v})\} = \text{Null}(T-S)$.

b. If the determinant of a 5×5 matrix A is 4, then $1 \leq \text{rank}(A) \leq 4$.

False $\det A = 4 \Rightarrow A$ is invertible

$\Rightarrow \text{rank}\, A = 5$.

Typo - missing parentheses.

c. Of $\vec{v}_1, \ldots, \vec{v}_n$ are orthogonal (column) vectors in $\mathbb{R}^n$ then the $n \times n$ matrix $(\vec{v}_1, \ldots, \vec{v}_n)$ is an orthogonal matrix.

False. $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ is not orthogonal: $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^T = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

d. For all 2×2 matrices A, B, we have $\det(A + B) = \det(A) + \det(B)$.

False. $\det\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right) = \det\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 4$, $\det\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \det\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1 + 1 = 2$

e. There is a 2×3 matrix A such that the kernel (or null space) of A is the intersection of the planes $x + y - z = 0$ and $y = x + z$.

True ~~Yes~~ $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$.

In Problem 15, do all calculations by hand and show all steps. You may check your work with a calculator.

15. Let A be the matrix

$$\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}.$$

a. Find the characteristic polynomial $p_A(\lambda)$ of A.

$p(x) = x^2 - 5x + 6$

$= (x-2)(x-3)$

b. Find the eigenvalues of A.

$\lambda_1 = 2 \quad \lambda_2 = 3$

c. For each eigenvalue λ, find a basis for the eigenspace E_λ.

$E_{\lambda_1} = \left\{ \begin{pmatrix} -2 \\ 1 \end{pmatrix} t \;\middle|\; t \in \mathbb{R} \right\}$ $\quad E_{\lambda_2} = \left\{ \begin{pmatrix} -2 \\ 2 \end{pmatrix} t \;\middle|\; t \in \mathbb{R} \right\}$

basis $= \left\{ \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right\}$ $\quad$ basis $= \left\{ \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}$

d. Is A diagonalizable? If so, find an eigenbasis for A and a diagonal matrix D which is similar to A. If not, give your reasons.

~~E~~ Yes ~~Eigenvect~~ Eigenvectors forming a basis for $\mathbb{R}^2 = \left\{ \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}$

END OF EXAM

$D = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}, \; P = \begin{pmatrix} -2 & -1 \\ 1 & 1 \end{pmatrix}, \; P^{-1}AP = D$