SM261.1001. Extra Problem Set 1. Due 20100910.

Provide justification or explanation for all of your assertions.

1. Suppose that $A = [\vec{a}_1 \,|\, \vec{a}_2 \,|\, \vec{a}_3 \,|\, \vec{a}_4]$ is a 3×4 matrix whose column vectors are four <u>nonzero</u> vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3,$ and $\vec{a}_4$ in $\mathbf{R}^3$. Describe, with as much specificity as possible, A_{rref}, the row-reduced echelon form of A if the vectors

a. $\vec{a}_1, \vec{a}_2, \vec{a}_3,$ and $\vec{a}_4$ are coplanar and $\vec{a}_1$ and $\vec{a}_2$ are not collinear.

b. $\vec{a}_1, \vec{a}_2$ and $\vec{a}_3$ are not coplanar.

c. $\vec{a}_1, \vec{a}_2$ and $\vec{a}_3$ are not coplanar and $4\vec{a}_1 - 3\vec{a}_2 + 2\vec{a}_3 - \vec{a}_4 = \vec{0}$.

2. Suppose that $A = [\vec{a}_1 \,|\, \vec{a}_2 \,|\, \vec{a}_3 \,|\, \vec{a}_4]$ is a 4×4 matrix whose column vectors are four <u>nonzero</u> vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3,$ and $\vec{a}_4$ in $\mathbf{R}^4$. Describe, with as much specificity as possible, A_{rref}, the row-reduced echelon form of A if

a. $(\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4)$ is linearly independent.

b. $(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ is linearly independent and $(\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4)$ is linearly dependent.

c. $(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ and $(\vec{a}_1, \vec{a}_2, \vec{a}_4)$ are linearly independent and $(\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4)$ is linearly dependent.

3. Consider a linear system of equations summarized by the matrix equation $A\vec{x} = \vec{b}$ where A is an $m \times n$ matrix and $b \in \mathbf{R}^m$. Suppose that A has p pivots.

a. If $p = m$, what can be said about the number of solutions of $A\vec{x} = \vec{b}$?

b. If $p = n$, what can be said about the number of solutions of $A\vec{x} = \vec{b}$?

c. If $p = m = n$, what can be said about the number of solutions of $A\vec{x} = \vec{b}$?