

Special Problem B Solutions

$M = [\vec{v}_1 | \cdots | \vec{v}_n]$ is a $3 \times n$ matrix. Assume that each column vector of M is nonzero. Complete the following table and provide some justification for each of your entries.

n	What are the geometrical relationships among the column vectors of M ?	What is M_{ref} ?	How many solutions are there to $[\vec{v}_1 \cdots \vec{v}_{n-1}] \vec{x} = \vec{v}_n$?
2	The two column vectors of M are not collinear.	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$	There is no solution.
2	The two column vectors of M are collinear.	$\begin{bmatrix} 1 & a \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ where $a \neq 0$	There is exactly one solution.
3	All three column vectors of M are collinear.	$\begin{bmatrix} 1 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ where $ab \neq 0$	There are infinitely many solutions.
3	The first two column vectors of M are collinear but all three are not.	$\begin{bmatrix} 1 & a & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ where $a \neq 0$	There is no solution.
3	The three column vectors of M are non-coplanar.	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	There is no solution.
4	No three of the four column vectors of M are coplanar.	$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{bmatrix}$ where at least one of a, b, c is not zero	There is exactly one solution.
4	The first two column vectors of M are non-collinear and the remaining two are coplanar with those two.	$\begin{bmatrix} 1 & 0 & a & c \\ 0 & 1 & b & d \\ 0 & 0 & 0 & 0 \end{bmatrix}$ where $abcd \neq 0$	There are infinitely many solutions.
4	No two column vectors of M are collinear. The first three column vectors are coplanar.	$\begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ where $ab \neq 0$	There is no solution.