SM261.1001. Extra Problem Set 2. Due 20100913.
This is optional and counts for extra credit.

On the first day of this course, I stated that linear systems of equations may have no solutions, one solution, or infinitely many solutions; nothing else is possible. Linear systems of equations cannot have, for example, exactly two solutions or exactly 2013 solutions. Now, you should be able to demonstrate why this is true.

Any real linear system of m equations in n variables can be abbreviated as a single matrix equation of the form $A\vec{x} = \vec{b}$ where A is an $m \times n$ matrix, $\vec{b}$ is a vector in $\mathbf{R}^m$ and $\vec{x}$ is the vector of variables in $\mathbf{R}^n$. Suppose that this system has two distinct solutions labeled $\vec{y}$ and $\vec{z}$.

a. Show that $A\vec{x} = \vec{0}$ has infinitely many different solutions and list them.

b. From the previous result, show that $A\vec{x} = \vec{b}$ has infinitely many different solutions and list them.

c. Now show that $A\vec{x} = \vec{b}$ cannot have exactly p solutions if p is a positive integer greater than 1.