

# Advanced Thermodynamics Homework

Assignment: \_\_\_\_\_

Name: \_\_\_\_\_

Date: \_\_\_\_\_

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## 6.

A pumped hydro storage system is drained to extract stored energy on-demand. Water is drained toward a lower lake and a volumetric flow rate of  $Q = 500 \text{ m}^3/\text{s}$  with an  $\Delta z = 85 \text{ m}$  drop in elevation.

(a) **Maximum possible power with upper reservoir as a lake ( $V_1 = 0$ ):**

- **Apply the Energy Equation:**

$$\frac{V_1^2}{2} + gz_1 = \frac{V_2^2}{2} + gz_2 + w_{\text{turbine}}$$

where:

$V_1$  = velocity at upper reservoir (lake) = 0

$V_2$  = velocity at lower lake (assume 0)

$z_1$  = elevation of upper reservoir

$z_2$  = elevation of lower lake

$g = 9.81 \text{ m/s}^2$

- **Solve for work per unit mass:**

$$w_{\text{turbine}} = g(z_1 - z_2)$$

- **Calculate total power:**

$$\dot{W}_{\text{turbine}} = \dot{m} \cdot w_{\text{turbine}}$$

$$\dot{m} = \rho Q = 1000 \text{ kg/m}^3 \times 500 \text{ m}^3/\text{s} = 500,000 \text{ kg/s}$$

$$w_{\text{turbine}} = 9.81 \text{ m/s}^2 \times 85 \text{ m} = 833.85 \text{ J/kg}$$

$$\dot{W}_{\text{turbine}} = 500,000 \text{ kg/s} \times 833.85 \text{ J/kg} = 416,925,000 \text{ W} = 416.9 \text{ MW}$$

(b) **Maximum possible power with upper reservoir as a river ( $V_1 = 3.75 \text{ m/s}$ ):**

- **Include kinetic energy at inlet:**

$$w_{\text{turbine}} = \frac{V_1^2}{2} + g(z_1 - z_2)$$

$$= \frac{(3.75 \text{ m/s})^2}{2} + 9.81 \text{ m/s}^2 \times 85 \text{ m}$$

$$= 7.03 \text{ J/kg} + 833.85 \text{ J/kg} = 840.88 \text{ J/kg}$$

$$\dot{W}_{\text{turbine}} = 500,000 \text{ kg/s} \times 840.88 \text{ J/kg} = 420,440,000 \text{ W} = 420.4 \text{ MW}$$

- **Percent increase in power:**

$$\% \text{ increase} = \frac{420.4 - 416.9}{416.9} \times 100\% = 0.84\%$$

- **Physical explanation:**

The increase in possible power output is due to the additional kinetic energy of the water entering the turbine when the upper reservoir is a river (nonzero velocity). This kinetic energy is converted to useful work by the turbine, resulting in a slightly higher power output compared to the case where the upper reservoir is a still lake.

## 7.

A piston-cylinder system is filled with 3.8 kg of  $\text{H}_2$  gas ( $R = 4.124 \text{ kJ/kg} \cdot \text{K}$ ) and is initially at 250 kPa and 900 K. Heat is removed from the system until it reaches 400 K.  $C_v(T)$  and  $C_p(T)$  are plotted as functions of temperature for this gas.

**Part A: Find the change in volume ( $\text{m}^3$ ) of the system.**

- Use the ideal gas law to find the initial volume:

$$\begin{aligned} PV &= mRT \\ V_1 &= \frac{mRT_1}{P} \\ &= \frac{(3.8 \text{ kg})(4.124 \text{ kJ/kg} \cdot \text{K})(900 \text{ K})}{250 \text{ kPa}} \\ &= 56.416 \text{ m}^3 \end{aligned}$$

- For a constant pressure process,  $\frac{T_1}{V_1} = \frac{T_2}{V_2}$ :

$$\begin{aligned} \frac{900}{56.416} &= \frac{400}{V_2} \\ V_2 &= \frac{400 \times 56.416}{900} = 25.073 \text{ m}^3 \end{aligned}$$

- Change in volume:

$$\Delta V = V_2 - V_1 = 25.073 - 56.416 = -31.343 \text{ m}^3$$

The negative sign indicates the volume decreased as the gas cooled.

**Part B: Find the magnitude (kJ) and direction (in or out of system) of the boundary work during this process. Defend why your answer makes sense.**

- For a constant pressure process, boundary work is:

$$\begin{aligned}
 W &= P\Delta V = P(V_2 - V_1) \\
 &= 250 \text{ kPa} \times (25.073 - 56.416) \text{ m}^3 \\
 &= 250 \text{ kPa} \times (-31.343 \text{ m}^3) \\
 &= -7,836 \text{ kJ}
 \end{aligned}$$

- The negative sign indicates work is done **by** the system (the gas does work on the surroundings as it contracts). If you define work done **on** the system,  $W = +7,836 \text{ kJ}$  into the system. This makes sense because as heat is removed at constant pressure, the gas contracts and the surroundings do work on the gas.

**Part C: Calculate the change in specific enthalpy (kJ/kg) and internal energy (kJ/kg) of the gas during this process using the given property tables.**

- **Change in specific enthalpy ( $\Delta h$ ):**

$$\begin{aligned}
 \Delta h &= \int_{T_1}^{T_2} C_p(T) dT \\
 &\approx \bar{C}_p \times (T_2 - T_1)
 \end{aligned}$$

Where  $\bar{C}_p$  is the average of  $C_p$  at 900 K and 400 K (read from the graph):

$$\begin{aligned}
 \bar{C}_p &\approx \frac{14.35 + 14.93}{2} = 14.64 \text{ kJ/kg} \cdot \text{K} \\
 \Delta h &= 14.64 \text{ kJ/kg} \cdot \text{K} \times (400 - 900) \text{ K} \\
 &= 14.64 \times (-500) = -7,320 \text{ kJ/kg}
 \end{aligned}$$

- **Change in specific internal energy ( $\Delta u$ ):**

$$\begin{aligned}
 \Delta u &= \int_{T_1}^{T_2} C_v(T) dT \\
 &\approx \bar{C}_v \times (T_2 - T_1)
 \end{aligned}$$

Where  $\bar{C}_v$  is the average of  $C_v$  at 900 K and 400 K (read from the graph, e.g. 10.2 and 10.9):

$$\begin{aligned}
 \bar{C}_v &\approx \frac{10.2 + 10.9}{2} = 10.55 \text{ kJ/kg} \cdot \text{K} \\
 \Delta u &= 10.55 \text{ kJ/kg} \cdot \text{K} \times (400 - 900) \text{ K} \\
 &= 10.55 \times (-500) = -5,275 \text{ kJ/kg}
 \end{aligned}$$

- Both  $\Delta h$  and  $\Delta u$  are negative, indicating the gas lost enthalpy and internal energy as it cooled from 900 K to 400 K.

8.

A centrifugal pump is operated steadily and consumes 10 kW of shaft work to move water ( $1000 \text{ kg/m}^3$ ,  $C = 4.2 \text{ kJ/kg} \cdot \text{K}$ ) at a mass flow rate of  $0.05 \text{ kg/s}$ . The pump is not very well insulated. Temperature is measured at the inlet ( $20^\circ\text{C}$ ) and the outlet ( $45^\circ\text{C}$ ) of the pump. The cross sectional area at the inlet and the outlet of the pump is the same ( $A_2 = A_3$ ).

**Part A: Prove that the change in kinetic energy is negligible**

- **Given:**

$$\dot{m} = 0.05 \text{ kg/s}$$

$$A_2 = A_3$$

$$\rho = 1000 \text{ kg/m}^3$$

- **Conservation of mass for steady flow:**

$$\dot{m} = \rho A_2 v_2 = \rho A_3 v_3$$

$$\Rightarrow v_2 = v_3$$

- **Change in kinetic energy:**

$$\Delta KE = \frac{1}{2} m (v_3^2 - v_2^2) = 0$$

- **Conclusion:** The change in kinetic energy is negligible (zero) for the water in this pump.

**Part B: Estimate the heat loss (kW) from the pump**

- **Given:**

$$\dot{W}_s = 10 \text{ kW}$$

$$C = 4.2 \text{ kJ/kg} \cdot \text{K}$$

$$\dot{m} = 0.05 \text{ kg/s}$$

$$T_1 = 20^\circ\text{C}$$

$$T_2 = 45^\circ\text{C}$$

- **Steady-flow energy equation (SFEE):**

$$\dot{Q}_{\text{loss}} - \dot{W}_s = \dot{m} [\Delta h + \Delta KE + \Delta PE]$$

- **Assumptions:**

$$- \Delta KE = 0 \text{ (from Part A)}$$

- $\Delta PE \approx 0$  (not significant)
- For incompressible liquid:  $\Delta h = C\Delta T$

- **Energy balance:**

$$\begin{aligned}\dot{Q}_{\text{loss}} - \dot{W}_s &= \dot{m} \cdot C \cdot (T_2 - T_1) \\ \dot{m} \cdot C \cdot \Delta T &= 0.05 \times 4.2 \times (45 - 20) \\ &= 0.05 \times 4.2 \times 25 \\ &= 5.25 \text{ kW}\end{aligned}$$

- **Solve for heat loss:**

$$\begin{aligned}\dot{Q}_{\text{loss}} &= \dot{W}_s - \dot{m} \cdot C \cdot \Delta T \\ &= 10 - 5.25 = 4.75 \text{ kW}\end{aligned}$$

- **Answer:** The heat loss from the pump is approximately 4.75 kW (rounded: 4.8 kW).



## 9.

A well-insulated steam turbine is operating reversibly with 3 kg/s of steam at a pressure of 3 MPa. The majority of the steam is exhausted at 50 kPa and 100°C. However, 12% of this flow is diverted within the steam turbine to provide preheating for other cycle components once the inlet stream was expanded to 500 kPa. Determine the power produced by this turbine, in kW.

### Problem Setup

- **Given:**

$$\begin{aligned}\dot{m}_1 &= 3 \text{ kg/s} \\ P_1 &= 3 \text{ MPa} \\ P_2 &= 500 \text{ kPa} \\ P_3 &= 50 \text{ kPa} \\ T_3 &= 100^\circ\text{C}\end{aligned}$$

Extraction flow fraction: 0.12

- **Mass Flow Rates:**

$$\begin{aligned}\dot{m}_2 &= 0.12 \times \dot{m}_1 = 0.36 \text{ kg/s} \\ \dot{m}_3 &= \dot{m}_1 - \dot{m}_2 = 2.64 \text{ kg/s}\end{aligned}$$

- **Steady Flow Energy Equation (SFEE) for the turbine:**

$$\dot{W}_{\text{out}} = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$$

where:

- $h_1$  = specific enthalpy at inlet (3 MPa, isentropic assumption from inlet state)
- $h_2$  = specific enthalpy at extraction point (500 kPa)
- $h_3$  = specific enthalpy at exhaust (50 kPa, 100°C)

- **Values to Look Up from Steam Tables:**

- State 1 (Inlet):  $P_1 = 3 \text{ MPa}$ . Need to determine if saturated or superheated. Assuming isentropic process:  $s_1 = s_2 = s_3$  (for reversible operation). Look up:  $h_1, s_1$ .
- State 3 (Exhaust):  $P_3 = 50 \text{ kPa}, T_3 = 100^\circ\text{C}$ . Look up:  $h_3, s_3$ .
- State 2 (Extraction):  $P_2 = 500 \text{ kPa}, s_2 = s_1$  (isentropic). Look up:  $h_2$  at 500 kPa with entropy  $s_2$ .

- **Power Calculation Setup:**

$$\begin{aligned}\dot{W}_{\text{out}} &= (3 \text{ kg/s})h_1 - (0.36 \text{ kg/s})h_2 - (2.64 \text{ kg/s})h_3 \\ &= 3h_1 - 0.36h_2 - 2.64h_3 \quad [\text{kW}]\end{aligned}$$

- **Note:** Use steam tables to look up  $h_1$ ,  $h_2$ , and  $h_3$  as needed. If isentropic, use  $s_1 = s_2 = s_3$  to find the correct enthalpy values at each state.

## 10.

A steam turbine is operating steadily and expands a 2.5 kg/s stream of steam/water from 3 MPa and 400 °C to 100 kPa. Assume the turbine is well-insulated. The manufacturer performance curves indicate that under these conditions, the turbine would operate with an isentropic efficiency of 0.85.

**Given:**

- $\dot{m} = 2.5 \text{ kg/s}$
- $P_1 = 3 \text{ MPa}, T_1 = 400 \text{ °C}$
- $P_2 = 100 \text{ kPa}$
- $\eta_s = 0.85$
- Steady flow, well-insulated ( $\dot{Q} = 0$ ), negligible  $\Delta KE$  and  $\Delta PE$

**PART A: Isentropic Operation**

- **Goal:** Find  $T_{2s}, h_{2s}, x_{2s}$  (quality),  $s_{2s}$
- **Process:**
  1. Look up inlet state properties (superheated steam at  $P_1 = 3 \text{ MPa}, T_1 = 400^\circ\text{C}$ ):  $h_1, s_1$  (from steam tables)
  2. Apply isentropic condition:  $s_{2s} = s_1$
  3. At  $P_2 = 100 \text{ kPa}, s_{2s} = s_1$ : compare  $s_{2s}$  with  $s_f$  and  $s_g$  at 100 kPa
  4. If  $s_f < s_{2s} < s_g$ , two-phase mixture:  $x_{2s} = (s_{2s} - s_f)/s_{fg}$
  5. Find enthalpy:  $h_{2s} = h_f + x_{2s}h_{fg}$
  6. Temperature:  $T_{2s} = T_{sat}$  at 100 kPa
- **Summary for Part A:**
  - $T_{2s} = \text{ _ K}$
  - $h_{2s} = \text{ _ kJ/kg}$
  - $x_{2s} = \text{ _}$
  - $s_{2s} = \text{ _ kJ/kg-K}$

**PART B: Real Operation**

- **Goal:** Find  $T_2, h_2, x_2, s_2$  (actual outlet conditions)
- **Process:**
  1. Use isentropic efficiency:  $\eta_s = (h_1 - h_2)/(h_1 - h_{2s}) = 0.85$

2. Solve for  $h_2$ :  $h_2 = h_1 - \eta_s(h_1 - h_{2s})$
3. At  $P_2 = 100$  kPa,  $h_2$  known: compare  $h_2$  with  $h_f$  and  $h_g$  at 100 kPa
4. If  $h_f < h_2 < h_g$ , two-phase mixture:  $x_2 = (h_2 - h_f)/h_{fg}$
5. Find entropy:  $s_2 = s_f + x_2 s_{fg}$
6. Temperature:  $T_2 = T_{sat}$  at 100 kPa

• **Summary for Part B:**

- $T_2 = \_ \text{ K}$
- $h_2 = \_ \text{ kJ/kg}$
- $x_2 = \_$
- $s_2 = \_ \text{ kJ/kg-K}$

**PART C: Work Comparison**

• **Goal:** Calculate and compare turbine work output

• **Process:**

1. Isentropic work:  $w_s = h_1 - h_{2s}$
2. Actual work:  $w_{actual} = h_1 - h_2$
3. Power output:  $\dot{W}_s = \dot{m}w_s$ ,  $\dot{W}_{actual} = \dot{m}w_{actual}$
4. Compare difference:  $\Delta\dot{W} = \dot{W}_s - \dot{W}_{actual}$
5. Explanation: Real turbine produces less work due to irreversibilities (friction, turbulence, heat transfer within fluid). Energy lost to entropy generation. Verify:  
 $w_{actual} = \eta_s \times w_s$

**PART D: Entropy Generation Rate**

• **Goal:** Calculate  $\dot{S}_{gen}$  using entropy balance

• **Given entropy balance:**

$$\sum \frac{\dot{Q}}{T_{boundary}} + \sum \dot{m}s_{in} - \sum \dot{m}s_{out} + \dot{S}_{gen} = \frac{dS_{sys}}{dt}$$

• **For steady flow, well-insulated turbine:**

- $\frac{dS_{sys}}{dt} = 0$  (steady state)
- $\sum \frac{\dot{Q}}{T_{boundary}} = 0$  (adiabatic)
- Single inlet and outlet stream

- **Simplified:**

$$0 + \dot{m}s_1 - \dot{m}s_2 + \dot{S}_{gen} = 0$$

$$\Rightarrow \dot{S}_{gen} = \dot{m}(s_2 - s_1)$$

- **Check:**  $\dot{S}_{gen} > 0$  for real process (confirms 2nd law)

### PART E: Comparison and Benefit

- **Goal:** Compare A vs B properties and identify benefit of non-ideal turbine

- **Process:**

1. Create comparison table:

Property	Isentropic (A)	Real (B)	Difference
$h_2$ (kJ/kg)	-	-	-
$s_2$ (kJ/kg-K)	-	-	-
$x_2$ (quality)	-	-	-
$T_2$ (K)	-	-	-

2. Key observation: Real turbine has higher quality ( $x_2 > x_{2s}$ ), higher entropy ( $s_2 > s_1$ ), higher enthalpy at exit ( $h_2 > h_{2s}$ )
3. Benefit: Less liquid content in exhaust (higher quality = more vapor), reduces water droplet erosion on turbine blades. Non-ideal operation trades work output for blade longevity; small sacrifice in efficiency protects equipment.

### Solution Steps Summary

1. Look up inlet properties from steam tables ( $P_1, T_1$ )
2. Part A: Use  $s_{2s} = s_1$ , find two-phase properties at  $P_2$
3. Part B: Apply  $\eta_s$  to find  $h_2$ , then find two-phase properties
4. Part C: Calculate work from enthalpy drops, compare
5. Part D: Apply entropy balance with given equation
6. Part E: Compare moisture content (quality), explain blade erosion benefit

## 11.

A free piston-cylinder device is filled with 3 kg of a saturated liquid-vapor mixture of water/steam at 200 kPa. The exterior of the tank may be assumed to be well-insulated. **Initially, 40% of the mass is in the liquid phase** (so  $x_1 = 0.60$ ). Heat is supplied via an electric resistance heater until all the liquid in the tank is evaporated. **Determine the specific entropy change ( $\Delta s$ , kJ/kg-K) of the steam inside the tank during this process.**

### Given Information

- Mass:  $m = 3$  kg
- Pressure:  $P = 200$  kPa (constant)
- Initial quality:  $x_1 = 0.60$  (40% liquid  $\rightarrow$  60% vapor)
- Final quality:  $x_2 = 1.0$  (all liquid evaporated  $\rightarrow$  saturated vapor)

### Relevant Equations

- For specific entropy in a two-phase mixture:

$$s = s_f + x \cdot s_{fg}$$

- Specific entropy change:

$$\Delta s = s_2 - s_1$$

### Setup with Numbers

- State 1 (initial):

$$\begin{aligned} s_1 &= s_f + x_1 \cdot s_{fg} \\ &= s_f + (0.60) \cdot s_{fg} \end{aligned}$$

- State 2 (final):

$$s_2 = s_g \quad (\text{or equivalently: } s_2 = s_f + (1.0) \cdot s_{fg})$$

- Change in specific entropy:

$$\begin{aligned} \Delta s &= s_2 - s_1 \\ &= s_g - [s_f + (0.60) \cdot s_{fg}] \end{aligned}$$

### Process Steps

1. Look up steam table properties at  $P = 200$  kPa (saturated pressure table):  $s_f$ ,  $s_{fg}$ ,  $s_g$

2. Calculate initial specific entropy ( $s_1$ ):  $s_1 = s_f + (0.60) \cdot s_{fg}$
3. Identify final specific entropy ( $s_2$ ):  $s_2 = s_g$
4. Calculate the change:  $\Delta s = s_2 - s_1$
5. Check your answer (units should be kJ/kg·K)

## 12.

A piston–cylinder device is well insulated and contains 4.8 L of saturated liquid ( $x = 0$ ) water at 150 kPa. The piston moves freely within the cylinder. An electric resistance heater inside the cylinder is suddenly turned on, and 1700 kJ of energy is delivered to the steam. **Determine the total entropy change ( $\Delta S$ , kJ/kg) of the steam in the device during this process.**

### Given Information

- Initial volume:  $V_1 = 4.8 \text{ L} = 0.0048 \text{ m}^3$
- Initial state: Saturated liquid water,  $x_1 = 0$
- Initial pressure:  $P_1 = 150 \text{ kPa}$
- Constant pressure process ( $P_2 = P_1$ )
- Energy input:  $Q_{in} = 1700 \text{ kJ}$

### Assumptions

- Well-insulated system  $\rightarrow$  no heat loss to surroundings
- Piston moves freely  $\rightarrow$  constant pressure process
- Neglect kinetic and potential energy changes
- Quasi-equilibrium process

### Relevant Equations

- Energy Balance (First Law):

$$Q_{in} - W_{out} = \Delta U = m(u_2 - u_1)$$

For constant pressure:

$$\begin{aligned} W_{out} &= P(V_2 - V_1) = mP(v_2 - v_1) \\ Q_{in} &= m(u_2 - u_1) + mP(v_2 - v_1) = m(h_2 - h_1) \end{aligned}$$

- Entropy Change:

$$\Delta S = m(s_2 - s_1)$$

$$\Delta s = s_2 - s_1$$

### Solution Process

#### 1. Find initial state properties at State 1:

- Look up saturated liquid properties at  $P_1 = 150$  kPa:
- $v_1 = v_f$  @ 150 kPa ( $\text{m}^3/\text{kg}$ )
- $h_1 = h_f$  @ 150 kPa ( $\text{kJ}/\text{kg}$ )
- $s_1 = s_f$  @ 150 kPa ( $\text{kJ}/\text{kg}\cdot\text{K}$ )
- $T_{sat}$  @ 150 kPa ( $^{\circ}\text{C}$ )

#### 2. Calculate mass of water:

$$m = \frac{V_1}{v_1} = \frac{0.0048 \text{ m}^3}{v_f @ 150 \text{ kPa}}$$

#### 3. Apply energy balance to find State 2 enthalpy:

$$h_2 = h_1 + \frac{Q_{in}}{m} = h_f @ 150 \text{ kPa} + \frac{1700 \text{ kJ}}{m}$$

#### 4. Determine State 2 condition:

- Knowing  $P_2 = 150$  kPa and  $h_2$  from above, compare  $h_2$  to  $h_f$  and  $h_g$  at 150 kPa:
- If  $h_f < h_2 < h_g \rightarrow$  two-phase mixture:

$$x_2 = \frac{h_2 - h_f}{h_{fg}}$$

- Then find entropy:

$$s_2 = s_f + x_2 \cdot s_{fg}$$

- If  $h_2 > h_g \rightarrow$  superheated vapor:
  - Use superheated steam tables at  $P_2 = 150$  kPa to interpolate for  $s_2$  at the calculated  $h_2$ .

#### 5. Calculate entropy change:

$$\Delta s = s_2 - s_1 = s_2 - s_f @ 150 \text{ kPa}$$

(Result in  $\text{kJ}/\text{kg}\cdot\text{K}$ )



## 13.

Two solar receivers are proposed for storing concentrated solar energy as heat in either solid ceramic particles (CARBOBEAD CP,  $\rho = 3270 \text{ kg/m}^3$ ,  $C \approx 1.3 \text{ kJ/kg}\cdot\text{K}$  up to  $1100^\circ\text{C}$ ) or compressed air. Both receivers are designed to heat their respective materials from  $250^\circ\text{C}$  to  $1100^\circ\text{C}$ . Compressed air is introduced into the one receiver at a mass flow rate of  $0.75 \text{ kg/s}$ ,  $250^\circ\text{C}$ , and  $4000 \text{ kPa}$ . The heated air in the receiver is then expanded reversibly through a well-insulated turbine to an environmental pressure of  $100 \text{ kPa}$ . Assume table A-17 is populated with a reference state of  $0 \text{ K}$  and  $100 \text{ kPa}$  and the gas constant for air is  $R = 0.287 \text{ kJ/kg}\cdot\text{K}$ .

**Make sure you clearly label additional assumptions used to solve the problem.**

**PART A: Change in specific entropy (kJ/kg·K) of the particle curtain from 1  $\rightarrow$  2**

For an incompressible substance with constant specific heat:

$$\Delta s_{1 \rightarrow 2} = C \ln \left( \frac{T_2}{T_1} \right)$$

Where  $C = 1.3 \text{ kJ/kg}\cdot\text{K}$ ,  $T_1$  and  $T_2$  in K.

*Assumption:* Initial and final temperatures of the ceramic particles must be specified or reasonably assumed.

**PART B: Change in specific entropy (kJ/kg·K) of the compressed air stream from 3  $\rightarrow$  4**

For an ideal gas using Table A-17:

$$\Delta s_{3 \rightarrow 4} = s^\circ(T_4) - s^\circ(T_3) - R \ln \left( \frac{P_4}{P_3} \right)$$

Where  $R = 0.287 \text{ kJ/kg}\cdot\text{K}$ .

*Assumption:*  $P_4$  at the receiver outlet is likely  $\approx P_3$  if pressure drop is negligible.

**PART C: Specific entropy (kJ/kg·K) of the compressed air stream for state 4**

$$s_4 = s^\circ(T_4) - R \ln \left( \frac{P_4}{P_0} \right)$$

Where  $P_0 = 100 \text{ kPa}$  (reference pressure).

*Assumption:* Use  $P_4$  from Part B.

**PART D: Work (kW) produced via the turbine from 4  $\rightarrow$  5**

For a reversible adiabatic turbine, using SFEE:

$$\dot{W}_{turb} = \dot{m}(h_4 - h_5)$$

Where  $\dot{m} = 0.75 \text{ kg/s}$ .

*Process:*

1. Look up  $h_4$  at  $T_4 = 1373$  K from Table A-17
2. For reversible (isentropic) expansion:  $s_5 = s_4$
3. With  $s_5 = s_4$  and  $P_5 = 100$  kPa, find  $T_5$  such that  $s^\circ(T_5) = s_4$
4. Look up  $h_5$  at  $T_5$  from Table A-17
5. Calculate  $h_4 - h_5$
6. Multiply by mass flow rate to get power in kW

### Summary of Solution Process

1. Identify all state points and list known properties (T, P) for each
2. State assumptions clearly (e.g.,  $P_4 \approx P_3$ , reversible processes, reference states)
3. Use Table A-17 for all air property lookups ( $s^\circ$ ,  $h$ )
4. For incompressible substances:  $\Delta s = C \ln(T_2/T_1)$
5. For ideal gases:  $s = s^\circ(T) - R \ln(P/P_0)$
6. For turbine work: Apply SFEE with isentropic constraint ( $s_5 = s_4$ )
7. Check signs: Heating  $\rightarrow +\Delta s$ , Cooling  $\rightarrow -\Delta s$ ; Turbine  $\rightarrow +W$  (work out)
8. Units: Keep everything in kJ, kg, K, kPa throughout

## 14.

The performance of an ideal concentrated solar power plant can be modeled as the combined efficiency of a (1) solar receiver and (2) Carnot heat engine. The solar receiver absorbs concentrated solar resources and delivers them as heat to a power cycle. The total ideal system efficiency is the product of the receiver efficiency and the heat engine efficiency in converting incident solar resources into work.

### Given Information

- Concentration ratios ( $C$ ): 100, 500, 1000, 2000, 3000 suns
- Solar irradiance ( $I$ ): 1000 W/m<sup>2</sup>
- Stefan-Boltzmann constant ( $\sigma$ ):  $5.67 \times 10^{-8}$  W/(m<sup>2</sup>·K<sup>4</sup>)
- Cold reservoir temperature ( $T_L$ ): Assume ambient  $\approx$  300 K
- Hot reservoir temperature range ( $T_H$ ): 300 K to 2600 K

### PART A: Plot Total Efficiency

#### Relevant Equations

- Receiver Efficiency:

$$\eta_{receiver} = 1 - \frac{\sigma T_H^4}{C \cdot I}$$

- Carnot Heat Engine Efficiency:

$$\eta_{Carnot,HE} = 1 - \frac{T_L}{T_H}$$

- Total System Efficiency:

$$\begin{aligned} \eta_{Total} &= \eta_{receiver} \cdot \eta_{Carnot,HE} \\ &= \left(1 - \frac{5.67 \times 10^{-8} \cdot T_H^4}{C \cdot 1000}\right) \cdot \left(1 - \frac{300}{T_H}\right) \end{aligned}$$

### Process for Part A

1. For each concentration ratio  $C$  (100, 500, 1000, 2000, 3000):
  - Create array of  $T_H$  values from 300 K to 2600 K (increment 25–50 K)
  - For each  $T_H$ , calculate:
    - $\eta_{receiver} = 1 - (5.67 \times 10^{-8} \times T_H^4)/(C \times 1000)$
    - $\eta_{Carnot,HE} = 1 - (300/T_H)$

$$- \eta_{Total} = \eta_{receiver} \times \eta_{Carnot, HE}$$

2. Plot  $\eta_{Total}$  (y-axis) vs  $T_H$  (x-axis) with 5 curves (one for each  $C$  value)
3. Label axes, add legend showing  $C$  values
4. Submit plot and code

### **PART B: Identify Optimal Receiver Temperature**

1. From the plots in Part A, identify the  $T_H$  that gives maximum  $\eta_{Total}$  for each  $C$  value
2. Comment on trends:
  - How does optimal  $T_H$  change with concentration ratio  $C$ ?
  - What physical trade-offs are happening?
    - Higher  $T_H \rightarrow$  Better Carnot efficiency BUT worse receiver efficiency (more radiation losses)
    - Higher  $C \rightarrow$  Can operate at higher  $T_H$  before radiation losses dominate
3. Explain why there's an optimum (competing effects of the two efficiency terms)

#### **Physical Insight**

The total efficiency is a product of two competing terms:

- Receiver efficiency decreases with  $T_H$  (radiation losses scale as  $T^4$ )
- Carnot efficiency increases with  $T_H$  (better thermodynamic limit)

The optimum occurs where the product is maximized. Higher concentration  $C$  allows higher optimal  $T_H$  because concentrated sunlight offsets radiation losses.

## **A Appendix**