

# Advanced Thermodynamics

## Homework Assignment #1

### Assignment Overview

Problems 1-14: Fundamental thermodynamic concepts, energy balances, entropy analysis, steam turbines, Carnot cycles, and concentrated solar power optimization

**Student:** Dominic

**Submitted:** February 1, 2026

### Computational Tools Developed

- CoolProp CLI for multi-fluid property lookup
- Property solver for iterative thermodynamic calculations
  - Carnot cycle optimization analysis
  - Steam turbine performance analysis

## Contents

<b>General Theory</b>	<b>2</b>
<b>1 Problem 6: Hydro Storage System Energy Analysis</b>	<b>3</b>
<b>Problem 6: Hydro Storage System</b>	<b>3</b>
<b>2 Problem 7: Piston-Cylinder Energy Balance</b>	<b>4</b>
<b>Problem 7: Piston-Cylinder System</b>	<b>4</b>
<b>Centrifugal Pump Energy Balance</b>	<b>6</b>
<b>Isentropic Steam Turbine</b>	<b>8</b>
<b>Real Devices and Entropy Balances</b>	<b>9</b>
<b>Entropy Change of a Pure Substance</b>	<b>12</b>
<b>Total Entropy Change in a Piston-Cylinder</b>	<b>13</b>
<b>Entropy Changes: Incompressible Substances and Ideal Gases</b>	<b>14</b>
<b>Carnot Cycle Heat Engines</b>	<b>17</b>
<b>A Appendix</b>	<b>23</b>

**1.**

It is assumed the system is a simple compressible system, therefore there are two degrees of freedom and the rest of the intensive properties can be calculated using an equation of state, or found in a table of common fluids.

**2.**

- Closed system: (a)
- Open system: (c)
- Isolated system: (b)

**3.****Answer:**

- a. internal energy
- b. kinetic energy
- c. potential energy

**Notes:**

- f. Electrical energy could potentially be considered if the mass crossing a system boundary is electrically charged, but it would be a rare circumstance.
- e. Flow work is a summation of other components, not necessarily carried by the mass itself.
- c.  $E = mc^2$ , so an argument could be made but it is not relevant here.

**4.**

**False.** Only solids resist shear stress by static deflection. Fluids continuously deform under shear stress.

**5.**

(c), assuming  $V_n$  is the velocity normal to  $A_c$ , then the final units are kg/s.

# 1 Problem 6: Hydro Storage System Energy Analysis

Given:

- Volumetric flow rate:  $\dot{V} = 500 \text{ m}^3/\text{s}$
- Elevation drop:  $\Delta z = 85 \text{ m}$
- Water density:  $\rho = 1000 \text{ kg/m}^3$  (assumed)

(a) Maximum possible power with upper reservoir as a lake ( $V_1 = 0$ ):

- Apply the Energy Equation:

$$\frac{V_1^2}{2} + gz_1 = \frac{V_2^2}{2} + gz_2 + w_{\text{turbine}}$$

where:

$V_1$  = velocity at upper reservoir (lake) = 0

$V_2$  = velocity at lower lake (assume 0)

$z_1$  = elevation of upper reservoir

$z_2$  = elevation of lower lake

$$g = 9.81 \text{ m/s}^2$$

- Solve for work per unit mass:

$$w_{\text{turbine}} = g(z_1 - z_2)$$

- Calculate total power:

$$\dot{W}_{\text{turbine}} = \dot{m} \cdot w_{\text{turbine}}$$

$$\dot{m} = \rho Q = 1000 \text{ kg/m}^3 \times 500 \text{ m}^3/\text{s} = 500,000 \text{ kg/s}$$

$$w_{\text{turbine}} = 9.81 \text{ m/s}^2 \times 85 \text{ m} = 833.85 \text{ J/kg}$$

$$\dot{W}_{\text{turbine}} = 500,000 \text{ kg/s} \times 833.85 \text{ J/kg} = 416,925,000 \text{ W} = 416.9 \text{ MW}$$

(b) Maximum possible power with upper reservoir as a river ( $V_1 = 3.75 \text{ m/s}$ ):

- Include kinetic energy at inlet:

$$\begin{aligned} w_{\text{turbine}} &= \frac{V_1^2}{2} + g(z_1 - z_2) \\ &= \frac{(3.75 \text{ m/s})^2}{2} + 9.81 \text{ m/s}^2 \times 85 \text{ m} \\ &= 7.03 \text{ J/kg} + 833.85 \text{ J/kg} = 840.88 \text{ J/kg} \end{aligned}$$

$$\dot{W}_{\text{turbine}} = 500,000 \text{ kg/s} \times 840.88 \text{ J/kg} = 420,440,000 \text{ W} = 420.4 \text{ MW}$$

- Percent increase in power:

$$\% \text{ increase} = \frac{420.4 - 416.9}{416.9} \times 100\% = 0.84\%$$

- Physical explanation:

The increase in possible power output is due to the additional kinetic energy of the water entering the turbine when the upper reservoir is a river. This kinetic energy is converted to useful work by the turbine, resulting in a slightly higher power output compared to the case where the upper reservoir is a still lake.

## 2 Problem 7: Piston-Cylinder Energy Balance

Given:

- Mass of H<sub>2</sub> gas:  $m = 3.8 \text{ kg}$
- Gas constant:  $R = 4.124 \text{ kJ/kg}\cdot\text{K}$
- Initial state:  $P_1 = 250 \text{ kPa}$ ,  $T_1 = 900 \text{ K}$
- Final temperature:  $T_2 = 400 \text{ K}$
- Process: Constant pressure (heat removal)

**Part A: Find the change in volume (m<sup>3</sup>) of the system.**

Using the ideal gas law to find the initial volume:

$$\begin{aligned} PV &= mRT \\ V_1 &= \frac{mRT_1}{P} \\ &= \frac{(3.8 \text{ kg})(4.124 \text{ kJ/kg}\cdot\text{K})(900 \text{ K})}{250 \text{ kPa}} \\ &= 56.416 \text{ m}^3 \end{aligned}$$

For a constant pressure process,  $\frac{T_1}{V_1} = \frac{T_2}{V_2}$ :

$$\begin{aligned} \frac{900}{56.416} &= \frac{400}{V_2} \\ V_2 &= \frac{400 \times 56.416}{900} = 25.073 \text{ m}^3 \end{aligned}$$

The change in volume is:

$$\Delta V = V_2 - V_1 = 25.073 - 56.416 = -31.343 \text{ m}^3$$

The negative sign indicates the volume decreased as the gas cooled.

**Part B: Find the magnitude (kJ) and direction (in or out of system) of the boundary work during this process. Defend why your answer makes sense.**

For a constant pressure process, the boundary work is:

$$\begin{aligned} W &= P\Delta V = P(V_2 - V_1) \\ &= 250 \text{ kPa} \times (25.073 - 56.416) \text{ m}^3 \\ &= 250 \text{ kPa} \times (-31.343 \text{ m}^3) \\ &= -7,836 \text{ kJ} \end{aligned}$$

The negative sign indicates work is done **by** the system (the gas does work on the surroundings as it contracts). If you define work done **on** the system,  $W = +7,836 \text{ kJ}$  into the system. This makes sense because as heat is removed at constant pressure, the gas contracts and the surroundings do work on the gas.

**Part C: Calculate the change in specific enthalpy (kJ/kg) and internal energy (kJ/kg) of the gas during this process using the given property tables.**

- Change in specific enthalpy ( $\Delta h$ ):

$$\begin{aligned} \Delta h &= \int_{T_1}^{T_2} C_p(T) dT \\ &\approx \bar{C}_p \times (T_2 - T_1) \end{aligned}$$

Where  $\bar{C}_p$  is the average of  $C_p$  at 900 K and 400 K (read from the graph):

$$\begin{aligned} \bar{C}_p &\approx \frac{14.35 + 14.93}{2} = 14.64 \text{ kJ/kg} \cdot \text{K} \\ \Delta h &= 14.64 \text{ kJ/kg} \cdot \text{K} \times (400 - 900) \text{ K} \\ &= 14.64 \times (-500) = -7,320 \text{ kJ/kg} \end{aligned}$$

- Change in specific internal energy ( $\Delta u$ ):

$$\begin{aligned} \Delta u &= \int_{T_1}^{T_2} C_v(T) dT \\ &\approx \bar{C}_v \times (T_2 - T_1) \end{aligned}$$

Where  $\bar{C}_v$  is the average of  $C_v$  at 900 K and 400 K (read from the graph, e.g. 10.2 and 10.9):

$$\begin{aligned} \bar{C}_v &\approx \frac{10.2 + 10.9}{2} = 10.55 \text{ kJ/kg} \cdot \text{K} \\ \Delta u &= 10.55 \text{ kJ/kg} \cdot \text{K} \times (400 - 900) \text{ K} \\ &= 10.55 \times (-500) = -5,275 \text{ kJ/kg} \end{aligned}$$

- Both  $\Delta h$  and  $\Delta u$  are negative, indicating the gas lost enthalpy and internal energy as it cooled from 900 K to 400 K.

## Problem 8: Centrifugal Pump Energy Balance

**Given:**

- Shaft work:  $\dot{W}_s = 10 \text{ kW}$
- Mass flow rate:  $\dot{m} = 0.05 \text{ kg/s}$
- Water properties:  $\rho = 1000 \text{ kg/m}^3$ ,  $C = 4.2 \text{ kJ/kg} \cdot \text{K}$
- Inlet temperature:  $T_1 = 20^\circ\text{C}$
- Outlet temperature:  $T_2 = 45^\circ\text{C}$
- Cross-sectional areas:  $A_{inlet} = A_{outlet}$

**Part A: Prove that the change in kinetic energy is negligible**

- **Given:**

$$\dot{m} = 0.05 \text{ kg/s}$$

$$A_2 = A_3$$

$$\rho = 1000 \text{ kg/m}^3$$

- **Conservation of mass for steady flow:**

$$\begin{aligned}\dot{m} &= \rho A_2 v_2 = \rho A_3 v_3 \\ \Rightarrow v_2 &= v_3\end{aligned}$$

- **Change in kinetic energy:**

$$\Delta KE = \frac{1}{2} m (v_3^2 - v_2^2) = 0$$

- **Conclusion:** The change in kinetic energy is negligible (zero) for the water in this pump.

**Part B: Estimate the heat loss (kW) from the pump**

Applying the flow energy equation:

$$\dot{Q}_{loss} - \dot{W}_s = \dot{m} [\Delta h + \Delta KE + \Delta PE]$$

---

With the assumptions that  $\Delta KE = 0$  (from Part A),  $\Delta PE \approx 0$  (not significant), and for incompressible liquid  $\Delta h = C\Delta T$ , the energy balance becomes:

$$\begin{aligned}\dot{Q}_{\text{loss}} - \dot{W}_s &= \dot{m} \cdot C \cdot (T_2 - T_1) \\ \dot{m} \cdot C \cdot \Delta T &= 0.05 \times 4.2 \times (45 - 20) \\ &= 0.05 \times 4.2 \times 25 \\ &= 5.25 \text{ kW}\end{aligned}$$

Solving for the heat loss:

$$\begin{aligned}\dot{Q}_{\text{loss}} &= \dot{W}_s - \dot{m} \cdot C \cdot \Delta T \\ &= 10 - 5.25 = 4.75 \text{ kW}\end{aligned}$$

Therefore, the heat loss from the pump is approximately 4.75 kW.

## Problem 9: Isentropic Steam Turbine with Extraction

**Given:**

- Steam inlet:  $\dot{m}_1 = 3 \text{ kg/s}$  at  $P_1 = 3 \text{ MPa}$  (saturated vapor)
- Extraction: 12% of flow at  $P_2 = 500 \text{ kPa}$
- Exhaust: 88% of flow at  $P_3 = 50 \text{ kPa}$ ,  $T_3 = 100^\circ\text{C}$
- Process: Isentropic (reversible adiabatic)

**Solution:**

The mass flow rates are  $\dot{m}_2 = 0.12 \times \dot{m}_1 = 0.36 \text{ kg/s}$  and  $\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 2.64 \text{ kg/s}$ .  
The steady flow energy equation for the turbine is:

$$\dot{W}_{\text{out}} = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$$

where  $h_1$  is the specific enthalpy at inlet (3 MPa, saturated vapor),  $h_2$  is at extraction (500 kPa), and  $h_3$  is at exhaust (50 kPa, 100°C).

**Steam Properties from CoolProp:**

- **State 1 (Inlet):**  $P_1 = 3 \text{ MPa}$ , saturated vapor

$$T_1 = 507.0 \text{ K} = 234^\circ\text{C}$$

$$h_1 = 2803.2 \text{ kJ/kg}$$

$$s_1 = 6185.6 \text{ J/kg} \cdot \text{K} = 6.186 \text{ kJ/kg} \cdot \text{K}$$

- **State 2 (Extraction):**  $P_2 = 500 \text{ kPa}$ ,  $s_2 = s_1$  (isentropic)

$$s_2 = 6185.6 \text{ J/kg} \cdot \text{K}$$

$$h_2 = 2478.2 \text{ kJ/kg}$$

- **State 3 (Exhaust):**  $P_3 = 50 \text{ kPa}$ ,  $T_3 = 100^\circ\text{C}$

$$h_3 = 2682.4 \text{ kJ/kg}$$

The power output is:

$$\begin{aligned}\dot{W}_{\text{out}} &= \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 \\ &= (3.0)(2803.2) - (0.36)(2478.2) - (2.64)(2682.4) \\ &= 8409.6 - 892.2 - 7081.5 \\ &= 435.9 \text{ kW}\end{aligned}$$

## Problem 10: Real Steam Turbine Entropy Analysis

For this real steam turbine analysis, we have a mass flow rate of  $\dot{m} = 2.5 \text{ kg/s}$  with inlet conditions of  $P_1 = 3 \text{ MPa}$  and  $T_1 = 400^\circ\text{C}$  (superheated steam). The steam expands to an outlet pressure of  $P_2 = 100 \text{ kPa}$  with an isentropic efficiency of  $\eta_s = 0.85$ . The process is adiabatic expansion ( $\dot{Q} = 0$ ) with negligible kinetic and potential energy changes.

### PART A: Isentropic Operation

- **Goal:** Find  $T_{2s}$ ,  $h_{2s}$ ,  $x_{2s}$  (quality),  $s_{2s}$

- **Process:**

1. Look up inlet state properties (superheated steam at  $P_1 = 3 \text{ MPa}$ ,  $T_1 = 400^\circ\text{C}$ ):  $h_1$ ,  $s_1$  (from steam tables)
2. Apply isentropic condition:  $s_{2s} = s_1$
3. At  $P_2 = 100 \text{ kPa}$ ,  $s_{2s} = s_1$ : compare  $s_{2s}$  with  $s_f$  and  $s_g$  at 100 kPa
4. If  $s_f < s_{2s} < s_g$ , two-phase mixture:  $x_{2s} = (s_{2s} - s_f)/s_{fg}$
5. Find enthalpy:  $h_{2s} = h_f + x_{2s}h_{fg}$
6. Temperature:  $T_{2s} = T_{sat}$  at 100 kPa

- **Summary for Part A:**

- $h_1 = 3231.7 \text{ kJ/kg}$ ,  $s_1 = 6.923 \text{ kJ/kg-K}$  (inlet properties)
- $T_{2s} = 372.8 \text{ K} = 99.6^\circ\text{C}$
- $h_{2s} = 2512.6 \text{ kJ/kg}$
- $x_{2s} = 0.928$  (92.8% vapor)
- $s_{2s} = 6.923 \text{ kJ/kg-K}$  (isentropic)

### PART B: Real Operation

- **Goal:** Find  $T_2$ ,  $h_2$ ,  $x_2$ ,  $s_2$  (actual outlet conditions)

- **Process:**

1. Use isentropic efficiency:  $\eta_s = (h_1 - h_2)/(h_1 - h_{2s}) = 0.85$
2. Solve for  $h_2$ :  $h_2 = h_1 - \eta_s(h_1 - h_{2s})$
3. At  $P_2 = 100 \text{ kPa}$ ,  $h_2$  known: compare  $h_2$  with  $h_f$  and  $h_g$  at 100 kPa
4. If  $h_f < h_2 < h_g$ , two-phase mixture:  $x_2 = (h_2 - h_f)/h_{fg}$
5. Find entropy:  $s_2 = s_f + x_2s_{fg}$
6. Temperature:  $T_2 = T_{sat}$  at 100 kPa

- **Summary for Part B:**

- Isentropic work:  $w_s = h_1 - h_{2s} = 3231.7 - 2512.6 = 719.1 \text{ kJ/kg}$
- Actual work:  $w_{actual} = \eta_s \times w_s = 0.85 \times 719.1 = 611.2 \text{ kJ/kg}$
- Actual outlet enthalpy:  $h_2 = h_1 - w_{actual} = 3231.7 - 611.2 = 2620.5 \text{ kJ/kg}$
- $T_2 = 372.8 \text{ K} = 99.6^\circ\text{C}$
- $h_2 = 2620.5 \text{ kJ/kg}$
- $x_2 = 0.976$  (97.6% vapor)
- $s_2 = 7.213 \text{ kJ/kg-K}$

### PART C: Work Comparison

To calculate and compare the turbine work output, we first find the isentropic work:  $w_s = h_1 - h_{2s} = 3231.7 - 2512.6 = 719.1 \text{ kJ/kg}$ . The actual work is  $w_{actual} = h_1 - h_2 = 3231.7 - 2620.5 = 611.2 \text{ kJ/kg}$ .

The corresponding power outputs are:

$$\begin{aligned}\dot{W}_s &= \dot{m}w_s = 2.5 \times 719.1 = 1797.8 \text{ kW} \\ \dot{W}_{actual} &= \dot{m}w_{actual} = 2.5 \times 611.2 = 1528.0 \text{ kW}\end{aligned}$$

The difference is  $\Delta\dot{W} = \dot{W}_s - \dot{W}_{actual} = 1797.8 - 1528.0 = 269.8 \text{ kW}$ . This verifies the calculation since  $w_{actual} = \eta_s \times w_s = 0.85 \times 719.1 = 611.2 \text{ kJ/kg}$ .

The real turbine produces 15% less work due to irreversibilities (friction, turbulence, heat transfer within fluid), with the energy lost to entropy generation.

### PART D: Entropy Generation Rate

- **Goal:** Calculate  $\dot{S}_{gen}$  using entropy balance
- **Given entropy balance:**

$$\sum \frac{\dot{Q}}{T_{boundary}} + \sum \dot{m}s_{in} - \sum \dot{m}s_{out} + \dot{S}_{gen} = \frac{dS_{sys}}{dt}$$

- **For steady flow, well-insulated turbine:**

- $\frac{dS_{sys}}{dt} = 0$  (steady state)
- $\sum \frac{\dot{Q}}{T_{boundary}} = 0$  (adiabatic)
- Single inlet and outlet stream

- **Simplified:**

$$\begin{aligned}0 + \dot{m}s_1 - \dot{m}s_2 + \dot{S}_{gen} &= 0 \\ \Rightarrow \dot{S}_{gen} &= \dot{m}(s_2 - s_1)\end{aligned}$$

- **Calculation:**

$$\begin{aligned}\dot{S}_{gen} &= \dot{m}(s_2 - s_1) = 2.5 \times (7.213 - 6.923) \\ &= 2.5 \times 0.290 = 0.725 \text{ kW/K}\end{aligned}$$

- **Check:**  $\dot{S}_{gen} = 0.725 > 0$  (confirms 2nd law for real process)

### PART E: Comparison and Benefit

Comparing the isentropic (A) versus real (B) turbine properties:

Property	Isentropic (A)	Real (B)	Difference
$h_2$ (kJ/kg)	2512.6	2620.5	+107.9
$s_2$ (kJ/kg-K)	6.923	7.213	+0.290
$x_2$ (quality)	0.928	0.976	+0.048
$T_2$ (K)	372.8	372.8	0

The key observation is that the real turbine has higher quality ( $x_2 > x_{2s}$ ), higher entropy ( $s_2 > s_1$ ), and higher enthalpy at exit ( $h_2 > h_{2s}$ ). This provides a benefit: less liquid content in the exhaust (higher quality means more vapor), which reduces water droplet erosion on turbine blades. The non-ideal operation trades work output for blade lifespan.

## Problem 11: Steam Quality Change in Piston-Cylinder

**Given:**

- System: Free piston-cylinder (constant pressure process)
- Working fluid:  $m = 3 \text{ kg}$  of water/steam mixture
- Pressure:  $P = 200 \text{ kPa}$  (constant throughout process)
- Initial state:  $x_1 = 0.60$  (40% liquid, 60% vapor)
- Final state:  $x_2 = 1.0$  (all liquid evaporated to saturated vapor)
- Process: Adiabatic heating with electric resistance heater

### Relevant Equations

- For specific entropy in a two-phase mixture:

$$s = s_f + x \cdot s_{fg}$$

- Specific entropy change:

$$\Delta s = s_2 - s_1$$

For this entropy calculation, the initial state (1) has specific entropy  $s_1 = s_f + x_1 \cdot s_{fg} = s_f + (0.60) \cdot s_{fg}$ , while the final state (2) has  $s_2 = s_g$  since it's saturated vapor. The change in specific entropy is therefore  $\Delta s = s_2 - s_1 = s_g - [s_f + (0.60) \cdot s_{fg}]$ .

### Solution

From saturated steam properties at  $P = 200 \text{ kPa}$ :  $s_f = 1.530 \text{ kJ/kg}\cdot\text{K}$ ,  $s_g = 7.127 \text{ kJ/kg}\cdot\text{K}$ , and  $s_{fg} = s_g - s_f = 5.597 \text{ kJ/kg}\cdot\text{K}$ .

The initial specific entropy is:

$$\begin{aligned} s_1 &= s_f + x_1 \cdot s_{fg} = 1.530 + (0.60)(5.597) \\ &= 1.530 + 3.358 = 4.888 \text{ kJ/kg}\cdot\text{K} \end{aligned}$$

The final specific entropy is  $s_2 = s_g = 7.127 \text{ kJ/kg}\cdot\text{K}$  for saturated vapor.

Therefore, the specific entropy change is:

$$\Delta s = s_2 - s_1 = 7.127 - 4.888 = \boxed{2.239 \text{ kJ/kg}\cdot\text{K}}$$

The positive  $\Delta s$  reflects increased molecular disorder as liquid evaporates to vapor.

## Problem 12: Piston-Cylinder Total Entropy Change

**Given:**

- System: Well-insulated piston-cylinder with freely moving piston
- Working fluid: Water (saturated liquid initially)
- Initial state:  $V_1 = 4.8 \text{ L}$ ,  $P_1 = 150 \text{ kPa}$ ,  $x_1 = 0$
- Energy input:  $Q_{in} = 1700 \text{ kJ}$  from electric resistance heater
- Process: Constant pressure heating (piston moves freely)

### Assumptions

- Well-insulated system  $\rightarrow$  no heat loss to surroundings
- Piston moves freely  $\rightarrow$  constant pressure process
- Neglect kinetic and potential energy changes
- Quasi-equilibrium process

### Relevant Equations

- Energy Balance (First Law):

$$Q_{in} - W_{out} = \Delta U = m(u_2 - u_1)$$

For constant pressure:

$$\begin{aligned} W_{out} &= P(V_2 - V_1) = mP(v_2 - v_1) \\ Q_{in} &= m(u_2 - u_1) + mP(v_2 - v_1) = m(h_2 - h_1) \end{aligned}$$

- Entropy Change:

$$\begin{aligned} \Delta S &= m(s_2 - s_1) \\ \Delta s &= s_2 - s_1 \end{aligned}$$

### Solution

The initial state properties at 150 kPa saturated liquid are:  $h_1 = h_f = 467.1 \text{ kJ/kg}$ ,  $s_1 = s_f = 1.434 \text{ kJ/kg}\cdot\text{K}$ ,  $\rho_1 = 949.9 \text{ kg/m}^3$ , and specific volume  $v_1 = 1/\rho_1 = 0.001053 \text{ m}^3/\text{kg}$ .

The mass of water is:

$$m = \frac{V_1}{v_1} = \frac{0.0048 \text{ m}^3}{0.001053 \text{ m}^3/\text{kg}} = 4.558 \text{ kg}$$

Applying the energy balance for constant pressure:

$$\begin{aligned} Q_{in} &= m(h_2 - h_1) \\ h_2 &= h_1 + \frac{Q_{in}}{m} = 467.1 + \frac{1700}{4.558} \\ &= 467.1 + 373.0 = 840.1 \text{ kJ/kg} \end{aligned}$$

Since  $h_1 = 467.1 < h_2 = 840.1 < h_g = 2693.1 \text{ kJ/kg}$  (saturated vapor enthalpy), the final state is a two-phase mixture with quality  $x_2 = 0.168$  (16.8% vapor) and final entropy  $s_2 = 2.404 \text{ kJ/kg}\cdot\text{K}$ .

The entropy changes are:

$$\begin{aligned} \Delta s &= s_2 - s_1 = 2.404 - 1.434 = 0.970 \text{ kJ/kg}\cdot\text{K} \\ \Delta S &= m \times \Delta s = 4.558 \times 0.970 = \boxed{4.42 \text{ kJ/K}} \end{aligned}$$

## Problem 13: Solar Energy Storage Entropy Analysis

**Given:**

- Ceramic Particles (CARBOBEAD CP):

- Density:  $\rho = 3270 \text{ kg/m}^3$
- Specific heat:  $C = 1.3 \text{ kJ/kg}\cdot\text{K}$
- Temperature range:  $T_1 = 250^\circ\text{C}$  to  $T_2 = 1100^\circ\text{C}$

- Compressed Air System:

- Mass flow rate:  $\dot{m} = 0.75 \text{ kg/s}$
- Inlet conditions:  $T_3 = 250^\circ\text{C}$ ,  $P_3 = 4000 \text{ kPa}$
- Heating:  $T_4 = 1100^\circ\text{C}$  at constant pressure
- Turbine expansion: Isentropic to  $P_5 = 100 \text{ kPa}$

- Air Properties: Gas constant  $R = 0.287 \text{ kJ/kg}\cdot\text{K}$

**Assumptions:**

- Ceramic Particles (CARBOBEAD CP):

- Incompressible solid with constant specific heat ( $C = 1.3 \text{ kJ/kg}\cdot\text{K}$ )
- Uniform heating from  $250^\circ\text{C}$  to  $1100^\circ\text{C}$
- Negligible thermal expansion effects

- Compressed Air System:

- Ideal gas behavior with constant gas constant ( $R = 0.287 \text{ kJ/kg}\cdot\text{K}$ )
  - Negligible pressure drop in solar receiver ( $P_4 = P_3 = 4000 \text{ kPa}$ )
  - Constant pressure heating process (3→4)
  - Reversible adiabatic (isentropic) turbine expansion (4→5)
  - Steady flow conditions throughout
- **Reference State:** Table A-17 reference at 0 K and 100 kPa
  - **Negligible Effects:** Kinetic and potential energy changes, heat losses

**PART A: Change in specific entropy (kJ/kg·K) of ceramic particles from 1 → 2**

For an incompressible substance with constant specific heat:

$$\begin{aligned}\Delta s_{1 \rightarrow 2} &= C \ln \left( \frac{T_2}{T_1} \right) \\ &= 1.3 \text{ kJ/kg} \cdot \text{K} \times \ln \left( \frac{1373.15}{523.15} \right) \\ &= 1.3 \times \ln(2.625) = 1.3 \times 0.965 \\ &= \boxed{1.254 \text{ kJ/kg} \cdot \text{K}}\end{aligned}$$

**PART B: Change in specific entropy (kJ/kg·K) of compressed air from 3 → 4**

Using CoolProp for air property calculations with state 3 at  $T_3 = 523.15 \text{ K}$ ,  $P_3 = 4000 \text{ kPa}$  and state 4 at  $T_4 = 1373.15 \text{ K}$ ,  $P_4 = 4000 \text{ kPa}$  (constant pressure heating):

$$\begin{aligned}\Delta s_{3 \rightarrow 4} &= s_4 - s_3 \\ s_3 &= 3389.9 \text{ J/kg} \cdot \text{K} = 3.390 \text{ kJ/kg} \cdot \text{K} \\ s_4 &= 4468.7 \text{ J/kg} \cdot \text{K} = 4.469 \text{ kJ/kg} \cdot \text{K} \\ \Delta s_{3 \rightarrow 4} &= 4.469 - 3.390 = \boxed{1.079 \text{ kJ/kg} \cdot \text{K}}\end{aligned}$$

**PART C: Specific entropy (kJ/kg·K) of air at state 4**

From Part B calculation:

$$s_4 = \boxed{4.469 \text{ kJ/kg} \cdot \text{K}}$$

**PART D: Work (kW) produced via turbine from 4 → 5**

For isentropic expansion ( $s_5 = s_4$ ), we solve for  $T_5$  at  $P_5 = 100 \text{ kPa}$  using property\_solver.py to find  $T_5 = 529.7 \text{ K}$  where  $s_5 = 4468.7 \text{ J/kg}\cdot\text{K}$ .

Using CoolProp for enthalpy property lookup:

$$h_4 = 1612.8 \text{ kJ/kg} \quad (\text{at } 1373.15 \text{ K, } 4000 \text{ kPa})$$

$$h_5 = 660.1 \text{ kJ/kg} \quad (\text{at } 529.7 \text{ K, } 100 \text{ kPa})$$

$$w_t = h_4 - h_5 = 1612.8 - 660.1 = 952.7 \text{ kJ/kg}$$

$$\dot{W}_{turb} = \dot{m} \times w_t = 0.75 \times 952.7 = \boxed{714.5 \text{ kW}}$$

### Solution Summary

1. **Ceramic Particles:**  $\Delta s_{1 \rightarrow 2} = 1.254 \text{ kJ/kg}\cdot\text{K}$  (heating from 250°C to 1100°C)
2. **Air Entropy Change:**  $\Delta s_{3 \rightarrow 4} = 1.079 \text{ kJ/kg}\cdot\text{K}$  (constant pressure heating)
3. **Air Entropy at State 4:**  $s_4 = 4.469 \text{ kJ/kg}\cdot\text{K}$
4. **Turbine Work:**  $\dot{W}_{turb} = 714.5 \text{ kW}$  (isentropic expansion from 4000 kPa to 100 kPa)

### Key Tools Used:

- **CoolProp** For accurate air property calculations instead of Table A-17 approximations
- **Property Solver:** Iterative tool to find  $T_5$  where  $s_5 = s_4$  for isentropic expansion
- **Incompressible Substance Formula:**  $\Delta s = C \ln(T_2/T_1)$  for ceramic particles

## Problem 14: Concentrated Solar Power Carnot Analysis

Given:

- **Solar Concentrator System:**
  - Concentration ratios:  $C = 100, 500, 1000, 2000, 3000$  suns
  - Solar irradiance:  $I = 1000 \text{ W/m}^2$
  - Stefan-Boltzmann constant:  $\sigma = 5.67 \times 10^{-8} \text{ W/(m}^2 \cdot \text{K}^4)$
- **Carnot Heat Engine:**
  - Cold reservoir:  $T_L = 300 \text{ K}$  (ambient temperature)
  - Hot reservoir:  $T_H$  varies (300 K to 2600 K)
- **Analysis Scope:** Combined receiver-engine efficiency optimization

**Assumptions:**

- **Ideal Carnot cycle:** Reversible processes, no irreversibilities in heat engine
- **Steady-state operation:** Constant solar irradiance and ambient conditions
- **Perfect solar concentration:** All concentrated solar energy reaches receiver surface
- **Black-body radiation:** Receiver emits according to Stefan-Boltzmann law
- **Ambient cold reservoir:**  $T_L = 300 \text{ K}$  (environmental heat sink)
- **Negligible convection losses:** Only radiative losses from receiver considered
- **Uniform receiver temperature:** Isothermal hot reservoir at  $T_H$

**PART A: Plot Total Efficiency**

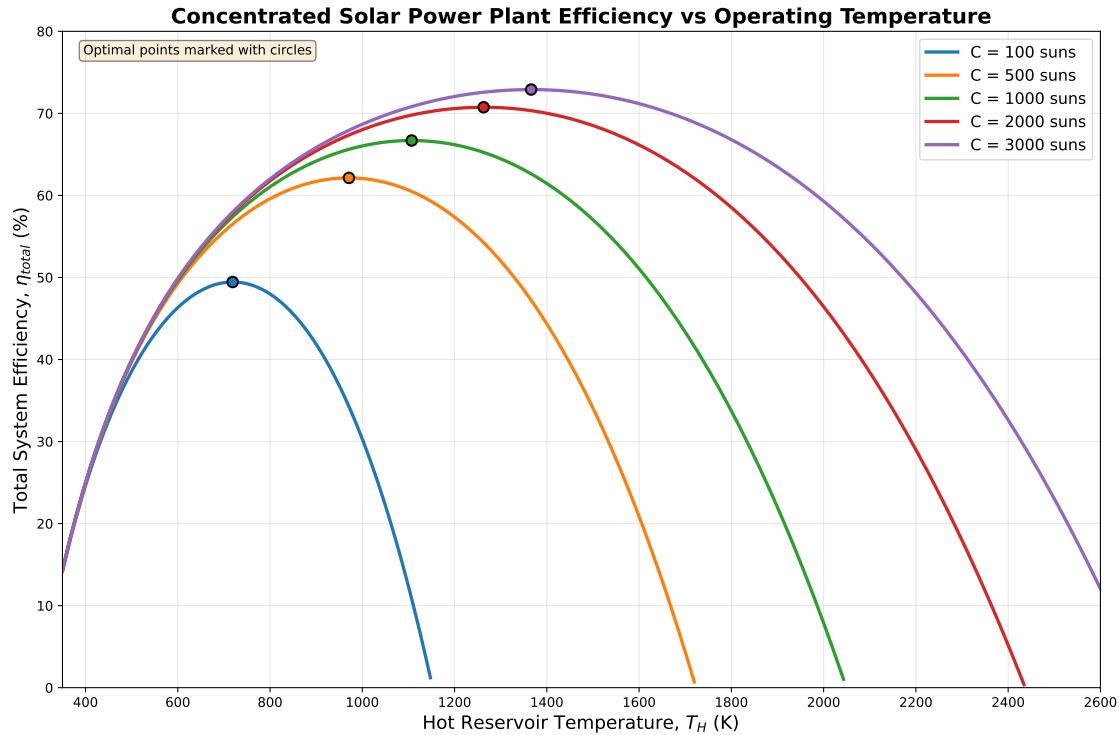


Figure 1: Total system efficiency vs hot reservoir temperature for different concentration ratios. Optimal operating points are marked with circles.

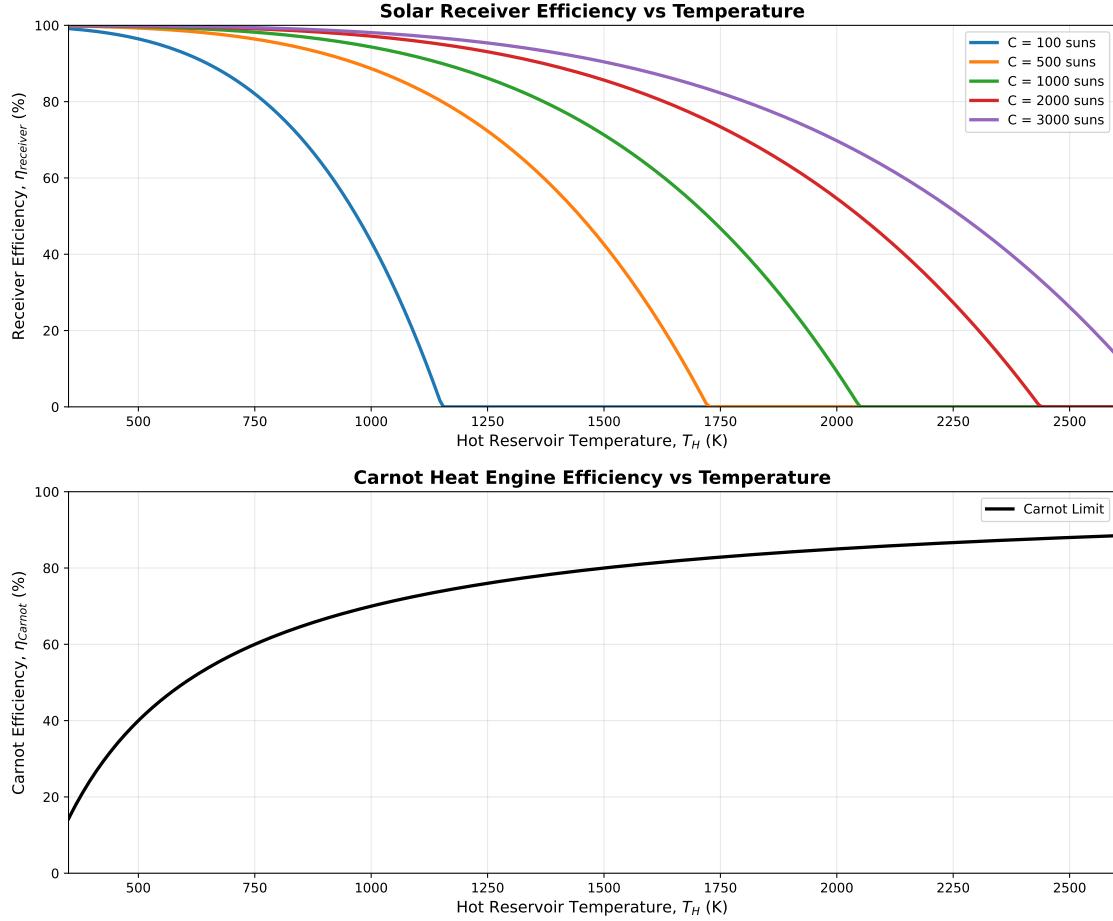


Figure 2: Component efficiencies: (top) Solar receiver efficiency decreases with temperature due to radiation losses, (bottom) Carnot heat engine efficiency increases with temperature.

### Relevant Equations

- Receiver Efficiency:

$$\eta_{receiver} = 1 - \frac{\sigma T_H^4}{C \cdot I}$$

- Carnot Heat Engine Efficiency:

$$\eta_{Carnot,HE} = 1 - \frac{T_L}{T_H}$$

- Total System Efficiency:

$$\begin{aligned}\eta_{Total} &= \eta_{receiver} \cdot \eta_{Carnot,HE} \\ &= \left(1 - \frac{5.67 \times 10^{-8} \cdot T_H^4}{C \cdot 1000}\right) \cdot \left(1 - \frac{300}{T_H}\right)\end{aligned}$$

## Solution using Python Analysis

- **Python Scripts:** carnot\_simple.py for optimization, carnot\_plotter.py for visualization
- **Method:** Temperature optimization scan to find maximum  $\eta_{total}$  for each concentration ratio

## Results - Optimal Operating Conditions:

$C$ [suns]	$T_{H,opt}$ [K]	$T_{H,opt}$ [°C]	$\eta_{total,max}$	$\eta_{receiver}$	$\eta_{Carnot}$
100	719	446	49.4%	84.8%	58.3%
500	971	698	62.1%	89.9%	69.1%
1000	1107	834	66.7%	91.5%	72.9%
2000	1263	990	70.7%	92.8%	76.2%
3000	1366	1093	72.9%	93.4%	78.0%

## Sample Data for Plotting:

$T_H$ [K]	$C = 100$	$C = 500$	$C = 1000$	$C = 2000$	$C = 3000$
500	0.386	0.397	0.399	0.399	0.400
750	0.492	0.578	0.589	0.595	0.596
1000	0.303	0.621	0.660	0.680	0.687
1250	—	0.550	0.655	0.707	0.725
1500	—	0.341	0.570	0.685	0.723
1750	—	—	0.388	0.608	0.682
2000	—	—	0.079	0.464	0.593

## PART B: Identify Optimal Receiver Temperature

### Key Trends Observed:

#### 1. Optimal Temperature Increases with Concentration:

- $C = 100$  suns:  $T_{H,opt} = 719$  K (446°C)
- $C = 3000$  suns:  $T_{H,opt} = 1366$  K (1093°C)
- Higher concentration allows operation at higher temperatures

#### 2. Maximum Efficiency Increases with Concentration:

- $C = 100$  suns:  $\eta_{max} = 49.4\%$
- $C = 3000$  suns:  $\eta_{max} = 72.9\%$
- Improvement of 23.5 percentage points

#### 3. Physical Trade-offs:

- Carnot efficiency increases with  $T_H$ :  $\eta_{Carnot} = 1 - T_L/T_H$
- Receiver efficiency decreases with  $T_H$ :  $\eta_{receiver} = 1 - \sigma T_H^4/(C \cdot I)$
- Optimal point balances these competing effects
- Higher concentration shifts optimum to higher temperatures and efficiencies

## Python Code Implementation

The analysis was performed using custom Python scripts for accurate calculations:

```

#!/usr/bin/env python3
"""
Problem 14: Carnot Cycle Heat Engines - Analysis
"""

import math
import numpy as np
import matplotlib.pyplot as plt

# Physical constants
SIGMA = 5.67e-8 # Stefan-Boltzmann constant [W/(m^2\cdot K^4)]
I = 1000 # Solar irradiance [W/m^2]
T_L = 300 # Cold reservoir temperature [K]

# Concentration ratios [suns]
concentration_ratios = [100, 500, 1000, 2000, 3000]

def receiver_efficiency(T_H, C):
    """Calculate receiver efficiency: eta_receiver = 1 - sigma*T_H^4/(C*I)"""
    return 1 - (SIGMA * T_H**4) / (C * I)

def carnot_efficiency(T_H, T_L):
    """Calculate Carnot heat engine efficiency: eta_Carnot = 1 - T_L/T_H"""
    return 1 - T_L / T_H

def total_efficiency(T_H, C, T_L):
    """Calculate total system efficiency: eta_total = eta_receiver x eta_Carnot"""
    eta_recv = receiver_efficiency(T_H, C)
    eta_carnot = carnot_efficiency(T_H, T_L)
    if eta_recv <= 0: # Invalid operating regime
        return 0
    return eta_recv * eta_carnot

def find_optimal_temperature(C, T_L):
    """Find optimal temperature by scanning the valid range"""
    T_range = np.arange(T_L + 50, 2601, 1) # 350K to 2600K

    max_efficiency = -1
    optimal_T_H = T_L + 50

    for T_H in T_range:
        eta = total_efficiency(T_H, C, T_L)

```

```

if eta > max_efficiency:
    max_efficiency = eta
    optimal_T_H = T_H

return float(optimal_T_H), max_efficiency

# Find optimal conditions for each concentration ratio
for C in concentration_ratios:
    T_opt, eta_opt = find_optimal_temperature(C, T_L)
    print(f"C = {C:4d} suns: T_opt = {T_opt:6.1f} K, "
          f"eta_max = {eta_opt:5.1%}")

```

### Command Line Execution:

```

$ python carnot_simple.py      # Analysis and optimization
$ python carnot_plotter.py     # Generate efficiency plots

```

### Key Insights from Analysis:

#### 1. Physical Trade-offs:

- **Carnot efficiency** increases with  $T_H$  (better thermodynamic limit)
- **Receiver efficiency** decreases with  $T_H$  (radiation losses  $\propto T_H^4$ )
- Optimum occurs where product is maximized

#### 2. Concentration Effects:

- Higher  $C$  shifts optimum to higher temperatures
- Concentrated sunlight ( $C \cdot I$ ) can overcome larger radiation losses ( $\sigma T_H^4$ )
- Engineering trade-off: system complexity vs. performance

### Physical Interpretation

The total efficiency is the product of two competing effects:

$$\eta_{total}(T_H) = \underbrace{\left(1 - \frac{\sigma T_H^4}{C \cdot I}\right)}_{\text{decreases with } T_H} \times \underbrace{\left(1 - \frac{T_L}{T_H}\right)}_{\text{increases with } T_H}$$

At the optimum:  $\frac{d\eta_{total}}{dT_H} = 0$ , which yields the balance between thermodynamic benefits and radiation losses.

### Python Command Used:

- `python carnot_simple.py` → Complete analysis with optimization

## A Appendix