Advanced Thermodynamics

Homework Assignment #1

Assignment Overview

Problems 1-14: Fundamental thermodynamic concepts, energy balances, entropy analysis, steam turbines, Carnot cycles, and concentrated solar power optimization

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Computational Tools Developed

- CoolProp CLI for multi-fluid property lookup
- Property solver for iterative thermodynamic calculations
- Carnot cycle optimization analysis
- Steam turbine performance analysis

Contents

1.

It is assumed the system is a simple compressible system, therefore there are two degrees of freedom and the rest of the intensive properties can be calculated using an equation of state, or found in a table of common fluids.

2.

- Closed system: (a)
- Open system: (c)
- Isolated system: (b)

3.

a. Internal energy (U): The microscopic kinetic and potential energy of the molecules within the mass.

b. Kinetic energy (KE): Energy associated with the macroscopic velocity of the mass, given by $\frac{1}{2}mv^2$.

e. Potential energy (PE): Energy associated with the position of the mass in a gravitational or other force field, given by mgz.

Notes:

f. Electrical energy could potentially be considered if the mass crossing a system boundary is electrically charged, but it would be a rare circumstance.

e. Flow work is a summation of other components, not necessarily carried by the mass itself.

c. $E = mc^2$, so an argument could be made but it is not relevant here.

4.

False! Only solids resist shear stress by static deflection. Fluids continuously deform under shear stress.

5.

(c), assuming V_n is the velocity normal to A_c, then the final units are kg/s.

1 Problem 6: Hydro Storage System Energy Analysis

Given:

- Volumetric flow rate: $\dot{V} = 500\ \mathrm{m^3/s}$
- Elevation drop: $\Delta z = 85\ \mathrm{m}$
- Water density: $\rho = 1000\ \mathrm{kg/m^3}$ (assumed)

(a) **Maximum possible power with upper reservoir as a lake ($V_1 = 0$):**

- **Apply the Energy Equation:**

$$\frac{V_1^2}{2} + gz_1 = \frac{V_2^2}{2} + gz_2 + w_{\text{turbine}}$$

where:

$$\begin{aligned} V_1 &= \text{velocity at upper reservoir (lake)} = 0 \\ V_2 &= \text{velocity at lower lake (assume 0)} \\ z_1 &= \text{elevation of upper reservoir} \\ z_2 &= \text{elevation of lower lake} \\ g &= 9.81\ \mathrm{m/s^2} \end{aligned}$$

- **Solve for work per unit mass:**

$$w_{\text{turbine}} = g(z_1 - z_2)$$

- **Calculate total power:**

$$\begin{aligned} \dot{W}_{\text{turbine}} &= \dot{m} \cdot w_{\text{turbine}} \\ \dot{m} &= \rho Q = 1000\ \mathrm{kg/m^3} \times 500\ \mathrm{m^3/s} = 500{,}000\ \mathrm{kg/s} \\ w_{\text{turbine}} &= 9.81\ \mathrm{m/s^2} \times 85\ \mathrm{m} = 833.85\ \mathrm{J/kg} \\ \dot{W}_{\text{turbine}} &= 500{,}000\ \mathrm{kg/s} \times 833.85\ \mathrm{J/kg} = 416,925,000\ \mathrm{W} = 416.9\ \mathrm{MW} \end{aligned}$$

(b) **Maximum possible power with upper reservoir as a river ($V_1 = 3.75$ m/s):**

- **Include kinetic energy at inlet:**

$$\begin{aligned} w_{\text{turbine}} &= \frac{V_1^2}{2} + g(z_1 - z_2) \\ &= \frac{(3.75\ \mathrm{m/s})^2}{2} + 9.81\ \mathrm{m/s^2} \times 85\ \mathrm{m} \\ &= 7.03\ \mathrm{J/kg} + 833.85\ \mathrm{J/kg} = 840.88\ \mathrm{J/kg} \\ \dot{W}_{\text{turbine}} &= 500{,}000\ \mathrm{kg/s} \times 840.88\ \mathrm{J/kg} = 420,440,000\ \mathrm{W} = 420.4\ \mathrm{MW} \end{aligned}$$

- **Percent increase in power:**

$$\% \text{ increase} = \frac{420.4 - 416.9}{416.9} \times 100\% = 0.84\%$$

- **Physical explanation:**
The increase in possible power output is due to the additional kinetic energy of the water entering the turbine when the upper reservoir is a river (nonzero velocity). This kinetic energy is converted to useful work by the turbine, resulting in a slightly higher power output compared to the case where the upper reservoir is a still lake.

2 Problem 7: Piston-Cylinder Energy Balance

Given:

- Mass of H gas: $m = 3.8$ kg
- Gas constant: $R = 4.124$ kJ/kg·K
- Initial state: $P_1 = 250$ kPa, $T_1 = 900$ K
- Final temperature: $T_2 = 400$ K
- Process: Constant pressure (heat removal)

Part A: Find the change in volume (m^3) of the system.

- Use the ideal gas law to find the initial volume:

$$\begin{aligned} PV &= mRT \\ V_1 &= \frac{mRT_1}{P} \\ &= \frac{(3.8 \text{ kg})(4.124 \text{ kJ/kg} \cdot \text{K})(900 \text{ K})}{250 \text{ kPa}} \\ &= 56.416 \text{ m}^3 \end{aligned}$$

- For a constant pressure process, $\frac{T_1}{V_1} = \frac{T_2}{V_2}$:

$$\begin{aligned} \frac{900}{56.416} &= \frac{400}{V_2} \\ V_2 &= \frac{400 \times 56.416}{900} = 25.073 \text{ m}^3 \end{aligned}$$

- Change in volume:

$$\Delta V = V_2 - V_1 = 25.073 - 56.416 = -31.343 \text{ m}^3$$

The negative sign indicates the volume decreased as the gas cooled.

Part B: Find the magnitude (kJ) and direction (in or out of system) of the boundary work during this process. Defend why your answer makes sense.

- For a constant pressure process, boundary work is:

$$\begin{aligned} W &= P\Delta V = P(V_2 - V_1) \\ &= 250 \text{ kPa} \times (25.073 - 56.416) \text{ m}^3 \\ &= 250 \text{ kPa} \times (-31.343 \text{ m}^3) \\ &= -7,836 \text{ kJ} \end{aligned}$$

- The negative sign indicates work is done **by** the system (the gas does work on the surroundings as it contracts). If you define work done **on** the system, $W = +7,836$ kJ into the system. This makes sense because as heat is removed at constant pressure, the gas contracts and the surroundings do work on the gas.

Part C: Calculate the change in specific enthalpy (kJ/kg) and internal energy (kJ/kg) of the gas during this process using the given property tables.

- **Change in specific enthalpy (Δh):**

$$\begin{aligned} \Delta h &= \int_{T_1}^{T_2} C_p(T)\, dT \\ &\approx \bar{C}_p \times (T_2 - T_1) \end{aligned}$$

  Where $\bar{C}_p$ is the average of C_p at 900 K and 400 K (read from the graph):

$$\begin{aligned} \bar{C}_p &\approx \frac{14.35 + 14.93}{2} = 14.64 \text{ kJ/kg} \cdot \text{K} \\ \Delta h &= 14.64 \text{ kJ/kg} \cdot \text{K} \times (400 - 900) \text{ K} \\ &= 14.64 \times (-500) = -7,320 \text{ kJ/kg} \end{aligned}$$

- **Change in specific internal energy (Δu):**

$$\begin{aligned} \Delta u &= \int_{T_1}^{T_2} C_v(T)\, dT \\ &\approx \bar{C}_v \times (T_2 - T_1) \end{aligned}$$

  Where $\bar{C}_v$ is the average of C_v at 900 K and 400 K (read from the graph, e.g. 10.2 and 10.9):

$$\begin{aligned} \bar{C}_v &\approx \frac{10.2 + 10.9}{2} = 10.55 \text{ kJ/kg} \cdot \text{K} \\ \Delta u &= 10.55 \text{ kJ/kg} \cdot \text{K} \times (400 - 900) \text{ K} \\ &= 10.55 \times (-500) = -5,275 \text{ kJ/kg} \end{aligned}$$

- Both Δh and Δu are negative, indicating the gas lost enthalpy and internal energy as it cooled from 900 K to 400 K.

8.

A centrifugal pump is operated steadily and consumes 10 kW of shaft work to move water (1000 kg/m^3, $C = 4.2$ kJ/kg $\cdot$ K) at a mass flow rate of 0.05 kg/s. The pump is not very well insulated. Temperature is measured at the inlet (20 °C) and the outlet (45 °C) of the pump. The cross sectional area at the inlet and the outlet of the pump is the same ($A_2 = A_3$).

Part A: Prove that the change in kinetic energy is negligible

- **Given:**

$$\begin{aligned} \dot{m} &= 0.05 \text{ kg/s} \\ A_2 &= A_3 \\ \rho &= 1000 \text{ kg/m}^3 \end{aligned}$$

- **Conservation of mass for steady flow:**

$$\begin{aligned} \dot{m} &= \rho A_2 v_2 = \rho A_3 v_3 \\ \Rightarrow v_2 &= v_3 \end{aligned}$$

- **Change in kinetic energy:**

$$\Delta KE = \frac{1}{2} m (v_3^2 - v_2^2) = 0$$

- **Conclusion:** The change in kinetic energy is negligible (zero) for the water in this pump.

Part B: Estimate the heat loss (kW) from the pump

- **Given:**

$$\begin{aligned} \dot{W}_s &= 10 \text{ kW} \\ C &= 4.2 \text{ kJ/kg} \cdot \text{K} \\ \dot{m} &= 0.05 \text{ kg/s} \\ T_1 &= 20 \text{ °C} \\ T_2 &= 45 \text{ °C} \end{aligned}$$

- **Steady-flow energy equation (SFEE):**

$$\dot{Q}_{\text{loss}} - \dot{W}_s = \dot{m} \left[\Delta h + \Delta KE + \Delta PE \right]$$

- **Assumptions:**
  - $\Delta KE = 0$ (from Part A)

  - $\Delta PE \approx 0$ (not significant)
  - For incompressible liquid: $\Delta h = C\Delta T$

- **Energy balance:**

$$\begin{aligned}
\dot{Q}_{\text{loss}} - \dot{W}_s &= \dot{m} \cdot C \cdot (T_2 - T_1) \\
\dot{m} \cdot C \cdot \Delta T &= 0.05 \times 4.2 \times (45 - 20) \\
&= 0.05 \times 4.2 \times 25 \\
&= 5.25 \text{ kW}
\end{aligned}$$

- **Solve for heat loss:**

$$\begin{aligned}
\dot{Q}_{\text{loss}} &= \dot{W}_s - \dot{m} \cdot C \cdot \Delta T \\
&= 10 - 5.25 = 4.75 \text{ kW}
\end{aligned}$$

- **Answer:** The heat loss from the pump is approximately 4.75 kW (rounded: 4.8 kW).

9.

A well-insulated steam turbine is operating reversibly with 3 kg/s of steam at a pressure of 3 MPa. The majority of the steam is exhausted at 50 kPa and 100°C. However, 12% of this flow is diverted within the steam turbine to provide preheating for other cycle components once the inlet stream was expanded to 500 kPa. Determine the power produced by this turbine, in kW.

Problem Setup

- **Given:**

$$\dot{m}_1 = 3 \text{ kg/s}$$
$$P_1 = 3 \text{ MPa}$$
$$P_2 = 500 \text{ kPa}$$
$$P_3 = 50 \text{ kPa}$$
$$T_3 = 100°\text{C}$$
$$\text{Extraction flow fraction: } 0.12$$

- **Mass Flow Rates:**

$$\dot{m}_2 = 0.12 \times \dot{m}_1 = 0.36 \text{ kg/s}$$
$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 2.64 \text{ kg/s}$$

- **Steady Flow Energy Equation (SFEE) for the turbine:**

$$\dot{W}_{\text{out}} = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$$

  where:

  - h_1 = specific enthalpy at inlet (3 MPa, isentropic assumption from inlet state)
  - h_2 = specific enthalpy at extraction point (500 kPa)
  - h_3 = specific enthalpy at exhaust (50 kPa, 100°C)

- **Steam Properties from CoolProp CLI:**

  - **State 1 (Inlet):** $P_1 = 3$ MPa, saturated vapor

$$T_1 = 507.0 \text{ K} = 234°\text{C}$$
$$h_1 = 2803.2 \text{ kJ/kg}$$
$$s_1 = 6185.6 \text{ J/kg} \cdot \text{K} = 6.186 \text{ kJ/kg} \cdot \text{K}$$

- **State 2 (Extraction):** $P_2 = 500$ kPa, $s_2 = s_1$ (isentropic)

$$s_2 = 6185.6 \text{ J/kg} \cdot \text{K}$$
$$h_2 = 2478.2 \text{ kJ/kg}$$

- **State 3 (Exhaust):** $P_3 = 50$ kPa, $T_3 = 100°\text{C}$

$$h_3 = 2682.4 \text{ kJ/kg}$$

- **Power Calculation:**

$$\begin{aligned}\dot{W}_{\text{out}} &= \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 \\ &= (3.0)(2803.2) - (0.36)(2478.2) - (2.64)(2682.4) \\ &= 8409.6 - 892.2 - 7081.5 \\ &= 435.9 \text{ kW}\end{aligned}$$

- **CLI Commands Used:**
  - `python steam_cli.py T P 3000000 Q 1` $\rightarrow T_1 = 507.0$ K
  - `python steam_cli.py H P 3000000 Q 1` $\rightarrow h_1 = 2803.2$ kJ/kg
  - `python steam_cli.py S P 3000000 Q 1` $\rightarrow s_1 = 6185.6$ J/kg·K
  - `python steam_cli.py H P 500000 S 6185.583126654898` $\rightarrow h_2 = 2478.2$ kJ/kg
  - `python steam_cli.py H P 50000 T 373.15` $\rightarrow h_3 = 2682.4$ kJ/kg

10.

A steam turbine is operating steadily and expands a 2.5 kg/s stream of steam/water from 3 MPa and 400 °C to 100 kPa. Assume the turbine is well-insulated. The manufacturer performance curves indicate that under these conditions, the turbine would operate with an isentropic efficiency of 0.85.

Given:

- $\dot{m} = 2.5$ kg/s
- $P_1 = 3$ MPa, $T_1 = 400$ °C
- $P_2 = 100$ kPa
- $\eta_s = 0.85$
- Steady flow, well-insulated ($\dot{Q} = 0$), negligible ΔKE and ΔPE

PART A: Isentropic Operation

- **Goal:** Find T_{2s}, h_{2s}, x_{2s} (quality), s_{2s}
- **Process:**
  1. Look up inlet state properties (superheated steam at $P_1 = 3$ MPa, $T_1 = 400$°C): h_1, s_1 (from steam tables)
  2. Apply isentropic condition: $s_{2s} = s_1$
  3. At $P_2 = 100$ kPa, $s_{2s} = s_1$: compare s_{2s} with s_f and s_g at 100 kPa
  4. If $s_f < s_{2s} < s_g$, two-phase mixture: $x_{2s} = (s_{2s} - s_f)/s_{fg}$
  5. Find enthalpy: $h_{2s} = h_f + x_{2s} h_{fg}$
  6. Temperature: $T_{2s} = T_{sat}$ at 100 kPa
- **Summary for Part A:**
  - $h_1 = 3231.7$ kJ/kg, $s_1 = 6.923$ kJ/kg-K (inlet properties)
  - $T_{2s} = 372.8$ K $= 99.6$°C
  - $h_{2s} = 2512.6$ kJ/kg
  - $x_{2s} = 0.928$ (92.8% vapor)
  - $s_{2s} = 6.923$ kJ/kg-K (isentropic)
- **CLI Commands Used:**
  - `python steam_cli.py H P 3000000 T 673.15` → $h_1 = 3231.7$ kJ/kg
  - `python steam_cli.py S P 3000000 T 673.15` → $s_1 = 6923.4$ J/kg·K

- `python steam_cli.py H P 100000 S 6923.448666758736` → $h_{2s} = 2512.6$ kJ/kg
- `python steam_cli.py T P 100000 S 6923.448666758736` → $T_{2s} = 372.8$ K
- `python steam_cli.py Q P 100000 S 6923.448666758736` → $x_{2s} = 0.928$

PART B: Real Operation

- **Goal:** Find T_2, h_2, x_2, s_2 (actual outlet conditions)
- **Process:**
  1. Use isentropic efficiency: $\eta_s = (h_1 - h_2)/(h_1 - h_{2s}) = 0.85$
  2. Solve for h_2: $h_2 = h_1 - \eta_s(h_1 - h_{2s})$
  3. At $P_2 = 100$ kPa, h_2 known: compare h_2 with h_f and h_g at 100 kPa
  4. If $h_f < h_2 < h_g$, two-phase mixture: $x_2 = (h_2 - h_f)/h_{fg}$
  5. Find entropy: $s_2 = s_f + x_2 s_{fg}$
  6. Temperature: $T_2 = T_{sat}$ at 100 kPa
- **Summary for Part B:**
  - Isentropic work: $w_s = h_1 - h_{2s} = 3231.7 - 2512.6 = 719.1$ kJ/kg
  - Actual work: $w_{actual} = \eta_s \times w_s = 0.85 \times 719.1 = 611.2$ kJ/kg
  - Actual outlet enthalpy: $h_2 = h_1 - w_{actual} = 3231.7 - 611.2 = 2620.5$ kJ/kg
  - $T_2 = 372.8$ K $= 99.6$°C
  - $h_2 = 2620.5$ kJ/kg
  - $x_2 = 0.976$ (97.6% vapor)
  - $s_2 = 7.213$ kJ/kg-K
- **CLI Commands Used:**
  - `python steam_cli.py T P 100000 H 2620500` → $T_2 = 372.8$ K
  - `python steam_cli.py Q P 100000 H 2620500` → $x_2 = 0.976$
  - `python steam_cli.py S P 100000 H 2620500` → $s_2 = 7212.8$ J/kg·K

PART C: Work Comparison

- **Goal:** Calculate and compare turbine work output
- **Process:**
  1. Isentropic work: $w_s = h_1 - h_{2s} = 3231.7 - 2512.6 = 719.1$ kJ/kg
  2. Actual work: $w_{actual} = h_1 - h_2 = 3231.7 - 2620.5 = 611.2$ kJ/kg

3. Power output:

$$\dot{W}_s = \dot{m}w_s = 2.5 \times 719.1 = 1797.8 \text{ kW}$$
$$\dot{W}_{actual} = \dot{m}w_{actual} = 2.5 \times 611.2 = 1528.0 \text{ kW}$$

4. Compare difference: $\Delta\dot{W} = \dot{W}_s - \dot{W}_{actual} = 1797.8 - 1528.0 = 269.8$ kW
5. Verification: $w_{actual} = \eta_s \times w_s = 0.85 \times 719.1 = 611.2$ kJ/kg ✓
6. Explanation: Real turbine produces 15% less work due to irreversibilities (friction, turbulence, heat transfer within fluid). Energy lost to entropy generation.

PART D: Entropy Generation Rate

- **Goal:** Calculate $\dot{S}_{gen}$ using entropy balance
- **Given entropy balance:**

$$\sum \frac{\dot{Q}}{T_{boundary}} + \sum \dot{m}s_{in} - \sum \dot{m}s_{out} + \dot{S}_{gen} = \frac{dS_{sys}}{dt}$$

- **For steady flow, well-insulated turbine:**
  - $\frac{dS_{sys}}{dt} = 0$ (steady state)
  - $\sum \frac{\dot{Q}}{T_{boundary}} = 0$ (adiabatic)
  - Single inlet and outlet stream
- **Simplified:**

$$0 + \dot{m}s_1 - \dot{m}s_2 + \dot{S}_{gen} = 0$$
$$\Rightarrow \dot{S}_{gen} = \dot{m}(s_2 - s_1)$$

- **Calculation:**

$$\dot{S}_{gen} = \dot{m}(s_2 - s_1) = 2.5 \times (7.213 - 6.923)$$
$$= 2.5 \times 0.290 = 0.725 \text{ kW/K}$$

- **Check:** $\dot{S}_{gen} = 0.725 > 0$ ✓ (confirms 2nd law for real process)

PART E: Comparison and Benefit

- **Goal:** Compare A vs B properties and identify benefit of non-ideal turbine
- **Process:**

1. Create comparison table:

Property	Isentropic (A)	Real (B)	Difference
h_2 (kJ/kg)	2512.6	2620.5	+107.9
s_2 (kJ/kg-K)	6.923	7.213	+0.290
x_2 (quality)	0.928	0.976	+0.048
T_2 (K)	372.8	372.8	0

2. Key observation: Real turbine has higher quality ($x_2 > x_{2s}$), higher entropy ($s_2 > s_1$), higher enthalpy at exit ($h_2 > h_{2s}$)
3. Benefit: Less liquid content in exhaust (higher quality = more vapor), reduces water droplet erosion on turbine blades. Non-ideal operation trades work output for blade longevity; small sacrifice in efficiency protects equipment.

Solution Steps Summary

1. Look up inlet properties from steam tables (P_1, T_1)
2. Part A: Use $s_{2s} = s_1$, find two-phase properties at P_2
3. Part B: Apply η_s to find h_2, then find two-phase properties
4. Part C: Calculate work from enthalpy drops, compare
5. Part D: Apply entropy balance with given equation
6. Part E: Compare moisture content (quality), explain blade erosion benefit

11.

A free piston-cylinder device is filled with 3 kg of a saturated liquid-vapor mixture of water/steam at 200 kPa. The exterior of the tank may be assumed to be well-insulated. **Initially, 40% of the mass is in the liquid phase** (so $x_1 = 0.60$). Heat is supplied via an electric resistance heater until all the liquid in the tank is evaporated. **Determine the specific entropy change (Δs, kJ/kg-K) of the steam inside the tank during this process.**

Given Information

- Mass: $m = 3$ kg
- Pressure: $P = 200$ kPa (constant)
- Initial quality: $x_1 = 0.60$ (40% liquid $\rightarrow$ 60% vapor)
- Final quality: $x_2 = 1.0$ (all liquid evaporated $\rightarrow$ saturated vapor)

Relevant Equations

- For specific entropy in a two-phase mixture:

$$s = s_f + x \cdot s_{fg}$$

- Specific entropy change:

$$\Delta s = s_2 - s_1$$

Setup with Numbers

- State 1 (initial):

$$\begin{aligned} s_1 &= s_f + x_1 \cdot s_{fg} \\ &= s_f + (0.60) \cdot s_{fg} \end{aligned}$$

- State 2 (final):

$$s_2 = s_g \quad \text{(or equivalently: } s_2 = s_f + (1.0) \cdot s_{fg}\text{)}$$

- Change in specific entropy:

$$\begin{aligned} \Delta s &= s_2 - s_1 \\ &= s_g - [s_f + (0.60) \cdot s_{fg}] \end{aligned}$$

Solution

1. **Look up saturated steam properties at $P = 200$ kPa using CLI:**

- `python steam_cli.py S P 200000 Q 0` $\rightarrow s_f = 1530.2$ J/kg·K $= 1.530$ kJ/kg·K
- `python steam_cli.py S P 200000 Q 1` $\rightarrow s_g = 7126.9$ J/kg·K $= 7.127$ kJ/kg·K
- Calculate: $s_{fg} = s_g - s_f = 7.127 - 1.530 = 5.597$ kJ/kg·K

2. **Calculate initial specific entropy** ($x_1 = 0.60$):

$$s_1 = s_f + x_1 \cdot s_{fg} = 1.530 + (0.60)(5.597)$$
$$= 1.530 + 3.358 = 4.888 \text{ kJ/kg} \cdot \text{K}$$

Verification: `python steam_cli.py S P 200000 Q 0.6` $\rightarrow s_1 = 4888.2$ J/kg·K ✓

3. **Identify final specific entropy** ($x_2 = 1.0$, saturated vapor):

$$s_2 = s_g = 7.127 \text{ kJ/kg} \cdot \text{K}$$

4. **Calculate the specific entropy change:**

$$\Delta s = s_2 - s_1 = 7.127 - 4.888 = \boxed{2.239 \text{ kJ/kg} \cdot \text{K}}$$

5. **Physical interpretation:** Positive Δs reflects increased molecular disorder as liquid evaporates to vapor.

12.

A piston–cylinder device is well insulated and contains 4.8 L of saturated liquid ($x = 0$) water at 150 kPa. The piston moves freely within the cylinder. An electric resistance heater inside the cylinder is suddenly turned on, and 1700 kJ of energy is delivered to the steam. **Determine the total entropy change (ΔS, kJ/kg) of the steam in the device during this process.**

Given Information

- Initial volume: $V_1 = 4.8$ L $= 0.0048$ m^3
- Initial state: Saturated liquid water, $x_1 = 0$
- Initial pressure: $P_1 = 150$ kPa
- Constant pressure process ($P_2 = P_1$)
- Energy input: $Q_{in} = 1700$ kJ

Assumptions

- Well-insulated system $\rightarrow$ no heat loss to surroundings
- Piston moves freely $\rightarrow$ constant pressure process

- Neglect kinetic and potential energy changes
- Quasi-equilibrium process

Relevant Equations

- Energy Balance (First Law):

$$Q_{in} - W_{out} = \Delta U = m(u_2 - u_1)$$

For constant pressure:

$$\begin{aligned} W_{out} &= P(V_2 - V_1) = mP(v_2 - v_1) \\ Q_{in} &= m(u_2 - u_1) + mP(v_2 - v_1) = m(h_2 - h_1) \end{aligned}$$

- Entropy Change:

$$\begin{aligned} \Delta S &= m(s_2 - s_1) \\ \Delta s &= s_2 - s_1 \end{aligned}$$

Solution

1. **Find initial state properties at 150 kPa saturated liquid using CLI:**
   - `python steam_cli.py H P 150000 Q 0` $\rightarrow h_1 = h_f = 467.1$ kJ/kg
   - `python steam_cli.py S P 150000 Q 0` $\rightarrow s_1 = s_f = 1433.7$ J/kg·K $= 1.434$ kJ/kg·K
   - `python steam_cli.py D P 150000 Q 0` $\rightarrow \rho_1 = 949.9$ kg/m^3
   - Specific volume: $v_1 = 1/\rho_1 = 1/949.9 = 0.001053$ m^3/kg
2. **Calculate mass of water:**

$$m = \frac{V_1}{v_1} = \frac{0.0048 \text{ m}^3}{0.001053 \text{ m}^3/\text{kg}} = 4.558 \text{ kg}$$

3. **Apply energy balance to find final enthalpy:**

$$\begin{aligned} Q_{in} &= m(h_2 - h_1) \quad \text{(constant pressure)} \\ h_2 &= h_1 + \frac{Q_{in}}{m} = 467.1 + \frac{1700}{4.558} \\ &= 467.1 + 373.0 = 840.1 \text{ kJ/kg} \end{aligned}$$

4. **Determine final state condition at 150 kPa:**
   - `python steam_cli.py H P 150000 Q 1` $\rightarrow h_g = 2693.1$ kJ/kg
   - Since $h_1 = 467.1 < h_2 = 840.1 < h_g = 2693.1 \rightarrow$ **two-phase mixture**

- `python steam_cli.py Q P 150000 H 840100` → $x_2 = 0.168$ (16.8% vapor)
- `python steam_cli.py S P 150000 H 840100` → $s_2 = 2403.7$ J/kg·K = 2.404 kJ/kg·K

5. **Calculate entropy changes:**

$$\Delta s = s_2 - s_1 = 2.404 - 1.434 = 0.970 \text{ kJ/kg·K}$$
$$\Delta S = m \times \Delta s = 4.558 \times 0.970 = \boxed{4.42 \text{ kJ/K}}$$

13.

Two solar receivers are proposed for storing concentrated solar energy as heat in either solid ceramic particles (CARBOBEAD CP, $\rho = 3270$ kg/m^3, $C \approx 1.3$ kJ/kg·K up to 1100°C) or compressed air. Both receivers are designed to heat their respective materials from 250°C to 1100°C. Compressed air is introduced into the one receiver at a mass flow rate of 0.75 kg/s, 250°C, and 4000 kPa. The heated air in the receiver is then expanded reversibly through a well-insulated turbine to an environmental pressure of 100 kPa. Assume table A-17 is populated with a reference state of 0 K and 100 kPa and the gas constant for air is $R = 0.287$ kJ/kg·K.

Make sure you clearly label additional assumptions used to solve the problem. State Points and Assumptions:

- Ceramic particles: State 1 (250°C = 523.15 K) → State 2 (1100°C = 1373.15 K)
- Air: State 3 (250°C, 4000 kPa) → State 4 (1100°C, $P_4 \approx P_3$) → State 5 (isentropic expansion to 100 kPa)
- *Assumptions:* Negligible pressure drop in receiver ($P_4 = P_3$), reversible turbine process, steady flow

PART A: Change in specific entropy (kJ/kg·K) of ceramic particles from 1 → 2

For an incompressible substance with constant specific heat:

$$\begin{aligned}
\Delta s_{1\to 2} &= C \ln\left(\frac{T_2}{T_1}\right) \\
&= 1.3 \text{ kJ/kg}\cdot\text{K} \times \ln\left(\frac{1373.15}{523.15}\right) \\
&= 1.3 \times \ln(2.625) = 1.3 \times 0.965 \\
&= \boxed{1.254 \text{ kJ/kg}\cdot\text{K}}
\end{aligned}$$

PART B: Change in specific entropy (kJ/kg·K) of compressed air from 3 → 4

Using CoolProp for air property calculations:

- **State 3:** $T_3 = 523.15$ K, $P_3 = 4000$ kPa
- **State 4:** $T_4 = 1373.15$ K, $P_4 = 4000$ kPa (constant pressure heating)

$$\begin{aligned}\Delta s_{3\to4} &= s_4 - s_3\\ s_3 &= 3389.9 \text{ J/kg}\cdot\text{K} = 3.390 \text{ kJ/kg}\cdot\text{K}\\ s_4 &= 4468.7 \text{ J/kg}\cdot\text{K} = 4.469 \text{ kJ/kg}\cdot\text{K}\\ \Delta s_{3\to4} &= 4.469 - 3.390 = \boxed{1.079 \text{ kJ/kg}\cdot\text{K}}\end{aligned}$$

CLI Commands:

- `python coolprop_cli.py S P 4000000 T 523.15 Air` → $s_3 = 3389.9$ J/kg·K
- `python coolprop_cli.py S P 4000000 T 1373.15 Air` → $s_4 = 4468.7$ J/kg·K

PART C: Specific entropy (kJ/kg·K) of air at state 4
From Part B calculation:

$$s_4 = \boxed{4.469 \text{ kJ/kg}\cdot\text{K}}$$

PART D: Work (kW) produced via turbine from 4 → 5
For isentropic expansion ($s_5 = s_4$), solve for T_5 at $P_5 = 100$ kPa:

- **Solve using property solver:** Find T_5 where $s_5 = 4468.7$ J/kg·K at $P_5 = 100$ kPa
- `python property_solver.py S 4468.7 P 100000 T 300 1500 Air` → $T_5 = 529.7$ K

Calculate turbine work:

$$\begin{aligned}h_4 &= 1612.8 \text{ kJ/kg} \quad \text{(at 1373.15 K, 4000 kPa)}\\ h_5 &= 660.1 \text{ kJ/kg} \quad \text{(at 529.7 K, 100 kPa)}\\ w_t &= h_4 - h_5 = 1612.8 - 660.1 = 952.7 \text{ kJ/kg}\\ \dot{W}_{turb} &= \dot{m} \times w_t = 0.75 \times 952.7 = \boxed{714.5 \text{ kW}}\end{aligned}$$

CLI Commands:

- `python coolprop_cli.py H P 4000000 T 1373.15 Air` → $h_4 = 1612783$ J/kg
- `python coolprop_cli.py H P 100000 T 529.688 Air` → $h_5 = 660057$ J/kg

Solution Summary

1. **Ceramic Particles:** $\Delta s_{1\to2} = 1.254$ kJ/kg·K (heating from 250°C to 1100°C)
2. **Air Entropy Change:** $\Delta s_{3\to4} = 1.079$ kJ/kg·K (constant pressure heating)
3. **Air Entropy at State 4:** $s_4 = 4.469$ kJ/kg·K

4. **Turbine Work:** $\dot{W}_{turb} = 714.5$ kW (isentropic expansion from 4000 kPa to 100 kPa)

Key Tools Used:

- **CoolProp CLI:** For accurate air property calculations instead of Table A-17 approximations
- **Property Solver:** Iterative tool to find T_5 where $s_5 = s_4$ for isentropic expansion
- **Incompressible Substance Formula:** $\Delta s = C \ln(T_2/T_1)$ for ceramic particles

14.

The performance of an ideal concentrated solar power plant can be modeled as the combined efficiency of a (1) solar receiver and (2) Carnot heat engine. The solar receiver absorbs concentrated solar resources and delivers them as heat to a power cycle. The total ideal system efficiency is the product of the receiver efficiency and the heat engine efficiency in converting incident solar resources into work.

Given Information

- Concentration ratios (C): 100, 500, 1000, 2000, 3000 suns
- Solar irradiance (I): 1000 W/m^2
- Stefan-Boltzmann constant (σ): 5.67×10^{-8} W/(m^2·K^4)
- Cold reservoir temperature (T_L): Assume ambient $\approx$ 300 K
- Hot reservoir temperature range (T_H): 300 K to 2600 K

PART A: Plot Total Efficiency
Relevant Equations

- Receiver Efficiency:

$$\eta_{receiver} = 1 - \frac{\sigma T_H^4}{C \cdot I}$$

- Carnot Heat Engine Efficiency:

$$\eta_{Carnot,HE} = 1 - \frac{T_L}{T_H}$$

- Total System Efficiency:

$$\begin{aligned} \eta_{Total} &= \eta_{receiver} \cdot \eta_{Carnot,HE} \\ &= \left(1 - \frac{5.67 \times 10^{-8} \cdot T_H^4}{C \cdot 1000}\right) \cdot \left(1 - \frac{300}{T_H}\right) \end{aligned}$$

Solution using Python Analysis

- **Python Script:** Created `carnot_simple.py` to calculate optimal operating points
- **Method:** Scanned temperature range 350-2600 K to find maximum η_{total} for each concentration ratio

Results - Optimal Operating Conditions:

C [suns]	$T_{H,opt}$ [K]	$T_{H,opt}$ [°C]	$\eta_{total,max}$	$\eta_{receiver}$	η_{Carnot}
100	719	446	49.4%	84.8%	58.3%
500	971	698	62.1%	89.9%	69.1%
1000	1107	834	66.7%	91.5%	72.9%
2000	1263	990	70.7%	92.8%	76.2%
3000	1366	1093	72.9%	93.4%	78.0%

Sample Data for Plotting:

T_H [K]	$C = 100$	$C = 500$	$C = 1000$	$C = 2000$	$C = 3000$
500	0.386	0.397	0.399	0.399	0.400
750	0.492	0.578	0.589	0.595	0.596
1000	0.303	0.621	0.660	0.680	0.687
1250	–	0.550	0.655	0.707	0.725
1500	–	0.341	0.570	0.685	0.723
1750	–	–	0.388	0.608	0.682
2000	–	–	0.079	0.464	0.593

PART B: Identify Optimal Receiver Temperature
Key Trends Observed:

1. **Optimal Temperature Increases with Concentration:**
   - $C = 100$ suns: $T_{H,opt} = 719$ K (446°C)
   - $C = 3000$ suns: $T_{H,opt} = 1366$ K (1093°C)
   - Higher concentration allows operation at higher temperatures
2. **Maximum Efficiency Increases with Concentration:**
   - $C = 100$ suns: $\eta_{max} = 49.4\%$
   - $C = 3000$ suns: $\eta_{max} = 72.9\%$
   - Improvement of 23.5 percentage points
3. **Physical Trade-offs:**
   - **Carnot efficiency** increases with T_H (better thermodynamic limit)
   - **Receiver efficiency** decreases with T_H (radiation losses $\propto T_H^4$)
   - Optimum occurs where product is maximized
4. **Concentration Effects:**
   - Higher C shifts optimum to higher temperatures
   - Concentrated sunlight ($C \cdot I$) can overcome larger radiation losses (σT_H^4)
   - Engineering trade-off: system complexity vs. performance

Physical Interpretation

The total efficiency is the product of two competing effects:

$$\eta_{total}(T_H) = \underbrace{\left(1 - \frac{\sigma T_H^4}{C \cdot I}\right)}_{\text{decreases with } T_H} \times \underbrace{\left(1 - \frac{T_L}{T_H}\right)}_{\text{increases with } T_H}$$

At the optimum: $\frac{d\eta_{total}}{dT_H} = 0$, which yields the balance between thermodynamic benefits and radiation losses.

Python Command Used:

- `python carnot_simple.py` → Complete analysis with optimization

A Appendix