Task 1 (3.0 marks)
Consider the relational schemas given below and the respective sets of functional dependencies valid in the schemas.

$R(P,Q,R,S,T,U,V,W)$
F$unctional\ Dependency$:$RW \rightarrow V, P \rightarrow QR, Q \rightarrow RUW, T \rightarrow P, U \rightarrow TV$

(i) Find all the minimal super keys of the relational table R . List the derivations of all minimal keys.

Functional Dependencies (FDs):
RW -> V
P -> QR
Q -> RUW
T -> P
U -> TV

Closure of U:

U -> T, V (Given FD) [U -> T , U -> V] (via decomposition)
T -> P (Given FD)
U -> T -> P = U -> P (transitivity axiom)
P -> Q, R (Given FD)
U -> P -> Q = U -> Q(transitivity axiom)
U -> P -> R = U -> R(transitivity axiom)
Q -> R, U, W (Given FD)
Q -> W (via decomposition)
U -> Q -> R = U -> R (transitivity axiom)

RW -> V -> already have R, W -> V already derived

U+ = {U, T, V, P, Q, R, W}
Add S -> US+ = {U, T, V, P, Q, R, W, S} = all attributes
US is a candidate key

Closure of T:

T -> P
P -> QR
Q -> RUW
U -> TV
RW -> V
T+ = {T, P, Q, R, U, W, V}
T^+ missing S
{T, S}+ = {T, P, Q, R, U, V, W, S}
TS is a candidate key

Closure of Q:

Q -> RUW

RW -> V
U -> TV
T -> P
P -> QR
Q+ = {Q, R, U, W, V, T, P }
{Q, S}+ = {Q, R, U, W, V, T, P, S}
QS is a candidate key

Closure of P:

P -> QR
Q -> RUW
RW -> V
U -> TV
P+ = {P, Q, R, U, W, V, T}
{P, S}+ = {P, Q, R, U, W, V, T, S}
PS is a candidate key

Check for minimality: removing any single attribute breaks full closure

Answer:
Minimal Superkeys (Candidate Keys):US,TS,QS,PS

(ii) Identify the highest normal form of the relational table R. Remember that the identification of a normal form requires analysis of the valid functional dependencies.

Assume 1NF is satisfied (atomic values).

2NF Check:
All attributes must be fully functionally dependent on the entire key
There are no partial dependencies from any minimal key subset to non-prime attributes

use {U, S} as the reference candidate key

Prime Attributes (Part of any Candidate Key):

- P, Q, T, U, S

Non-Prime Attributes:

- R, V, W

RW → V → not a subset of any candidate key

P → QR → P is part of {P, S}, not a subset of any other key

Q → RUW → Q is part of {Q, S}, not partial for others

T → P → T is part of {T, S}

U → TV → U is part of {U, S}

No partial dependencies found
Therefore, the table is in 2NF

2NF is satisfied

3NF Check:
No transitive dependency should exist on a non-prime attribute.

RW → V

- RW is not a super key
- V is non-prime (Violates 3NF)

P → QR

- P is not a super key
- Q and R: Q is prime, R is not (Violates 3NF due to R)

Q → RUW

- Q is not a super key
- R, U, W: U is prime, R and W are not (Violates 3NF)

T → P

- T is not a super key
- P is prime (Acceptable)

U → TV

- U is not a super key
- T is prime, V is not (Violates 3NF)

V is transitively dependent on U
Transitive dependency detected

Answer: Highest Normal Form: 2NF

(iii)Decompose the relational table R into minimal number of normalized relational tables in BCNF. Remember to indicate the primary key and foreign keys (if any).

We decompose R into BCNF by removing violating FDs step-by-step.

R(P, Q, R, S, T, U, V, W)

1. RW -> V: RW is not a super key
Decompose:
- R1(R, W, V), key = RW
- R2(P, Q, S, T, U)

2. In R2: P -> QR:
P is not a super key
Decompose:
- R3(P, Q, R), key = P
- R4(S, T, U)

3. In R4: U -> TV:
U is not a super key.
Decompose:
- R5(U, T, V), key = U
- R6(S)

Final set of BCNF Relations:

- R1(R, W, V), PK = RW
- R3(P, Q, R), PK = P
- R5(U, T, V), PK = U
- R6(S), PK = S

All resulting relations are in BCNF