

CSCI235 – Database Systems

Normalization In Practice

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Example 1

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C$

What is the minimal super key?

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A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C$

What is the minimal super key?

Closure of attributes AB:

$$\{AB\}^+ = \{AB\}$$

$$= \{ABC\} \quad \text{Since } AB \rightarrow C \subseteq AB^+$$

AB^+ derives the attribute C, hence, AB is the **candidate key** (or **minimal super key**).

Example 1

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Functional dependencies: $AB \rightarrow C$

What is the normal form?

Example 1

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C$

What is the normal form?

- There is **NO** partial dependency in R . Hence, the relational schema R is in 2NF.
- There is **NO** transitive dependency in R . Hence, the relational schema R is in 3NF.
- There is **NO** non-trivial dependency in R . Hence, the relational schema R has no BCNF violation.
- \therefore The relational schema R is in **BCNF**.

Example 2

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C, C \rightarrow B$

What is the minimal super key?

Example 2

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C, C \rightarrow B$

What is the minimal super key?

Closure of attribute AB:

$$\begin{aligned}\{AB\}^+ &= \{AB\} \\ &= \{ABC\}\end{aligned}$$

AB^+ derives the attribute C, hence, AB is a **candidate key** (or **minimal super key**).

Example 2

Closure of attribute C:

$$\begin{aligned}\{C\}^+ &= \{C\} \\ &= \{CB\}\end{aligned}$$

Closure of attribute A:

$$\{A\}^+ = \{A\}$$

Example 2

Since none of the determinants $\{C\}^+$ and $\{A\}^+$ closure sets derive all the attributes, hence, we can combine AC, using composition rule, and get the closure:
 $\{AC\}^+ = \{ABC\}$

Since $\{AC\}^+$ derives all the attribute, hence, AC is a **candidate key** (or a **minimal super key**).

Example 2

The relational schema $R = (A, B, C)$ has two minimal super key, and they are $\{AB\}$ and $\{AC\}$.

Example 2

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C, C \rightarrow B$

What is the normal form?

Example 2

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C, C \rightarrow B$

What is the normal form? (Scenario 1)

- Since (A, B) is a minimal super key, there is **NO** partial dependency and **NO** transitive dependency, but there **exist** non-trivial dependency $C \rightarrow B$. Existence of non-trivial dependency is a violation of BCNF requirement. Hence, the relational schema **CANNOT** be in BCNF.
- \therefore the relational schema $R = (A, B, C)$ is in **3NF**.

Example 2

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C, C \rightarrow B$

What is the normal form? (Scenario 2)

- Since (A, C) is a minimal super key, there **exist** a **partial dependency** $C \rightarrow B$, a violation of 2NF requirement. There is **NO** transitive dependency, and there is **NO** non-trivial dependency.
- \therefore the relational schema $R = (A, B, C)$ is in **1NF**.
- Question: Why the relational schema is not in **3NF** or **BCNF**?

Example 2

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C, C \rightarrow B$

How to transform the relational schema R into BCNF?
(Scenario 1)

Example 2

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C, C \rightarrow B$

How to transform the relational schema R into BCNF?
(Scenario 1)

- Remove the non-trivial dependency $C \rightarrow B$ from the relational schema R , and decompose it into two relational schema, $R1 = (A, B)$ and $R2 = (C, B)$.
- In relational schema $R1$, the minimal super key is (A, B) .
 - There is NO partial dependency, NO transitive dependency, and NO non-trivial dependency. Hence, the relational schema $R1$ is in **BCNF**.

Example 2

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C, C \rightarrow B$

How to transform the relational schema R into BCNF?
(Scenario 1)

- Remove the non-trivial dependency $C \rightarrow B$ from the relational schema R , and decompose it into two relational schema, $R1 = (A, B)$ and $R2 = (C, B)$.
- In relational schema $R2$, the minimal super key is (C) .
 - There is NO partial dependency, NO transitive dependency, and NO non-trivial dependency. Hence, the relational schema $R2$ is in BCNF.

Example 2

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C, C \rightarrow B$

How to transform the relational schema R into BCNF?
(Scenario 2)

Example 2

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C, C \rightarrow B$

How to transform the relational schema R into BCNF?
(Scenario 2)

- In scenario 2, $AC \rightarrow AB$. Remove the **partial dependency** $C \rightarrow B$ from the relational schema R , and decompose it into two relational schema, $R1 = (A, C)$ and $R2 = (C, B)$.
- In relational schema $R1$, the minimal super key is (AC) .
 - There is **NO partial dependency**, **NO transitive dependency**, and **NO non-trivial dependency**. Hence, the relational schema $R1$ is in **BCNF**.

Example 2

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C, C \rightarrow B$

How to transform the relational schema R into BCNF?
(Scenario 2)

- In scenario 2, $AC \rightarrow AB$. Remove the partial dependency $C \rightarrow B$ from the relational schema R , and decompose it into two relational schema, $R1 = (A, C)$ and $R2 = (C, B)$.
- In relational schema $R2$, the minimal super key is (C) .
 - There is NO partial dependency, NO transitive dependency, and NO non-trivial dependency. Hence, the relational schema $R2$ is in **BCNF**.

Example 3

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C, C \rightarrow B, C \rightarrow A$

What is the minimal super key?

Example 3

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C$, $C \rightarrow B$, $C \rightarrow A$

What is the minimal super key?

- If $AB \rightarrow C$ is valid in R , and it covers the entire relational schema, then the left hand side of the functional dependency, (A, B) is a minimal super key.
- If $C \rightarrow B$ and $C \rightarrow A$, then through **union rule** $C \rightarrow AB$.
- If $C \rightarrow AB$ is valid in R and it covers the entire relational schema, then the left hand side of the functional dependency, C is a minimal super key.
- Hence, the minimal super keys of the relational schema R are (A, B) and (C) .

Example 3

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C, C \rightarrow B, C \rightarrow A$

What is the normal form?

Example 3

A relational schema $R = (A, B, C)$

Functional dependencies: $AB \rightarrow C, C \rightarrow B, C \rightarrow A$

What is the normal form?

- There is **NO** partial dependency in the relational schema R . Hence, the relational schema R is in **2NF**.
- There is **NO** transitive dependency in the relational schema R . Hence, the relational schema R is in **3NF**.
- There is **NO** non-trivial dependency violation in the relational schema R . Hence the relational schema R does not have **BCNF** violation.
- \therefore The relational schema R is in **BCNF**.

Example 4

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B$

What is the minimal super key?

Example 4

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B$

What is the minimal super key?

- If $A \rightarrow B$ is valid in R , then through **augmentation rule**, $AC \rightarrow BC$.
- If $AC \rightarrow BC$ is valid in R and it covers the entire relational schema then the left hand side of the functional dependency (A, C) is a minimal key.
- Hence, the minimal super keys of the relational schema R is (A, C) .

Example 4

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B$

What is the normal form?

Example 4

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B$

What is the normal form?

- Since (A, C) is a minimal super key, and $A \rightarrow B$, there exist a partial dependency. Hence, the relational schema R CANNOT be in 2NF.
- There is NO transitive dependency and NO non-trivial dependency in the relational schema R .
- Hence, the highest normal form of the relational schema R is in 1NF.

Example 4

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B$

How to transform the relational schema R into BCNF?

Example 4

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B$

How to transform the relational schema R into BCNF?

- Since there exists a partial dependency in the relational schema R , to transform the relational schema to BCNF, we need to remove the partial dependency, and split it into two relational schemas $R1 = (A, C)$ and $R2 = (A, B)$.

Example 4

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B$

How to transform the relational schema R into BCNF?

- In the relational schema $R1 = (A, C)$, the minimal super key is (A, C) .
 - There is **NO** partial dependency nor transitive dependency in the relational schema $R1$. In addition, there is also **NO** non-trivial dependency exist. Hence, the relational schema $R1$ is in **BCNF**.
- In the relational schema $R2 = (A, B)$, the minimal super key is (A) .

Example 4

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B$

How to transform the relational schema R into BCNF?

- In the relational schema $R2 = (A, B)$, the minimal super key is (A) .
 - There is **NO** partial dependency nor transitive dependency. In addition, there is also **NO** non-trivial dependency. Hence, the relational schema $R2$ is in **BCNF**.

Example 5

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow A$

What is the minimal super key?

Example 5

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow A$

What is the minimal super key?

- If $A \rightarrow B$ then through **augmentation rule**, $AC \rightarrow BC$.
- If $AC \rightarrow BC$ is valid in R and it covers the entire relational schema, then the left-hand-side of the functional dependency (AC) is a minimal super key.
- If $B \rightarrow A$ then through **augmentation rule**, $BC \rightarrow AC$.
- If $BC \rightarrow AC$ is valid in R and it covers the entire relational schema, then the left-hand-side of the functional dependency (BC) is a minimal super key.

Example 5

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow A$

What is the normal form?

Example 5

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow A$

What is the normal form?

- Since (A, C) is a minimal super key, and $A \rightarrow B$, there exist a partial dependency (violate 2NF requirement). In addition, the functional dependency $B \rightarrow A$, the determinant B is not a minimal super key and A is part of a minimal super key (A, C) , this forms a non-trivial dependency (violate BCNF requirement). Hence, the relational schema is in 1NF.

Example 5

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow A$

What is the normal form?

- Similarly, if (B, C) is a minimal super key, then $B \rightarrow A$ forms a partial dependency (this violates 2NF requirement). Hence, the relational schema **CANNOT** be in 2NF. In addition, the functional dependency $A \rightarrow B$, the determinant A is not a minimal super key and B is part of a minimal super key (B, C) , this forms a non-trivial dependency (violate BCNF requirement). Hence, the relational schema is in 1NF.
- Hence, the relational schema $R = (A, B, C)$ is in **1NF**.

Example 5

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow A$

How to transform the relational schema R into BCNF?

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Example 5

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow A$

How to transform the relational schema R into BCNF?

- Since there exists a **partial dependency** in the relational schema R , to transform the relational schema to BCNF, we need to remove the **partial dependency**, and split it into two relational schemas $R1 = (A, C)$ and $R2 = (A, B)$.

Example 5

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow A$

How to transform the relational schema R into BCNF?

- In the relational schema $R1 = (A, C)$, the minimal super key is (A, C) .
 - There is **NO** partial dependency (No violation of 2NF).
 - There is **NO** transitive dependency (No violation of 3NF).
 - There is **NO** non-trivial dependency (No violation of BCNF).
 - \therefore the relational schema $R1$ is in **BCNF**.

Example 5

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow A$

How to transform the relational schema R into BCNF?

- In the relational schema $R2 = (A, B)$, the minimal super key can be (A) or (B) .
 - There is **NO** partial dependency (No violation of 2NF).
 - There is **NO** transitive dependency (No violation of 3NF).
 - There is **NO** non-trivial dependency (No violation of BCNF).
 - \therefore the relational schema $R2$ is in **BCNF**.

Example 6

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow C$.

What is the minimal super key?

Example 6

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow C$.

What is the minimal super key?

- If $A \rightarrow B$ and $B \rightarrow C$ then through transitivity rule, $A \rightarrow C$.
- If $A \rightarrow B$ and $A \rightarrow C$ then through union rule, $A \rightarrow BC$
- If $A \rightarrow BC$ is valid in R and it covers the entire relational schema then, the left-hand-side of the functional dependency (A) is a minimal super key.

Example 6

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow C$.

What is the normal form?

Example 6

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow C$.

What is the normal form?

- With the functional dependency $A \rightarrow BC$, there is **NO** partial dependency.
- However, there exist a transitive dependency $B \rightarrow C$ (a violation of 3NF requirement), hence, the relational schema $R = (A, B, C)$ is in **2NF**.

Example 6

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow C$.

How to transform the relational schema R into BCNF?

Example 6

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow C$.

How to transform the relational schema R into BCNF?

- With the functional dependency $A \rightarrow BC$, there exist a transitive dependency $B \rightarrow C$ (a violation of 3NF requirement).
- To transform the relational schema R to BCNF, we need to remove the transitive dependency violation and decompose the relational schema R into two relational schemas $R1 = (A, B)$ and $R2 = (B, C)$.

Example 6

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow C$.

How to transform the relational schema R into BCNF?

- In the relational schema $R1 = (A, B)$, the minimal super key is (A) and there is NO partial dependency, transitive dependency, and non-transitive dependency, the relational schema $R1$ is in **BCNF**.

Example 6

A relational schema $R = (A, B, C)$

Functional dependencies: $A \rightarrow B, B \rightarrow C$.

How to transform the relational schema R into BCNF?

- In the relational schema $R2 = (B, C)$, the minimal super key is (B) and there is **NO** partial dependency, transitive dependency, and non-transitive dependency, the relational schema $R2$ is also in **BCNF**.

Example 7

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow D$

What is the minimal super key?

Example 7

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow D$

What is the minimal super key?

- If $A \rightarrow B$ and $A \rightarrow C$ then through **union rule** $A \rightarrow BC$.
- If $A \rightarrow B$ and $B \rightarrow D$ then through **transitivity rule** $A \rightarrow D$.
- If $A \rightarrow BC$ and $A \rightarrow D$ then through **union rule** $A \rightarrow BCD$.
- If $A \rightarrow BCD$ is valid in R and it covers the entire relational schema, then the left-hand-side of the functional dependency (A) is a minimal super key.

Example 7

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow D$

What is the normal form?

Example 7

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow D$

What is the normal form?

- In the functional dependency $A \rightarrow BCD$, there is **NO partial dependency**. However, there is a **transitive dependency** $B \rightarrow D$ (a violation of 3NF.)
- \therefore the relational schema $R = (A, B, C, D)$ is in **2NF**.

Example 7

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow D$

How to transform the relational schema R into BCNF?

Example 7

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow D$

How to transform the relational schema R into BCNF?

- With the functional dependency $A \rightarrow BCD$, there is no partial dependency, but there exists a transitive dependency $B \rightarrow D$ (a violation of 3NF.) To transform the relational schema to BCNF, we need to remove the transitive dependency and split the relational schema R into two schemas $R1 = (A, B, C)$ and $R2 = (B, D)$.

Example 7

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow D$

How to transform the relational schema R into BCNF?

- In $R1 = (A, B, C)$, the minimal super key is (A) and there are **NO** partial dependency, transitive dependency, and non-trivial dependency violations. Hence, the relational schema $R1 = (A, B, C)$ is in **BCNF**.
- In $R2 = (B, D)$, the minimal super key is (B) and there are **NO** partial dependency, transitive dependency, and non-trivial dependency violations. Hence, the relational schema $R2 = (B, D)$ is in **BCNF**.

Example 8

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow D, C \rightarrow B$

What is the minimal super key?

Example 8

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow D, C \rightarrow B$

What is the minimal super key?

- If $A \rightarrow B$ and $B \rightarrow D$ then through **transitivity rule** $A \rightarrow D$.
- If $A \rightarrow D$ and $A \rightarrow B$ then through **union rule** $A \rightarrow BD$.
- If $A \rightarrow BD$ then through **augmentation rule** $AC \rightarrow BCD$.
- If $AC \rightarrow BCD$ is valid in R and it covers the entire relational schema, then the left-hand-side of the functional dependency (AC) is a minimal super key.

Example 8

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow D, C \rightarrow B$

What is the minimal super key?

- If $C \rightarrow B$ and $B \rightarrow D$ then through **transitivity rule** $C \rightarrow D$.
- If $C \rightarrow D$ and $C \rightarrow B$ then through **union rule** $C \rightarrow BD$.
- If $C \rightarrow BD$ then through **augmentation rule** $AC \rightarrow ABD$.
- If $AC \rightarrow ABD$ is valid in R and it covers the entire relational schema, then the left-hand-side of the functional dependency (AC) is a minimal super key.

Example 8

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow D, C \rightarrow B$

What is the normal form?

Example 8

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow D, C \rightarrow B$

What is the normal form?

- In $AC \rightarrow BCD$, there exists a partial dependency $A \rightarrow B$ and $C \rightarrow B$, hence, a violation of 2NF requirement.
- There exists also a transitive dependency $B \rightarrow D$, hence, a violation of 3NF requirement.
- \therefore the relational schema is in 1NF.

Example 8

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow D, C \rightarrow B$

What is the normal form?

- In $AC \rightarrow ABD$, there exists a partial dependency $A \rightarrow B$ and $C \rightarrow B$, hence, a violation of 2NF requirement.
- There exists also a transitive dependency $B \rightarrow D$, hence, a violation of 3NF requirement.
- \therefore the relational schema is in **1NF**.

Example 8

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow D, C \rightarrow B$

How to transform the relational schema R into BCNF?

Example 8

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow D, C \rightarrow B$

How to transform the relational schema R into BCNF?

- In $AC \rightarrow BCD$, there exists a partial dependency $A \rightarrow B$ and $C \rightarrow B$, hence, a violation of 2NF requirement.
- There exists also a transitive dependency $B \rightarrow D$, hence, a violation of 3NF requirement.
- To transform the relational schema to BCNF, we need to first remove all partial dependencies from the relational schema $AC \rightarrow BCD$, and we have $R1 = (A, B, D)$, $R2 = (C, B)$, $R3 = (A, C)$.

Example 8

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow D, C \rightarrow B$

How to transform the relational schema R into BCNF?

- In the relational schema $R1 = (A, B, D)$, (A) is the minimal super key. There is **NO** partial dependency, but there exist a transitive dependency $B \rightarrow D$, (a violation of 3NF.)
- To transform the relational schema $R1 = (A, B, D)$ to BCNF, we need to remove the transitive dependency $B \rightarrow D$ by splitting $R1$ into $R1 = (A, B)$ and $R4 = (B, D)$.

Example 8

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow D, C \rightarrow B$

How to transform the relational schema R into BCNF?

- In relational schema $R1 = (A, B)$, the minimal super key is (A) . There are NO partial dependency, no transitive dependency, and no non-trivial dependency. Hence, $R1 = (A, B)$ is in **BCNF**.
- In relational schema $R4 = (B, D)$, the minimal super key is (B) . There are NO partial dependency, no transitive dependency, and no non-trivial dependency. Hence, $R4 = (B, D)$ is in **BCNF**.

Example 8

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow D, C \rightarrow B$

How to transform the relational schema R into BCNF?

- In relational schema, $R2 = (C, B)$, the minimal super key is (C) . There are NO partial dependency, no transitive dependency, and no non-trivial dependency. Hence, $R2 = (C, B)$ is in **BCNF**.
- In relational schema, $R3 = (A, C)$, the minimal super key is (AC) . There are NO partial dependency, no transitive dependency, and no non-trivial dependency. Hence, $R3 = (A, C)$ is in **BCNF**.

Example 9

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow A, B \rightarrow C$

What is the minimal super key?

Example 9

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow A, B \rightarrow C$

What is the minimal super key?

- If $A \rightarrow B$ and $A \rightarrow C$, then through **union rule**, $A \rightarrow BC$
- If $A \rightarrow BC$ then through **augmentation rule** $AD \rightarrow BCD$.
- If $AD \rightarrow BCD$ is valid in R and it covers the entire relational schema, then the left-hand-side of the functional dependency (A, D) is a minimal super key.

Example 9

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow A, B \rightarrow C$

What is the minimal super key?

- If $B \rightarrow A$ and $B \rightarrow C$, then through **union rule**, $B \rightarrow AC$
- If $B \rightarrow AC$ then through **augmentation rule** $BD \rightarrow ACD$.
- If $BD \rightarrow ACD$ is valid in R and it covers the entire relational schema, then the left-hand-side of the functional dependency (B, D) is a minimal super key.

Example 9

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow A, B \rightarrow C$

What is the normal form?

Example 9

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow A, B \rightarrow C$

What is the normal form?

- In the functional dependency $AD \rightarrow BCD$, there exist:
 - Partial dependencies $A \rightarrow B$ and $A \rightarrow C$. These partial dependencies are a violation of 2NF requirement.
 - Transitive dependency $B \rightarrow C$. This transitive dependency is a violation of 3NF requirement.
 - Non-trivial dependency $B \rightarrow A$. This non-trivial dependency is a violation of BCNF.
- Hence, the relational schema R is in 1NF.

Example 9

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow A, B \rightarrow C$

What is the normal form?

- In the functional dependency $BD \rightarrow ACD$, there exist:
 - Partial dependencies $B \rightarrow A$ and $B \rightarrow C$. These partial dependencies are a violation of 2NF requirement.
 - Transitive dependency $A \rightarrow C$. This transitive dependency is a violation of 3NF requirement.
 - Non-trivial dependency $B \rightarrow A$. This non-trivial dependency is a violation of BCNF.
- Hence, the relational schema R is in 1NF.

Example 9

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow A, B \rightarrow C$

How to transform the relational schema R into BCNF?

Example 9

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow A, B \rightarrow C$

How to transform the relational schema R into BCNF?

- With $AD \rightarrow BCD$, we need to remove the partial dependencies $A \rightarrow BC$ and split the relational schema R into two relational schemas $R1 = (A, D)$ and $R2 = (A, B, C)$
 - In relational schema $R1 = (A, D)$, the minimal super key is (AD) , and the relational schema has **NO** partial dependency, **NO** transitive dependency, and **NO** non-trivial dependency. Hence, the relational schema $R1$ is in **BCNF**.

Example 9

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow A, B \rightarrow C$

How to transform the relational schema R into BCNF?

- In relational schema $R2 = (A, B, C)$, the minimal super key is (A) , and the relational schema has **NO** partial dependency, but there exists a transitive dependency $B \rightarrow C$, and a non-trivial dependency $B \rightarrow A$. To transform the relational schema to BCNF, we need to remove both the transitive dependency $B \rightarrow C$, and the non-trivial dependency $B \rightarrow A$. Split the relational schema $R2$ into $R3 = (A, B)$, and $R4 = (B, C)$.

Example 9

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, A \rightarrow C, B \rightarrow A, B \rightarrow C$

How to transform the relational schema R into BCNF?

- In relational schema $R3 = (A, B)$, the minimal super key is (A) , and the relational schema has **NO** partial dependency, **NO** transitive dependency and **NO** non-trivial dependency. Hence the relational schema $R3$ is in **BCNF**.
- In relational schema $R4 = (B, C)$, the minimal super key is (B) , and the relational schema has **NO** partial dependency, **NO** transitive dependency and **NO** non-trivial dependency. Hence the relational schema $R4$ is in **BCNF**.

Example 10

A relational schema $R = (A, B, C, D)$

Functional dependencies: $AB \rightarrow C, C \rightarrow D, D \rightarrow A, D \rightarrow B$

What is the minimal super key? (Scenario 1)

Example 10

A relational schema $R = (A, B, C, D)$

Functional dependencies: $AB \rightarrow C, C \rightarrow D, D \rightarrow A, D \rightarrow B$

What is the minimal super key? (Scenario 1)

- If $AB \rightarrow C$ and $C \rightarrow D$ then through **transitivity axiom**, $AB \rightarrow D$.
- If $AB \rightarrow D$ and $AB \rightarrow C$ then through **union rule**, $AB \rightarrow CD$.
- If $AB \rightarrow CD$ is valid in R and it covers the entire relational schema then the left-hand-side of the functional dependency (A, B) is a minimal key.

Example 10

A relational schema $R = (A, B, C, D)$

Functional dependencies: $AB \rightarrow C, C \rightarrow D, D \rightarrow A, D \rightarrow B$

What is the minimal super key? (Scenario 2)

- If $D \rightarrow A$ and $D \rightarrow B$ then through **union rule**, $D \rightarrow AB$.
- If $D \rightarrow AB$ and $AB \rightarrow C$ then through **transitivity axiom**, $D \rightarrow C$.
- If $D \rightarrow C$ and $D \rightarrow AB$ then $D \rightarrow ABC$.
- If $D \rightarrow ABC$ is valid in R and it covers the entire relational schema, then the left-hand-side of the functional dependency (D) is a minimal key.

Example 10

A relational schema $R = (A, B, C, D)$

Functional dependencies: $AB \rightarrow C, C \rightarrow D, D \rightarrow A, D \rightarrow B$

What is the minimal super key? (Scenario 3)

- If $C \rightarrow D$ and $D \rightarrow AB$ then through **transitivity axiom**, $C \rightarrow AB$.
- If $C \rightarrow D$ and $C \rightarrow AB$ then through **union rule**, $C \rightarrow ABD$.
- If $C \rightarrow ABD$ is valid in R and it covers the entire relational schema, then the left-hand-side of the functional dependency (C) is a minimal key.

Example 10

A relational schema $R = (A, B, C, D)$

Functional dependencies: $AB \rightarrow C, C \rightarrow D, D \rightarrow A, D \rightarrow B$

What is the normal form? (Scenario 1)

Example 10

A relational schema $R = (A, B, C, D)$

Functional dependencies: $AB \rightarrow C, C \rightarrow D, D \rightarrow A, D \rightarrow B$

What is the normal form? (Scenario 1)

- In the functional dependency $AB \rightarrow CD$, there is **NO partial dependency**, and **NO** transitive dependency (Note, the functional dependency $C \rightarrow D \rightarrow AB$ is not a transitive dependency because C is a minimal super key.) There is also no non-trivial functional dependency as well. Hence, the relational schema R is in Boyce-Codd normal form (BCNF).
- \therefore the relational schema $R = (A, B, C, D)$ is in *BCNF* (Boyce – Codd normal form).

Example 10

A relational schema $R = (A, B, C, D)$

Functional dependencies: $AB \rightarrow C, C \rightarrow D, D \rightarrow A, D \rightarrow B$

What is the normal form? (Scenario 2)

- In the functional dependency $D \rightarrow ABC$, there is **NO partial dependency**, and **NO** transitive dependency (Note, the functional dependency $AB \rightarrow C \rightarrow D$ is not a transitive dependency because AB is a minimal super key.) There is also no non-trivial functional dependency as well. Hence, the relational schema R is in Boyce-Codd normal form (BCNF).
- \therefore the relational schema $R = (A, B, C, D)$ is in *BCNF* (Boyce – Codd normal form).

Example 10

A relational schema $R = (A, B, C, D)$

Functional dependencies: $AB \rightarrow C, C \rightarrow D, D \rightarrow A, D \rightarrow B$

What is the normal form? (Scenario 3)

- In the functional dependency $C \rightarrow DAB$, there is **NO partial dependency**, and **NO** transitive dependency (Note, the functional dependency $D \rightarrow AB \rightarrow C$ is not a transitive dependency because D is a minimal super key.) There is also no non-trivial functional dependency as well. Hence, the relational schema R is in Boyce-Codd normal form (BCNF).
- \therefore the relational schema $R = (A, B, C, D)$ is in *BCNF* (Boyce – Codd normal form).

Example 10

A relational schema $R = (A, B, C, D)$

Functional dependencies: $AB \rightarrow C, C \rightarrow D, D \rightarrow A, D \rightarrow B$

How to transform the relational schema R into BCNF?
(Scenario 1)

Example 10

A relational schema $R = (A, B, C, D)$

Functional dependencies: $AB \rightarrow C, C \rightarrow D, D \rightarrow A, D \rightarrow B$

How to transform the relational schema R into BCNF?
(Scenario 1)

- The relational schema is already in BCNF, hence, there is no need for further transformation.

$R(A, B, C, D)$

$PK: AB$

$MSK1: C$

$MSK2: D$

Example 10

A relational schema $R = (A, B, C, D)$

Functional dependencies: $AB \rightarrow C, C \rightarrow D, D \rightarrow A, D \rightarrow B$

How to transform the relational schema R into BCNF?
(Scenario 2)

- The relational schema is already in BCNF, hence, there is no need for further transformation.

$R(A, B, C, D)$

$PK: D$

$MSK1: AB$

$MSK2: C$

Example 10

A relational schema $R = (A, B, C, D)$

Functional dependencies: $AB \rightarrow C, C \rightarrow D, D \rightarrow A, D \rightarrow B$

How to transform the relational schema R into BCNF?
(Scenario 3)

- The relational schema is already in BCNF, hence, there is no need for further transformation.

$R(A, B, C, D)$

$PK: C$

$MSK1: D$

$MSK2: AB$

Example 10

- Note: In the Example 10, the transformation of the relational schema to BCNF is needed (done) only using one of the three possible scenario.

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

What is the minimal super key? (Scenario 1)

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

What is the minimal super key? (Scenario 1)

- If $A \rightarrow B$ and $B \rightarrow C$ then through **transitivity axiom**, $A \rightarrow C$.
- If $A \rightarrow C$ and $C \rightarrow D$ then through **transitivity axiom**, $A \rightarrow D$.
- If $A \rightarrow B$ and $A \rightarrow D$ then through **union rule** $A \rightarrow BCD$.
- If $A \rightarrow BCD$ is valid in R and it covers the entire relational schema, then the left-hand-side of the functional dependency (A) is a minimal super key.

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

What is the minimal super key? (Scenario 2)

- If $B \rightarrow C$ and $C \rightarrow D$ then through **transitivity axiom**, $B \rightarrow D$.
- If $B \rightarrow D$ and $D \rightarrow A$ then through **transitivity axiom**, $B \rightarrow A$.
- If $B \rightarrow C$ and $B \rightarrow D$ and $B \rightarrow A$ then through **union rule** $B \rightarrow ACD$.
- If $B \rightarrow ACD$ is valid in R and it covers the entire relational schema, then the left-hand-side of the functional dependency (B) is a minimal super key.

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

What is the minimal super key? (Scenario 3)

- If $C \rightarrow D$ and $D \rightarrow A$ then through **transitivity axiom**, $C \rightarrow A$.
- If $C \rightarrow A$ and $A \rightarrow B$ then through **transitivity axiom**, $C \rightarrow B$.
- If $C \rightarrow A$ and $C \rightarrow B$ and $C \rightarrow D$ then through **union rule** $C \rightarrow ABD$.
- If $C \rightarrow ABD$ is valid in R and it covers the entire relational schema, then the left-hand-side of the functional dependency (C) is a minimal super key.

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

What is the minimal super key? (Scenario 4)

- If $C \rightarrow D$ and $D \rightarrow A$ then through **transitivity axiom**, $C \rightarrow A$.
- If $C \rightarrow A$ and $A \rightarrow B$ then through **transitivity axiom**, $C \rightarrow B$.
- If $C \rightarrow A$ and $C \rightarrow B$ and $C \rightarrow D$ then through **union rule** $C \rightarrow ABD$.
- If $C \rightarrow ABD$ is valid in R and it covers the entire relational schema, then the left-hand-side of the functional dependency (C) is a minimal super key.

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

What is the normal form? (Scenario 1)

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

What is the normal form? (Scenario 1)

- In the functional dependency $A \rightarrow BCD$, there is **NO partial dependency**. Hence, the relational table is qualified to be in 2NF.
- There are other functional dependencies $B \rightarrow C, C \rightarrow D$, and $D \rightarrow A$ exist in the relational schema R. However, the determinants, B, C and D are all candidate keys (according to Scenario 2, Scenario 3, and Scenario 4), these functional dependencies does not constitute as transitive functional dependencies. Hence, the relational table is qualified to be in 3NF.

Example 11

- Since all the determinants A, B, C, and D are minimal super-keys (according to Scenario 2, Scenario 2, Scenario 3, and Scenario 4), the relational schema R does not violate the Boyce-Codd normal form requirements.
- \therefore the relational schema $R = (A, B, C, D)$ is in *BCNF*.

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

What is the normal form? (Scenario 2)

- In the functional dependency $B \rightarrow ACD$, there is **NO partial dependency**, Hence, the relational table is qualified to be in 2NF.
- There are other functional dependencies $A \rightarrow B, C \rightarrow D$, and $D \rightarrow A$ exist in the relational schema R. However, the determinants, A, C and D are all candidate keys (according to Scenario 1, Scenario 3, and Scenario 4), these functional dependencies does not constitute as transitive functional dependencies. Hence, the relational table is qualified to be in 3NF.

Example 11

- Since all the determinants A, B, C, and D are minimal super-keys (according to Scenario 1, Scenario 2, Scenario 3, and Scenario 4), the relational schema R does not violate the Boyce-Codd normal form requirements.
- \therefore the relational schema $R = (A, B, C, D)$ is in *BCNF*.

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

What is the normal form? (Scenario 3)

- In the functional dependency $C \rightarrow ABD$, there is **NO partial dependency**, Hence, the relational table is qualified to be in 2NF.
- There are other functional dependencies $A \rightarrow B, B \rightarrow C$, and $D \rightarrow A$ exist in the relational schema R. However, the determinants, A, B and D are all candidate keys (according to Scenario 1, Scenario 2, and Scenario 4), these functional dependencies does not constitute as transitive functional dependencies. Hence, the relational table is qualified to be in 3NF.

Example 11

- Since all the determinants A, B, C, and D are minimal super-keys (according to Scenario 1, Scenario 2, Scenario 3, and Scenario 4), the relational schema R does not violate the Boyce-Codd normal form requirements.
- \therefore the relational schema $R = (A, B, C, D)$ is in *BCNF*.

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

How to transform the relational schema R into BCNF? (Scenario 1)

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

How to transform the relational schema R into BCNF? (Scenario 1)

- The relational schema R is already in BCNF. No transformation is required.

$R(A, B, C, D)$

$PK: A$

$CK1: B$

$CK2: C$

$CK3: D$

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

How to transform the relational schema R into BCNF? (Scenario 2)

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

How to transform the relational schema R into BCNF? (Scenario 2)

- The relational schema R is already in BCNF. No transformation is required.

$R(A, B, C, D)$

$PK: B$

$CK1: A$

$CK2: C$

$CK3: D$

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

How to transform the relational schema R into BCNF? (Scenario 3)

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

How to transform the relational schema R into BCNF? (Scenario 3)

- The relational schema R is already in BCNF. No transformation is required.

$R(A, B, C, D)$

$PK: C$

$CK1: A$

$CK2: B$

$CK3: D$

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

How to transform the relational schema R into BCNF? (Scenario 3)

Example 11

A relational schema $R = (A, B, C, D)$

Functional dependencies: $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$

How to transform the relational schema R into BCNF? (Scenario 3)

- The relational schema R is already in BCNF. No transformation is required.

$R(A, B, C, D)$

$PK: C$

$CK1: A$

$CK2: B$

$CK3: D$

Example 11

- Note: In the Example 11, the transformation of the relational schema to BCNF is needed (done) only using one of the three possible scenario.

References

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