Statistical Mean Reversion of Airline and Oil Company Stock Prices

CSC 265 Final Project

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Source Code Available At:
https://github.com/DominickH20/statistical-mean-reversion

May 7, 2021

Abstract

This paper demonstrates the development and analysis of two forms of statistical mean reversion (statistical arbitrage) strategies for trading airline and oil company stocks. The first strategy is a pairwise model that leverages correlations among pairs of stocks to trade. It is fit using OLS and performs rather well out of sample. The second strategy is a basket model that leverages correlations among baskets of securities. To handle the multicollinearity problem within a basket, a principal component decomposition is applied to the data before fitting with OLS. The basket strategy suffers from higher volatility and noise in the signal, but still demonstrates the presence of a signal. The deterioration found in the basket strategy relative to the pairwise strategy is likely attributable to the higher dimensionality of the data and distributional nuances in the data that prohibit effective feature selection.

1 Introduction

The intent of this paper is to showcase a reliable statistical mean reversion trading strategy on correlated baskets of stocks, specifically airline and oil company stocks. Airline companies are exposed to very similar pools of risk, and the same can be said for oil companies. Because of this, one would expect their stock prices to move together when these underlying risk factors change. Because airlines are large consumers of oil, one might expect the airline and oil sectors to exhibit some relationship as well.

1.1 Statistical Mean Reversion

This paper aims to develop a model for how these companies behave in relation to one another. This model can be thought of as the "mean" in this mean reversion concept. I utilize this model in the context of a trading strategy by computing the expected stock return at time t, call it e_t. The expected return can be compared to the actual return, a_t at that same time period t. This generates a residual, $r_t = a_t - e_t$. If this residual is sufficiently positive, then the strategy concludes that the stock in question has grown too much over the period, so it would sell. Likewise, if the r_t is sufficiently negative, then the strategy judges that stock has underperformed over the period relative to its peers, so it would buy. In both cases the strategy is betting on convergence to the statistical relationship between the securities. In this paper, I outline such strategies on pairs of securities as well as baskets of securities, using principal component analysis to handle the multicollinearity problem for regression models implemented on such data.

1.2 Data

1.2.1 Scope

The data used in the subsequent analyses were obtained from the Alpaca Markets historical market data API. The securities examined in this study are the following: American Airlines (AAL), Delta Airlines (DAL), Southwest Airlines (LUV), Allegiant Airlines (ALGT), United Airlines (UAL), Chevron (CVX), Marathon Oil (MRO), Murphy Oil (MUR), Exxon Mobil (XOM), Devon Energy (DVN), SPDR S&P 500 ETF, and iShares Treasury Bond ETF (TLT). There are 5 airline companies, 5 oil companies, and 2 market factors (SPY and TLT). The motivation behind including the market factors in this study is that they help parameterize what is going on in equity and debt markets at any given point in time. The data range from January 1st 2016 to May 1st 2021. The years 2020 and 2021 are held out as test data while the rest of the data is used to train the models. Given the occurrence of the pandemic in 2020, this is a particularly challenging test set.

1.2.2 Data Format

The data is organized in a *bar* format. Bars are a common way of aggregating financial data and are structured as follows:

$$B_S = \begin{bmatrix}\text{bar}_0, & \text{bar}_1, & ..., & \text{bar}_n\end{bmatrix}$$

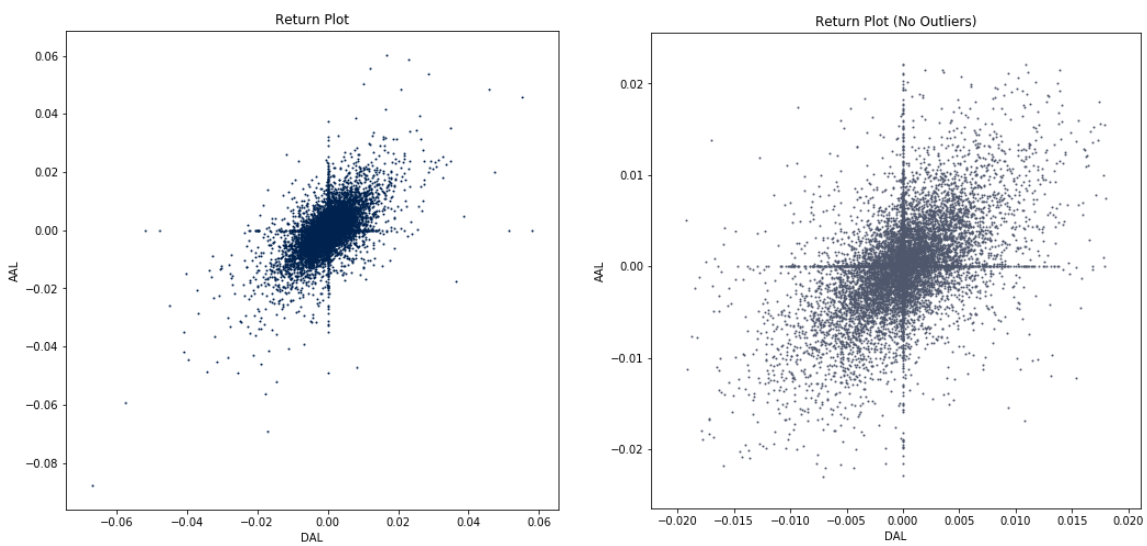
$$\text{bar}_i = \begin{bmatrix}\text{datetime}_i & \text{open}_i & \text{high}_i & \text{low}_i & \text{close}_i & \text{volume}_i\end{bmatrix}$$

Where S is some security, and B_S is the bar set for that security. Each bar summarizes a section of trading activity. The open and close are the first and last trades occurring within the timeframe of the bar, while the high and low are the highest and lowest trade prices observed within the timeframe of the bar. The volume represents the number of shares that traded within the bar. For this analysis, only the closing prices are used.

1.2.3 Data Cleaning

The data set was cleaned and imputed before use in any models. There were missing bars for several securities, so the most abundant time series (SPY) was used as a baseline for bar availability. For every bar in SPY, if there was not an analagous bar present in each other security, S, then the most recent bar in S was backfilled in its place.

There were also several outliers present in the data that would obstruct the fitting of linear models. In order for the models to be able to learn the usual relationship between the securities, I used the middle 99% of data *for training only*. The models were still evaluated on train and test data that included outliers. The outlier removal was solely done to improve the fit of the linear models.



(a) Returns with Outliers

(b) Returns with Outliers Removed

Figure 1: Impact of Outliers on Data

1.3 Returns and Markouts

1.3.1 Returns

Asset returns represent a scaled first difference in the data, so they eliminate the serial component in the time series data along with its associated complications. The actual return as mentioned above can be defined as follows:

$$a_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

Normal approximations have been used for asset returns on the monthly time scale, however, over short intervals this assumption breaks down. The hourly returns used in this study are abundant close to 0 and extreme returns occur with greater frequency than under normality assumptions. Despite this fact, OLS is a $BLUE$ estimator, so we can still confidently estimate a line despite the distribution not being normal.

1.3.2 Markouts

Markouts are a way of computing hypothetical trade profits or losses. At every point along the time series, we hypothetically enter a trade and hold for some predefined number of time periods, after which we exit the trade. Note that the entry can be a selling (short selling) or buying trade, while the exit is simply the opposite side of the entry transaction. To specify this more rigorously, we can write:

$$M_{t,k} = \begin{cases} P_{t+k} - P_t \text{ if Buy} \\ P_t - P_{t+k} \text{ if Sell} \end{cases}$$

This is the markout at time t, held for k periods. Notice that $M^B_{t,k} = -M^S_{t,k}$, meaning that the profit or loss of a buying and selling trade at a given point in time are exact opposites (since the asset can only move one direction). A good trading signal is one that is able to separate positive from negative markouts.

2 Pairwise Mean Reversion

As mentioned in the introduction, airline companies are exposed to similar pools of risk. Oil companies are also exposed to similar pools of risk. Airline and oil companies are both exposed to the price risk of oil itself, so it would stand to reason that the stock prices of companies in this sector would move together. This assumption underpins the pairwise mean reversion models presented in this section.

2.1 Pairwise Correlations

To get an idea of the relationships present among the companies in the data set, a pairwise correlation analysis was conducted on the hourly returns of each of the securities. We can see from Figure 1 above

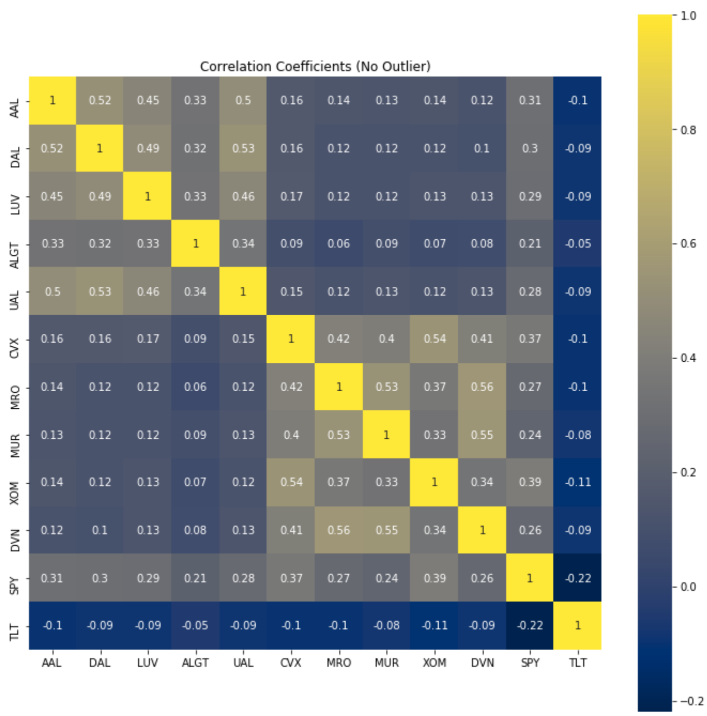
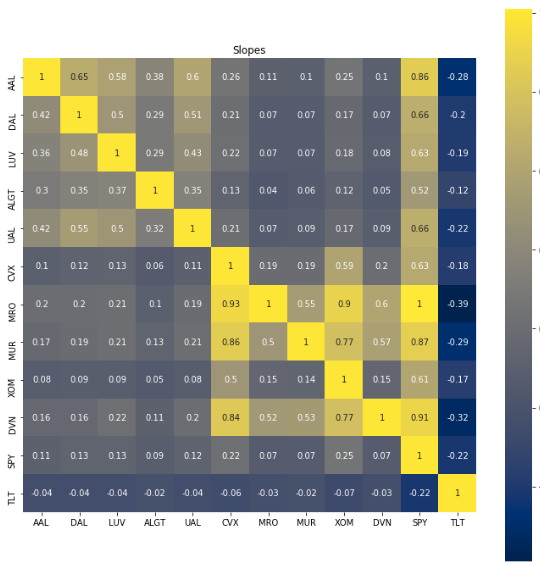


Figure 2: Pairwise Correlation - Outliers Removed

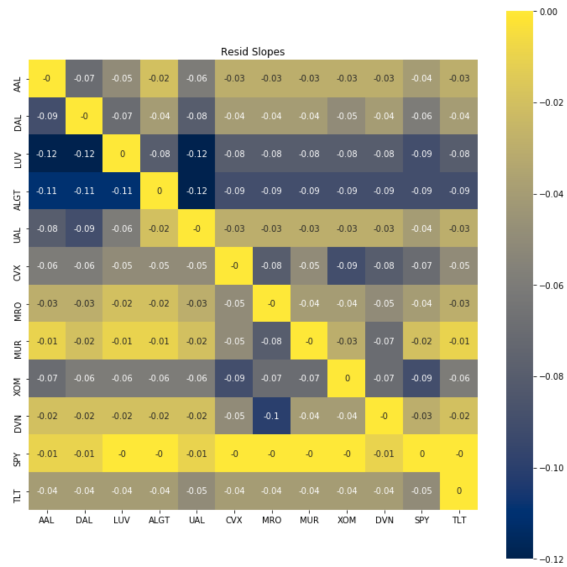
and section 6.3 in the appendix that the outliers in the data set had a noticeable positive impact on the correlations between the securities, so for training purposes, it was appropriate to remove them. There are clear sections of correlatedness in Figure 2 above. We can see that the 5 airline companies are together correlated, while the same can be said for the 5 oil companies in the data set. There is also some positive correlation between the companies in the data set and the S&P 500 as well as some negative correlation with price of bonds.

2.2 Pairwise OLS

With this understanding of the correlations between the data as well as the fact that the outlier free return data exhibits ellipsoidal linear relationships, it became clear that OLS would be a good model for this data. OLS was able to fit lines to the data well, however, because the data fail to satisfy some assumptions of OLS, inference on the OLS parameters was not possible. The coefficients of the pairwise OLS model can be found in Figure 3(a). We see that the the strength of the slope coefficients



(a) Model Slope Coefficient



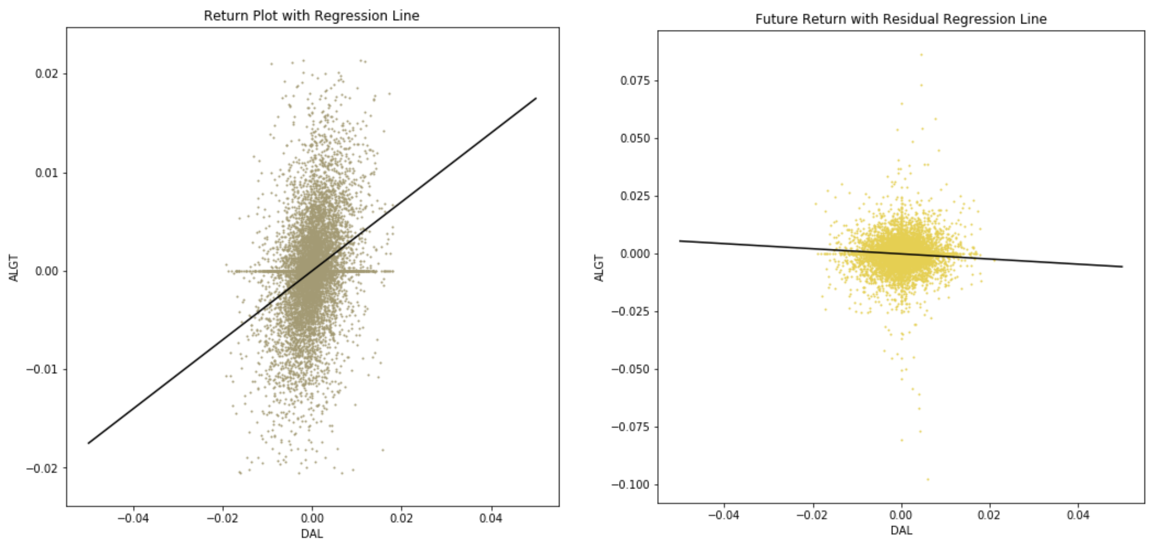
(b) Residual Slope Coefficient

Figure 3: Pairwise OLS, Y Axis Response

lines up well with the strength of the pairwise correlation between the assets. The intercept terms for these models were essentially all zero. This can be seen in section 6.5. As defined in section 1.1, the idea behind a statistical mean reverting trading strategy is to make bets on convergence to a forecasted mean return. In this case, the mean return is the return for company Y predicted at time t using company X's return under the pairwise model. The model makes money if when there is a negative residual on the model, the future return is positive, and likewise if there is a positive residual on the model, the future return is negative. In order to test if this is possible, we can run a regression on model residuals to future returns. The results of this can be found in Figure 3(b). We see that there are some pairs with negative coefficients, which indicates that there is a relationship present. Such an example is shown in Figure 4. On the left we see the regression model over the training set, while on the right we see the future returns regression model. There is a distinct downward slope in the model, but again, because the data fails OLS assumptions, we cannot tell whether the coefficient is significant. Analysis on whether this strategy is good or not is presented in the following section.

2.3 Profit and Loss Analysis

In order to assess whether the trading strategy is profitable, I introduce the notion of a Profit and Loss (Pnl) Curve derived from cumulative markouts of trades taken according to some signal parameter. Recalling the definition of markouts from section 1.3.2, as well as the fact that our trading signal for



(a) OLS Fit on No Outlier Data (b) Future Returns Regressed on Model residuals

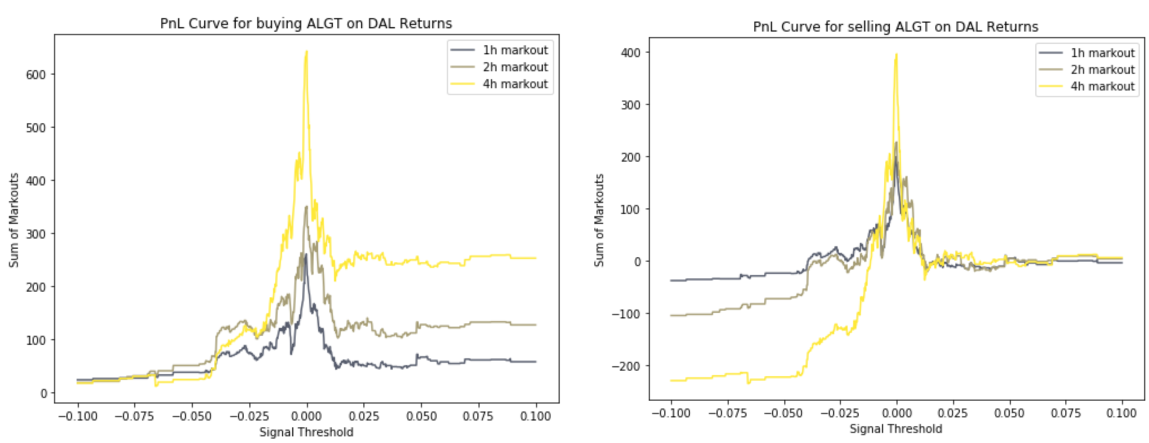
Figure 4: Return Plots with Regression

this model is the size of the model residual, we can generate a PnL curve on the *test* set. We define this as follows:

$$\mathrm{PnL}(\lambda)_k^B = \sum_{t=1}^{n} \Phi_B(\lambda, r_t) M_{t,k}^B \quad \Phi_B(\lambda, r_t) = \begin{cases} 1 \text{ if } r_t < \lambda \\ 0 \text{ if } r_t > \lambda \end{cases}$$

$$\mathrm{PnL}(\lambda)_k^S = \sum_{t=1}^{n} \Phi_S(\lambda, r_t) M_{t,k}^S \quad \Phi_S(\lambda, r_t) = \begin{cases} 1 \text{ if } r_t > \lambda \\ 0 \text{ if } r_t < \lambda \end{cases}$$

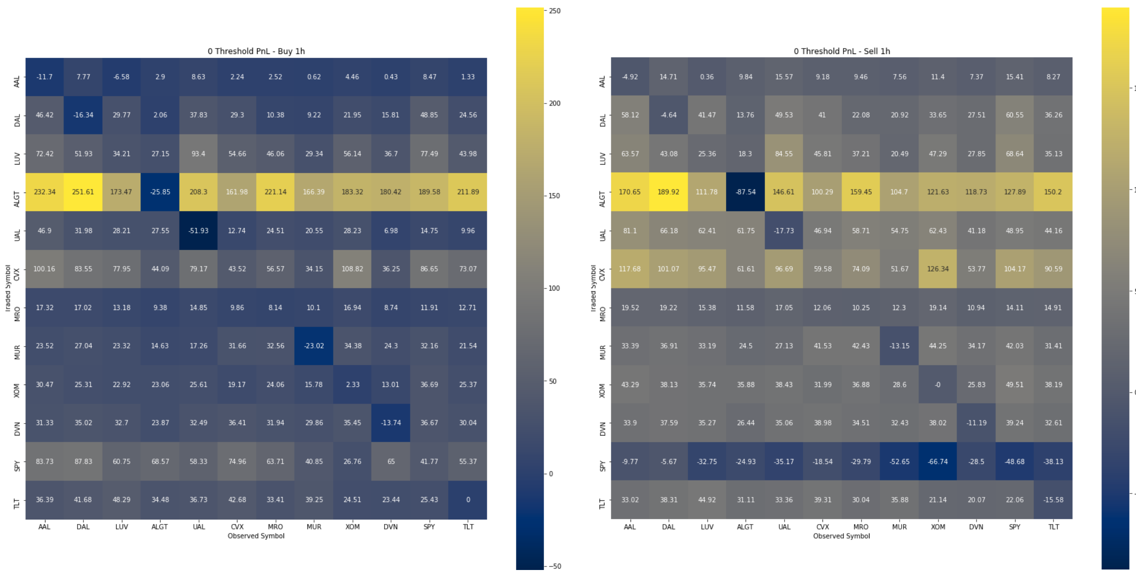
Where Φ is an indicator function to determine how the residual relates to the signal threshold λ. The curves for buying and selling PnL can be seen below in Figure 5. I chose a signal threshold of $\lambda = 0$ by observing the peaks of these curves on *train* data, and they are applied here on *test* data. The shape



(a) Buy Side PnL (b) Sell Side PnL

Figure 5: Pairwise Pnl Curves - ALGT

of the curve follows from intuition, at the left end of the buying curve, there are no data points with a sufficiently negative residual for us to buy, so the PnL is 0, while at the right end, we are making every possible trade. Because the market went up for a large part of 2020 and 2021 (except for the March 2020 crash), it makes sense that we come out positive if we buy all the time. We can see that there are noticeable peaks at 0 for both buying and selling PnL, which further justifies the choice of 0 as a threshold parameter. The PnL for a 0 threshold is reported in Figure 6 below. Surprisingly,



(a) Buy Side PnL

(b) Sell Side PnL

Figure 6: 1-Hour Markouts

most of the pairs are profitable, with Allegiant Airlines standing out as one of the most profitable companies to trade, likely due to its volatility, which can be seen in Figure 4(a). The ellipse is very steep, meaning that for returns in Delta Airlines stock, Allegiant stock is quite volatile.

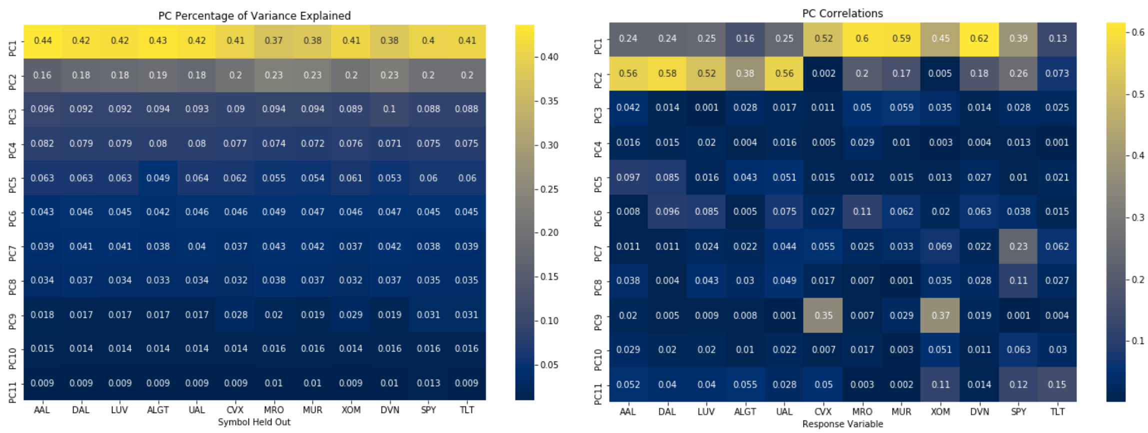
3 Basket Mean Reversion using Principal Components

Having seen the success of the pairwise analyses and the positive markouts for a $\lambda = 0$ signal, one might wonder what improvements could be had by including more predictors in the regression models. However, the returns of the stocks in the data set are correlated (Figure 2) and in some cases would be prohibitively multicollinear under an OLS model. In order to solve this problem, we can transform the returns into orthogonal principal components and regress on the transformed data.

3.1 Orthogonal Transformation

In order to prevent data leakage, the target variable was held out of the data set before a transformation was applied. I examined many possible subsets of data over which diagonalization and regression was performed (airlines on airlines, oil on oil, all on all). The best results from an RMSE perspective mandated that all of the stocks (except for the target) be included in the principal component transformation. This can be seen in section 6.1 and will be discussed further in section 3.2. The following analysis applies to the principal components extracted over all the data.

In Figure 7, we can see the results of the principal component decomposition. As expected, due to the correlations in the data, we see that the first principal component explains a significant proportion of the variance. For smaller subsets of data, say airlines, the first principal component



(a) PCA Percentage of Variance Explained (b) Principal Component Correlation with Response

Figure 7: PCA Metrics

explained upwards of 60% of the variance. The interesting observation however, is that the returns of the held out security do not always have the highest correlation with the first principal component. For example, Chevron (CVX) and Exxon Mobil (XOM) seem to correlate with the 9th principal component. This is especially interesting because from a business standpoint, CVX and XOM are very similar. In Figure 8 above, we can see the plot of American Airlines returns against the first 4

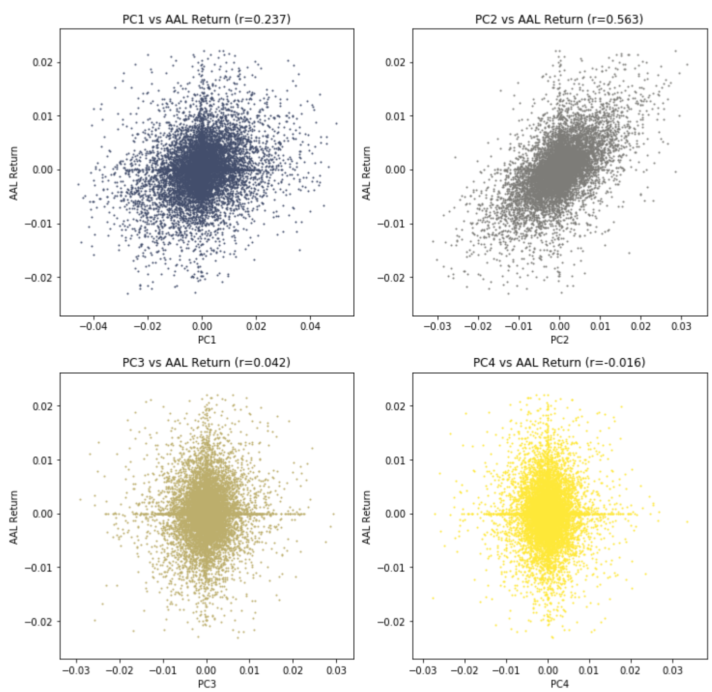


Figure 8: Decomposition Plot

principal components of the other 11 stocks. We can also see there that there appears to be a stronger relationship between AAL and PC2 than AAL and PC1.

3.2 Basket OLS

After fitting the principal componenst on *train* data, and OLS model was run with the held out symbol as response and the principal components as predictors. The coefficients of the principal components can be seen below in Figure 9. We can also see that the intercept is not meaningfully different from 0. An important thing to note is that this plot above, as well as the correlation plot found in Figure

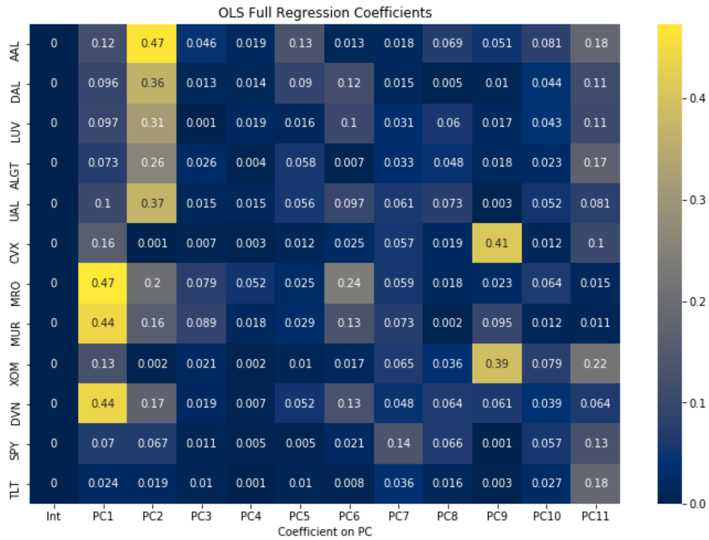


Figure 9: Full PC OLS Regression

7 display absolute values of coefficients and correlations. Because principal component analysis yields unique vectors, but not unique directions, we can flip the signs of the values for easier visual analysis.

3.2.1 Basket Lasso for Feature Selection

As mentioned in the start of section 3.1, I examined several subsets of data and settled on the full principal component set as the best set of predictors for the model. The model can be thought of as follows:

$$\hat{x}_{target} = f(T(\mathbf{X}_{-target})) \quad \mathbf{X} = \left[x_{target}, \mathbf{X}_{-target}\right]$$

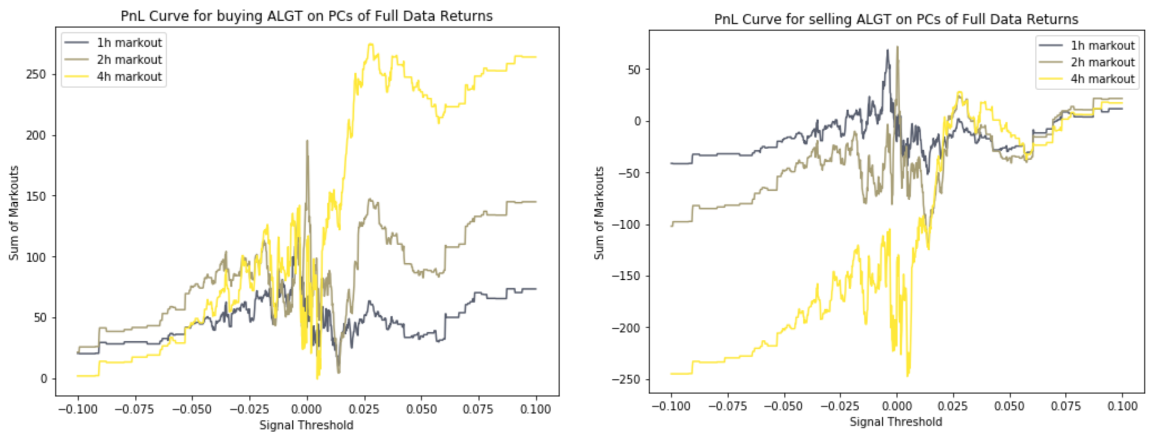
where T is the orthogonal transformation applied to the predictors and f is the function learned by OLS. Section 6.1 outlines several choices of $\mathbf{X}$ and their corresponding RMSE. I also applied a lasso regression to the data with a cross validated tuning parameter for the impact of the L1 penalty, however, I found that the optimal loss penalty was very small and that the RMSE of the original OLS model was better in nearly every case. With this understanding, we can see that the best set of features is the full set, and that the best model to use is in fact OLS.

3.3 Profit and Loss Analysis

The application of the PnL analysis in this scenario is the same as in section 2.3, and the formulas remain the same. However there is the caveat that now:

$$r_t = a_t - f(T(\mathbf{x}_{-target,t}))$$

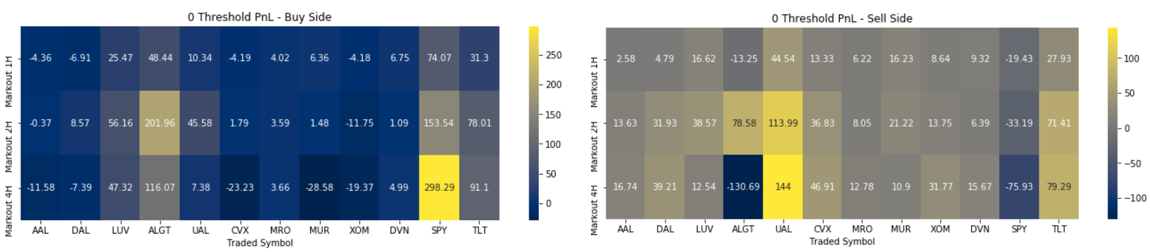
We are using a transformed set of all the predictors to forecast the expected return for an asset. It is important to note that the PCA fit and OLS fit were carried out on *train* data, and that the same parameters were carried over to evaluate the *test* data and produce the plots that are seen below.



(a) Buy Side PnL (b) Sell Side PnL

Figure 10: Basket Pnl Curves - ALGT

We can see in Figure 10 the PnL curves for Allegiant Airlines using all of the principal component predictors. They are substantially more noisy when compared to the plots in Figure 5, especially as the markout horizon grows longer. We can see the results for a 0 threshold PnL for all of the securities in Figure 11 below. It is clear that there is still some activity at $\lambda = 0$ occuring in the data, however



(a) Buy Side PnL (b) Sell Side PnL

Figure 11: Basket Model PnL

it is a lot more noisy. The threshold of 0 was maintained for ease of comparison to the other model, however it is not necessarily the case that $\lambda = 0$ is an optimal threshold for this model, since it has different characteristics. Further discussion can be found below.

4 Discussion

The unexpected element of this analysis is the fact that the basket model with more data and more meticulous preprocessing of that data happens to perform worse than the comparatively simple pair-wise model. There are several possible explanations for this and I will touch on a handful below.

4.1 The Compounding Outlier Problem

The breakdown of the full principal component model could be attributed to what I would call the "compounding outlier problem". The test set contains data from 2020 and 2021, and 2020 was a very volatile year with several extreme price moves for airline and oil companies. These were driven primarily by the shutdown of travel due to the pandemic and the brief period in April 2020 when the price of oil was negative. However, even apart from the pandemic setting, the models in this study were fitted on outlier trimmed data. When evaluated on a pairwise basis, there is some chance of encountering an outlier at any given point in time in the predictor series. However, when using k predictor series in the model, there are 2^k possible scenarios in which outliers might play a role at any time t. These outliers are thus more likely to be present in the larger model and can throw off the predictions substantially more often.

4.2 Poor Feature Selection

Another possible explanation for the lackluster performance of the basket model could be attributed to poor feature selection. Based on the correlation matrix found in Figure 2, it is hard to claim that each company's stock depends on all of the other companies. The TLT ETF is a great example - it doesn't seem to relate well to any of companies. Yet, after a principal component analysis, the left over principal components with rather poor correlation to the target are not dropped by Lasso Regression. This could be due to the distribution of the data. There is and abundance of hourly returns centered around or exactly equal to 0, and the outliers that were not removed (but rightfully deemed outliers) drive the fit to such an extent that Lasso Regression neglects to shrink their coefficients.

4.3 Assumption Breakdown

It is perhaps always the case that assumption breakdown is at least in part responsible for poor model performance. In this context, I don't refer to statistical assumptions, though the outliers may have had an impact as mentioned in section 4.1. Instead I refer to the stationarity assumptions made about the correlations between asset classes. This style of statistical mean reversion trading strategies relies on a continued statistical relationship between assets, which may cease to exist at any point in time. 2020 was a great example of this because long standing correlations between assets (eg. inverse correlation of S&P 500 and Gold) broke down. It could very well be the case that it is not reasonable to make convergence bets in the period defined by the test set.

5 Conclusion

This paper highlights two statistical mean reversion trading strategies and evaluates them on their peak signal profit or loss. The simpler pairwise trading strategy outperformed the basket strategy that utilized a larger set of features and principal component analysis to preprocess and select these features. On the surface, this is a counterintuitive result because one might believe that more data allows for better models. However, the deterioration of the basket strategy indicates the potentially problematic effects of higher dimensional data. In order to achieve comparable quality and interpretability in results, one needs exponentially cleaner data that is analyzed in a substantially more rigourous fashion.

This type of mean reverting signal shows promise, and the next steps for this analysis will likely be an implementation of this strategy in a backtest or simulation context. It is important to note that any positive results in this paper do not guarantee positive results while trading live, so readers should exercise caution if they decide to implement any of the ideas discussed in this paper.

6 Appendix

6.1 RMSE for PC Regression Models

Table 1: Airline PC Regression RMSE

Ticker	**PC1**	**Full**	**Lasso**
AAL	0.003519	0.003487	0.003487
DAL	0.002762	0.002725	0.002725
LUV	0.002839	0.002819	0.002819
ALGT	0.003561	0.003554	0.003554
UAL	0.002931	0.002902	0.002902

Table 2: Oil PC Regression RMSE

Ticker	**PC1**	**Full**	**Lasso**
CVX	0.002338	0.002136	0.002136
MRO	0.004708	0.004647	0.004647
MUR	0.004573	0.004536	0.004536
XOM	0.002240	0.002047	0.002047
DVN	0.004302	0.004281	0.004284

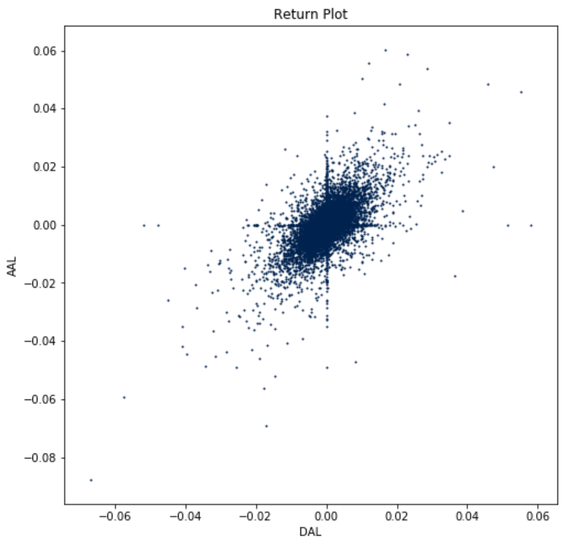
Table 3: Airline and Oil PC Regression RMSE

Ticker	**PC1**	**Full**	**Lasso**
AAL	0.004306	0.003475	0.003478
DAL	0.003450	0.002721	0.002722
LUV	0.003363	0.002809	0.002809
ALGT	0.003865	0.003552	0.003553
UAL	0.003602	0.002896	0.002896
CVX	0.002320	0.002124	0.002125
MRO	0.004851	0.004640	0.004641
MUR	0.004661	0.004530	0.004530
XOM	0.002231	0.002044	0.002044
DVN	0.004414	0.004275	0.004278

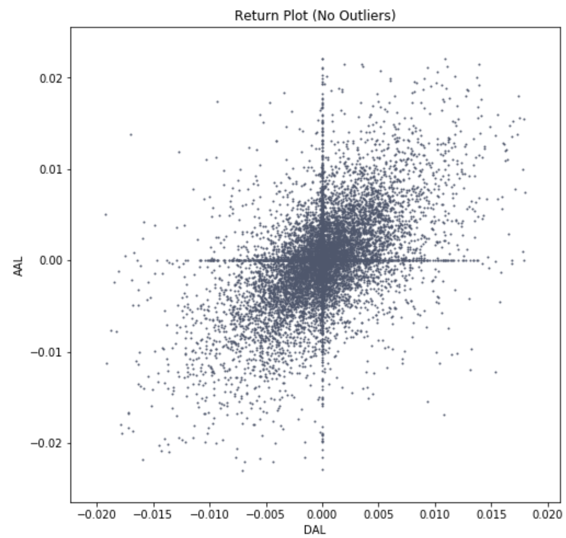
Table 4: Full PC Regression RMSE

Ticker	**PC1**	**Full**	**Lasso**
AAL	0.004301	0.003458	0.003458
DAL	0.003446	0.002713	0.002713
LUV	0.003359	0.002801	0.002801
ALGT	0.003863	0.003543	0.003545
UAL	0.003598	0.002891	0.002891
CVX	0.002316	0.002104	0.002105
MRO	0.004854	0.004634	0.004635
MUR	0.004666	0.004530	0.004531
XOM	0.002227	0.001996	0.001996
DVN	0.004418	0.004271	0.004275
SPY	0.001469	0.001329	0.001330
TLT	0.001574	0.001546	0.001546

6.2 Return Plots

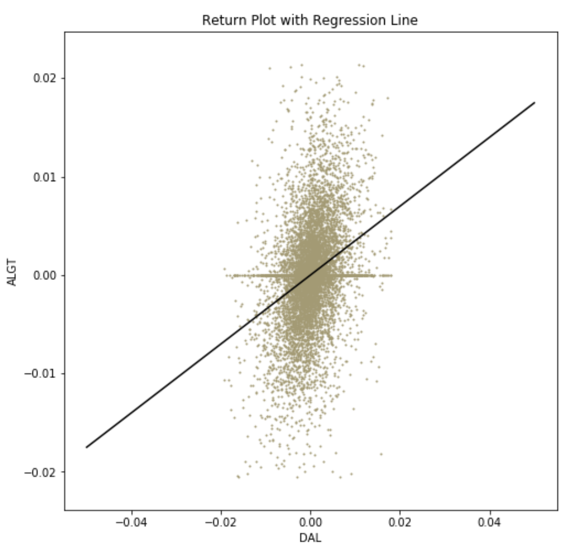


(a) Returns with Outliers

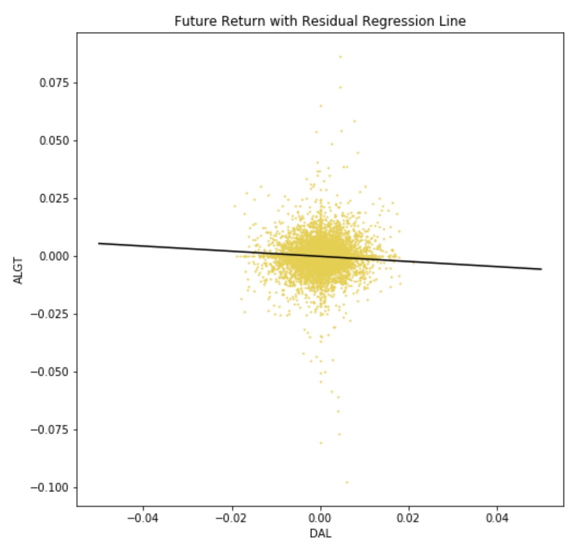


(b) Returns with Outliers Removed

Figure 12: Impact of Outliers on Data



(a) OLS Fit on No Outlier Data



(b) Future Returns Regressed on Model residuals

Figure 13: Return Plots with Regression

6.3 Correlations

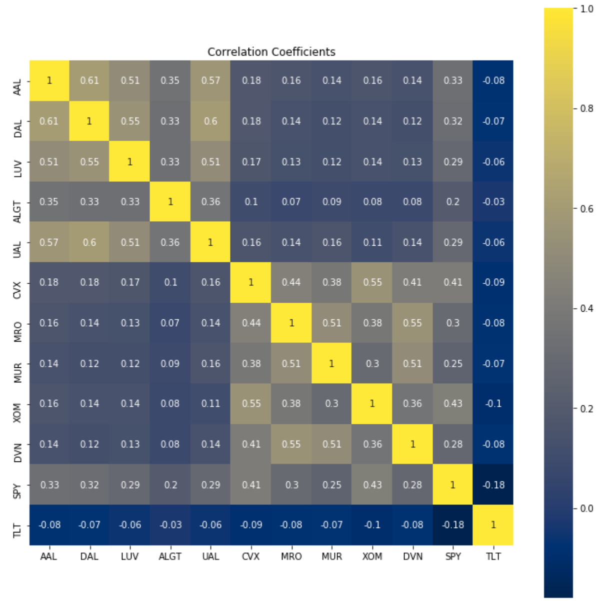


Figure 14: Pairwise Correlation - Outliers Present

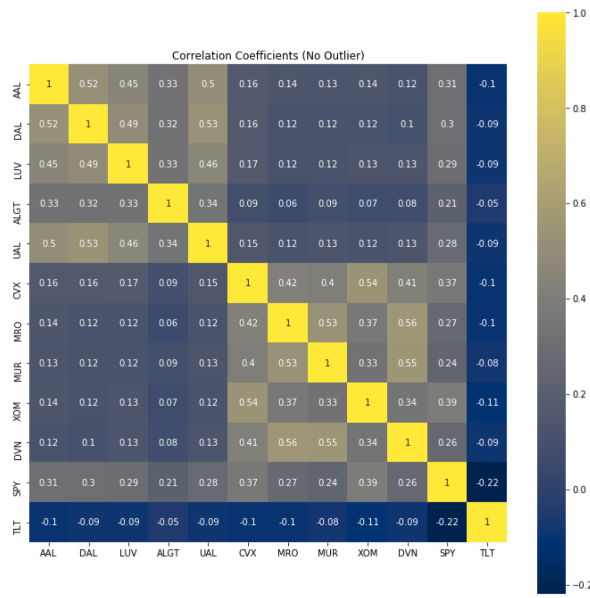


Figure 15: Pairwise Correlation - Outliers Removed

6.4 Principal Component Analysis

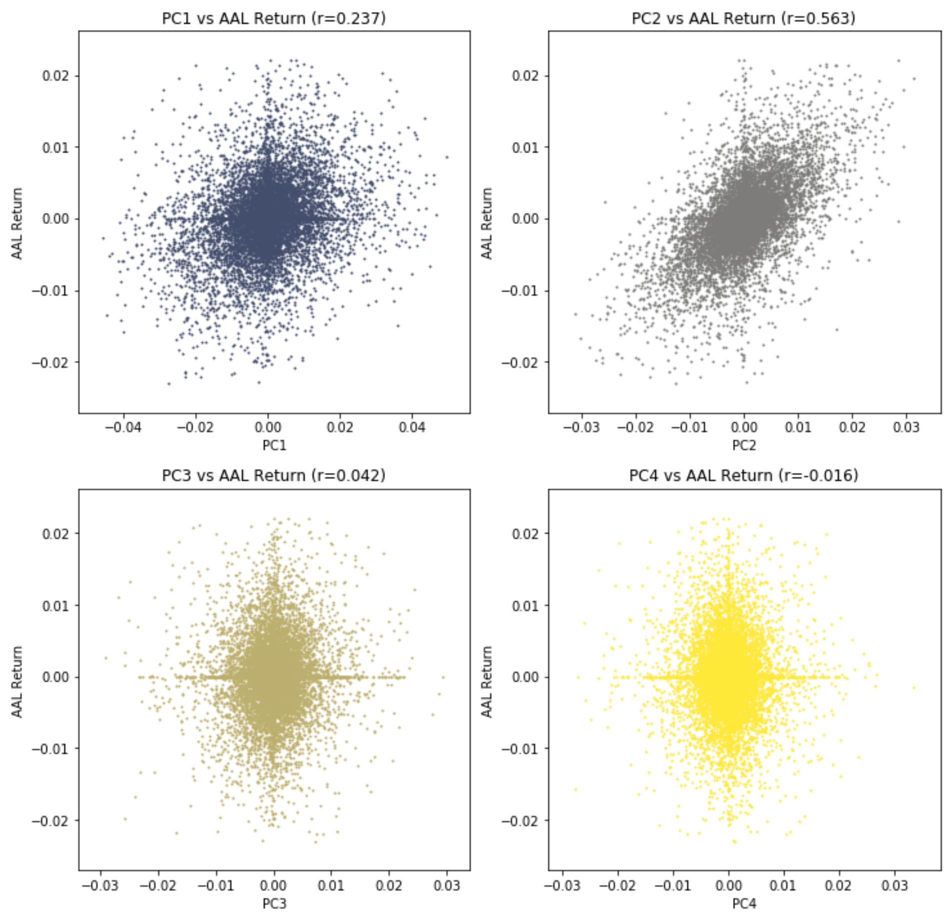
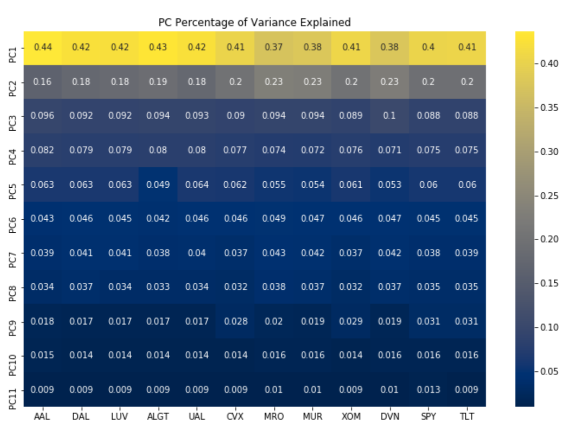
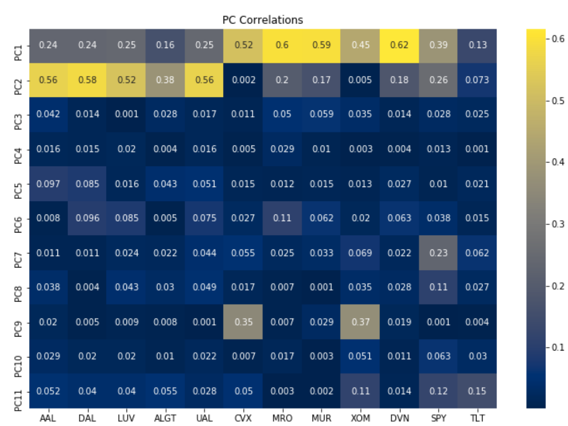


Figure 16: Decomposition Plot



(a) PCA Percentage of Variance Explained

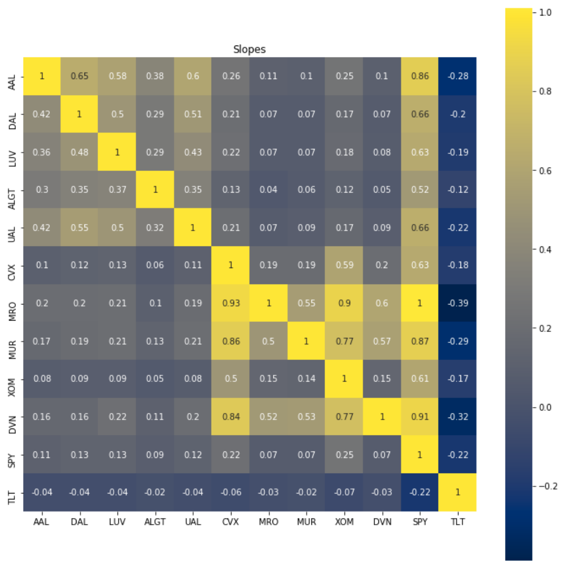


(b) Principal Component Correlation with Response

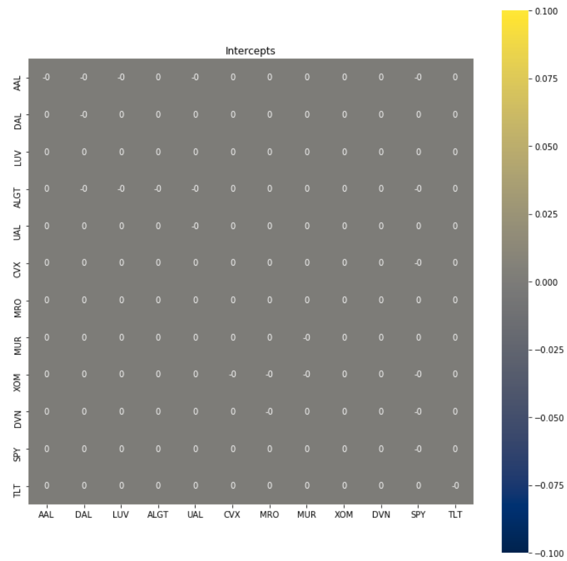
Figure 17: PCA Metrics

6.5 Regression Coefficients

6.5.1 Pairwise Regression



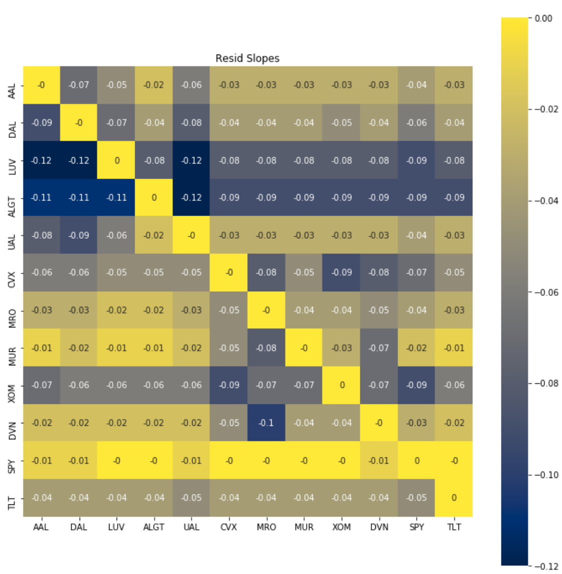
(a) Slope Coefficient



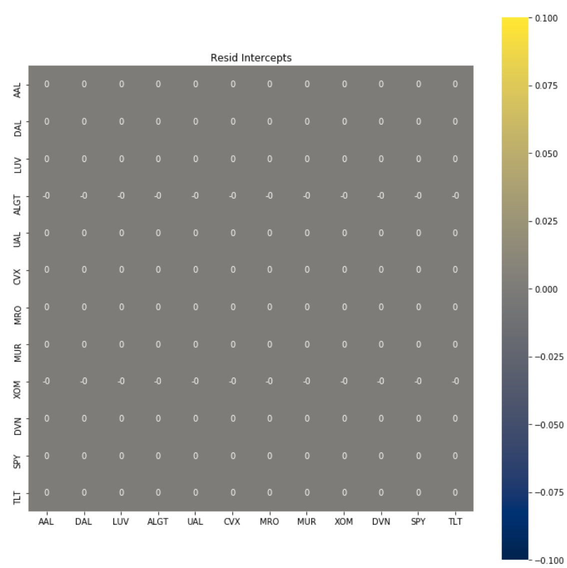
(b) Intercept

Figure 18: Pairwise Regression

6.5.2 Pairwise Residual Regression



(a) Slope Coefficient



(b) Intercept

Figure 19: Pairwise Residual Regression

6.5.3 Principal Component Regression

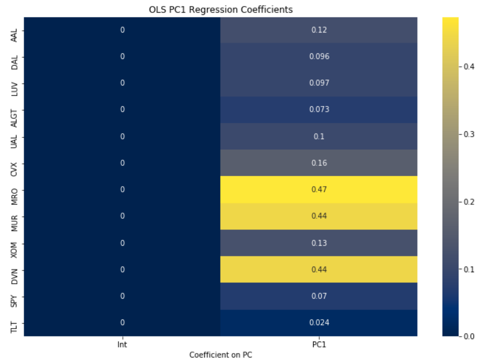


Figure 20: PC1 OLS Regression

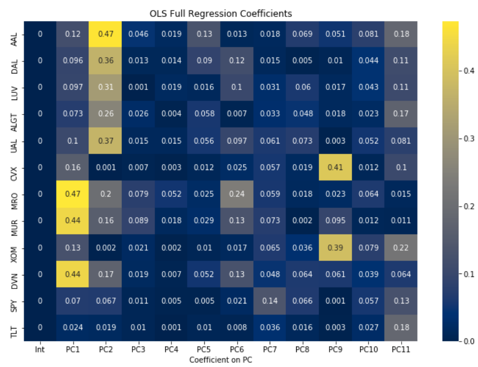


Figure 21: Full PC OLS Regression

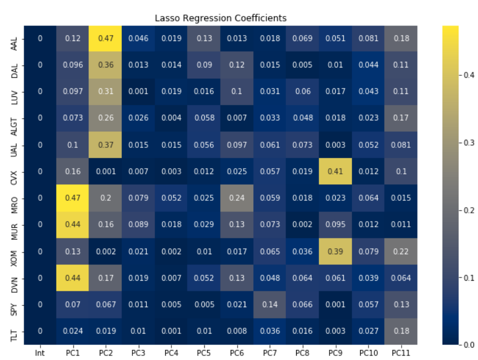
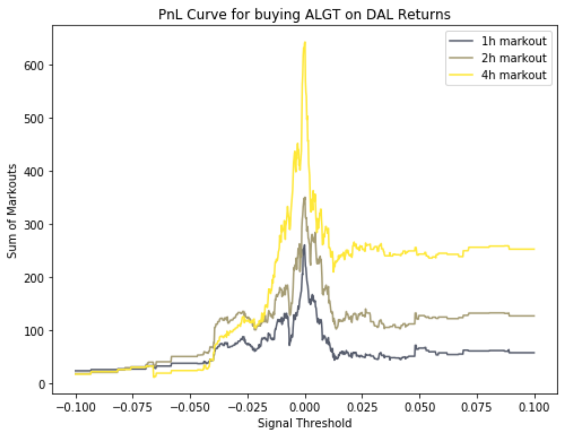
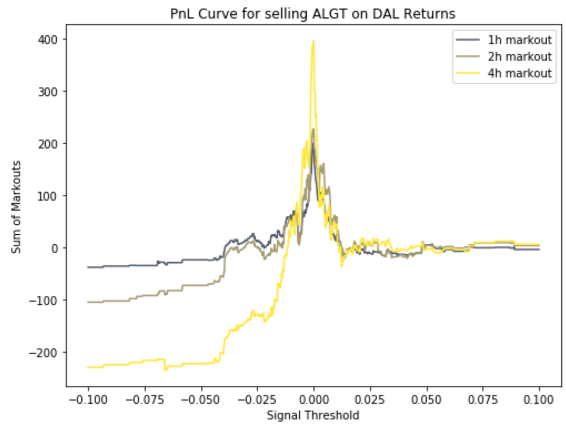


Figure 22: Lasso PC Regression

6.6 Profit and Loss Signal Curves

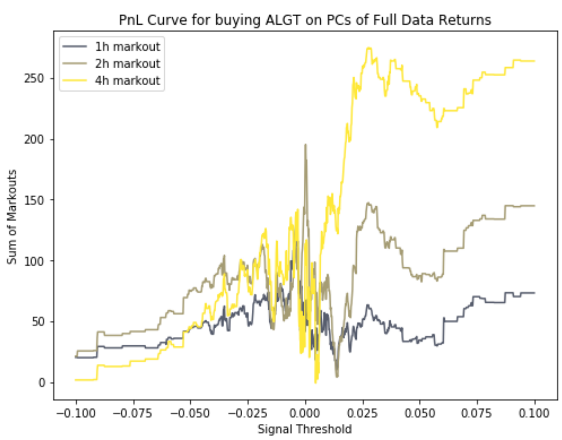


(a) Buy Side PnL

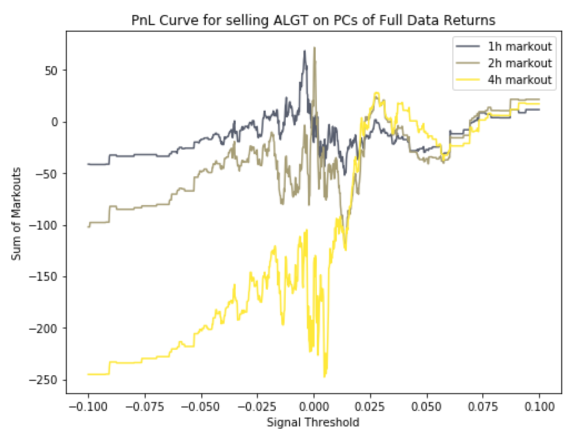


(b) Sell Side PnL

Figure 23: Pairwise Pnl Curves - ALGT



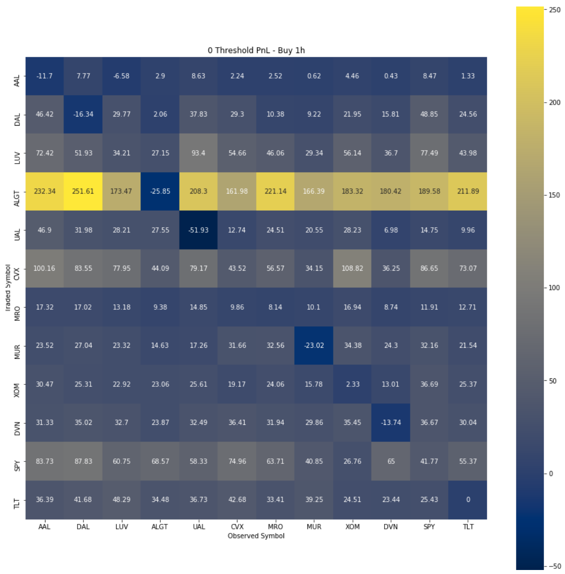
(a) Buy Side PnL



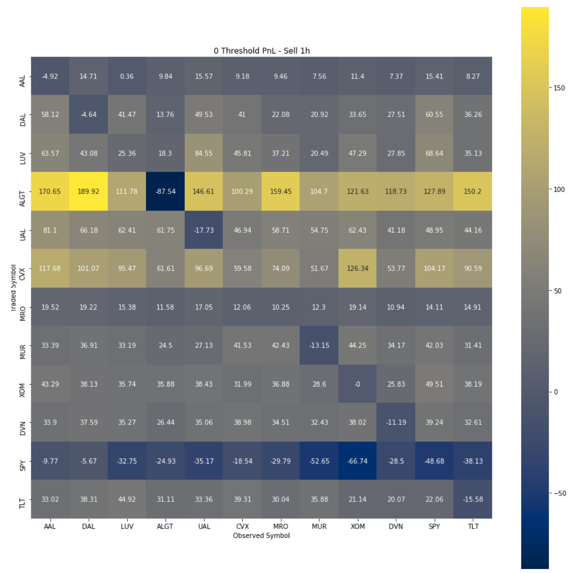
(b) Sell Side PnL

Figure 24: Basket Pnl Curves - ALGT

6.7 Profit and Loss over Markout Horizons

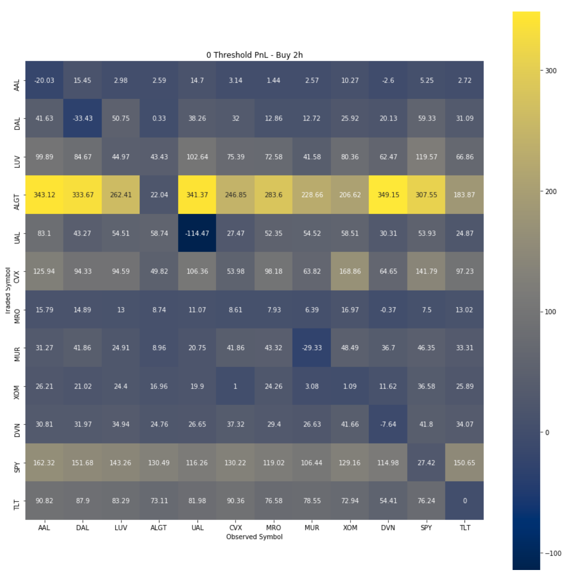


(a) Buy Side PnL

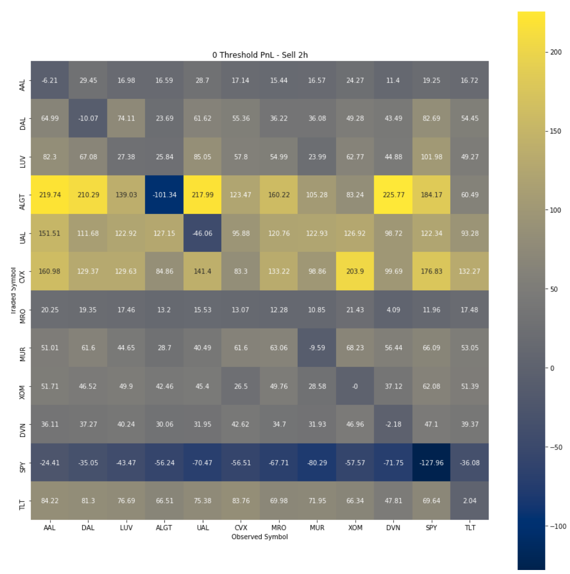


(b) Sell Side PnL

Figure 25: 1-Hour Markouts

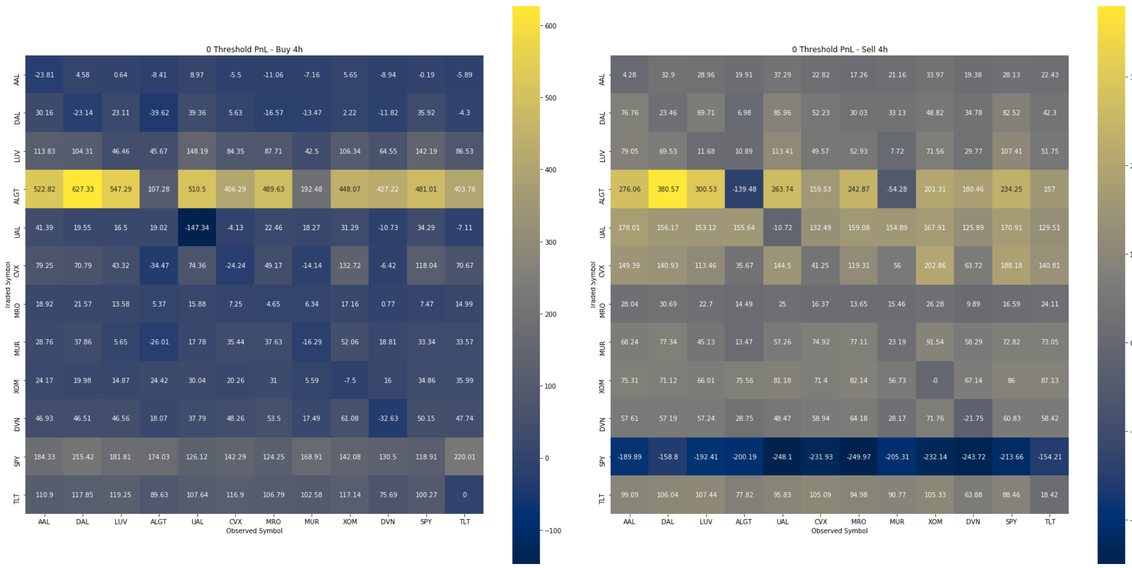


(a) Buy Side PnL



(b) Sell Side PnL

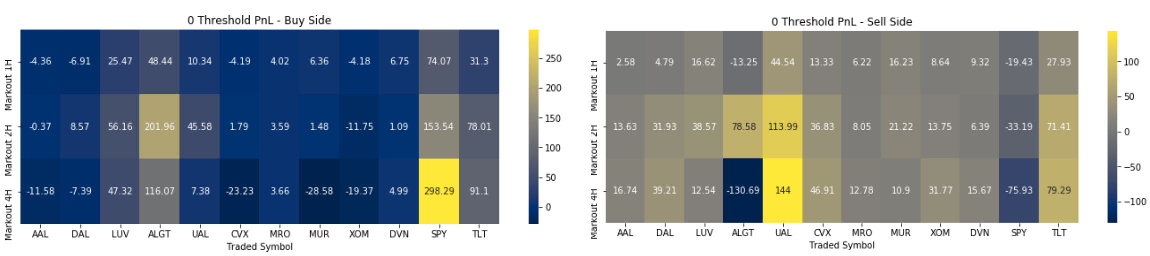
Figure 26: 2-Hour Markouts



(a) Buy Side PnL

(b) Sell Side PnL

Figure 27: 4-Hour Markouts



(a) Buy Side PnL

(b) Sell Side PnL

Figure 28: Basket Model PnL