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Magnetotelluric data analysis: removal of bias

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Two new techniques for analyzing 4-channel magnetotelluric (MT) data are described. These techniques produce estimates of the elements Z_{ij} of the impedance tensor that are unbiased by noise in the autopowers of the electric and magnetic fields. Effectively, each technique uses one field channel as a reference signal that can be correlated with the other three channels. Method 1 obtains estimates for the Z_{ij} in terms of crosspowers of the Fourier components of the electric and magnetic fields $E_x(\omega)$, $E_y(\omega)$, $H_x(\omega)$, and $H_y(\omega)$. Method 2 is a generalization of method 1, and obtains estimates for Z_{ij} in terms of weighted crosspowers. Both methods fail when the geology is one-dimensional, or two-dimensional with one electrode oriented along the strike direction. To obtain results that are stable for any geology and that are unbiased by autopower noise, at least five channels of data are required. To also minimize bias by correlated noises, one needs six channels of data, two channels of which are for fields measured at a site that is remote from the base MT station. The analysis of MT data using a remote magnetometer as a reference is discussed.

INTRODUCTION

Magnetotelluric (MT) data consist of simultaneous records of naturally occurring, time dependent magnetic field fluctuations measured at the surface of the earth, and of fluctuating electric fields induced in the ground by the magnetic fields. In the conventional MT technique, one records simultaneously the two orthogonal horizontal components of the fluctuating electric $[E_x(t)$ and $E_y(t)]$ and magnetic $[H_x(t)$ and $H_y(t)]$ fields (Figure 1). In addition, it is sometimes useful to record fluctuations in the vertical component of the magnetic field $H_z(t)$. The Fourier transforms of the fields $E_x(\omega)$, $E_y(\omega)$, $H_x(\omega)$, and $H_y(\omega)$ are related by an impedance tensor $\mathbf{Z}$ defined by

$$E_x = Z_{xx}H_x + Z_{xy}H_y, \qquad (1)$$

and

$$E_y = Z_{yx}H_x + Z_{yy}H_y. \qquad (2)$$

The goal of MT is to determine the elements of $\mathbf{Z}$ as a function of frequency, and ultimately to relate these elements to the resistivity of the ground. As with any physical measurements, the measured electric and magnetic fields contain noise that introduces uncertainties in the impedance tensor. Because neither the spatial extent nor the location of the sources producing MT signals are known, one can, in a general sense, think of noise as any electromagnetic disturbance that cannot be modeled as a plane wave incident normally on the earth. This definition includes instrumental noise.

One approach (Kao and Rankin, 1977) to determine the impedance elements is to multiply equations (1) and (2) in turn by E_x^*, E_y^*, H_x^*, and H_y^* (* denotes complex conjugate), and to average the equations over the N field values. One thus obtains eight independent equations relating the four impedance elements. These equations can be written as follows:

$$\overline{|E_x|^2} = Z_{xx}b^* + Z_{xy}c^*, \qquad (3)$$

$$a = Z_{xx}d^* + Z_{xy}e^*, \qquad (4)$$

$$b = Z_{xx}\overline{|H_x|^2} + Z_{xy}f^*, \qquad (5)$$

$$c = Z_{xx}f + Z_{xy}\overline{|H_y|^2}, \qquad (6)$$

$$\overline{|E_y|^2} = Z_{yx}d^* + Z_{yy}e^*, \qquad (7)$$

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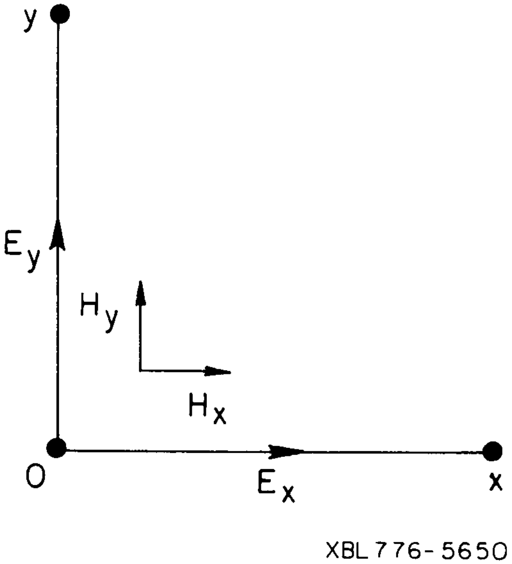

FIG. 1. Conventional 4-channel MT configuration. The three electrodes 0, x, and y are used to measure the electric field fluctuations E_x and E_y. A magnetometer measures the magnetic field fluctuations H_x and H_y.

$$d = Z_{yx}\,\overline{|H_x|^2} + Z_{yy} f^*, \tag{8}$$

$$e = Z_{yx} f + Z_{yy}\,\overline{|H_y|^2}, \tag{9}$$

and

$$a^* = Z_{yx} b^* + Z_{yy} c^*. \tag{10}$$

Here a, b, c, d, e, and f are crosspowers defined by $a = \overline{E_x E_y^*}$, $b = \overline{E_x H_x^*}$, $c = \overline{E_x H_y^*}$, $d = \overline{E_y H_x^*}$, $e = \overline{E_y H_y^*}$, and $f = \overline{H_x H_y^*}$, and the bar denotes an average. In principle, one can use any pair of equations (3) through (6) to obtain values for Z_{xx} and Z_{xy}, and any pair of equations (7) through (10) to obtain values for Z_{yx} and Z_{yy}. For example, from equations (3) and (4) one obtains

$$Z_{xy} = \frac{\overline{|E_x|^2} d^* - ab^*}{d^* c^* - e^* b^*}, \tag{11}$$

while from equations (5) and (6) one obtains

$$Z_{xy} = \frac{c\,\overline{|H_x|^2} - bf}{\overline{|H_x|^2}\,\overline{|H_y|^2} - |f|^2}. \tag{12}$$

In the absence of noise, all methods of estimating the impedance are equivalent. However, in the presence of noise, each estimate is different. The different values arise from the combination of random errors in the crosspower and autopower densities, and systematic bias errors produced by noise power in the autopowers. In equation (11) $|Z_{xy}|$ is biased upward by the noise power in $\overline{|E_x|^2}$, while in equation (12) $|Z_{xy}|$ is biased downward by the noise power in $\overline{|H_y|^2}$. The influence of random and bias errors on impedance estimates has been discussed by several authors (for example, Swift, 1967; Sims et al, 1971; and Reddy et al, 1976).

Since random errors average to zero for a sufficiently large collection of data, it is the bias errors that eventually dominate the uncertainty in MT data analysis. While there are several possible sources of bias errors (Bendat and Piersol, 1971), we assume here that bias results predominantly from noise in the measured fields, and not from systematic errors in instrumentation and data processing. For the standard methods of MT analysis (Swift, 1967) it is necessary to know the noise power in the various electric and/or magnetic channels in order to estimate the bias errors. Since these noise powers are usually unknown, one is restricted to using data for which the total noise power [determined from the coherency between the measured electric fields and electric fields predicted from the computed impedance and the measured magnetic fields (Vozoff, 1972)] is sufficiently small that the bias errors are known to be no greater than the random errors. This is a rather severe constraint in that the signal power in some of the interesting portions of the frequency spectrum is often comparable to or less than the noise power.

In the present paper we describe new approaches we have developed to obtain estimates of the impedance tensor unbiased by noise in the autopowers. We also discuss a means of minimizing possible bias by correlated noises. We first discuss methods that are applicable to the usual 4-channel MT data, with uncorrelated noise in each channel. Method 1 is a solution of the 8 simultaneous equations (3)–(10) in which the four impedance elements Z_{xx}, Z_{xy}, Z_{yx}, and Z_{yy}, and the four autopowers $\overline{|E_x|^2}$, $\overline{|E_y|^2}$, $\overline{|H_x|^2}$, and $\overline{|H_y|^2}$ are expressed entirely in terms of crosspowers. Method 2 is a generalization of method 1, and expresses the impedance in terms of weighted average crosspowers rather than linear average crosspowers. Effectively, both techniques use one field channel as a reference signal that can be correlated with the other three channels. Thus, the reference channel provides a means of lock-in detecting MT signals. As we shall see, both methods are unstable when Z_{xx} or Z_{yy} is zero; for example, when the geology is one-dimensional, $Z_{xx} = Z_{yy} = 0$, $Z_{xy} = -Z_{yx}$, and when it is two-dimensional, $Z_{xx} + Z_{yy} = 0$, with one electrode in the strike direction. Nevertheless, under the appropriate circumstances, each of the two methods may be a useful technique for handling 4-channel data.

The difficulties associated with certain geologies and electrode orientations can be circumvented if one introduces a fifth independent signal channel. We discuss the analysis of data obtained with three electric field channels and two magnetic field channels, assuming that the noise in each channel is uncorrelated with that in the other channels. We present expressions for the impedance tensor that are unbiased by noise in autopower densities, but that may be biased by noise that is correlated between two or more channels. To minimize correlated noises, one should use reference fields that are measured at a location remote from the base station. We discuss the analysis of MT data using a remote magnetometer.

ANALYSIS OF 4-CHANNEL DATA

In this section we discuss two schemes (methods 1 and 2) that can be used to analyze magnetotelluric data obtained with two electric field channels and two magnetic field channels. We assume throughout that the noise in each channel is uncorrelated with the noise in any other channel.

Method 1: Determination of impedance from average crosspowers

It is apparent from equations (3) to (10) that an estimate of the impedance tensor, not biased by the autopower estimates, should be possible since there are sufficient equations (i.e., eight) to allow one to treat the four autopowers, $\overline{|E_x|^2}$, $\overline{|E_y|^2}$, $\overline{|H_x|^2}$, $\overline{|H_y|^2}$, and the four impedance elements, Z_{xx}, Z_{xy}, Z_{yx}, Z_{yy}, as unknown quantities, and to solve for these quantities in terms of the crosspowers a, b, c, d, e, and f. Kao and Rankin (1977) were able to reduce bias in the impedance estimates by an iterative solution of these equations. In this section we present the analytic solution of this problem.

It is easiest to solve first for $\overline{|H_x|^2}$ and $\overline{|H_y|^2}$, and to express all of the other unknown quantities in terms of these two autopowers. We begin by eliminating Z_{xx} and Z_{xy} from equation (4) by finding expressions for them from equations (5) and (6):

$$Z_{xx} = \frac{b\,\overline{|H_y|^2} - cf^*}{\overline{|H_x|^2}\,\overline{|H_y|^2} - |f|^2}, \tag{13}$$

and

$$Z_{xy} = \frac{c\,\overline{|H_x|^2} - bf}{\overline{|H_x|^2}\,\overline{|H_y|^2} - |f|^2}. \tag{14}$$

Substituting equations (13) and (14) into equation (4), we find

$$|a|^2(\overline{|H_x|^2}\,\overline{|H_y|^2} - |f|^2 = (e^*ca^*)\,\overline{|H_x|^2} + d^*ba^*\,\overline{|H_y|^2} - a^*(d^*cf^* + e^*bf) \tag{15}$$

Similarly, by substituting into equation (10) solutions for Z_{yx} and Z_{yy} obtained from equations (8) and (9), one finds

$$|a|^2(\overline{H_x^2}\,\overline{H_y^2} - |f| = ec^*a\,\overline{|H_x|^2} + db^*a\,\overline{H_y^2} - a(dc^*f + eb^*f^*).| \tag{16}$$

Subtracting equation (16) from (15), we obtain the relationship between $\overline{H_x^2}$ and $\overline{H_y^2}$:

$$\overline{|H_x|^2} = \frac{1}{\mathrm{Im}(e^*ca^*)}\{-\,\mathrm{Im}(d^*ba^*)\overline{|H_y|^2} + \mathrm{Im}[a^*(d^*cf^* + e^*bf)]\}, \tag{17}$$

where $\mathrm{Im}(x)$ is the imaginary part of x. Substituting this result for $\overline{|H_x|^2}$ into equation (15) we find a quadratic equation for $\overline{|H_y|^2}$,

$$(\overline{|H_y|^2})^2 - 2u\overline{|H_y|^2} - w = 0, \tag{18}$$

where

$$u = \mathrm{Im}[d^*c^*eb + a^*(d^*cf^* + e^*bf)]/2\mathrm{Im}(d^*ba^*),$$

and

$$w = -\{\mathrm{Im}[ec^*(d^*cf^* + e^*bf)] + |f|^2\mathrm{Im}(e^*ca^*)\}/\mathrm{Im}(d^*ba^*).$$

The solution to equation (18) is

$$\overline{|H_y|^2} = u[1 \pm (1 + w/u^2)^{1/2}]. \tag{19}$$

The calculated autopower $\overline{|H_y|^2}$ will be real only if $1 + w/u^2 \geq 0$. Data leading to complex values of $\overline{|H_y|^2}$ should be rejected since complex autopowers are not physically possible. $\overline{|H_x|^2}$ is obtained by substituting equation (19) into (17). In terms of $\overline{|H_x|^2}$ and $\overline{|H_y|^2}$, the impedance elements and electric field autopowers are given by

$$Z_{xx} = (b\,\overline{|H_y|^2} - cf^*)/D, \tag{20}$$

$$Z_{xy} = (c\,\overline{|H_x|^2} - bf)/D, \tag{21}$$

$$Z_{yx} = (d\,\overline{|H_y|^2} - ef^*)/D, \tag{22}$$

$$Z_{yy} = (e\,\overline{|H_x|^2} - fd)/D, \tag{23}$$

$$\overline{|E_x|^2} = [|c|^2\,\overline{|H_x|^2} + |b|^2\,\overline{|H_y|^2} - 2\mathrm{Re}(c^*bf)]/D, \tag{24}$$

and

$$\overline{|E_y|^2} = [|e|^2\,\overline{|H_x|^2} + |d|^2\,\overline{|H_y|^2} - 2\mathrm{Re}(e^*fd)]/D, \tag{25}$$

where $D = \overline{|H_x|^2}\,\overline{|H_y|^2} - |f|^2$, and $\mathrm{Re}(x)$ is the real part of x.

In equation (19), there are two possible solutions for $\overline{|H_y|^2}$ corresponding to the positive and negative values of the square root. The remaining problem is to determine which of the two solutions is correct. It is evident from equation (17) that $\overline{|H_x|^2}$ is real when $\overline{|H_y|^2}$ is real and, therefore, that the electric field autopowers obtained from equations (24) and (25) are also real when $\overline{|H_y|^2}$ is real. Consequently, no information regarding the selection of the correct root in equation (19) is obtained from the imaginary parts of equations (3) and (7), since they are identically zero. All of the information in equations (3) to (10) has been utilized.

In the absence of noise, it is obvious that one can determine the correct value for $\overline{|H_y|^2}$ by comparing the calculated and measured autopowers. In the presence of noise, the situation is, in general, rather complicated. To determine criteria for the correct choice of the sign in equation (19) we generated MT data on a PDP-11/20 computer for a known impedance tensor. A varying amount of noise was superimposed on each electric and magnetic channel (the Appendix gives details of the simulation). We found that if there is noise on one channel only (for example, H_x), the autopowers for the remaining (noise-free) channels (e.g., E_x, E_y, H_y) calculated using the correct solution of equation (19) agree *exactly* with the measured autopowers. If there is noise on more than one channel, all of the calculated autopowers are influenced by noise, and none agrees exactly with the measured value. In this general case, we have found that the following criteria, listed in order of decreasing priority, enabled us to obtain unbiased estimates of the impedance elements and the autopowers.

A) Compute all autopowers using both signs in equation (19). If one sign leads to autopowers that are all positive, and the other leads to one or more negative autopowers, assume that the former sign is correct. If each sign leads to one or more negative autopowers, the data should be rejected.

B) If both signs lead to positive autopowers, compute the absolute values of the logarithms of $\overline{|E_x|^2_c}/\overline{|E_x|^2_m}$, $\overline{|E_y|^2_c}/\overline{E_y|^2_m}$, $\overline{|H_x|^2_c}/\overline{H_x|^2_m}$, and $\overline{|H_y|^2_c}/\overline{|H_y|^2_m}$ for each sign, where the subscripts c and m denote calculated and measured quantities. The sign in equation (19) that produces the *smallest* absolute value of any of the logarithms is assumed to be correct. This procedure ensures that we obtain the correct root in the case where there is significant noise in only one channel.

C) If the value of the calculated autopower is significantly higher than the measured autopower, there is a significant error due to random noise. Data that meet criterion (B) can be further screened by rejecting those for which the ratios of calculated to measured autopowers are significantly greater than a cut-off value $S(\geq 1)$. The value of S can be selected at will, but must be the same for all channels to avoid biasing the impedance tensor. As the cut-off value is made closer to unity, the computed impedance tensor becomes more accurate, but fewer sets of data pass the criterion.

As an example of this technique, we used our computer simulation on the two-dimensional impedance tensor $Z_{xx} = -Z_{yy} = 2(1 - i)$ and $Z_{xy} = -Z_{yx} = 3(1 - i)$. The values of H_x and H_y were chosen randomly, and the appropriate values of E_x and E_y were calculated (Appendix). Noise was added to each channel. The noise-to-signal power ratios were 1.5 for the electric channels, and 1.0 for the magnetic channels. The impedance tensor was calculated from equations (20) to (23) using the criteria (A) to (C) to determine $\overline{|H_y|^2}$. There were 256 crossproducts in each estimate of the average crosspowers. The calculation was repeated 256 times, using new data each time for the electric and magnetic fields. For each element (Z_{ij}) of the impedance tensor, we computed the average value,

$$\bar{Z}_{ij} = K^{-1}\sum_{\ell=1}^{K} Z_{ij}^{(\ell)},$$

and the sample variance

$$\sigma^{ij} = \left[K^{-1}\sum_{\ell=1}^{K} |Z_{ij} - \bar{Z}_{ij}|^2\right]^{1/2},$$

where K is the number of data points for which all selection criteria were satisfied. The expected standard deviation of the mean value of $|\bar{Z}_{ij}|$, $\Delta\bar{Z}_{ij}$, is approximately $\pm\sigma_{ij}/K^{1/2}$. In Table 1 we show the results for a cut-off value $S = 1.5$. Of the 256 repetitions of the calculation, the selection criteria were satisfied 110 times, so that $K = 110$. We see that the discrepancy between the true and calculated values of the impedance tensor (ΔZ_{ij}) is generally within one standard deviation, and hence that there is no significant bias.

The ability to express the impedance and autopowers entirely in terms of crosspowers is due to the correlation between E_x and E_y. In equation (3), E_y acts as a reference signal (in the sense of lock-in detection) for E_x, while in equation (10) E_x acts as a reference signal for E_y. If Z_{xx} or $Z_{yy} = 0$ the solution to equations (3)–(10) becomes indeterminate. Thus, an unbiased estimate of the impedance tensor cannot be obtained when Z_{xx} or Z_{yy} is zero; for example, when the geology is one-dimensional ($Z_{xx} = Z_{yy} = 0$, $Z_{xy} = -Z_{yx}$), or when the geology is two-dimensional ($Z_{xx} + Z_{yy} = 0$) with one electrode in the strike direction ($Z_{xx} = Z_{yy} = 0$). To avoid the instability for a two-dimensional geology, one should first roughly locate the strike direction, and make sure that neither electric field measurement is parallel to the strike. Ideally, one would choose the orientation so that $|Z_{xy}| \approx |Z_{yx}|$.

Method 2: Determination of impedance from weighted averages of crosspowers

In the previous section, we obtained an unbiased estimate of $\mathbf{Z}$ by multiplying equations (1) and (2) in turn by a single field component, and solving the resulting eight equations for the impedance elements in terms of the average crosspowers a, b, c, d, e, f. Each estimate made use of all the information contained in the crosspowers, and hence was the only possible unbiased estimate. In this section, we multiply equations (1) and (2) by more complicated functions of the various fields to obtain estimates of the impedance elements in terms of *weighted* averages of crosspowers. As we shall see, this technique yields an infinite number of estimates which are unbiased by autopowers. Method 1 can be regarded as a special case of method 2. Method 2 avoids introducing autopowers, and thus eliminates the problem of determining the sign in equation (19). Further, method 1 fails completely if all the data were rejected by the selection criteria, whereas method 2 could produce impedance estimates from the same set of data.

Table 1. Calculation of impedance tensor elements from computer-simulated 4-channel MT data using crosspower analysis (method 1). The noise-to-signal power ratios were 1.5 and 1.0 for the electric and magnetic channels, respectively, and $S = 1.5$.

Element	True value	Calculated value, $\bar{Z}_{ij}$	σ_{ij}	$\Delta\bar{Z}_{ij}$
Z_{xx}	$2(1-i)$	$2.15 - 2.04i$	± 1.19	± 0.12
Z_{xy}	$3(1-i)$	$3.08 - 3.14i$	± 1.83	± 0.18
Z_{yx}	$-3(1-i)$	$-3.03 + 3.11i$	± 1.67	± 0.17
Z_{yy}	$-2(1-i)$	$-1.99 + 2.01i$	± 1.21	± 0.12

Consider again equation (1). Let λ and λ' be two distinct, but as yet unspecified functions. For the ith values of E_x, H_x, and H_y, λ and λ' take the values λ_i and λ_i'. If one multiplies equation (1) in turn by λ and λ', and averages over all N data points, one obtains

$$\overline{\lambda E_x} = Z_{xx}\,\overline{\lambda H_x} + Z_{xy}\,\overline{\lambda H_y}, \tag{26}$$

and

$$\overline{\lambda' E_x} = Z_{xx}\,\overline{\lambda' H_x} + Z_{xy}\,\overline{\lambda' H_y}, \tag{27}$$

where

$$\overline{\lambda E_x} = (1/N)\sum_{i=1}^{N} \lambda_i E_x^{(i)}, \ldots .$$

These equations are linearly independent, provided the determinant $\overline{\lambda H_x}\,\overline{\lambda' H_y} - \overline{\lambda' H_x}\,\overline{\lambda H_y} \neq 0$, in which case they can be solved for Z_{xx} and Z_{xy}:

$$Z_{xx} = \frac{\overline{\lambda E_x}\,\overline{\lambda' H_y} - \overline{\lambda H_y}\,\overline{\lambda' E_x}}{\overline{\lambda H_x}\,\overline{\lambda' H_y} - \overline{\lambda' H_x}\,\overline{\lambda H_y}}, \tag{28}$$

and

$$Z_{xy} = \frac{\overline{\lambda H_x}\,\overline{\lambda' E_x} - \overline{\lambda' H_x}\,\overline{\lambda E_x}}{\overline{\lambda H_x}\,\overline{\lambda' H_y} - \overline{\lambda' H_x}\,\overline{\lambda H_y}}. \tag{29}$$

In a similar way, provided $\overline{\xi H_x}\,\overline{\xi' H_y} - \overline{\xi' H_x}\,\overline{\xi H_y} \neq 0$, one obtains expressions for Z_{yx} and Z_{yy} from equation (2):

$$Z_{yx} = \frac{\overline{\xi E_y}\,\overline{\xi' H_y} - \overline{\xi H_y}\,\overline{\xi' E_y}}{\overline{\xi H_x}\,\overline{\xi' H_y} - \overline{\xi' h_x}\,\overline{\xi H_y}}, \tag{30}$$

and

$$Z_{yy} = \frac{\overline{\xi H_x}\,\overline{\xi' E_y} - \overline{\xi' H_x}\,\overline{\xi E_y}}{\overline{\xi H_x}\,\overline{\xi' H_y} - \overline{\xi' H_x}\,\overline{\xi H_y}} \tag{31}$$

Table 2. Examples of three sets of weighting functions for weighted average method (method 2).

Trial	λ	λ'	ξ	ξ'
(a)	E_y^*	$E_y^*E_y^*H_x$	E_x^*	$E_x^*E_x^*H_x$
(b)	E_y^*	$E_y^*E_y^*H_y$	E_x^*	$E_x^*E_x^*H_y$
(c)	$E_y^*E_y^*H_x$	$E_y^*E_y^*H_y$	$E_x^*E_x^*H_x$	$E_x^*E_x^*H_y$

where ξ and ξ' are again unspecified functions, and

$$\overline{\xi E_y} = (1/N)\sum_{i=1}^{N} \xi_i E_{yy}^{(i)}, \ldots .$$

In the absence of noise, there are no constraints on λ, λ', ξ, ξ'. These functions could depend on the electric and magnetic fields, but equally well could be sequences of random numbers. In the presence of noise, certain restrictions must be imposed to obtain stable, unbiased estimates of the impedance elements from equations (28)–(31). The estimates are stable, provided the denominators do not tend to zero as $N \to \infty$. This requirement implies that λ, λ', ξ, and ξ' must be functions of the electric and/or magnetic fields, since otherwise all the averages (for example, $\overline{\lambda E_x}$) tend to zero as $N \to \infty$. The estimates will be unbiased if all the weighted averages approach the noise-free weighted averages in the limit $N \to \infty$. If the impedance elements are to be both stable and unbiased, it is straightforward to show that λ and λ' must be of the form

$$\lambda = \rho E_y^*, \tag{32}$$

and

$$\lambda' = \eta E_y^*, \tag{33}$$

where ρ and η are either unity or any combination of $E_xE_y^*$, $H_xE_y^*$, $H_yE_y^*$, and $E_yE_y^*$. Thus, the weighted averages $\overline{\lambda E_x}$, $\overline{\lambda' E_x}$, etc., becomes $\overline{\rho E_xE_y^*}$, $\overline{\eta E_xE_y^*}$, etc., where ρ and η represent weighting functions for the crosspowers $E_xE_y^*$, $H_xE_y^*$, and $H_yE_y^*$. The weighting function $E_yE_y^*$ contains the noise power in E_y, but it can be shown that this does not introduce bias into equations (26) and (27), provided the noise in E_y is uncorrelated with that in E_x, H_x, and H_y.

By similar arguments, one can show that ξ and ξ' must be of the form

$$\xi = \mu E_x^*, \tag{34}$$

and

$$\xi' = \nu E_x^*, \tag{35}$$

where μ and ν are unity or any combination of $E_yE_x^*$, $H_xE_x^*$, $H_yE_x^*$, and $E_xE_x^*$. The quantities μ and ν are weighting functions for the crosspowers $E_yE_x^*$, $H_xE_x^*$, and $H_yE_x^*$.

We tested the method of weighted averages using computer-simulated MT data with the three different sets of weighting functions shown in Table 2. We used the same impedance elements, noise-to-signal power ratios, and number of crossproducts in the average crosspowers as in the test of method 1. We performed 100 independent calculations for each impedance element. The results obtained for the functions (a) are shown in Table 3, and should be compared with those obtained using method 1 in Table 1. The weighted average technique yields unbiased impedance elements, but the sample variances are three to four times greater than those obtained with method 1. We also computed the impedance elements using weighting functions (b) and (c) in Table 2, and the same random numbers for the signal and noise fields as were used with (a), and found that the calculated impedances and standard deviations were similar to those of trial (a). By averaging-together the results for the three sets of weighting functions, we reduced the standard deviations shown in Table 3 by roughly a factor $\sqrt{3}$.

As with method 1, the weighted average technique for obtaining unbiased estimates for the impedance tensor requires that one of the two orthogonal electric channels be used as a reference channel. Consequently, the technique is unstable over one- and two-dimensional geology if one of the channels is in the strike direction. We suspect that the success of the method with real MT data will depend on how the signal and noise levels vary from one data segment to the next. If the signals vary while the noise levels remain roughly constant, one might expect the weighted average method to be somewhat better than is indicated by our computer simulation, because the analy-

Table 3. Calculation of impedance tensor elements from computer simulated 4-channel MT data using weighted averages, and the weighting functions (a) from Table 2. The noise-to-signal power ratios were 1.5 and 1.0 for the electric and magnetic fields, respectively. One-hundred independent calculations were used to obtain the average values and standard deviations.

Element	True value	Calculated value, $\overline{Z}_{ij}$	σ_{ij}	$\Delta \overline{Z}_{ij}$
Z_{xx}	$2(1-i)$	$2.10 - 2.43i$	± 4.2	± 0.42
Z_{xy}	$3(1-i)$	$3.18 - 2.37i$	± 6.4	± 0.64
Z_{yx}	$-3(1-i)$	$-3.47 + 1.92i$	± 8.3	± 0.83
Z_{yy}	$-2(1-i)$	$-1.74 + 2.65i$	± 6.0	± 0.60

sis will weight more heavily those terms in the crosspower averages that have the largest amplitudes. Although method 1 gave better results than method 2 for this particular simulation, we emphasize that this is not necessarily always the case. For a given set of MT data, it is not clear a priori which of the two analysis techniques will be better. To obtain the best estimates for the impedance elements, one should probably try both methods.

ANALYSIS OF MT DATA USING ADDITIONAL REFERENCES

Both analysis methods described above for four signal channels become unstable for a one-dimensional and two-dimensional impedance tensor whose diagonal elements are zero. Thus, while the methods may be applicable to some practical situations, they are not completely general. To obtain an impedance estimate that is unbiased by autopower noise and that is stable for any geology, it is necessary to introduce a fifth (electric or magnetic) reference channel. In this section we discuss the analysis of MT data when a third electric channel is used. The analysis is valid only for uncorrelated noises. To minimize the influence of correlated noises on estimates of the impedance tensor, we show that one needs six channels of MT data. Two of these channels (electric and/or magnetic) are used as references and are measured at a site remote from the MT base station. We discuss below the analysis of MT data when a remote magnetometer is used. We derive an expression for the impedance tensor that is stable for any geology, is unbiased by autopower noise, and is unbiased by correlated noises, provided the noise in the reference fields is not coherent with the noise in the fields at the base station.

Reference channel crosspower analysis 1

Sims and Bostick (1969) proposed that one could obtain an unbiased estimate of the impedance tensor if one made two additional independent measurements of the electric field, E_{xr} and E_{yr}. One could replace the autopowers $\overline{|E_x|^2}$ and $\overline{|E_y|^2}$ with the crosspowers $\overline{E_x E_{xr}^*}$ and $\overline{E_y E_{yr}^*}$ [for example, in equation (11)], to obtain expressions that are unbiased by noise in the electric field. We first discuss an analysis scheme that is based on this suggestion, but that requires a total of only three electric field channels. A convenient configuration for the third reference channel E_r is shown in Figure 2, where

$$E_r = E_{xr} \cos\theta + E_{yr} \sin\theta. \qquad (36)$$

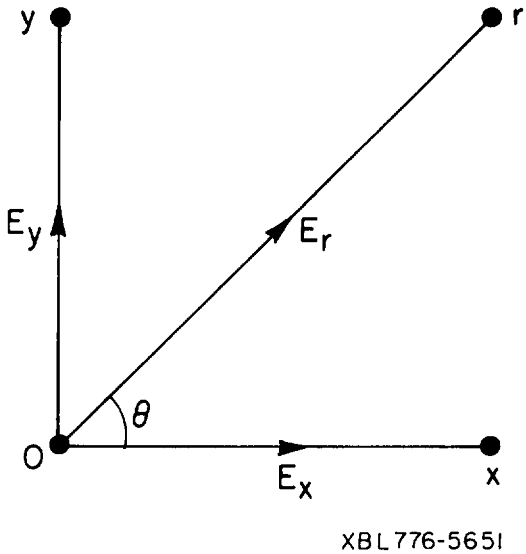

FIG. 2. Electrode configuration used to measure the three electric fields E_x, E_y, and E_r.

In the absence of noise, and if the ground is homogeneous over the array, we have $E_{xr} = E_x$ and $E_{yr} = E_y$. We can estimate E_{xr} by approximating E_{yr} by E_y in equation (36) to obtain

$$E_{xr} \approx (E_r - E_y \sin\theta)/\cos\theta. \qquad (37)$$

Since equation (37) does not depend explicitly on the measured field E_x, one can replace $\overline{|E_x|^2}$ in equation (3) with $\overline{E_x E_{xr}^*}$ to obtain unbiased estimates of Z_{xx} and Z_{xy} from equations (3) and (4). Similarly, one can obtain unbiased estimates of Z_{yx} and Z_{yy} from equations (7) and (10) by replacing $\overline{|E_y|^2}$ with $\overline{E_y E_{yr}^*}$, where

$$E_{yr} = (E_r - E_x \cos\theta)/\sin\theta. \qquad (38)$$

However, there are disadvantages with this technique. If θ is not known accurately, fractions of E_y and E_x appear as noise in E_{xr} and E_{yr}, respectively. Furthermore, even if the locations of the electrodes are known precisely, inhomogeneities in the ground cause a similar mixing of the field components. Since the magnitudes of E_x and E_y often differ by more than an order of magnitude, the noise induced in the channel with the smaller signal can be substantial.

Reference channel crosspower analysis 2

The above problems can be circumvented, if one multiplies equations (1) and (2) in turn by E_r^* and by the complex conjugate of the orthogonal component of the electric field and averages over all data segments to obtain:

$$\overline{E_x E_r^*} = Z_{xx} \overline{H_x E_r^*} + Z_{xy} \overline{H_y E_r^*}, \quad (39)$$

$$\overline{E_x E_y^*} = Z_{xx} \overline{H_x E_y^*} + Z_{xy} \overline{H_y E_y^*}, \quad (40)$$

$$\overline{E_y E_r^*} = Z_{yx} \overline{H_x E_r^*} + Z_{yy} \overline{H_y E_r^*}, \quad (41)$$

and

$$\overline{E_y E_x^*} = Z_{yx} \overline{H_x E_x^*} + Z_{yy} \overline{H_y E_x^*}. \quad (42)$$

These equations contain no autopowers and are true for *any* E_r^*. Therefore, they yield unbiased estimates of the impedance elements that are uninfluenced by inaccurate placement of the reference electrode.

The impedance tensor derived from equations (39)–(42) is not the only unbiased estimate possible with five data channels. One may relate any two fields (electric or magnetic) A and B to any other two fields P and Q by a transconductance tensor $\mathbf{G}$ defined by

$$A = G_{11} P + G_{12} Q, \quad (43)$$

and

$$B = G_{21} P + G_{22} Q. \quad (44)$$

If R is the field from a fifth channel one can obtain an unbiased estimate of $\mathbf{G}$ by multiplying in turn equation (43) by B^* and R^*, and equation (44) by A^* and R^*, and averaging each of the equations over all data to find

$$\overline{AB^*} = G_{11} \overline{PB^*} + G_{12} \overline{QB^*}, \quad (45)$$

$$\overline{AR^*} = G_{11} \overline{PR^*} + G_{12} \overline{QR^*}, \quad (46)$$

$$\overline{BA^*} = G_{21} \overline{PA^*} + G_{22} \overline{QA^*}, \quad (47)$$

and

$$\overline{BR^*} = G_{21} \overline{PR^*} + G_{22} \overline{QR^*}. \quad (48)$$

Once $\mathbf{G}$ is known, $\mathbf{Z}$ can be computed. For five data channels, one can show that there are six independent pairs of equations (43) and (44) leading to six independent estimates for $\mathbf{Z}$ that contain no autopowers. If $A = E_x$, $B = E_y$, $P = H_x$, $Q = H_y$, and $R = E_r$, one has $\mathbf{G} = \mathbf{Z}$, whereas if $A = H_x$, $B = H_y$, $P = E_x$, $Q = E_y$, and $R = E_r$, $\mathbf{G}$ is the admittance.

With five data channels it is apparent that one can readily obtain an estimate for $\mathbf{Z}$ that is unbiased by autopower noise. Furthermore, the estimate should be stable for any geology since E_r will always be correlated with both E_x and E_y. The fifth channel enables one to express impedance elements in terms of average crosspowers without resorting to the solution of nonlinear simultaneous equations as in the case with method 1. In general, it should not be necessary to use weighted averages of crosspowers (method 2). However, there may exist situations in which the weighted average technique yields better estimates of the impedances. Method 2 is then easily adapted to include data from the fifth channel. For example, in equations (28)–(31), λ, λ', ξ, and ξ' can be written in the form VE_r^*, where V is any combination of fields that does not introduce bias and for which the weighted averages do not tend to zero.

In the analysis of the 5-channel data, we utilized two of the five fields as references for the remaining three fields. If the noise in either of the two references is correlated with the noise in the other three channels, the impedance estimates will, in general, be biased. To minimize the possibility of bias by coherent noises, one should use two reference fields that are measured at a site that is spatially remote from the MT base station. For 5-channel data, one can use only a single remote reference because the other reference is, of necessity, one of the base station fields. In order to have two remote references, one needs six channels of data. We discuss below the use of a remote magnetometer to provide the reference signals.

Crosspower analysis utilizing two remote reference channels

As we discussed above, we need two remote reference channels to minimize the influence of correlated noises on estimates of the impedance tensor. The reference channels can, in principle, be electric and/or magnetic. However, since the amplitudes of the telluric signals are strongly dependent on local geology, magnetic references are probably superior. Morrison has shown that the magnetic fields are coherent over many kilometers (Zelwer and Morrison, 1972).

If H_{xr} and H_{yr} are the magnetic field components measured with a remote magnetometer, then the impedance tensor can be easily determined by multiplying equations (1) and (2) in turn by H_{xr}^* and H_{yr}^* to yield

$$\overline{E_x H_{xr}^*} = Z_{xx} \overline{H_x H_{xr}^*} + Z_{xy} \overline{H_y H_{xr}^*}, \quad (49)$$

$$\overline{E_x H_{yr}^*} = Z_{xx} \overline{H_x H_{yr}^*} + Z_{xy} \overline{H_y H_{yr}^*}, \quad (50)$$

$$\overline{E_y H_{xr}^*} = Z_{yx} \overline{H_x H_{xr}^*} + Z_{yy} \overline{H_y H_{xr}^*}, \quad (51)$$

and

$$\overline{E_y H_{yr}^*} = Z_{yy} \overline{H_x H_{yr}^*} + Z_{yy} \overline{H_y H_{yr}^*}. \quad (52)$$

The solutions for the impedance elements from equations (49)–(52) contain no autopowers. Furthermore, the only crosspowers are between the reference fields and the fields at the base station. Thus, the impedance estimates will be unbiased by noise, provided noise in the reference fields is uncorrelated with noise in the fields at the base station. Thus, with two remote references, one should be able to lock-in detect MT signals.

SUMMARY

We have discussed two methods of analyzing 4-channel MT data: crosspowers and weighted crosspowers. For a given set of MT data, either of these techniques may give an estimate of the impedance tensor that is unbiased by electric and magnetic noise, if the noises in the various channels are uncorrelated. Both methods are unstable when the geology is one-dimensional ($Z_{xx} = Z_{yy} = 0$, $Z_{xy} + Z_{yx} = 0$), and when it is two-dimensional ($Z_{xx} + Z_{yy} = 0$) with one electrode in the strike direction. The problems associated with particular geologies can be removed by the introduction of a third electric channel. If the electric noises are uncorrelated, a straightforward crosspower analysis that uses the third channel as a reference source yields estimates of the impedance tensor that are unbiased by autopower noise. One can minimize the influence of correlated noises by using two reference fields measured at a site that is spatially separated from the MT base station. We suggest the use of a remote magnetometer to provide the reference signals.

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APPENDIX
COMPUTER SIMULATION OF MAGNETOTELLURIC DATA

Magnetotelluric data were simulated for a known impedance tensor $\mathbf{Z}$ by choosing random numbers uniformly distributed over the range (-1 to 1) to represent the real and imaginary parts of the incident magnetic field signals H_{x0} and H_{y0}. Because the range of values of the random numbers was fixed, the probablility distribution for the simulated fields was constant, and hence the signals for the model calculation, unlike real MT signals, were stationary (Bendat and Piersol, 1971). Electric field signals were calculated from H_{x0} and H_{y0} using the relations

$$E_{x0} = Z_{xx} H_{x0} + Z_{xy} H_{y0}, \quad \text{(A-1)}$$

and

$$E_{y0} = Z_{yx} H_{x0} + Z_{yy} H_{y0}. \quad \text{(A-2)}$$

Noise was superimposed on the signal fields by adding random numbers χ to each field. The total fields H_x, H_y, E_x, and E_y are defined by

$$H_x = H_{x0} + \alpha_x \chi_{Hx}, \quad \text{(A-3)}$$

$$H_y = H_{y0} + \alpha_y \chi_{Hy}, \quad \text{(A-4)}$$

$$E_x = E_{x0} + \beta_x \chi_{Ex}, \quad \text{(A-5)}$$

and

$$E_y = E_{y0} + \beta_y \chi_{Ey}, \quad \text{(A-6)}$$

where the α and β are adjustable weighting factors for the random noise terms χ_{Hx}, χ_{Hy}, χ_{Ex}, and χ_{Ey}. The values of the α and β were adjusted empirically to give the desired values of the noise-to-signal power ratios determined from the computed quantities $\overline{|E_x|^2}/\overline{|E_{x0}|^2}$, etc.

We computed the crosspowers a, b, c, d, e, and f from the total fields by forming the appropriate products, and then summing over $i = 1$ to N. The impedance tensors were calculated from the cross-products with one of the two methods described in the text. We repeated the calculation, typically $K = 100$ times, using independent sets of random numbers each time to generate the signal and noise fields. The mean value $\bar{Z}_{ij}$ and the sample variance σ_{ij} for each impedance element were computed from the expressions

$$\bar{Z}_{ij} = K^{-1} \sum_{\ell=1}^{K} Z_{ij}^{(\ell)}, \tag{A-7}$$

and

$$\sigma_{ij} = \left\{ K^{-1} \sum_{\ell=1}^{K} |Z_{ij}^{(\ell)} - \bar{Z}_{ij}|^2 \right\}^{1/2}. \tag{A-8}$$

The expected variance in $|\bar{Z}_{ij}|$, ΔZ_{ij}, was taken as $\pm \sigma_{ij}/K^{1/2}$.