1. A dynamic system

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x) \end{aligned} \tag{1}$$

where

   (a) $[0,T]$ is an interval of time and $t \in [0,T]$.
   (b) $x:[0,T] \to \mathbb{R}^n$ is called *state trajectory.*
   (c) $\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t}$ denotes the derivative with respect to time.
   (d) $u:[0,T] \to \mathbb{R}^{m'}$ is called *known input.*
   (e) f is called right hand side of the system equation or simply *model.*
   (f) $h:\mathbb{R}^n \to \mathbb{R}^p$ is called *observation function.*
   (g) y is called *observable.*
   (h) n is the dimension of the state space, p is the dimension of the observation space.
   (i) For the sake of readability x denotes $x(t)$ whenever necessary.

The space of all possible states is called *state space.*

2. If we assume a true model we write

   (a) f^{true} and x^{true}.
   (b) y^{obs} is the *observation data.*
   (c) f is called the *nominal model* and f^{true} the *true model.*
   (d) $f^{\text{true}}(x(t),u(t)) - f(x(t),u(t)) =: w(t)$ where w is called *model error* or *hidden input.* The space of admissible hidden inputs is $\Omega \subseteq \mathbb{R}^m$.

3. The *dynamic elastic net (DEN)* reads

$$\begin{aligned} &\hat{w}^{\text{opt}} := \underset{\hat{w}\in\Omega}{\arg\min}\, J \\ &\text{subject to} \\ &\qquad J := ||y^{\text{obs}} - h(\hat{x})||_{L_2}^2 + \alpha_1 ||\hat{w}||_{L_1} + \frac{\alpha_2}{2}||\hat{w}||_{L_2}^2 \\ &\qquad \dot{\hat{x}} = f(\hat{x},u) + \hat{w} \quad , \quad \hat{x}(0) = x_0 \end{aligned} \tag{2}$$

and $\hat{x}^{\text{opt}}$ the predicted states according to $\hat{w}^{\text{opt}}$. Furthermore $\hat{y}^{\text{opt}} = h(\hat{x})$.

4. Spline regularization,

   (a) We define $\chi_1 := x$, $\chi_2 := w$ and $\chi_3 := \dot{w}$ or $\xi := w$ and $\eta = \dot{w}$.

(b) The *augmented state* is

$$\chi := \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix} = \begin{pmatrix} x \\ \xi \\ \eta \end{pmatrix} \tag{3}$$

(c)

$$F(\chi) := \begin{pmatrix} f(\chi_1, u) + \chi_2 \\ \chi_3 \\ 0 \end{pmatrix} \tag{4}$$

The augmented DEN reads

$$\hat{\vartheta}^{\text{opt}} := \underset{\hat{\vartheta} \in \mathbb{R}^m}{\arg\min}\, J$$

subject to

$$J := ||y^{\text{obs}} - h(\hat{\chi}_1)||_{L_2}^2 + \alpha_1 ||\hat{\chi}_2||_{L_1} + \frac{\alpha_2}{2} ||\hat{\vartheta}||_{L_2}^2 \tag{5}$$

$$\dot{\hat{\chi}} = F(\hat{\chi}) + \begin{pmatrix} 0 \\ 0 \\ \vartheta \end{pmatrix} \quad , \quad \hat{\chi}(0) = \chi_0 \quad .$$

5. Mathematical notations

(a) The gradient of a function $f : \mathbb{R}^n \to \mathbb{R}$ can be written as column or row vector,

$$\frac{\partial f}{\partial x} = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix} \quad \text{and} \quad \nabla f = \left(\frac{\partial f}{\partial x_1}, \quad \dots \quad , \frac{\partial f}{\partial x_n} \right) \quad . \tag{6}$$

(b) The Jacobian of a function $f : \mathbb{R}^n \to \mathbb{R}^m$ in x

$$\left. \frac{\partial f}{\partial x} \right|_x = \begin{pmatrix} \nabla f_1 \\ \vdots \\ \nabla f_m \end{pmatrix} \tag{7}$$