1 Example

Consider a linear dynamic system

$$\frac{\mathrm{d}x}{\mathrm{d}t}(t) = Ax(t) + Dw(t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

$$x(0) = 0 \tag{3}$$

where

$$A = \begin{pmatrix} 0 & 0 \\ -\frac{1}{2} & 0 \end{pmatrix} \quad , \quad D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} \frac{1}{2} & -1 \end{pmatrix} \quad . \tag{4}$$

The observability matrix

$$M := [CD, CAD] = \begin{bmatrix} \frac{1}{2} & -1 & \frac{1}{2} & 0 \end{bmatrix} \tag{5}$$

yields $\operatorname{rank} M = 1$ which shows that the system is target controllable.

Remark. The general solution

$$y(t) = \int_0^t C \exp\{A(t-\tau)\} D w(\tau)\, \mathrm{d}\tau \tag{6}$$

yields

$$y(t) = \int_0^t w_1(\tau) - w_2(\tau)\, \mathrm{d}\tau \quad . \tag{7}$$

We see that (7) is a Volterra equation of the first kind with kernel $k(t,\tau) \equiv 1$ and thus has a unique solution to any differentiable $y(t)$. Choosing $y = 0$ leads to

$$w_2 = w_1 \tag{8}$$

which shows, that it is not possible to identify the hidden inputs w_1 and w_2 solely from the observation.