1 Parameter-Tuning

Given a $\mathbf{R}^n$ nominal system (u not written)

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x) \tag{1}$$

with initial values $x(0) = x_0 \in \mathbb{R}^n$ an $t \in [0,T]$ and measuring function $h(x) = y \in \mathbb{R}^m$, a dataset

$$\mathscr{D} = \left\{(t[1], y[1]), (t[2], y[2]), \ldots, (t[N], y[N])\right\} \tag{2}$$

and the dynamic elastic net approach

$$\hat{f}(\hat{x}, w) = f(\hat{x}) + w \tag{3}$$

the cost-functional is

$$J[w] = \sum_{\mathscr{D}} ||y - h(\hat{x})||_Q^2 + \mathscr{R}[w] \quad , \tag{4}$$

where $\hat{x}$ solves (3). The matrix Q is the weight matrix and

$$\mathscr{R}[w] = \alpha_1 ||w||_1 + \frac{\alpha_2}{2} ||w||_2^2 \tag{5}$$

is the regularisation term.

2 Cascade-Model

Nominal model with statespace $\mathbb{R}^4$, measure function $h(x) = x$ and time interval $[0,T]$. Dependence of x and u on t not written. The initial values are $x(0) = 0$.

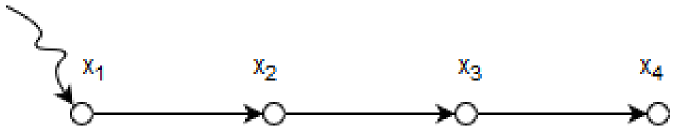
$$\begin{aligned} f(x) &= \left[\underbrace{\begin{pmatrix} 0 & 0 & 0 & 0 \\ \beta_1 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 \end{pmatrix}}_{\text{cascade}} + \underbrace{\begin{pmatrix} -\gamma_1 & 0 & 0 & 0 \\ 0 & -\gamma_2 & 0 & 0 \\ 0 & 0 & -\gamma_3 & 0 \\ 0 & 0 & 0 & -\gamma_4 \end{pmatrix}}_{\text{self-loops}} \right] \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} u & (6) \\ &= [A_1 + A_2]\, x + Bu \quad . & (7) \end{aligned}$$

Using the general solution

$$x(t) = \int_0^t \mathrm{e}^{(A_1 + A_2)(t-\tau)} B u(\tau) \mathrm{d}\tau \tag{8}$$

and with $t-\tau \to \tau$, $u(t-\tau) := u(\tau)$ and $\alpha_i = \alpha$, $\beta_i = \beta$ for simplicity we get

$$x_n(t) = \int_0^t e^{-\alpha\tau} b^{n-1} \frac{\tau^{n-1}}{(n-1)!} u(\tau) \mathrm{d}\tau \tag{9}$$



Here we will use

$$u(t) = \frac{a}{2\omega} \sin(\omega t) \cos(\omega t) \quad . \tag{10}$$

thus we get

$$x_1(t) = \frac{(1-\mathrm{e}^{-\alpha t})}{\alpha} \tag{11}$$

2.1 δ-peak

Delta-peaks are regularly used in electrical-engineering and as a mathematical tool in the theory of partial differential equations.

Perturbation function with $\mathcal{N} > 0$, $\Gamma > 0$, $t_0 \in (0, T)$

$$\delta(t) = \begin{cases} \mathcal{N}(2\Gamma)^{-1} & \text{if} \quad t_0 - \Gamma \le t \le t_0 + \Gamma \\ 0 \quad \text{else} \end{cases} \tag{12}$$

and perturbed system.

$$f^{\delta}(x,t) = f(x) + \left(0, 0, \delta(t), 0\right)^{\mathrm{T}} \quad . \tag{13}$$

2.2 Breit-Wigner-Resonance

The Breit-Wigner-resonance is the standard resonance curve in classical an quantum mechanics and also known as Cauchy-distribution (statistics) and Lorentz-curve (spectroscopy).

Perturbation function

$$F(t) = \mathcal{N}\pi^{-1} \frac{\Gamma}{\Gamma^2 + (t-t_0)^2} \tag{14}$$

and perturbed system

$$f^F(x,t) = f(x) + \left(0, 0, F(t), 0\right)^{\mathrm{T}} \quad . \tag{15}$$

3 Chaotic Systems

3.1 Lorentz-System

The nominal model is in $\mathbb{R}^3$, measure function $h(x) = x$ and $a, b, c > 0$.

$$f(x) = \begin{pmatrix} a(y-x) \\ bx - y - yz \\ xy - z \end{pmatrix} \tag{16}$$

3.2 Verhulst-Dynamic

A simple chaotic model, also known as logistic growth curve, with interesting behaviour when $r \sim 3.57$.

$$f(x) = rx(1-x) \quad . \tag{17}$$

4 Oscillations

4.1 harmonic oscillator

$$\ddot{x} + \omega^2 x = 0 \tag{18}$$