MAAE 3300: Lab D
Compressible Pipe Flow

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1 Summary

The objective of this lab to evaluate the losses associated with pipes carrying compressible fluids. This will be done by calculating the friction factor under two assumptions; adiabatic and isothermal flow. The experiment will consist of two slightly different setups consisting of a tank and duct. Air is pressurized within a tank and then released through a long duct. Each setup is fitted with thermocouples, pressure transducers, and flow meters to collect lab data. The two lab setups are slightly different, which means that two different methods of analysis are required to solve for the friction factors. This lab will present results such as experimental and theoretical friction factors, as well factors that cause results to be inaccurate.

2 Introduction

The primary source of losses within the pipes comes from the friction. The value of the friction can be determined using the friction factor and the roughness of the pipe. When it comes to engineering, pipe losses are critical when designing systems. Many engineering projects require some kind of movement of fluid, and pumps are the machines that do the moving. Generally, engineers strive to create systems that are as efficient as possible, so that the minimum power is required. Since the longer the pipe the more losses, engineers will design piping systems to be as simple as possible to reduce losses and lower pumping requirements. The objective of this lab is to determine pipe losses due to friction within a pipe under compressible flow assumptions. The objectives of this lab are met by using pressurized air reservoir and allowing the air to escape through a duct, measuring the pressure drop along the pipe and the rate of flow at the exit. Analysis consists of measuring the mass flow rate, finding the critical pressure of the pipe, and then using calculated values to determine the friction factor throughout the pipe.

3 Theory

The main calculation difference between the two tanks is the method of calculating mass flow rate. While the larger reservoir has a rotometer, the flow rate can be calculated using the following equation, where $\dot{V}$ is the volumetric flow rate measured by the rotometer.

$$\dot{m} = \rho_{atm}\dot{V} \tag{1}$$

The smaller reservoir does not have a flowmeter, and the mass flow rate must be calculated using a different method. The first step to solve is to find the static pressure at the inlet. To do this plot the static pressure measurements from each tap against the distance from the inlet. Using this plot estimate the pressure at the inlet, where the distance from the inlet is equal to zero. This can be done using software like excel using a polynomial line of best fit. Once the static pressure is known at the inlet, the following relationship can be used to find the mach number at the inlet.

$$\frac{P_o}{P_{in}} = \left(1 + \frac{k-1}{2} Ma_{in}^2\right)^{\frac{1}{k-1}} \tag{2}$$

Then use the stagnation temperature and mach number to determine the temperature at the inlet.

$$\frac{T_o}{T_{in}} = \left(1 + \frac{k-1}{2} Ma_{in}^2\right) \tag{3}$$

The next two equations are used in order to find the velocity of the air at the inlet, as well as the density of the air using compressible flow theory.

$$V_{in} = \sqrt{kRT_{in}} Ma_{in} \tag{4}$$

$$\rho_{in} = \frac{P_{in}}{RT_{in}} \tag{5}$$

Using all of the previous equations, the mass flow rate can be solved for using the following equation.

$$\dot{m} = \rho_{in} A_{in} V_{in} \tag{6}$$

When fluids flow through long ducts principles of thermodynamics dictate that flow will continue to accelerate until it reaches a mach number of 1. It will not accelerate past this speed, and this is also known as the critical point within pipe flow. All properties at the critical point will be denoted with a star symbol. Since there are now two mass flow rates determined for each pressure tank, the next step is to find the critical pressure for each duct. The derivation steps for this equation are listed below.

$$\frac{P}{P^\star} = \frac{1}{Ma}\left[\frac{k+1}{2+(k-1)Ma^2}\right]^{1/2} \tag{7}$$

$$\frac{P}{P^\star} = \frac{a}{v}\left[\frac{k+1}{2(1+\frac{k-1}{2}Ma^2)}\right]^{1/2}$$

$$\frac{P}{P^\star} = \frac{\rho A\sqrt{kRT}}{\dot{m}}\left[\frac{kRT(k+1)T}{2T_o}\right]^{1/2}$$

$$\frac{P}{P^\star} = \frac{PAR^{1/2}}{RT\dot{m}}\left[\frac{k(k+1)T^2}{2T_o}\right]^{1/2}$$

$$\frac{1}{P^\star} = \frac{A}{R^{1/2}\dot{m}}\left[\frac{k(k+1)}{2T_o}\right]^{1/2}$$

$$\frac{1}{P^\star} = \frac{A}{\dot{m}}\left[\frac{k(k+1)}{2RT_o}\right]^{1/2}$$

$$P^\star = \frac{\dot{m}}{A}\sqrt{\frac{2RT_o}{k(k+1)}} \tag{8}$$

This equation can be used in order to calculate the critical pressure value for both large and small tanks. Once the critical pressure value is found, use the value to calculate the ratio $\frac{P_n}{P^\star}$ for each tap, where n is the tap number. With the pressure ratio, evaluate equation 7 at the pressure ratio to the find the mach number at the tap. This mach number can now be used to determine the average friction factor between the tap and the critical point of the duct using the following formula.

$$\frac{\bar{f}L^\star}{D} = \frac{1-Ma^2}{kMa^2} + \frac{k+1}{2k} ln\frac{(k+1)Ma^2}{2+(k-1)Ma^2} \tag{9}$$

To determine the friction factor between two taps, use the following formula.

$$\frac{f\Delta L}{D} = \left(\frac{\bar{f}L^\star}{D}\right)_1 - \left(\frac{\bar{f}L^\star}{D}\right)_2 \tag{10}$$

By varying the values of 1 and 2 with the tap numbers, the average friction factor can be determined between each set of taps. The average friction factor can be calculated, which represents the experimental friction factor for the entire duct.

Up until this point of this section all calculations assumed adiabatic flow. Adiabatic flow assumes that there is no heat transfer between the system and its surroundings, therefore the temperature within the system changes and work done is due to the change in the internal energy [1]. This is a decent assumption for this experiment because the reservoir of air is brought to room temperature, and the duct is not very long. Flow expansion happens in the pipe, however the

duct is not very long so there is not much heat transfer. The limitation to this assumption is that there still is some heat transfer that affects the temperature within the system.
For this lab, flow will also be modelled using the isothermal assumption. An isothermal process is a thermodynamic process that occurs under a constant temperature [1]. This means that the temperature within the system changes, however heat transfer from outside the system will maintain the temperature at a constant value. However the main limitation of this assumption is that the change in temperature has to be relatively slow to allow ambient temperature to transfer heat to the system. This assumption should be fairly accurate for this experiment since the duct is not that long and there is not a significant drop in temperature. Since a new assumption is being used, the formula for calculating the friction factor is required, which is listed below.

$$\left(\frac{\dot{m}}{A}\right)^2 = \frac{P_1^2 - P_2^2}{RT\left[\frac{f_{12}L}{D} + 2ln\frac{P_1}{P_2}\right]} \tag{11}$$

Similar to equation 10, the numbers 1 and 2, corresponding to the tap numbers can be varied to find the friction factor between taps. Similar to the adiabatic process, the average friction factor can be calculated to approximate the friction factor of the entire duct.
This lab will compare experimental friction values to theoretical ones using the Moody chart. In order to use the Moody chart, two values need to be known, the relative roughness $\frac{\epsilon}{d}$ and the Reynolds number. The equation for the Reynolds number can be seen below.

$$Re = \frac{\dot{m}D}{A\mu} \tag{12}$$

With all the equations and derivations listed in this section, the friction factor of the duct can be calculated and compared to expected results using the Moody chart.

4 Apparatus and Procedure

4.1 Apparatus

The apparatus of this lab consists of two setups. One consists of a larger pressure tank filled to 70 psig, and the other has a smaller, 50 psig tank. Each tank is connected to a long duct open to the room at the end. Each duct is fitted with 5 pressure taps. There is also a pressure tap and thermocouple in the reservoir. On the larger pressure tank setup, a flowmeter is fitted to the end of the duct. On the samller pressure tank, a bellmouth feeds air from the reservoir to the inlet of the duct.

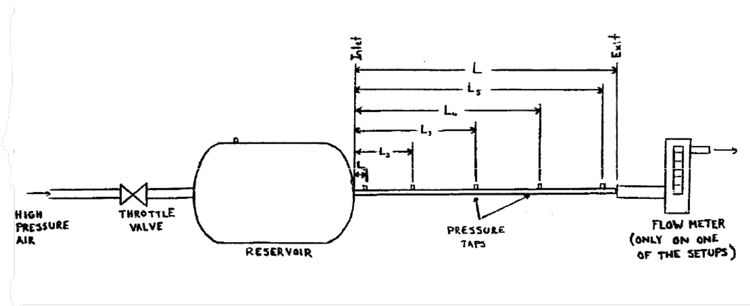

Figure 1: Experimental Setup [2]

4.2 Procedure

The first step of this experiment is to ensure that the reservoir pressure is at approximately 70 psig. Once the reservoir reaches 70 psig, leave the tank for a while in order to allow the air to reach steady state. The next step is to measure and record the reservoir pressure and temperature before releasing any air. Allow air to escape the reservoir through the pipe slowly. As the valve is opened, measure the pressure at each tap along the straight pipe. Also measure the flow rate using the rotometer fitted at the end of the pipe. After all measurements are recorded, remeasure the pressure at the reservoir. If the change in pressure is greater than 1 psi, the procedure must be repeated. If experiment successful, repeat the procedure using a reservoir pressure of 50 psig. As a note the second tank with 50 psig does not have a flowmeter.

5 Results

The pressure drop within the duct changes with the length of the pipe, and this trend can be plotted against the distance from the inlet. The following figure illustrates this as well as provides a polynomial line of best fit, which is useful in estimating the inlet static pressure.

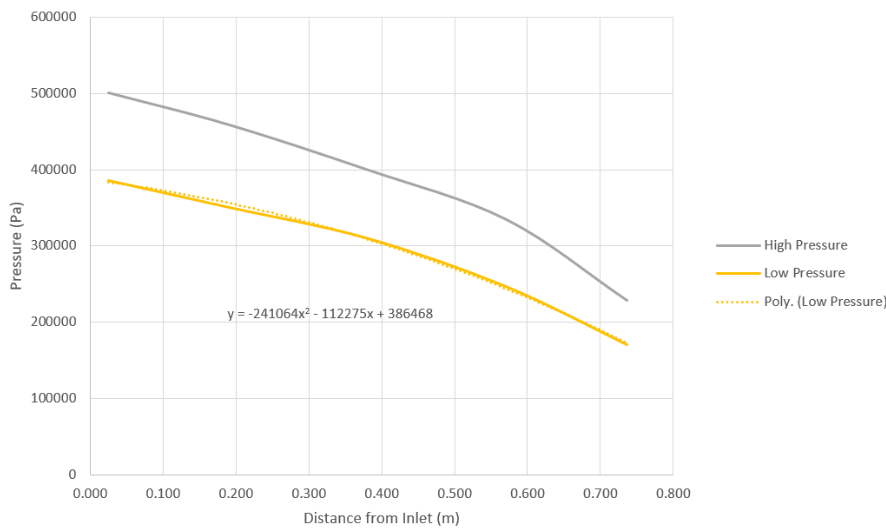

Figure 2: Pressure Change Within Duct

The critical pressure of the large and small tanks are 176775 kPa and 158934 kPa, respectively. Using the critical pressure, and the pressure at each tap, the mach number at each tap can be found using equation 7. Once the mach number is found use equation 9 to find the friction factor at each tap. Results are tabulated below.

Friction Term	High Pressure	Low Pressure
$\frac{f_1 L^\star}{D}$	2.69	1.66
$\frac{f_2 L^\star}{D}$	2.06	1.20
$\frac{f_3 L^\star}{D}$	1.31	0.756
$\frac{f_4 L^\star}{D}$	0.665	0.260
$\frac{f_5 L^\star}{D}$	0.073	0.005

Table 1: Friction Factors Determined Under Adiabatic Assumption

Once the friction factor at each tap is calculated, the difference between tap friction factors can be used to calculate the friction constant between those two taps. The following two tables represent the value of the friction factor for both adiabatic and isothermal assumptions

Friction Factor	High Pressure	Low Pressure
f_{12}	0.0111	0.0103
f_{23}	0.0115	0.0086
f_{34}	0.0098	0.0096
f_{45}	0.0105	0.0057

Table 2: Friction Factors Determined Under Adiabatic Assumption

Friction Factor	Low Pressure
f_{12}	0.0094
f_{23}	0.0075
f_{34}	0.0073
f_{45}	0.0003

Table 3: Friction Factors Determined Under Isothermal Assumption

Results of the lab are summarized in the table below. The average friction factor is the average of the four calculated friction factors. The theoretical friction factor is determined using the Moody Chart. To use the Moody Chart, two values are required; the relative roughness and the Reynolds number. To calculate the relative roughness simply divide the roughness of the material surface by the diameter of the pipe. For this experiment a roughness value of 0.0014 mm is chosen, which is the roughness of a new copper tube [3]. Once the Reynolds number is calculated the Moody Chart can be used to find the theoretical friction factor of the tube.

Experimental Setup	f_{avg}	f_{theory}	Percent Error (%)
High Pressure (adiabatic)	0.0107	0.0196	45.3
Low Pressure (adiabatic)	0.0085	0.0190	55.0
Low Pressure (isothermal)	0.0061	0.0190	67.9

Table 4: Comparison of Friction Factors for All Cases

As always, sample calculations, excel spreadsheets, graphs, and Moody Chart calculations can be found in the appendices, Section 8.

6 Discussion

6.1 Results

As tabulated in 4, there are significant discrepancies between evaluated friction factors and Moody chart values. These discrepancies cause percent errors of roughly 50% in each case. These discrepancies come from many different assumptions, calculation methods, and simplifications. Some of the discrepancies include assumption of the inlet pressure, the rotometer giving an average flow rate, and the Moody chart giving a non precise friction factor.

6.2 Assumptions

Isothermal and adiabatic assumptions are important in this lab in order to simplify many of the calculations. Limitations and requirements of both assumptions were discussed in the theory section, however in order for adiabatic flow to be assumed, there must not be any heat transfer to the system. This is actually a fairly good assumption for this lab only because the duct is not very long. Since the duct is not that long, not much of the ambient conditions will effect the expanding air. Although this may be a decent assumption, no system is perfect, and since none of the apparatus is insulated a lot of error will come from this assumption.
Isothermal conditions assume that the temperature of the system remains constant. For this to occur, fluid expansion must be relatively slow so that ambient temperature can transfer thermal energy to the system. However once again, this assumption is not ideal since the expansion does not occur slowly, and there is almost definitely some change in temperature within the system. For this reason this assumption creates some error. Between both assumptions it appears that the adiabatic assumption is more accurate for this experiment since the percent error is 12.9% higher.

6.3 Sources of Error

Like mentioned above some sources of error include different assumptions, calculation methods, and simplifications. Additionally, some more experimental error come from the lab setup itself, in that the pipes are not perfect and there may be leaks around the pressure transducers or the fittings. Unfortunately, this lab has very poor repeatability. This lab was conducted 7 times, and the extreme values were listed in a table. Just looking at a few of the variables, the temperature ranges by five degrees, while the volume flow rate varies by around 10 cubic feet per hour. Due to this, there is no guarantee as to what the actual experimental values should be, which adds a lot of error into the lab.
In order to reduce some of the error, experiment modifications would include a pressure transducer much closer to the inlet or even at the inlet itself. Another way of reducing the error in this experiment is to perform the experiment

many times, recording the data and performing the calculations for each run and finding average values to approximate a more accurate result.

7 Conclusions

The objective of this lab was to evaluate the losses due to friction of a pipe carrying compressed air from a reservoir to an outlet. The objectives of this lab were successfully met by calculating friction factors throughout the length of the pipe by producing pressure drop trends, calculating the mach number of the air and determining the critical pressure of the pipe. This lab consists of two setups, both including a tank and a duct, which is open to atmospheric conditions at one end. The procedure of this lab consists of releasing air from the constant pressure reservoir, and measuring the pressure at several locations along the pipe. One of the setups is fitted with a flowmeter to measure the volume flow rate of the air, while the other does not have such flowmeter. To summarize results, friction factors for the high pressure tank, low pressure tank, and low pressure tank under isothermal assumption were 0.0107, 0.0085, and 0.0061, respectively. There is a significant error between the experiment data and expected results of around 55%. In conclusion, this lab served the purpose of observing how compressed air behaves through a duct. It is interesting to see that flow accelerates in the pipe, which was counter-intuitive at first.

References

[1] F. M. White, *Fluid mechanics Eight Edition*. McGraw-Hill Education, 2016.

[2] Carleton, *Laboratory Requirments and Experiments*. Carleton University, May 2000.

[3] "Roughness of pipes." http://www.pressure-drop.com/Online-Calculator/rauh.html.

8 Appendix A: Sample Calculations

$$p^* = \frac{\dot{m}}{A}\sqrt{\frac{2RT_0}{k(k+1)}}$$

$$= \frac{0.0053\ \mathrm{kg/s}}{6.7\times10^{-6}\ \mathrm{m^2}}\sqrt{\frac{2(287)(244\,\mathrm{K})}{1.4(1.4+1)}}$$

$$= 176775\ \mathrm{Pa}$$

Figure 3: Critical Pressure

$$\frac{p}{p^*} = \frac{1}{Ma}\left[\frac{k+1}{2+(k-1)Ma^2}\right]$$

$$\frac{501235}{176775} = \frac{1}{Ma}\left[\frac{k+1}{2+(k-1)Ma^2}\right]$$

$$2.84 = \frac{1}{Ma}\left[\frac{k+1}{2+(k-1)Ma^2}\right]$$

*Use Maple to solve for Ma

$$Ma = \frac{\sqrt{2.84(1.4-1)\left(-2.84\sqrt{2.84^2+1.4^2-1}\right)}}{2.84(1.4-1)}$$

$$= 0.381$$

Figure 4: Tap Mach Number

$$\frac{\bar{f}_1 L^*}{D} = \frac{1 - Ma^2}{k\,Ma^2} + \frac{k+1}{2k} \ln \frac{(k+1)\,Ma^2}{2 + (k-1)\,Ma^2}$$

$$= \frac{1 - 0.381^2}{1.4 \cdot 0.381^2} + \frac{1.4+1}{2 \cdot 1.4} \ln \frac{(1.4+1) \cdot 0.381^2}{2 + (1.4-1)\,0.381^2}$$

$$= 2.687$$

Figure 5: Friction Term at Tap

$$\frac{f \Delta L}{D} = \left(\frac{\bar{f} L^*}{D}\right)_1 - \left(\frac{\bar{f} L^*}{D}\right)_2$$

$$\frac{f_{12}(0.191 - 0.025)}{0.00292} = 2.687 - 2.058$$

$$f_{12} = 0.0111$$

Figure 6: Adiabatic Friction Factor

$$\left(\frac{\dot{m}}{A}\right)^2 = \frac{P_1^2 - P_2^2}{RT\left[\frac{f_{12} \Delta L}{D} + 2 \ln \frac{P_1}{P_2}\right]}$$

$$\frac{0.0072}{1.07 \times 10^{-5}\, m^2} = \frac{386084^2 - 350924^2}{287 \cdot 293 \left[\frac{f_{12}(0.191 - 0.025)}{0.00368} + 2 \ln \frac{386084}{350924}\right]}$$

$$f_{12} = 0.0110$$

Figure 7: Isothermal Friction Factor

$$f_{av} = \frac{f_{12} + f_{23} + f_{34} + f_{45}}{4}$$

$$= \frac{0.0111 + 0.0115 + 0.0098 + 0.0105}{4}$$

$$= 0.0107$$

Figure 8: Average Friction Factor

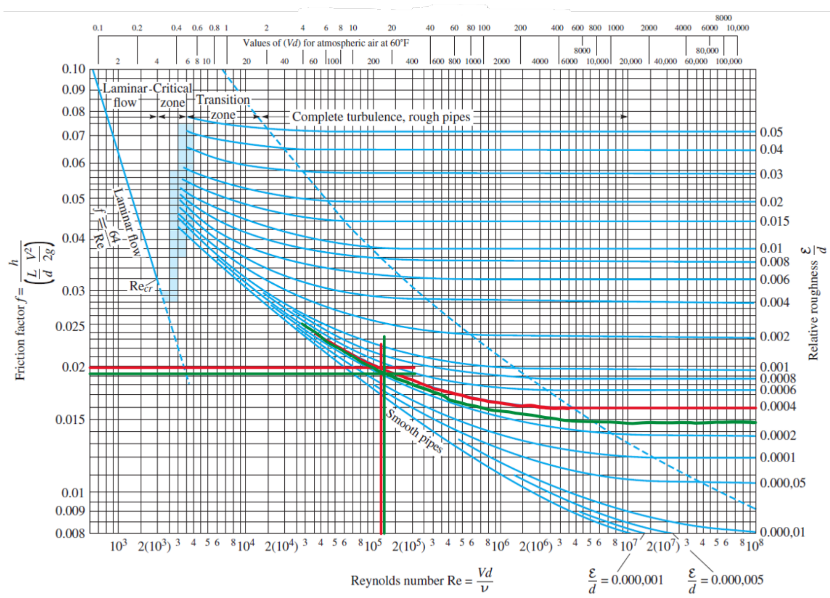

Figure 9: Theoretical Friction Factor

9 Appendix B: Original Data

Collected Data for MAAE 3300 Experiment 4 - Compressible Pipe Flow (Imperial)								
	Resevoir temp. (deg. C)	Resevoir Pressure (psig)	Tap 1 Static Pressure (psig)	Tap 2 Static Pressure (psig)	Tap 3 Static Pressure (psig)	Tap 4 Static Pressure (psig)	Tap 5 Static Pressure (psig)	Volume Flow Rate (measured by rotameter) (scfh)
Apparatus 1 (70 psig)*	21	70.0	58.0	51.9	43.3	33.8	18.4	560
Apparatus 2 (50 psig)	20	49.4	41.3	36.2	30.3	21.1	10.1	-

Figure 10: Original Lab Data

	Resevoir temp. (K)	Resevoir Pressure (Pa)	Inlet Pressure (Pa) From Chart	Tap 1 Static Pressure (Pa)	Tap 2 Static Pressure (Pa)	Tap 3 Static Pressure (Pa)	Tap 4 Static Pressure (Pa)	Tap 5 Static Pressure (Pa)	Volume Flow Rate (measured by rotameter) (m^3/s)	Diameter (m)	Area (m^2)
Apparatus 1 (High Pressure)*	294	583975	504438	501235	459176	399879	334376	228193	0.004405	0.00292	0.0000067
Apparatus 2 (Low Pressure)	293	441938	386468	386089	350924	310244	246810	170965	-	0.00368	0.0000107

Figure 11: Original Lab Data (Metric)

Tap Number	Location from Inlet (m)
1	0.025
2	0.191
3	0.381
4	0.572
5	0.737

Figure 12: Tap Distances

10 Appendix C: Spreadsheets

	Mach Number at Inlet	Inlet Temperature (K)	Inlet Velocity (m/s)	Inlet Density (kg/m^3)	Mass Flow Rate (kg/s)	P* (Pa)
Apparatus 1 (High Pressure)*	N/A	N/A	N/A	N/A	0.0053	176775
Apparatus 2 (Low Pressure)	0.44	282.0	148.8	4.78	0.0076	158934

Figure 13: Critical Pressure Calculations

	Tap 1	Tap 2	Tap 3	Tap 4	Tap 5
Apparatus 1 (High Pressure)*	2.84	2.60	2.26	1.89	1.29
Apparatus 2 (Low Pressure)	2.43	2.21	1.95	1.55	1.08

Figure 14: Tap Ratios

	Tap 1		Tap 2		Tap 3		Tap 4		Tap 5	
	Mach Number	Friction Term	Mach Number	Friction Term	Mach Number	Friction Term	Mach Number	Friction Term	Mach Number	Friction Term
Apparatus 1 (High Pressure)*	0.381	2.687	0.415	2.058	0.474	1.306	0.562	0.665	0.799	0.073
Apparatus 2 (Low Pressure)	0.442	1.661	0.485	1.200	0.545	0.756	0.675	0.260	0.939	0.005

Figure 15: Friction Factor Calculations I

	f12	f23	f34	f45	f average	f theory	Percent Error (%)	Re	Epsilon
Apparatus 1 (High Pressure)*	0.0111	0.0115	0.0098	0.0105	0.0107	0.0196	45.3	128003	0.000479
Apparatus 2 (Low Pressure)	0.0103	0.0086	0.0096	0.0057	0.0085	0.0190	55.0	145354	0.000380
Apparatus 2 (Low Pressure) Isothermal Case	0.0094	0.0075	0.0073	0.0003	0.0061	0.0190	67.9	137433	0.000380

Figure 16: Friction Factor Calculations II