Jet Pump

Dominik Parpura

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1 Summary

The purpose of this lab is to examine how a jet pump works. The flow will be analyzed and the experimental performance will be compared to the theoretical, ideal performance. A jet pump is a device that has no mechanical components and is important in applications where abrasive fluids can damage pump components. For this reason, it is important to develop models in order to quantify losses and what the performance required is. Through running two different flow rates, this lab is able to measure the loss of pressure along the mixing tube and determine how long the mixing tube should be. The jet pump apparatus is also essential to find the velocity profile within the tube at the outlet. All of these aspects help determine the difference between theoretical and experimental equations. As a note, flow rate 1 should be neglected because of unreliable data.

2 Nomenclature

2.1 Variables

q - Dynamic Pressure
V - Velocity
$\dot{m}$ - Mass Flow Rate
P - Pressure
A - Cross Sectional Area
F - Force
Q - Flow Rate

2.2 Constants

g = 9.81 m/s^2
C_q = 0.93
C_p = -0.045
ρ_{air} = 1.2 kg/m^3

$\rho_{water} = 1000$ kg/m^3
$P_{atm} = 101325$ Pa

3 Flow Analysis

The first thing that needs to be done is convert all the liquid levels from the manometer banks into pressure. The equation 1 can be used for both static taps and pitot tubes by finding the difference in heights compared to atmospheric pressure. This equation can also be used to determine P_{C1}, through derivation 2.

$$P_1 - P_2 = \rho g \Delta h \tag{1}$$

$$\begin{aligned} P_1 - P_2 &= \rho g \Delta h \\ P_1 &= \rho g \Delta h + P_2 \end{aligned} \tag{2}$$

To determine the primary flow velocity, use equation 3. Knowing all the values isolate for V_p, using equation 4 to get 5.

$$C_q = \frac{q}{P_{C1} - P_{C2}} \tag{3}$$

$$q = \frac{1}{2}\rho V_P^2 \tag{4}$$

$$\begin{aligned} C_q &= \frac{q}{P_{C1} - P_{C2}} \\ \frac{1}{2}\rho V_p^2 &= C_q(P_{C1} - P_{C2}) \\ V_p &= \sqrt{\frac{2C_q(P_{C1} - P_{C2}}{\rho_{air}}} \end{aligned} \tag{5}$$

To find the secondary flow velocity, apply the Bernoulli equation (6) to a point far away in space and the inlet of the bell mouth. Because there is no change in height and one dynamic pressure term goes to 0, velocity can be isolated for. The result is equation 7.

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h \tag{6}$$

$$P_{atm} + \frac{1}{2}\rho V_{atm}^2 + \rho gh = P_s + \frac{1}{2}\rho V_s^2 + \rho gh$$
$$P_{atm} = P_s + \frac{1}{2}\rho V_s^2$$
$$V_s = \sqrt{\frac{2(P_{atm} - P_s)}{\rho}} \tag{7}$$

Primary pressure can be calculated using equation 8. By isolating this equation, the primary flow pressure can be determined.

$$C_p = \frac{P_p - P_{C2}}{P_{C1} - P_{C2}} \tag{8}$$

$$C_p = \frac{P_p - P_{C2}}{P_{C1} - P_{C2}}$$
$$P_p = C_p(P_{C1} - P_{C2}) + P_{C2} \tag{9}$$

The pitot tube rake is placed at the end of the mixing tube to determine the gradient of velocity within the tube. A pitot tube measures the stagnation pressure, or the combination of static and dynamic pressures. The static pressure is already measured through the static tap at the wall of the tube. With this the velocity at any one of the pitot tubes can be measured by isolating for v in the dynamic pressure term.

$$P_{stagnation} = P_{static} + P_{dynamic} \tag{10}$$

$$P_{stagnation} = P_{static} + P_{dynamic}$$
$$P_{dynamic} = P_{stagnation} - P_{static}$$
$$\frac{1}{2}\rho V_P^2 = P_{stagnation} - P_{static}$$
$$V = \sqrt{\frac{2(P_{stag} - P_{static})}{\rho}} \tag{11}$$

To use the momentum equation, mass flow rates must be determined. This can be done using equation 12. Now, the momentum equation can be used. When analyzing the flow in the jet pump, choose a control volume that starts at the inlet of the primary flow, and ends right before the pitot tube rake. For the purposes of the lab, the only forces that are important are the forces in the x direction, that of the pressure.

$$\dot{m} = \rho V A \tag{12}$$

$$\sum F_x = (\dot{m}V)_{out} - (\dot{m}V)_{in} \tag{13}$$

On the right hand side of the equation, the out term consists of the outlet mass flow rate and velocity. Meanwhile the in term consists of the sum of the primary and secondary flow rates. The left hand side equation is the sum of the forces acting on the inlet and outlet. The forces can be written in terms of pressure and area with equation 14. By isolating for P_{out} in equation 15, the theoretical static pressure at the outlet can be determined.

$$F = PA \tag{14}$$

$$\begin{aligned} \sum F_x &= (\dot{m}V)_{out} - (\dot{m}V)_{in} \\ P_p A_p + P_s A_s - P_{out} A_{out} &= \dot{m}_{out} V_{out} - \dot{m}_p V_p - \dot{m}_s V_s \\ P_{out} &= \frac{\dot{m}_{out} V_{out} - \dot{m}_p V_p - \dot{m}_s V_s - P_s A_s + P_p A_p}{A_{out}} \end{aligned} \tag{15}$$

Because of the continuity equation, the theoretical velocity at the outlet can be determined since area is known and in velocities are known. See equation 17.

$$Q_{in} = Q_{out} \tag{16}$$

$$\begin{aligned} Q_{in} &= Q_{out} \\ V_p A_p + V_s A_s &= V_{out} A_{out} \end{aligned} \tag{17}$$

It is always to smart to check for error between theoretical and experimental results. This can be done by comparing the theoretical values of flow rate to the measurements taken at the pitot tubes. By integrating the flow rate formula 18 across all the pitot tubes, the flow rate passing across the tubes can be calculated.

$$Q = VA \tag{18}$$

$$\begin{aligned} Q &= VA \\ Q &= \int_0^r V(r) 2\pi r dr \\ Q &= 2\pi \int_0^r V(r) r dr \end{aligned} \tag{19}$$

4 Experimental Setup and Procedure

As the lab was performed online, the video watched is the only source of procedure available. Sample data was collected from the TA.

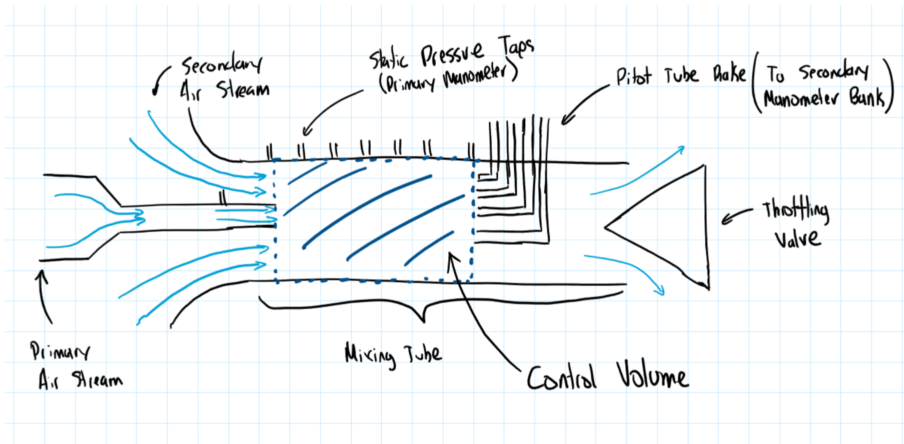

Figure 1: Jet Pump Diagram

5 Analysis and Discussion

5.1

The entire mixing tube can be analyzed through the momentum equation as long as the control volume is established correctly. Since the calibration constants and equations were given, it is a simple task of plugging numbers in correctly and isolating for unknowns. Please view section 7, for data and sample calculations.

Flow Rate	Theoretical	Experimental	Error
1	25.66 m/s	24.52 m/s	4.44%
2	21.96 m/s	7.225 m/s	67.1%

Table 1: Theoretical Vs. Experimental Velocity at Outlet

At any point in the mixing tube the velocity is a radial gradient. It is fastest in the middle and slowest at the walls. One of the things that contribute to this is the no-slip condition and friction. At a wall, the velocity is 0, and it slowly increases as it goes towards the middle. Meanwhile there is negligible friction in the middle and the velocity is at its highest. Because the velocity is a range of values, the average of the velocities was taken and compared to the theoretical velocity.

5.2

Static pressure changes along the length of the mixing tube, and this is demonstrated in figure 2, where the absolute static pressure is plotted against the tap number. This graph shows both flow rates and also marks the pressure at the outlet for each flow.

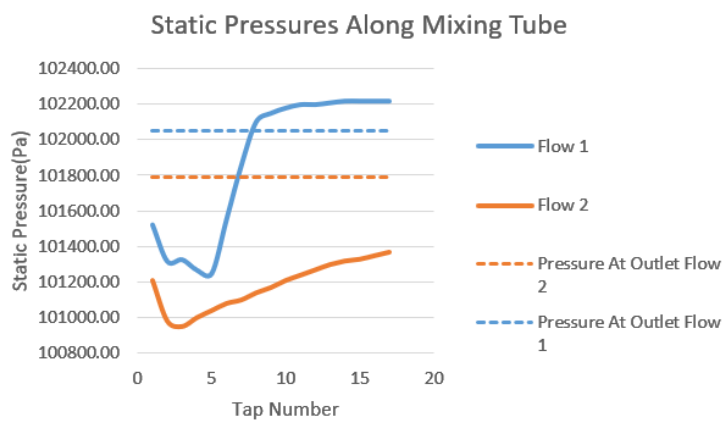

Figure 2: Static Pressure Along Mixing Tube

For flow 1, the data shows that the mixing process has completed, however the data is wrong and this should not be the case. To signify that the streams of air have mixed the pressure would be constant along the tube and match up with the theoretical pressure at the outlet. Sources of error are discussed in 5.1. However for flow 2, the trend is different and the pressure does not stabilize, which proves that the mixing process is not completed. In the second flow rate, there is a lot more turbulence, and the two different streams of air do not have time to mix into one. If they were to mix, a similar pattern would be observed, where the rate of change is close to zero. Instead the rate of change is positive and increasing, which means it still has a while to go before it reaches the theoretical pressure.

Comparing the experimental pressure at tap 17 and the theoretical pressure, Flow 1 has an error of 0.166%, while flow 2 has an error of 0.418%. For flow 1, the experimental pressure is higher than the theoretical pressure. This increase in pressure is due to the sources of error. However with such a low error, the pressure rise is predicted well. Flow 2 has not mixed properly, so the pressure can not be compared. If the mixing tube was longer, the two streams could mix together properly and the pressure could be compared.

Some of the issues with the way pressure is measured is that it relies on ideal equations like the Bernoulli equation. Factors like turbulence are not factored into the equation and for that reason it leads to errors. Instead it is best to use the Steady Flow Energy Equation, which compensates for losses due to energy converted and wasted.

5.3

Figure 3 shows the velocity of air at the outlet. A variety of velocities is essential because although in theory streamlines in 1 dimension all have the same velocity and direction, in reality that is not true. Because of the no-slip condition, velocity at the walls is greatly lower than velocity at the center line of the tube. In the graph, the measured velocities are graphed and the constant theoretical velocity is also plotted. A quadratic line is fitted onto each flow and shown on the graph.

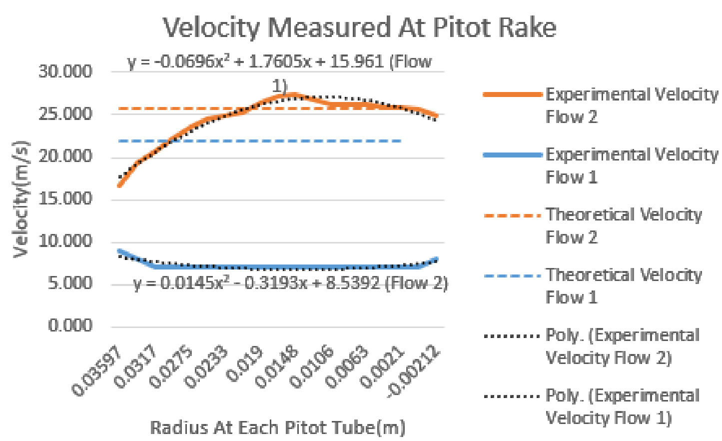

Figure 3: Velocity Gradient At Pitot Tubes

Referring to flow 2 the graph shows how the assumption that flow is uniform is incorrect. The velocity changes constantly and starts off slow at the walls and increases towards the centre of the tube. The one thing that the assumption was able to predict correctly was the velocity at the centre of the tube, with an error of 0.339%. Meanwhile for flow 1, the assumption failed completely, with an error of 68.1%, due to unreliable data.

5.4

In a fixed control volume, the sum of fluids entering the system is the same as the sum of the fluids exiting the system. At the inlet the flow rate is measured to be the sum of the primary and secondary streams. Through equation 16, the outlet flow rate is going to be equal to the inlet flow rate. This will be the theoretical outlet flow rate. By using equation 19, the sum of the flow rates can be calculated over the radius of the tube. The equation for the velocity as a function of radius is determined in 5.3.

Flow Rate	Theoretical	Experimental	Error
1	0.116 m^3/s	0.073m^3/s	37.1%
2	0.1 m^3/s	0.039m^3/s	61.0%

Table 2: Theoretical Vs. Experimental Velocity at Outlet

After integrating and comparing the two theoretical and experimental are significantly different. However this is expected because of sources of error and techniques for measuring flow rates. The main issue is that the equation for the velocity of air with respect to radius is off by a small amount, because there are gaps between the pitot tubes. Because of this, there are gaps where velocity is not measured, which are not accounted for in the equation. This skews the equation, and directly affects the flow rate. A more reliable way to measure would be to sum the dynamic pressure at each pitot tube and find flow rate that way.

The air that enters the tube must have energy to be transported across the tube to the outlet. But there are energy losses such as turbulence, friction, and thermal energy losses that cause a decrease in velocity and result in a lower experimental flow rate.

6 Conclusions

Analysis of the jet pump apparatus reveals that faster flow rates are more efficient than slower ones. However with the data from the experiment, this can not be proven. With a slow flow rate the fluids do not have to time to mix properly in the fixed length of tube. The first thing that was measured is velocity at the outlet. At the higher flow rate, theoretical data is accurate being 4.44% off of what is measured. For the slower flow, a greater error of 67.1& is observed.
From experimental data, at the faster flow rates, pressure stabilizes with a 0.166% error compared to the theoretical result. Flow 2 stabilizes with an error 0.418%.
The velocity gradient is measured at the outlet using a pitot rake, for the first flow, the measured velocity matches what is expected with a parabola starting slow at the wall, and increasing towards the centre line of the tube. Flow 2 consists of a rather flat gradient.
The flow rate at the outlet was compared to the theoretical value. By integrating along the pitot rake, a large loss of flow rate is observed. Flow 1 has an error value of 37.1%, while flow 2 has an error of 61.0%.
This lab is a good demonstration of losses in machines. It is concluded that to lower losses of energy, it is important to increase power provided to the machine. It may cost more, however it will be more efficient for process.

References

[1] Frank M. White (2016). *Fluid Mechanics*. New York: McGraw-Hill

7 Appendices

7.1 Spreadsheets

	Trial 2		Trial 1	
Tap	Height(m)	Static Pressure(Pa)	Height(m)2	Static Pressure(Pa)3
1(PC2)	0.109	101521.20	0.131	101207.28
2(Ps)	0.130	101315.19	0.154	100981.65
3	0.129	101325.00	0.157	100952.22
4	0.135	101266.14	0.152	101001.27
5	0.137	101246.52	0.148	101040.51
6	0.106	101550.63	0.144	101079.75
7	0.075	101854.74	0.142	101099.37
8	0.050	102099.99	0.138	101138.61
9	0.045	102149.04	0.135	101168.04
10	0.042	102178.47	0.131	101207.28
11	0.040	102198.09	0.128	101236.71
12	0.040	102198.09	0.125	101266.14
13	0.039	102207.90	0.122	101295.57
14	0.038	102217.71	0.12	101315.19
15	0.038	102217.71	0.119	101325.00
16	0.038	102217.71	0.117	101344.62
17(Pitot R	0.038	102217.71	0.115	101364.24
18	0.130	101325.00	0.118	101325
19	0.128	101325.00	0.119	101325
20(atm)	0.128	101325.00	0.119	101325

Figure 4: Static Pressure at Taps Along Mixing Tube

	Trial 2		Trial 1	
Tap	Height(m)	Stagnation Pressure(Pa)	Height(m)2	Stagnation Pressure(Pa)2
1	0.042	102168.66	0.089	101531.01
2	0.041	102178.47	0.083	101589.87
3	0.040	102188.28	0.080	101619.3
4	0.040	102188.28	0.076	101658.54
5	0.040	102188.28	0.072	101697.78
6	0.040	102188.28	0.069	101727.21
7	0.040	102188.28	0.068	101737.02
8	0.040	102188.28	0.067	101746.83
9	0.040	102188.28	0.063	101786.07
10	0.040	102188.28	0.061	101805.69
11	0.040	102188.28	0.06	101815.5
12	0.040	102188.28	0.062	101795.88
13	0.040	102188.28	0.064	101776.26
14	0.040	102188.28	0.064	101776.26
15	0.040	102188.28	0.064	101776.26
16	0.040	102188.28	0.065	101766.45
17	0.040	102188.28	0.065	101766.45
18	0.040	102188.28	0.066	101756.64
19	0.041	102178.47	0.068	101737.02
20(atm)	0.128	101325.00	0.110	101325.00

Figure 5: Stagnation Pressure at Pitot Tube Rake

Constant	Value
Primary Area(m^2)	3.87E-04
Secondary Area(m^2)(	4.17E-03
Cq	0.93
Cp	-0.045
PC1(Trial 1)	108957.18
PC1(Trial 2)	108682.5
Outlet Area(m^2)	4.56E-03
Outlet Radius(m)	0.03809
Delta H Flow 1	0.778
Delta H Flow 2	0.75

Figure 6: Constants Used Throughout Lab

7.2 Sample Calculations

Primary Pressure at inlet and any pressure from manometer bank - Equation 2

$$P_1 = \rho g \Delta h + P_2$$
$$P_1 = (1000\mathrm{kg/m^3})(9.81\mathrm{m/s^2})(0.778m) + 101325Pa$$
$$P_1 = 108957.18Pa$$

Primary Stream Velocity - Equation 5

$$V_p = \sqrt{\frac{2C_q(P_{C1} - P_{C2}}{\rho_{air}}}$$
$$V_p = \sqrt{\frac{2(0.93)(108957.18Pa - 101521.2Pa)}{1.2\mathrm{kg/m^3}}}$$
$$V_p = 107.35m/s$$

Primary Stream Pressure - Equation 9

$$P_p = C_p(P_{C1} - P_{C2}) + P_{C2}$$
$$P_p = -0.045(108957.18Pa - 101521.2Pa) + 101521.2Pa$$
$$P_p = 101186.6Pa$$

Mass Flow Rate - Equation 12

$$\dot{m} = \rho V A$$
$$\dot{m} = (1.2\mathrm{kg/m^3})(107.35m/s)(3.87 \times 10^{-4}\,\mathrm{m^2})$$
$$\dot{m} = 0.04986kg/s$$

Mass Flow Rate at outlet Theoretical - Equation 16

$$\dot{m}_{in} = \dot{m}_{out}$$
$$\dot{m}_p + \dot{m}_s = \dot{m}_{out}$$
$$\dot{m}_{out} = 0.04986kg/s + 0.090kg/s$$
$$\dot{m}_{out} = 0.140kg/s$$

Velocity at Outlet Theoretical - Equation 17

$$V_{out} = \frac{V_p A_p + V_s A_s}{A_{out}}$$
$$V_{out} = \frac{(107.35m/s)(3.87 \times 10^{-4}\,\mathrm{m^2}) + (18.08m/s)(4.17 \times 10^{-4}\,\mathrm{m^2})}{4.56 \times 10^{-4}\,\mathrm{m^2}}$$
$$V_{out} = 25.66m/s$$

Pressure at Outlet Theoretical - Equation 15

$$P_{out} = \frac{\dot{m}_{out}V_{out} - \dot{m}_p V_p - \dot{m}_s V_s - P_s A_s + P_{out}A_{out}}{A_p}$$

$$P_{out} = \frac{(0.140kg/s)(25.66m/s) - (0.04986kg/s)(107.35m/s) - (0.090kg/s)(18.08m/s) - (101315.19Pa)(4.17 \times}{4.56 \times 10^{-4}\,\mathrm{m}^2}$$

$$P_{out} = 102047.51Pa$$

Velocity at Each Pitot Tube Experimental - Equation 11

$$V = \sqrt{\frac{2(P_{stag} - P_{static})}{\rho}}$$

$$V = \sqrt{\frac{2(102168.66Pa - 102217.71Pa)}{1.2\mathrm{kg/m}^3}}$$

$$V = 9.042m/s$$

Flow Rate at The Outlet - Equation 19

$$Q = 2\pi \int_0^r V(r) r dr$$

$$Q = 2\pi \int_0^{0.03809m} (-0.0696r^2 + 1.7605r + 15.961) r dr$$

$$Q = 2\pi \int_0^{0.03809m} (-0.0696r^3 + 1.7605r^2 + 15.961r) dr$$

$$Q = 2\pi[(-0.0174r^4 + 0.5868r^3 + 7.9805r^2)]_0^{0.03809m}$$

$$Q = 0.07295\mathrm{m}^3/\mathrm{s}$$