6. domaća zadaća iz Neizrazitog, evolucijskog i neuroračunarstva

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1. Izvod postupka učenja

Prolaz unaprijed kroz mrežu:

$$\alpha_i^{(x)} = \frac{1}{1+e^{b_i^{(x)}\cdot(x-a_i^{(x)})}} \quad \alpha_i^{(y)} = \frac{1}{1+e^{b_i^{(y)}\cdot(y-a_i^{(y)})}} \quad \alpha_i = \alpha_i^{(x)} \cdot \alpha_i^{(y)}$$

$$o_k = \frac{\sum_{i=1}^{m} \alpha_i \cdot (p_i \cdot x_k + q_i \cdot y_k + r_i)}{\sum_{j=1}^{m} \alpha_j} \quad E_k = \frac{1}{2}\cdot(z_k - o_k)^2$$

Gradijent za p_i:

$$\frac{\partial E_k}{\partial p_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial p_i} \qquad \frac{\partial E_k}{\partial o_k} = -(z_k - o_k) \qquad \frac{\partial o_k}{\partial p_i} = \frac{\alpha_i \cdot x_k}{\sum_{j=1}^{m} \alpha_j}$$

$$\frac{\partial E_k}{\partial p_i} = -(z_k - o_k) \cdot \frac{\alpha_i \cdot x_k}{\sum_{j=1}^{m} \alpha_j}$$

Gradijent za q_i:

$$\frac{\partial E_k}{\partial q_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial q_i} \qquad \frac{\partial o_k}{\partial q_i} = \frac{\alpha_i \cdot y_k}{\sum_{j=1}^{m} \alpha_j} \qquad \frac{\partial E_k}{\partial q_i} = -(z_k - o_k) \cdot \frac{\alpha_i \cdot y_k}{\sum_{j=1}^{m} \alpha_j}$$

Gradijent za r_i:

$$\frac{\partial E_k}{\partial r_i}=\frac{\partial E_k}{\partial o_k}\cdot\frac{\partial o_k}{\partial r_i} \qquad \frac{\partial o_k}{\partial r_i}=\frac{\alpha_i}{\sum_{j=1}^{m}\alpha_j} \qquad \frac{\partial E_k}{\partial r_i}=-(z_k-o_k)\cdot\frac{\alpha_i}{\sum_{j=1}^{m}\alpha_j}$$

Ažuriranje parametara p_i, q_i, r_i :

	Koristeći pravi gradijent	Koristeći stohastičku varijantu
p	$p_i(t+1)=p_i(t)+\eta\sum_{k=1}^{N}(z_k-o_k)\cdot\frac{\alpha_i\cdot x_k}{\sum_{j=1}^{m}\alpha_j}$	$p_i(t+1)=p_i(t)+\eta(z_k-o_k)\cdot\frac{\alpha_i\cdot x_k}{\sum_{j=1}^{m}\alpha_j}$
q	$q_i(t+1)=q_i(t)+\eta\sum_{k=1}^{N}(z_k-o_k)\cdot\frac{\alpha_i\cdot y_k}{\sum_{j=1}^{m}\alpha_j}$	$q_i(t+1)=q_i(t)+\eta(z_k-o_k)\cdot\frac{\alpha_i\cdot y_k}{\sum_{j=1}^{m}\alpha_j}$
r	$r_i(t+1)=r_i(t)+\eta\sum_{k=1}^{N}(z_k-o_k)\cdot\frac{\alpha_i}{\sum_{j=1}^{m}\alpha_j}$	$r_i(t+1)=r_i(t)+\eta(z_k-o_k)\cdot\frac{\alpha_i}{\sum_{j=1}^{m}\alpha_j}$

Gradijent za $a_i^{(x)}$:

$a_i^{(x)}$ utječe na $\alpha_i^{(x)}$, $\alpha_i^{(x)}$ utječe na α_i , α_i utječe na o_k

- stoga $a_i^{(x)} \Rightarrow \alpha_i^{(x)} \Rightarrow \alpha_i \Rightarrow o_k$

$$\frac{\partial E_k}{\partial a_i^{(x)}} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial \alpha_i^{(x)}} \cdot \frac{\partial \alpha_i^{(x)}}{\partial a_i^{(x)}}$$

$$\frac{\partial o_k}{\partial \alpha_i} = \frac{\sum_{j=1, j \neq i}^{m} \alpha_j \cdot ((p_i \cdot x_k + q_i \cdot y_k + r_i) - (p_j \cdot x_k + q_j \cdot y_k + r_j))}{(\sum_{j=1}^{m} \alpha_j)^2} \qquad \frac{\partial \alpha_i}{\partial \alpha_i^{(x)}} = \alpha_i^{(y)}$$

$$\frac{\partial \alpha_i^{(x)}}{a_i^{(x)}} = b_i^{(x)} \cdot (1 - \alpha_i^{(x)}) \cdot \alpha_i^{(x)}$$

$$\frac{\partial E_k}{\partial a_i^{(x)}} = -(z_k - o_k) \cdot \frac{\sum_{j=1, j \neq i}^{m} \alpha_j \cdot ((p_i \cdot x_k + q_i \cdot y_k + r_i) - (p_j \cdot x_k + q_j \cdot y_k + r_j))}{(\sum_{j=1}^{m} \alpha_j)^2} \cdot \alpha_i^{(y)} \cdot b_i^{(x)} \cdot (1 - \alpha_i^{(x)}) \cdot \alpha_i^{(x)}$$

Gradijent za $b_i^{(x)}$:

$$\frac{\partial E_k}{\partial b_i^{(x)}} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial \alpha_i^{(x)}} \cdot \frac{\partial \alpha_i^{(x)}}{\partial b_i^{(x)}} \qquad \frac{\partial \alpha_i^{(x)}}{b_i^{(x)}} = -(x - a_i^{(x)}) \cdot \alpha_i^{(x)} \cdot (1 - \alpha_i^{(x)})$$

$$\frac{\partial E_k}{\partial b_i^{(x)}} = -(z_k - o_k) \cdot \frac{\sum_{j=1, j \neq i}^{m} \alpha_j \cdot ((p_i \cdot x_k + q_i \cdot y_k + r_i) - (p_j \cdot x_k + q_j \cdot y_k + r_j))}{(\sum_{j=1}^{m} \alpha_j)^2} \cdot \alpha_i^{(y)} \cdot -(x - a_i^{(x)}) \cdot \alpha_i^{(x)} \cdot (1 - \alpha_i^{(x)})$$

Gradijent za $a_i^{(y)}$:

$$\frac{\partial E_k}{\partial a_i^{(y)}}=\frac{\partial E_k}{\partial o_k}\cdot\frac{\partial o_k}{\partial \alpha_i}\cdot\frac{\partial \alpha_i}{\partial \alpha_i^{(y)}}\cdot\frac{\partial \alpha_i^{(y)}}{\partial a_i^{(y)}} \quad \frac{\partial \alpha_i}{\partial \alpha_i^{(x)}}=\alpha_i^{(y)} \quad \frac{\partial \alpha_i^{(y)}}{a_i^{(y)}}=b_i^{(y)}\cdot(1-\alpha_i^{(y)})\cdot\alpha_i^{(y)}$$

$$\frac{\partial E_k}{\partial a_i^{(y)}}=-(z_k-o_k)\cdot\frac{\sum_{j=1,j\neq i}^{m}\alpha_j\cdot((p_i\cdot x_k+q_i\cdot y_k+r_i)-(p_j\cdot x_k+q_j\cdot y_k+r_j))}{(\sum_{j=1}^{m}\alpha_j)^2}\cdot\alpha_i^{(x)}\cdot b_i^{(y)}\cdot(1-\alpha_i^{(y)})\cdot\alpha_i^{(y)}$$

Gradijent za $b_i^{(y)}$:

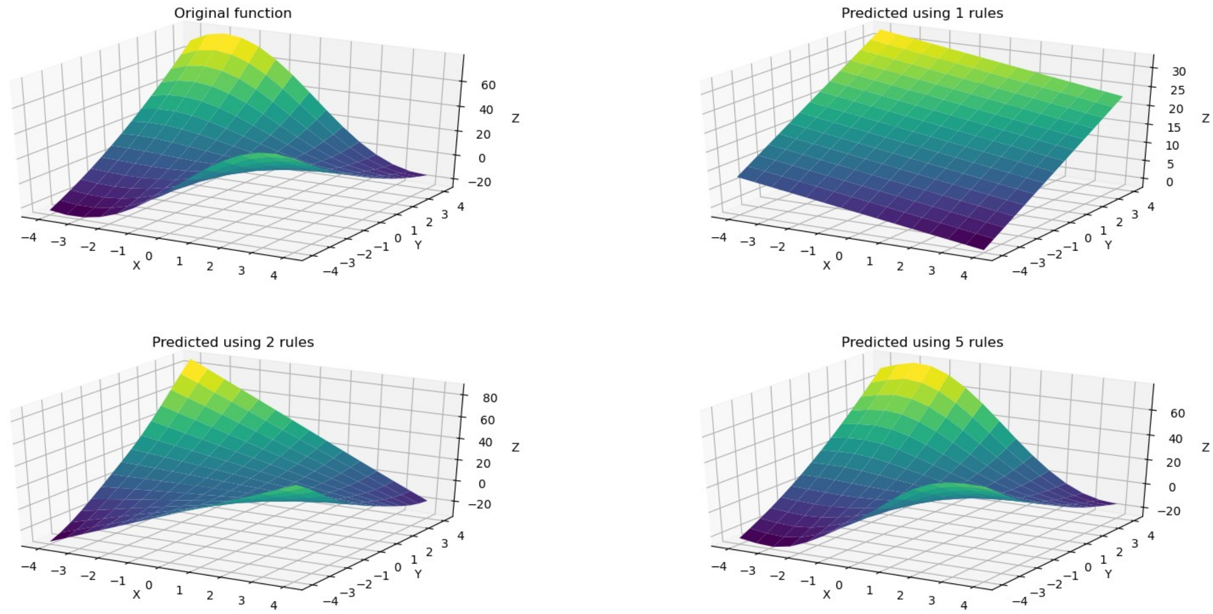
$$\frac{\partial E_k}{\partial b_i^{(y)}}=\frac{\partial E_k}{\partial o_k}\cdot\frac{\partial o_k}{\partial \alpha_i}\cdot\frac{\partial \alpha_i}{\partial \alpha_i^{(x)}}\cdot\frac{\partial \alpha_i^{(x)}}{\partial b_i^{(x)}} \quad \frac{\partial \alpha_i^{(y)}}{b_i^{(y)}}=-(x-a_i^{(y)})\cdot\alpha_i^{(y)}\cdot(1-\alpha_i^{(y)})$$

$$\frac{\partial E_k}{\partial b_i^{(y)}}=-(z_k-o_k)\cdot\frac{\sum_{j=1,j\neq i}^{m}\alpha_j\cdot((p_i\cdot x_k+q_i\cdot y_k+r_i)-(p_j\cdot x_k+q_j\cdot y_k+r_j))}{(\sum_{j=1}^{m}\alpha_j)^2}\cdot\alpha_i^{(x)}\cdot-(y-a_i^{(y)})\cdot\alpha_i^{(y)}\cdot(1-\alpha_i^{(y)})$$

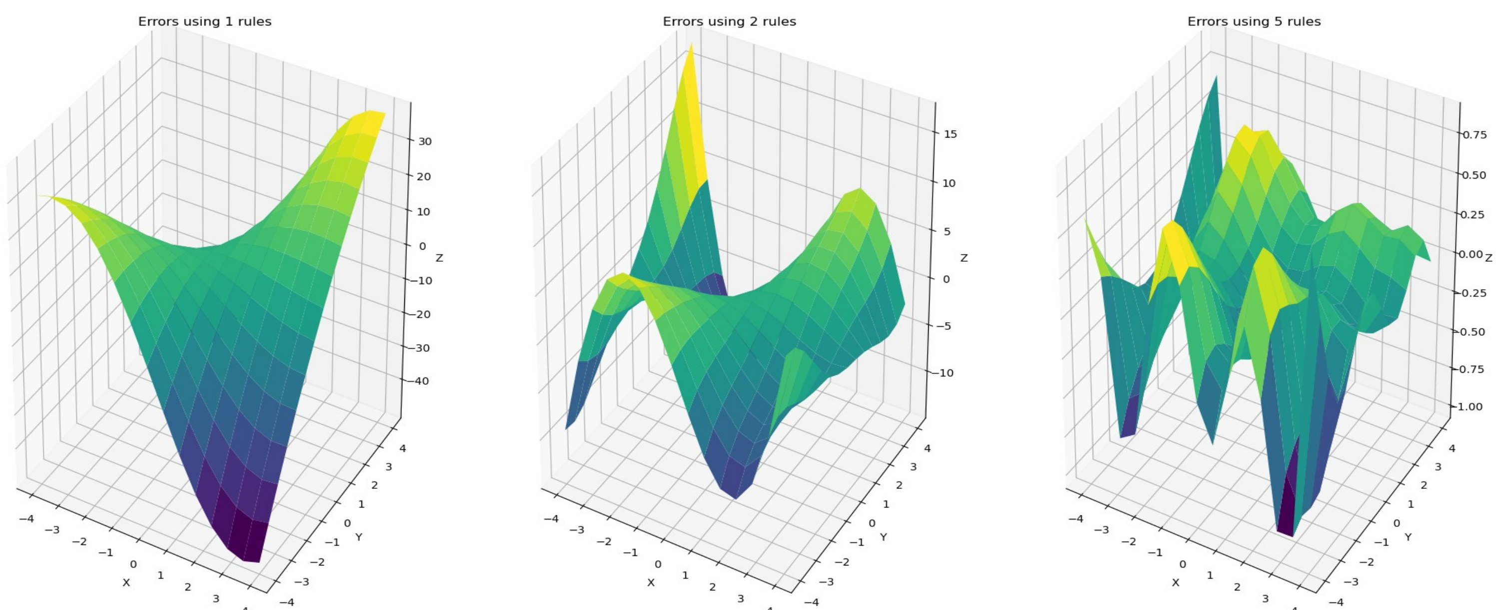
	Koristeći pravi gradijent
$a_i^{(x)}$	$a_i^{(x)}(t+1)=a_i^{(x)}(t)+\eta\cdot\sum_{k=1}^{N}(z_k-o_k)\cdot\frac{\sum_{j=1,\,j\neq i}^{m}\alpha_j\cdot((p_i\cdot x_k+q_i\cdot y_k+r_i)-(p_j\cdot x_k+q_j\cdot y_k+r_j))}{(\sum_{j=1}^{m}\alpha_j)^2}\cdot\alpha_i^{(y)}\cdot b_i^{(x)}\cdot(1-\alpha_i^{(x)})\cdot\alpha_i^{(x)}$
$a_i^{(x)}$	$b_i^{(x)}(t+1)=b_i^{(x)}(t)+\eta\cdot\sum_{k=1}^{N}(z_k-o_k)\cdot\frac{\sum_{j=1,\,j\neq i}^{m}\alpha_j\cdot((p_i\cdot x_k+q_i\cdot y_k+r_i)-(p_j\cdot x_k+q_j\cdot y_k+r_j))}{(\sum_{j=1}^{m}\alpha_j)^2}\cdot\alpha_i^{(y)}\cdot-(x-a_i^{(x)})\cdot\alpha_i^{(x)}\cdot(1-\alpha_i^{(x)})$
$a_i^{(y)}$	$a_i^{(y)}(t+1)=a_i^{(y)}(t)+\eta\cdot\sum_{k=1}^{N}(z_k-o_k)\cdot\frac{\sum_{j=1,\,j\neq i}^{m}\alpha_j\cdot((p_i\cdot x_k+q_i\cdot y_k+r_i)-(p_j\cdot x_k+q_j\cdot y_k+r_j))}{(\sum_{j=1}^{m}\alpha_j)^2}\cdot\alpha_i^{(x)}\cdot b_i^{(y)}\cdot(1-\alpha_i^{(y)})\cdot\alpha_i^{(y)}$
$b_i^{(y)}$	$b_i^{(y)}(t+1)=b_i^{(y)}(t)+\eta\cdot\sum_{k=1}^{N}(z_k-o_k)\cdot\frac{\sum_{j=1,\,j\neq i}^{m}\alpha_j\cdot((p_i\cdot x_k+q_i\cdot y_k+r_i)-(p_j\cdot x_k+q_j\cdot y_k+r_j))}{(\sum_{j=1}^{m}\alpha_j)^2}\cdot\alpha_i^{(x)}\cdot-(y-a_i^{(y)})\cdot\alpha_i^{(y)}\cdot(1-\alpha_i^{(y)})$

	Koristeći stohastičku varijantu
$a_i^{(x)}$	$a_i^{(x)}(t+1)=a_i^{(x)}(t)+\eta\cdot(z_k-o_k)\cdot\frac{\sum_{j=1,\,j\neq i}^{m}\alpha_j\cdot((p_i\cdot x_k+q_i\cdot y_k+r_i)-(p_j\cdot x_k+q_j\cdot y_k+r_j))}{(\sum_{j=1}^{m}\alpha_j)^2}\cdot\alpha_i^{(y)}\cdot b_i^{(x)}\cdot(1-\alpha_i^{(x)})\cdot\alpha_i^{(x)}$
$b_i^{(x)}$	$b_i^{(x)}(t+1)=b_i^{(x)}(t)+\eta\cdot(z_k-o_k)\cdot\frac{\sum_{j=1,\,j\neq i}^{m}\alpha_j\cdot((p_i\cdot x_k+q_i\cdot y_k+r_i)-(p_j\cdot x_k+q_j\cdot y_k+r_j))}{(\sum_{j=1}^{m}\alpha_j)^2}\cdot\alpha_i^{(y)}\cdot-(x-a_i^{(x)})\cdot\alpha_i^{(x)}\cdot(1-\alpha_i^{(x)})$
$a_i^{(y)}$	$a_i^{(y)}(t+1)=a_i^{(y)}(t)+\eta\cdot(z_k-o_k)\cdot\frac{\sum_{j=1,\,j\neq i}^{m}\alpha_j\cdot((p_i\cdot x_k+q_i\cdot y_k+r_i)-(p_j\cdot x_k+q_j\cdot y_k+r_j))}{(\sum_{j=1}^{m}\alpha_j)^2}\cdot\alpha_i^{(x)}\cdot b_i^{(y)}\cdot(1-\alpha_i^{(y)})\cdot\alpha_i^{(y)}$
$b_i^{(y)}$	$b_i^{(y)}(t+1)=b_i^{(y)}(t)+\eta\cdot(z_k-o_k)\cdot\frac{\sum_{j=1,\,j\neq i}^{m}\alpha_j\cdot((p_i\cdot x_k+q_i\cdot y_k+r_i)-(p_j\cdot x_k+q_j\cdot y_k+r_j))}{(\sum_{j=1}^{m}\alpha_j)^2}\cdot\alpha_i^{(x)}\cdot-(y-a_i^{(y)})\cdot\alpha_i^{(y)}\cdot(1-\alpha_i^{(y)})$

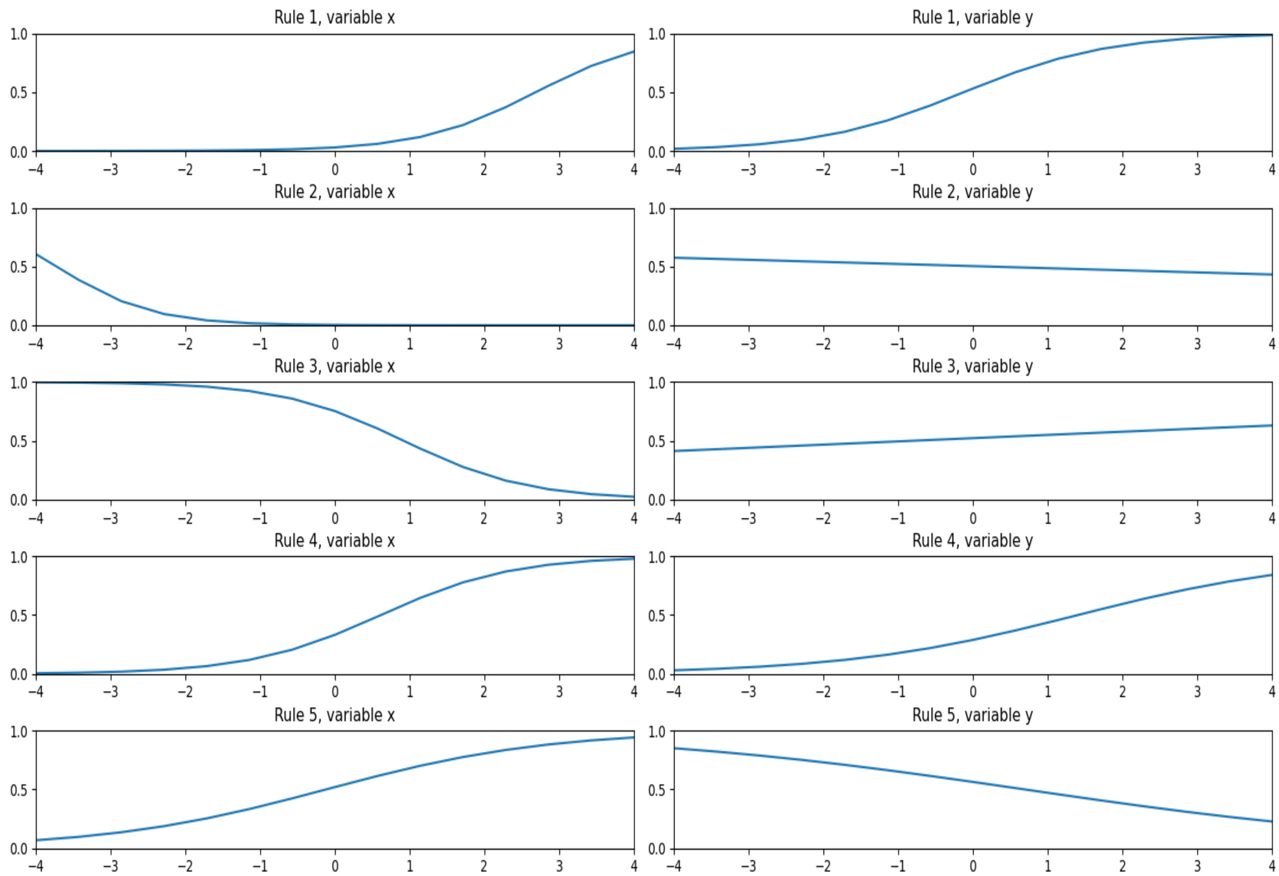
2. Aproksimacije funkcija za različit broj pravila



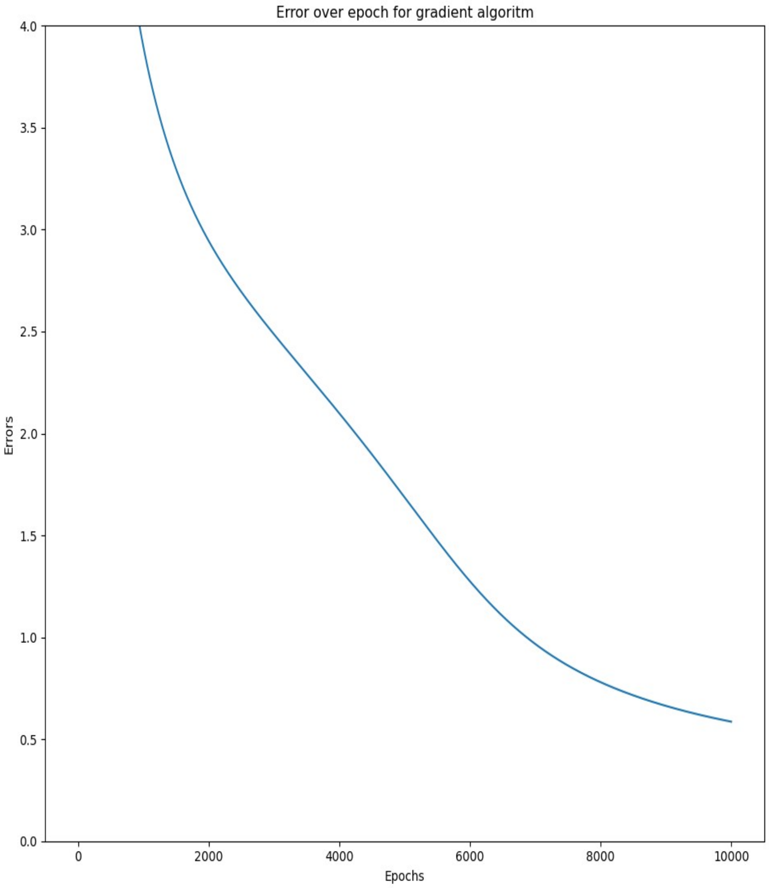
3. Greške aproksimacija

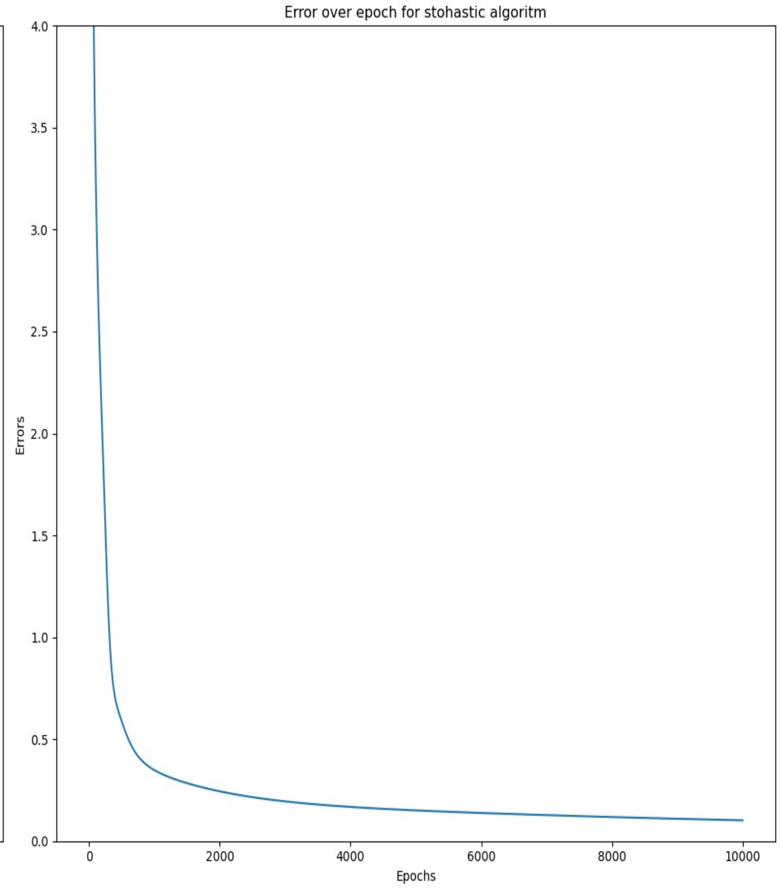


4. Funkcije pripadnosti po pravilima



5. Greške kroz epohe za stohastički i gradijentni algoritam





6. Greške kroz epohe za stohastički i gradijentni algoritam i različite stope učenja