Paper Offer Number 02ATT-97

Vehicle Speed Estimation Using Accelerometer and Wheel Speed Measurements

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ABSTRACT

This paper treats the problem of estimating the longitudinal velocity of a braking vehicle using measurements from an accelerometer and wheel speed data from standard antilock braking wheel speed sensors. We develop and experimentally test three velocity estimation algorithms of increasing complexity. The algorithm that works the best gives peak errors of less than 3 percent even when the accelerometer signal is significantly biased.

INTRODUCTION

WHY ESTIMATE VEHICLE SPEED?

One reason the absolute vehicle speed, *v*, is of interest is because it—along with the angular speed of the wheels, ω_i, and the radii of the tires, *r*—can be used to calculate the longitudinal slip ratio s_i at the vehicle's wheels:

$$s_i = \frac{r\omega_i - v}{v} \qquad i = 1,2,3,4 \tag{1}$$

Longitudinal slip is important because it is related to longitudinal tire force—and therefore the acceleration/deceleration of the vehicle—through tire models like the "Magic Formula Tire Model" (Bakker and Pacejka (1989)). Figure 1 shows a plot of normalized longitudinal tire force versus longitudinal slip ratio for traction on several road surfaces, obtained using the "Magic Formula." This "slip curve" shows that the tire force is typically an increasing function of slip ratio until a critical slip value—in this case about 15 percent—where there is a maximum. After this critical slip value, more slip leads to a decrease in tire force and wheel lock-up.

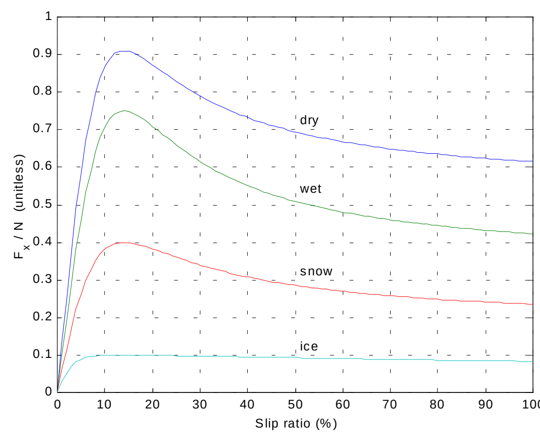

Figure 1: Simulated slip curves on several surfaces

Several automotive products use the slip curve concept. The most familiar of these are antilock braking systems (ABS) which look for the large slips and high wheel accelerations associated with impending lock-up and then lower the brake pressure to prevent it.

Other systems that exploit more subtle information from the slip curve might be feasible if it were possible to precisely measure slip (which is essentially the problem of precisely measuring velocity). For example, it might be possible to measure slip in order to warn the driver of overly aggressive driving before it causes the ABS to engage.

According to some research results, it may even be possible to estimate road coefficient of friction using slip information. Several researchers—for example, Dieckmann (1992), Gustafsson (1997), Yi *et al.* (1999) and Müller *et al.* (2002)—have demonstrated that there may be potential for a slip-based estimator of the maximum tire/road coefficient of friction, μ_{max}. A slip-based μ_{max} estimator attempts to use low slip, low tire force data from normal driving to determine the maximum amount of friction that is available to the driver. It therefore demands precise velocity estimation and slip calculation. Although this is possible for two wheel drive vehicles in traction using standard sensors (see Dieckmann (1992), for example), it is not yet possible for braking vehicles without a ground reference velocity sensor.

EXISTING SPEED ESTIMATION TECHNOLOGIES

Strategies to estimate vehicle speed fall into two main categories: ground reference techniques and non-ground reference techniques. Ground reference techniques tend to be more accurate, but they also tend to be more expensive than non-ground reference techniques. Some of them include:

1. *Optical cross-correlation sensors*: These measure the distance the vehicle travels in a given time period by correlating features on the ground. (See Corrsys in the references section, for example).
2. *Radar sensors*: When pointed forward and towards the ground, radars can detect a slight shift in the frequency of returning waves that is correlated with vehicle ground speed.
3. *5^{th} wheel sensors*: A bicycle-like wheel is mounted from a spring loaded arm on the back of the car. Because the wheel does not slip, and because its radius is well known, the velocity can be calculated.
4. *Global positioning systems*: They can calculate velocity directly (not just by differentiating position fixes). See Miller *et al.* (2001) and Bevly *et al.* (2001).

Frequently used non-ground reference techniques include using the maximum wheel speed multiplied by the effective tire radius, using Kalman Filters to combine acceleration and wheel speed estimates into an estimate (Kobayashi, Cheok and Watanabe (1995)), and using fuzzy logic to combine accelerometers and wheel speed sensors (Daiss and Kiencke (1995), Basset *et al.* (1997))

PAPER SUMMARY

The goal of the paper is to use measurements from a standard 50 tooth ABS sensor and measurements from a longitudinal accelerometer to get a high precision estimate of the vehicle's longitudinal speed.

For three main reasons, we have chosen to develop the algorithms using wheel speed data only from the left front wheel. First, such an approach makes it easier to examine the graphical results because it is possible to see a direct relationship between the behavior of the one wheel and the speed estimate. Several wheels would make this relationship harder to see. Second, this approach demonstrates a worst-case scenario for a velocity estimator. In such a worst case, all of the wheels but one lock up, and then the one remaining wheel locks, leaving the estimator with no data except for the accelerometer signal. Third, the simplicity gained by using only one wheel makes the essence of the velocity estimation problem stand out. It becomes possible to derive several interesting expressions and see clearly more clearly what the trade-offs are.

The obvious intuitive solution to the one wheel velocity estimation problem is: "When tire slip is low, calculate ground velocity using the wheel speed sensor; when tire slip is high, calculate the ground velocity by integrating the accelerometer."

The three velocity estimation algorithms that are presented are each based on this strategy, but differ in how they implement it. The first method is appealing because it is no more complex than the rule above. However, it has difficulties when the accelerometer is biased, when the wheel speed signal is noisy, and when the effective tire radius deviates from its nominal value. The second method uses a Kalman Filter and fuzzy logic to "fuse" the accelerometer and wheel speed measurements. The Kalman Filter method is effective for reducing noise, but it still has difficulties when the accelerometer is biased, or when the effective tire radius differs from its nominal value. The final method uses regression to simultaneously identify the effective tire radius and the accelerometer bias, and it is found to deliver very good velocity estimates, even in the presence of parameter changes.

All results in the paper are calculated using experimental data from straight-line braking maneuvers using a rear wheel drive test vehicle. The left front wheel speed signal and the accelerometer signal are the main sensors used for the velocity estimates. In addition to the accelerometer and wheel speed sensor, the vehicle is outfitted with a fifth wheel to provide a ground velocity reference.

ALGORITHM 1: SIMPLE ESTIMATOR

As a baseline for comparison, let us develop a simple, intuitive velocity estimator and analyze some difficulties it has.

Since the wheel speed signal is from the front wheel of a rear wheel drive car, it's slip is negligible whenever the car is not braking. Therefore, when the car is not braking (easily detected using the existing brake light circuit), we calculate the simple velocity estimate at time step k as $v_{simple}(k) = r_{est}\omega(k)$, where r_{est} is the estimate of the effective tire radius, and $\omega(k)$ is the angular speed of the wheel.

When the brakes are activated, we get the simple velocity estimate at time step *k* by numerically integrating the accelerometer according to

$$v_{simple}(k) = v_{simple}(k_{no\_brake}) + \sum_{i=k_{no\_brake}}^{k} a_{meas}(i) \cdot dt \quad (2)$$

where k_{no_brake} is the last time index at which there was no braking, $a_{meas}(\,)$ is the acceleration measurement, and dt is the sample time interval.

Figure 2 shows the performance of this simple velocity estimation algorithm. The maneuver was a straight line acceleration for approximately 5 seconds followed by braking of increasing intensity until the wheels locked at approximately 8 seconds. The true speed of the vehicle (from a 5th wheel) vs. time is shown by the thick gray line, and the speed of the braking wheel is shown by the thick line that drops abruptly to 0m/s at approximately 8s.

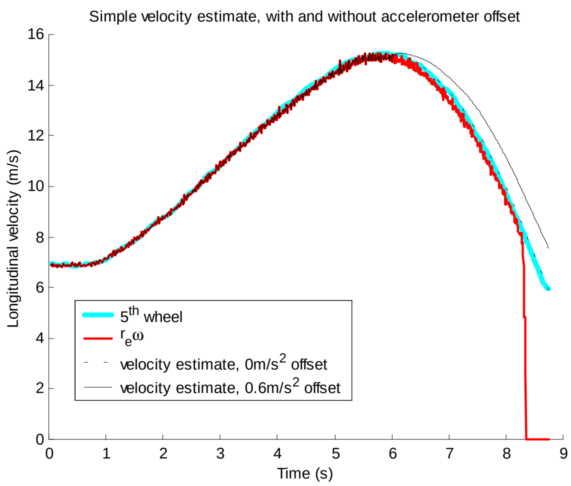

Figure 2: Performance of simple velocity estimation algorithm (algorithm 1). When there is no accelerometer bias, the velocity estimate (dotted line) is close to the true velocity from the 5th wheel. However, accelerometer bias makes the simple velocity estimate unacceptable (solid black line).

The two black lines show the estimation results. The dotted black line shows the estimate v_{simple} vs. time when the accelerometer has no offset—that is, the measured acceleration, $a_{meas}(t)$, equals the actual longitudinal acceleration $a(t)$ at each time instant t. In the case when the accelerometer measurement perfectly reflects the vehicle's acceleration, the estimate of the simple algorithm is quite good. The dotted line of Figure 3 shows that the percent velocity estimation error is less than 2 percent for the majority of the maneuver and only reaches a peak value of 4percent when the wheel is locked, the speed is low (making it easier for small absolute errors to translate to large percent errors).

The solid line of Figure 2 shows the estimate v_{simple} when the accelerometer is biased by +0.6m/s^2 (so that the measured acceleration is 0.6m/s^2 larger than the actual acceleration). As soon as the estimate starts to rely on the integrated accelerometer signal (at approximately 6 seconds), the velocity estimate diverges from the actual velocity. Figure 3, shows this same divergence in percentage terms. The solid line is the percentage error corresponding to the solid black line of Figure 2.

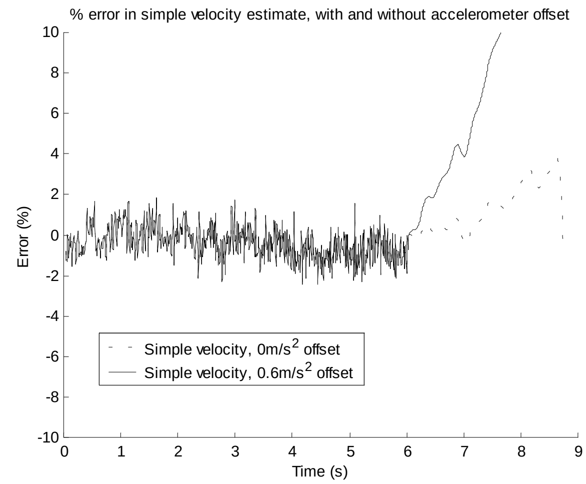

Figure 3: Percent errors for the velocity estimates of Figure 2, showing the simple estimation algorithm gives acceptable results when there is no accelerometer offset, but deteriorates when there is offset.

It quickly diverges to errors of 10 percent or more, rendering the simple velocity estimate inutile for most uses.

Unfortunately, biases do occur quite frequently in acceleration measurements, so the simple estimator's lack of robustness to them is a serious difficulty. Some sources of longitudinal accelerometer bias include road slope, temporary pitch angle changes resulting from longitudinal accelerations, and longer-term pitch angle changes due to vehicle loading, active/semi-active suspension behavior, and suspension aging. Of these factors, road slope is typically the most important. The 0.6m/s^2 accelerometer bias used here simulates the effect of a grade of 6 percent—a value that occasionally occurs on even the highest quality roads. (For tests correlating accelerometer bias and road slope, the reader is referred to Daiss and Kiencke (1995)).

Similarly, this simple velocity estimation algorithm is not robust to errors/changes in the dynamic tire radius. It is straightforward to show that during non-braking phases, the percent error in the simple velocity estimate is equal to

$$100 \times \left(\frac{r_{est} - r}{r} \right) \quad (3)$$

where r is the true effective tire radius, and r_{est} is, like above, the estimate of the effective tire radius used in the velocity estimator. Thus, a two percent error in the effective tire radius estimate results in a constant two percent error in the velocity estimate. Fortunately, the effective tire radius tends to change very little and very slowly under most circumstances.

A final difficulty with this simple velocity estimation algorithm is that the noise of the wheel speed measurement passes directly through to the velocity estimate during non-braking phases. This can be seen in Figure 2 and Figure 3 as high frequency noise on the estimates (or error) for times less than six seconds. This is not a serious problem in these particular tests because the wheel speed noise is reasonably small. However, it warrants attention for two reasons. First, it is a problem that grows directly with the sensor noise level, so it could become a problem when the sensor noise level is high. Second, it is a problem that can be remedied by utilizing redundant sensor information and standard theoretical techniques.

ALGORITHM 2: KALMAN FILTER/WEIGHTED AVERAGE VELOCITY ESTIMATOR

Thus, before returning to the radius and offset robustness problems, we take a moment in this section to develop a Kalman Filter that uses redundant sensor information to attenuate the wheel speed noise that found its way into the simple velocity estimate above.

We say that there is “redundant” sensor information during the non-braking phases of the maneuver because the vehicle speed could be calculated two different ways. First, because the wheel is not slipping, we can use the wheel speed signal, as in the simple velocity estimator of the previous section. Second, in theory at least, we could integrate the accelerometer signal starting at some known initial condition. This was the strategy taken during the braking phase for the simple estimator of the previous section.

Each of these velocity estimation methods has flaws. For example, the wheel speed signal is noisy, and is multiplied by an imperfect effective radius estimate to obtain velocity. As we saw in the previous section, the integrated accelerometer tends to diverge from the true velocity due to bias.

A standard technique that is sometimes used for “fusing” imperfect measurements and system models to make less noisy, more robust estimate of a system’s state (in our case, the velocity) is the Kalman Filter, which is documented in Anderson and Moore (1979), and Hayes (1996), among others. It is used in circumstances similar to ours in Daiss and Kiencke (1995) and Kobayashi, Cheok and Watanabe (1995).

We model the vehicle as a difference equation that performs a discrete integration of the measured vehicle acceleration *$a_{meas}(k)$* plus a noise term *$w(k)$* to arrive at the true velocity, *$v(k)$*. That is,

$$v(k) = [1]\cdot v(k-1) + [dt]\cdot a_{meas}(k) + w(k) \qquad (4)$$

At each sample interval, we have a noisy measurement *$\omega(k)$* of the wheel speed, which is formed from the actual velocity *$v(k)$* and the estimated effective radius *r_{est}* according to the measurement equation (assuming no wheel slip)

$$\omega(k) = [1/r_{est}]\cdot v(k) + n(k) \qquad (5)$$

If the noise processes *$w(k)$* and *$n(k)$* were white, then the least squares optimal estimate of the true velocity given the measurement *$\omega(k)$* and all of its predecessors would take the form

$$\begin{aligned}\hat{v}(k) &= [1]\cdot \hat{v}(k-1) + [dt]\cdot a_{meas}(k)\cdots \\ &+ K_1\left(\omega(k) - \frac{1}{r_{est}}\left([1]\cdot\hat{v}(k-1) + [dt]\cdot a_{meas}(k)\right)\right)\end{aligned} \qquad (6)$$

where $\hat{v}(k)$ is the optimal estimate and *K_1* is a specially chosen gain called the Kalman gain. Of course, the noise in our situation is not white, but this same general form taken from Kalman filtering theory still provides velocity estimates that tend to be better than the estimates obtained using just the accelerometer or just the wheel speed measurement.

A more intuitive way of looking at this Kalman Filter-like estimator can be obtained by defining *K_2* as *K_1/r_{est}* and rearranging for several lines to get

$$\hat{v}(k) \quad K_2 r_{est} \quad (k) \quad (1 \quad K_2)([1] \ \hat{v}(k \quad 1) \quad [dt] a_{meas}(k))$$

Following the intuition of the simple estimation algorithm, the weighting parameter, K_2, should not be the same under all circumstances. When the wheel slip is negligible—for example, when coasting—it should be closer to one than when braking hard. For the tests shown here we made K_2 vary between 0.09 when the wheel slip was low to 0.0 when the wheel slip was high.

To chose K_2 at each sample step, we calculated a slip $s(k)$ using the current wheel speed and the previous velocity estimate

$$s(k) = \frac{r_{est}\omega(k) - \hat{v}(k-1)}{\hat{v}(k-1)} \tag{8}$$

The slip value was then plugged into the K_2 vs. slip curve pictured in Figure 4. The Matlab Fuzzy Logic Toolbox (see The Mathworks (1999) for an accessible introduction to fuzzy logic and its implementation with the Matlab software) generated the surface according to the following rules:

	If the magnitude of the wheel slip is…		
	Near zero	Medium	High
Then K_2 is…	High	Medium	Low

As the K_2 vs. slip surface of Figure 4 indicates, the fuzzy sets corresponding to “near zero” and “medium” slip were very narrow bands centered around zero slip. Similarly, a “high” value of K_2 was only about 0.09.

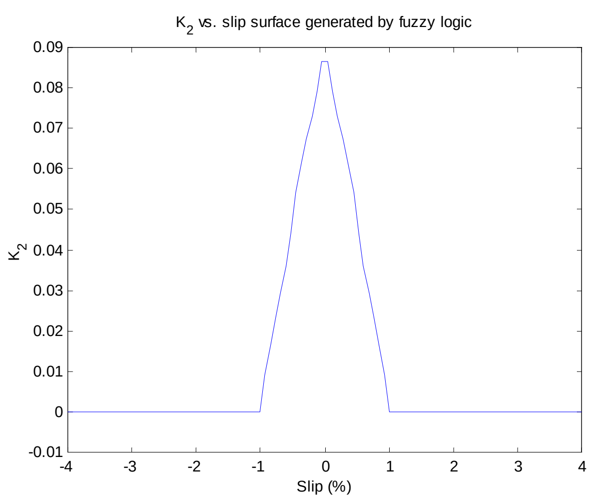

Figure 4: K_2 vs. slip surface generated by fuzzy logic.

Figure 5 shows velocity estimation results of the weighted average velocity estimator. The maneuver is the same straight line maneuver as that of Figure 2 of the previous section. As in Figure 2, the 5h wheel signal gives the true velocity reference, and the signal that drops to zero is the wheel speed signal multiplied by the effective radius estimate r_{est}.

When the accelerometer has no offset, the weighted average velocity estimation algorithm gives results with very little noise and good accuracy (dotted black line in Figure 5.) As Figure 6 shows, the velocity estimation error remains less than 2 percent for almost the entire maneuver. However, when an accelerometer offset of 0.6m/s^2 is introduced, the algorithm, which relies heavily on the accelerometer measurement, gives large errors (solid black lines in Figure 5 and Figure 6). When more weight was assigned to the wheel speed signal, the results were subjectively the same as those shown here.

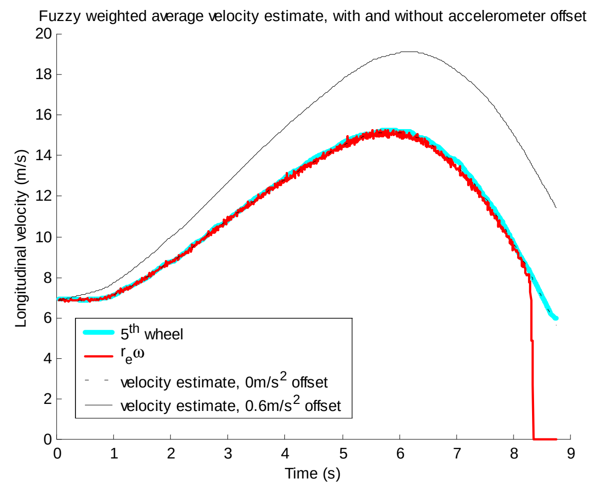

Figure 5: Performance of the weighted average/Kalman Filter velocity estimation algorithm (algorithm 2). It is acceptable when there is no accelerometer bias (dotted black line) but deteriorates in the presence of a bias (solid black line).

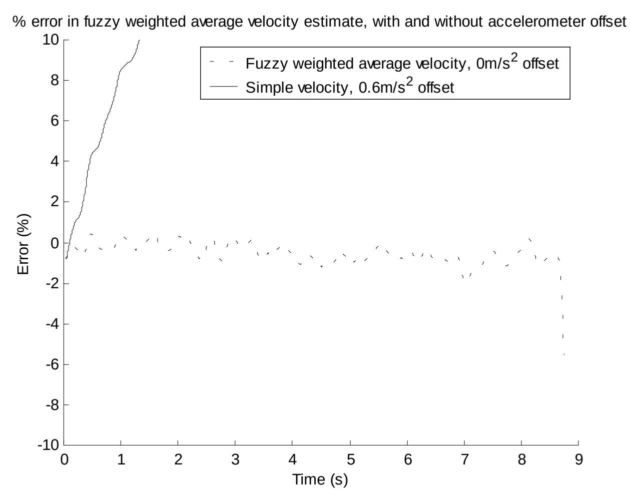

Figure 6: Percent error for the velocity estimates of the weighted average/Kalman Filter from Figure 5.

ALGORITHM 3: WEIGHTED AVERAGE WITH OFFSET AND RADIUS CORRECTION

We conclude that the weighted average algorithm successfully attenuates noise, but that it's heavy reliance on the accelerometer measurement makes it susceptible to accelerometer offset errors. Furthermore, the algorithm still relies on an accurate estimate of the effective tire radius r_{est}. To correct these problems, this section adds a parameter estimator to the weighted average algorithm.

The parameters that we would like to estimate are the accelerometer offset, ε, defined as

$$\varepsilon(k) = a_{meas}(k) - a(k) \tag{9}$$

and the effective wheel radius, *r*. The two measurements available to aid in the parameter estimation are the wheel speed *ω(k)* and the measured acceleration $a_{meas}(k)$. Whenever there is no slip at the wheel, and assuming that the accelerometer gain is correct, we can relate the unknown parameters to known quantities by the regression equation

$$\dot{\omega}(k) = \begin{bmatrix} a_{meas}(k) & -1 \end{bmatrix} \begin{bmatrix} 1/r \\ \varepsilon/r \end{bmatrix} \tag{10}$$

where $\dot{\omega}(k)$ is the (noisy, but still useful) numerical derivative *(ω(k)-ω(k-1))/dt*. If enough different acceleration values are collected over an *m* step time window so that the matrix

$$A(k) = \begin{bmatrix} a_{meas}(k) & -1 \\ a_{meas}(k-1) & -1 \\ \vdots & \vdots \\ a_{meas}(k-m+2) & -1 \\ a_{meas}(k-m+1) & -1 \end{bmatrix} \tag{11}$$

has rank two, we can solve for the parameters *1/r* and *ε/r* using either the standard least squares formula, recursive least squares, or a Kalman Filter.

The first two techniques have the advantage that they give least squares optimal solutions. However, the Kalman Filter has the advantage that it allows one to track time varying parameters and to incorporate *a priori* knowledge of their relative volatility.

In this particular problem, the parameters are time varying, and we understand their behavior fairly well. The radius *r* tends to change very little and very slowly, while the offset ε can change by much larger amounts in fairly short periods of time (when the traveling vehicle encounters a hill, for example). Therefore, we use a Kalman Filter to solve the regression problem. Similar to Gustafsson (1997), who discusses the use of Kalman Filters to solve regression problems in more detail, we assume the parameters evolve according to the difference equation

$$\begin{bmatrix} \frac{1}{r}(k) \\ \frac{\varepsilon}{r}(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{r}(k-1) \\ \frac{\varepsilon}{r}(k-1) \end{bmatrix} + \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix} \tag{12}$$

where w_1 and w_2 are white noise processes that cause the otherwise constant (due to the identity matrix) parameters to change.

We have a noisy measurement $\dot{\omega}(k)$ that is formed from the true parameters according to

$$\dot{\omega}(k) = \underbrace{\begin{bmatrix} a_{meas}(k) & -1 \end{bmatrix}}_{C(k)} \begin{bmatrix} 1/r \\ \varepsilon/r \end{bmatrix} + n(k) \tag{13}$$

where *n(k)* is the measurement noise (substantial, due to the numerical differentiation). Similar to the previous section, the Kalman Filter takes the form

$$\begin{bmatrix} \frac{\hat{1}}{r}(k) \\ \frac{\hat{\varepsilon}}{r}(k) \end{bmatrix} = I_{2\times2} \begin{bmatrix} \frac{\hat{1}}{r}(k-1) \\ \frac{\hat{\varepsilon}}{r}(k-1) \end{bmatrix} + K_{2\times1} \left(\dot{\omega}(k) - C(k) \begin{bmatrix} \frac{\hat{1}}{r}(k-1) \\ \frac{\hat{\varepsilon}}{r}(k-1) \end{bmatrix} \right) \tag{14}$$

where the hats denote estimates, and where K_{2x1} is a time varying gain matrix gotten from the standard Kalman Filter formulation (see Hayes (1996) for example). To reflect the fact that the differentiated angular velocity extremely noisy and that the accelerometer offset is far more volatile than the tire radius, the covariance of the noise process *n*(*k*) associated with the measurement was chosen to be very large compared to the covariances of the noise processes driving the parameters *ε/r* (small) and *1/r* (smaller still).

To ensure that data was only used when there was negligible slip at the wheel, estimation was frozen when the slip surpassed a threshold value of +/-2 percent (The brake light circuit could have also been used to freeze the estimation). Whenever the estimation was frozen, the parameters were set to their mean value over the past 2 seconds.

The weighted average velocity was then calculated in the previous section, but wherever $a_{meas}(k)$ was used, it was replaced by $a_{meas}(k) - \hat{\varepsilon}(k)$ and wherever r_{est} was used, it was replaced by $\hat{r}(k)$.

Figure 7 and Figure 8 show the velocity estimation results from the combined parameter estimator/weighted average combination. The dotted lines are the results with no accelerometer offset. The error is less than 2 percent for most of the test, and the high frequency noise in the estimate is attenuated. These results are comparable with the no-offset results from the previous two methods, so they show that parameter estimation did not degrade the velocity estimation performance. The solid lines are the results when the accelerometer offset is 0.6m/s^2. There is only a very minor degradation in performance, whereas there was a major breakdown in performance for the two previous algorithms. Peak error remains less than 3 percent for the duration of the test.

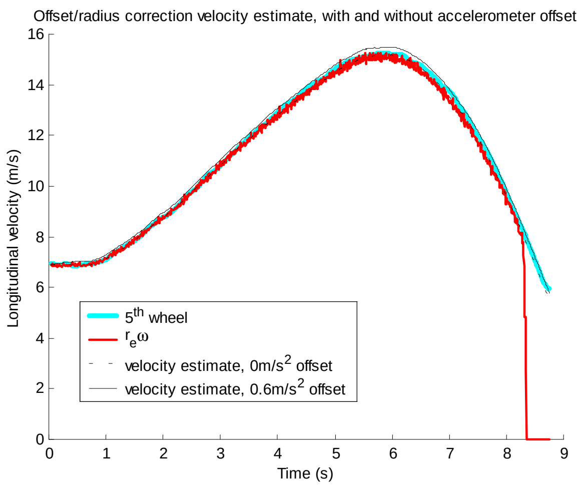

Figure 7: Performance of weighted average velocity estimator with radius and offset correction (algorithm 3). Even when there is accelerometer offset, the velocity estimate (solid black line) is very close to the 5th wheel velocity.

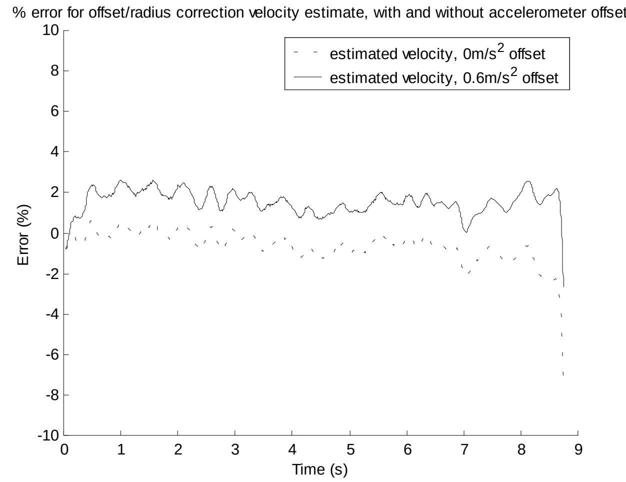

Figure 8: Percent error for the velocity estimation results of Figure 7.

Figure 9 shows the effective radius and accelerometer offset estimates corresponding to the data shown in the previous two figures. The two black lines near 0.3 are the tire radius estimation results. They make very small scale excursions from their (nearly correct) starting values. The light lines show that in both cases the accelerometer offset estimate was close to the correct value, but slightly high. This is consistent either with a slight positive underlying accelerometer bias that was not adjusted out before the tests.

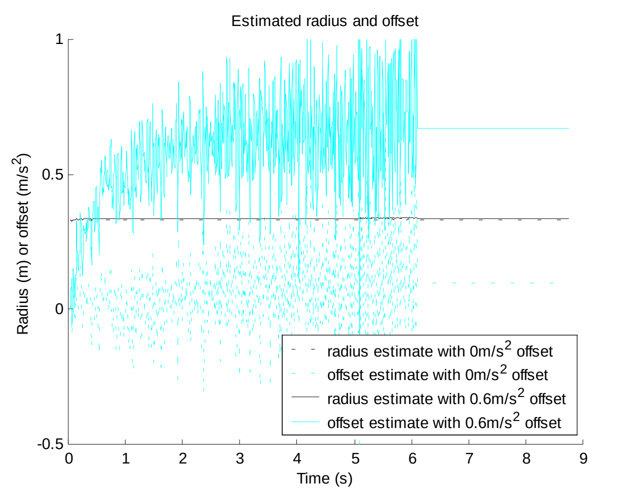

Figure 9: Radius and offset estimates corresponding to Figure 7. Radius estimates are nearly independent of the accelerometer offset. Offset estimates are close to the true accelerometer offset values.

CONCLUSION

The basic difficulty with the one wheel velocity estimation problem of this paper is that one must stop using wheel speed data to form the velocity estimate as soon as it is suspected that the wheel is slipping. The more precise the estimate needs to be, the less tolerable it is to use data from a slipping wheel. To take an extreme example, if the velocity estimate needs to be good to 20 percent precision, an accelerometer is almost not needed because the wheel angular velocity times the effective radius will almost always be within 20 percent of the vehicle velocity. If the velocity estimate needs to be good to 0.1 percent, then the wheel speed can not be used as soon as it reaches 0.1 percent.

Since we sought high precision velocity estimates, the wheel speed data was used very little to directly calculate the estimate. Instead, the successful approach used slip-free wheel speed data to identify the accelerometer offset, and then the corrected accelerometer signal was integrated to give a velocity estimate when the wheel was slipping. The peak estimation errors on the order of 3 percent would be suitable for medium fidelity applications.

Of course, most vehicles offer four wheel speeds for a velocity estimation algorithm to use, and this gives one the flexibility to choose the wheel with the least slip. However, when all four wheels are slipping (in a braking maneuver or on a four-wheel drive vehicle, for example), the basic difficulty of this paper may put a limit on the precision level that is available with just an accelerometer and wheel speed sensors.

ACKNOWLEDGMENTS

This work was partially supported by the Brain Korea 21 Project and the Korea Science/Engineering Foundation through the Research Center for Aircraft Parts Technology (ReCAPT), Gyeongsang National University, Korea. Additional financial support, as well as experimental facilities, were provided under MOU 388 of the California PATH (Partners for Advanced Transit and Highways) program. We thank Dan Empey, Paul Kretz and Dave Nelson of California PATH for their advice regarding the experimental setup.

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