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Adjustable box-wing model for solar radiation pressure impacting GPS satellites

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Abstract

One of the major uncertainty sources affecting Global Positioning System (GPS) satellite orbits is the direct solar radiation pressure. In this paper a new model for the solar radiation pressure on GPS satellites is presented that is based on a box-wing satellite model, and assumes nominal attitude. The box-wing model is based on the physical interaction between solar radiation and satellite surfaces, and can be adjusted to fit the GPS tracking data.

To compensate the effects of solar radiation pressure, the International GNSS Service (IGS) analysis centers employ a variety of approaches, ranging from purely empirical models based on in-orbit behavior, to physical models based on pre-launch spacecraft structural analysis. It has been demonstrated, however, that the physical models fail to predict the real orbit behavior with sufficient accuracy, mainly due to deviations from nominal attitude, inaccurately known optical properties, or aging of the satellite surfaces.

The adjustable box-wing model presented in this paper is an intermediate approach between the physical/analytical models and the empirical models. The box-wing model fits the tracking data by adjusting mainly the optical properties of the satellite's surfaces. In addition, the so called *Y*-bias and a parameter related to a rotation lag angle of the solar panels around their rotation axis (about 1.5° for Block II/IIA and 0.5° for Block IIR) are estimated. This last parameter, not previously identified for GPS satellites, is a key factor for precise orbit determination.

For this study GPS orbits are generated based on one year (2007) of tracking data, with the processing scheme derived from the Center for Orbit Determination in Europe (CODE). Two solutions are computed, one using the adjustable box-wing model and one using the CODE empirical model. Using this year of data the estimated parameters and orbits are analyzed. The performance of the models is comparable, when looking at orbit overlap and orbit prediction errors. Nevertheless, the models show important differences between orbits at the 1–2 cm level and total accelerations (up to 5×10^{-9} m/s^2). The differences are mainly due to the fact that the box-wing model is based on the physical interaction between solar radiation and satellite, while the CODE empirical model is not.

Keywords: GPS; Box-wing satellite model; Precise orbit determination; Solar radiation pressure

1. Introduction

Global Positioning System (GPS) satellites are at a distance from the Earth where the solar radiation pressure (SRP) is the main non-gravitational orbit perturbation. While the solar radiation impacting the satellites is simple to model, the perturbing acceleration depends on the structure of the satellite and on the optical properties of each surface facing the Sun. Furthermore, the satellite is constantly changing its orientation with respect to the Sun to maintain its nominal attitude, making the modeling of SRP a complex task. Several approaches (Sections 2 and 3) have been employed to model the SRP impacting the GPS satellites, both for orbit prediction purposes and for precise orbit determination. The last one is a key issue for geodetic and scientific applications of the Global Positioning System.

Within the IGS (International GNSS Service, Dow et al., 2009) the GPS final orbits have reached a precision

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of 2.5 cm, where the current SRP models have played an important role. Despite the high accuracy of the orbits, some orbit modeling deficiencies remain in the Satellite Laser Ranging (SLR) residuals, with a characteristic pattern first noted by Urschl et al. (2007). Additionally, orbit related frequencies were identified in geodetic time series, such as apparent geocenter motion by Hugentobler et al. (2010), and station displacements derived from GPS tracking data by Ray et al. (2008). Recently the impact of including an a priori Earth radiation model on the GPS orbits (Rodriguez-Solano et al., 2012) and position estimates (Rodriguez-Solano et al., 2011) was studied. In these works a perturbation in the orbits of 1–2 cm in radial direction (consistent with the findings of Ziebart et al., 2007) and few millimeters in along- and cross-track direction was found, together with a small reduction of the power spectrum at orbit related frequencies of GPS position estimates. The last examples show that orbit modeling deficiencies, in particular related to non-conservative forces, can still be found in the computed GPS orbits. The orbit mismodeling effects are not only noticeable in the orbits themselves but also in the geodetic parameters, highlighting the importance of further improvements in our understanding and modeling of the forces acting on GPS satellites, and GNSS satellites in general.

Non-conservative force modeling, in particular solar radiation pressure, also plays a key role in the precise orbit determination of other geodetic satellites. For GLONASS satellites, e.g. Ziebart and Dare (2001) have developed and tested a detail solar radiation pressure model. In the case of altimetry satellites (e.g. TOPEX/Poseidon, Jason-1 and Jason-2) several studies have developed and tested models for these forces. Marshall and Luthcke (1994) and Berthias et al. (2002) developed box-wing models for TOPEX/Poseidon and Jason-1 respectively. Moreover, they adjusted the optical properties of the box-wing models to reproduce as precise as possible the accelerations acting on the satellites. Further studies (Cerri et al., 2010; Lemoine et al., 2010; Zelensky et al., 2010; Flohrer et al., 2011) compared the performance of different box-wing (macro) models and detail physical satellite models for Jason-1 and Jason-2. In the case of satellites equipped with DORIS receivers (e.g. SPOTs, TOPEX/Poseidon, ENVISAT, Jason-1 and Jason-2), Gobinddass et al. (2009) found that different models of solar radiation pressure have a large impact on the geocenter time series derived from DORIS measurements.

To compensate the perturbing acceleration due to solar radiation acting on the GPS satellites, two types of models have been employed. (1) Empirical models which fit best the GPS global tracking data and which have led to a precision of 2.5 cm in the IGS final orbits. The main disadvantage of such models resides in the loss of physical understanding of the forces acting on the satellites, which can then result in non physical orbits and potentially introduce undesired systematic errors. (2) Analytical models based on the detailed structure of the satellites and surface properties measured on ground prior to launch. The principal problem of these models is that they cannot compensate accurately enough for the real on-orbit behavior of the satellites, e.g. due the change (aging) or uncertainty of the a priori optical properties of the satellite surfaces or deviations from nominal attitude. An interesting discussion of pros and cons between the two types of models is also given by Bar-Sever and Kuang (2004).

In Sections 2 and 3 the existing empirical and analytical models found in the literature are described, giving the basis for constructing an analytical box-wing model (satellite bus and solar panels, see Section 4). The box-wing model is based on the physical interaction between the solar radiation and the satellite surfaces, but it is also capable of fitting the GPS tracking data as an empirical model would do. As mentioned before, a similar approach was applied by Marshall and Luthcke (1994) and Berthias et al. (2002) to altimetry satellites in order to fulfill the high accuracy orbit requirements of these missions. More recently McMahon and Scheeres (2010) applied an analytical SRP model (described with only seven Fourier coefficients) to study the perturbation effects on the Gravity Recovery and Climate Experiment (GRACE, Tapley et al., 2004) satellites. The idea behind the model of McMahon and Scheeres (2010) is the following:

> "A precise, physics-based model of the SRP induced accelerations is combined with a perturbative theory of the accelerations on the orbit to create a system that combines a physical a priori model with a mathematical form that is conductive to studying the orbital effects and estimating the effect of SRP on the orbit."

In our case, the interest is not to study the effects of SRP on the orbits, but rather to fit the measurements of the orbits (GPS tracking data) with a model capable of compensating the SRP acting on the satellites, which is assumed to be the major error source affecting the GPS satellite orbits. By doing this an improvement in the orbits themselves is also expected. This is achieved by using the analytical box-wing model described in Sections 4 and 5. The estimated SRP box-wing parameters (on-orbit optical properties) and the impact of the model on the GPS orbits are analyzed in Sections 6 and 7. Finally the accelerations obtained with the box-wing model and existing SRP models are compared in Section 8.

2. Empirical SRP models

One of the first attempts to compensate for non-modeled forces affecting the GPS orbits was done by Colombo (1989) where one finds that:

> "Analytical orbit perturbation theory suggests that the errors in the ephemerides of the Global Positioning System (GPS) satellites should be mostly resonant effects that can be corrected by adjusting a few parameters in a simple empirical acceleration formula, despite of the

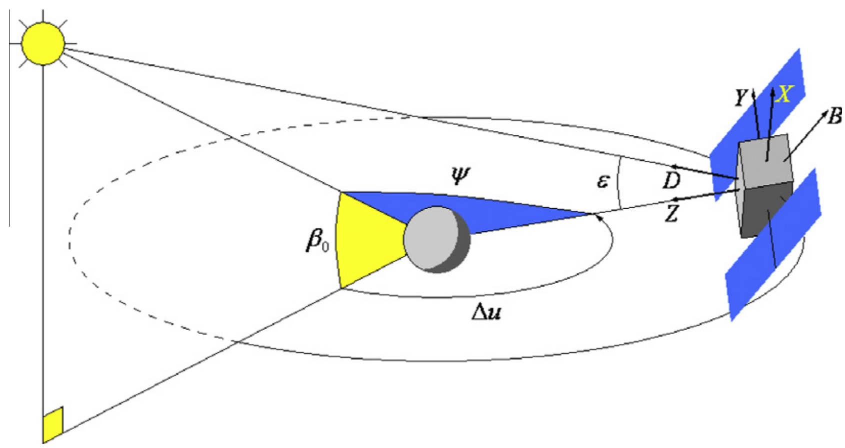

Fig. 1. Nominal attitude of GPS satellites as a function of the position of the Sun in the orbital plane, see also Eq. (5). Illustration of *DYB* (Sun-fixed) and *XYZ* (body-fixed) orthogonal frames.

> complexity of their causes (mismodeling of gravity, radiation pressure, etc.) at least for arcs free from orbits maneuvers."

The resonant frequencies of the GPS orbits found by Colombo (1989) are zero and once per revolution. Furthermore, Hill's equations were used to analytically formulate the orbit perturbations in radial, along- and cross-track directions, denoted by RSW. The nine proposed empirical parameters responsible for absorbing the orbit acceleration errors can be written as:

$$\begin{aligned} R(u) &= R_0 + R_C \cos u + R_S \sin u, \\ S(u) &= S_0 + S_C \cos u + S_S \sin u, \\ W(u) &= W_0 + W_C \cos u + W_S \sin u, \end{aligned} \tag{1}$$

where u is the argument of latitude of the satellite. As mentioned by Springer et al. (1999) the Colombo model considers the gravity field of the Earth as the major error source in the GPS orbits. Therefore there is not an explicit dependency of the model on the Sun position w.r.t. the satellite.

In Beutler et al. (1994) the extended orbit modeling techniques used by the Center for Orbit Determination in Europe (CODE) are described. To compensate the direct solar radiation pressure impacting the satellites, up to nine empirical parameters are estimated in a Sun-Earth oriented frame with directions denoted by *DYB* (Fig. 1). A description of this model is also found in Springer et al. (1999) and the estimated parameters are:

$$\begin{aligned} D(u) &= D_0 + D_C \cos u + D_S \sin u, \\ Y(u) &= Y_0 + Y_C \cos u + Y_S \sin u, \\ B(u) &= B_0 + B_C \cos u + B_S \sin u. \end{aligned} \tag{2}$$

The D direction gives the orientation of the normal to the solar panels in inertial space, the Y direction points along the solar panel beams and the B direction finally completes the right handed system. However, the B direction does not correspond to the orientation of the satellite bus and varies along the $+Z, +X$ and $-Z$ surfaces of the bus as shown in Fig. 1. Furthermore the argument of latitude u is not directly related to the attitude of the satellite. Additionally, in the direct radiation pressure model of Beutler et al. (1994) the acceleration provided by the SRP models (sometimes called ROCK models, see Section 3) developed by Fliegel et al. (1992) and Fliegel and Gallini (1996) can be included as a priori information.

In order to get an adequate a priori model of the real on-orbit solar radiation pressure impacting the satellites, Springer et al. (1999) developed a new model:

> "The performance of the new model is almost one order of magnitude better than that of the existing ROCK models. It also allows a reduction of the number of orbit parameters that have to be estimated."

This model was constructed by adjusting several years of CODE final orbits and can be written as:

$$\begin{aligned} D &= D_0, \\ Y &= Y_0, \\ B &= B_0, \\ Z(\Delta u) &= Z_1 \sin \Delta u, \\ X(\Delta u) &= X_1 \sin \Delta u + X_3 \sin 3\Delta u. \end{aligned} \tag{3}$$

The model has six main parameters, each of them consisting of other parameters with a periodic dependency on β_0, the elevation angle of the Sun above the orbital plane, in total the model has 18 different parameters. Two additional directions Z and X appear in the model, which correspond to the faces of the satellite illuminated by the Sun. The time argument of the periodic signals is related to Δu, the argument of latitude of the satellite w.r.t. the argument of latitude of the Sun in the orbital plane as shown in Fig. 1.

Nowadays CODE uses an updated version[1] of this last model as a priori information for the generation of its final

[1] ftp://ftp.unibe.ch/aiub/REPRO_2008/GEN/SATELLIT.I05, accessed on 18 May 2011.

orbits, additionally this IGS analysis center[2] estimates five empirical parameters from the model of Beutler et al. (1994), more specifically the D_0, Y_0, B_0, B_C and B_S parameters, to obtain a very good fit of the GPS orbits to the tracking data. Moreover, in order to compensate for inaccuracies in the model (a priori plus estimated) of the force field, pseudo-stochastic orbit parameters (instantaneous velocity changes) are estimated every 12 hours in the radial, along- and cross-track directions (Beutler et al., 2006).

With an approach similar to the one used by Springer et al. (1999) but using a different parametrization, Bar-Sever and Kuang (2004) fitted several years of JPL (Jet Propulsion Laboratory) final orbits to construct an improved GPS Solar Pressure Model (GSPM). The performance of the GSPM model as a priori acceleration plus estimated stochastic parameters was evaluated, against variants of the *DYB* model (without a priori acceleration) of Beutler et al. (1994), by Sibthorpe et al. (2011). The stochastic parameters used by JPL are: a constant Y-axis bias and a constant scale along the satellite to Sun direction, as well as time varying (stochastic) variations in model scale along the body-fixed spacecraft Z-and X-axes, and small stochastic changes along the Y-axis. This methodology can be regarded as "reduced-dynamic" approach. The evaluation between models (a priori models plus estimated parameters) was done by means of various internal (GIPSY-OASIS Software) and external metrics. Within the metrics favoring the use of the GSPM plus reduced-dynamics approach are: ambiguity resolution statistics, orbit overlaps, SLR tracking residuals, station repeatabilities, and GRACE K-band ranging statistics. However, clock overlaps between consecutive days and LOD (Length of Day) differences to IERS (International Earth Rotation and Reference Systems Service) Bulletin A seem to favor the *DYB* approach.

The GSPM model is basically a truncated harmonic expansion in the *XYZ* body-fixed frame:

$$\begin{aligned} X(\epsilon) &= X_1 \sin\epsilon + X_2 \sin 2\epsilon + X_3 \sin 3\epsilon + X_5 \sin 5\epsilon + X_7 \sin 7\epsilon, \\ Y(\epsilon) &= Y_1 \cos\epsilon + Y_2 \cos 2\epsilon, \\ Z(\epsilon) &= Z_1 \cos\epsilon + Z_3 \cos 3\epsilon + X_5 \cos 5\epsilon. \end{aligned} \tag{4}$$

The model has 10 parameters and some of them (X_2 and Y_1) also depend on the β_0 angle. It is very interesting to note the selection of the angle ϵ formed by Earth, satellite, and Sun (Fig. 1) as the main dependency of the model. The angle ϵ together with the *XYZ* directions contain the full information of the nominal attitude of the satellite and it can be derived from Fig. 1 as:

$$\cos\epsilon = -\cos\beta_0 \cos\Delta u \tag{5}$$

with $\epsilon = \pi - \psi$. The nominal attitude of GPS satellites is given by accomplishing at any time two conditions: the navigation antennas are pointing to the center of the Earth for transmitting the navigation signals, and the solar panels are pointing to the Sun for keeping the power supply. This is done by performing a rotation around the Z axis, such that the solar panels can rotate around the satellite bus to a perpendicular position w.r.t. the Sun. This kind of attitude control is called "yaw-steering". However, during eclipse seasons special maneuvers are required for the midnight and noon turns (Bar-Sever, 1996; Kouba, 2009).

Observing the development of the models to compensate for error sources in the GPS satellites orbits, one can note a tendency to create empirical models that can accommodate the solar radiation pressure. It becomes evident that errors in the determination of GPS satellite orbits are dominated by solar radiation pressure and that other forces acting on GPS satellites (e.g. gravitational forces) have nowadays a much lower contribution to the orbit error budget.

3. Analytical SRP models

Using an analytical approach, a priori solar radiation pressure models have been developed by considering the details of the satellite structure (e.g. small elements and shadowing or re-reflection effects between them), the known optical properties, the physical interaction of radiation with the satellite surfaces (including re-radiated heat effects), and nominal attitude. These models are based on information available on ground provided by the satellite manufactures.

The first available a priori analytical models for the Block I and Block II/IIA GPS satellites were the ROCK4 and ROCK42 models, developed by the spacecraft manufacturer, Rockwell International, and IBM. These models were improved by Fliegel et al. (1992), approximated by a simple Fourier series in the angle ϵ (Eq. (5)), and named T10 and T20 respectively. Fliegel and Gallini (1996) used the same approach to develop the T30 model for the Block IIR satellite based on a detailed spacecraft model by Martin Marietta, the spacecraft manufacturer. These models are based on the physical characteristics of the GPS satellites, like the optical properties, the dimensions and the interaction between radiation and satellite surfaces, as described in the two mentioned articles. However, these models have gradually lost favor in the IGS analysis centers, which no longer use them as a priori information. A summary of the different strategies to model solar radiation pressure within the IGS can be found in Froideval (2009). In particular Urschl et al. (2007) found that these models (at least for Block IIA satellites) introduce systematic errors in the SLR–GPS residuals.

In a later work, Marquis and Krier (2000) developed an improved radiation pressure model for the GPS Block IIR satellites. A very useful result from this study is the comparison of the different forces acting on the satellite, such that the following force contributions could be identified:

[2] ftp://igscb.jpl.nasa.gov/pub/center/analysis/code.acn, accessed on 15 December 2011.

- Solar radiation onto the vehicle surfaces.
- Radiation of thermal blankets.
- Thermal radiation from SV radiators. Includes the effect of radiation onto the solar arrays.
- Solar array thermal radiation.
- Thermal radiation of excess solar array power (shunt).
- Radiation pressure on the nadir surfaces from sunlight reflected off the Earth (albedo).
- Earth Infrared pressure on the nadir surfaces (EIR).

As pointed out by Marquis and Krier (2000):

> "The spectral or visible contribution from the Sun is the greatest contributor with nearly 100 percent of the total force."
> "Depending on the orbit position, albedo is the next highest contribution peaking 2.5 percent of the total force. The solar array and shunt thermal radiation forces are the next lowest, representing just under 1 percent of the total each, but provide a nearly constant value about the orbit. Although the magnitudes are similar, these forces are applied in opposite directions and nearly cancel each other out."

Additionally it is mentioned that the thermal part is small but important for the Y direction of the satellite and EIR is the lowest force acting on the satellite. The resulting model is available as a look up table[3] but not the dimensions or optical properties used for its construction. However, the dimensions and optical properties for the Block I, Block II/IIA and Block IIR are available from the T10, T20 and T30 models, which make them an ideal starting point for constructing the analytical box-wing model (Section 4).

More recently Ziebart et al. (2005) have developed precise models for dealing with the non-conservative forces acting on different types of low and high orbiting satellites. These models take into account further effects as compared to the T20 and T30 models, e.g. shadowing or re-reflection between surfaces. In the case of GPS satellites, a model was constructed for Block IIR considering similar forces as Marquis and Krier (2000).

The previously mentioned analytical satellite models are very important as a priori information but have the main problem that they cannot easily take into account the deviations of the models from reality. The deviations can be caused by aging of satellite surfaces, inaccurately known optical properties or not nominal attitude. An interesting example is the Y-bias acceleration reported for GPS satellites. For Block II/IIA, Fliegel et al. (1992) explain the Y-bias by a possible misalignment angle (0.5° to 1°) of the solar panels w.r.t. their nominal position. While for Block IIR, Marquis and Krier (2000) explain the Y-bias by internal heat radiated by the Y surfaces.

[3] ftp://sideshow.jpl.nasa.gov/pub/GPS_yaw_attitude/BlockIIR_srp_table, accessed on 20 May 2011.

4. Adjustable box-wing model

Based on the advantages and disadvantages from the previously exposed models, an analytical box-wing model has been derived based on the physical interaction between the direct solar radiation and a satellite consisting of a bus (box shape) and solar panels. Furthermore, some of the parameters of the box-wing model can be adjusted to fit the GNSS tracking data, namely the optical properties of the corresponding satellite surfaces. This kind of model is an intermediate approach between the physically correct analytical satellite models and the purely empirical ones. For constructing the box-wing model the following assumptions were made:

- The satellite structure can be simplified to a box shape bus and flat solar panels.
- Smaller structures (e.g. antennas or engines) contribute to the effective area, but their shape is not considered.
- No shadowing or re-reflection effects from one surface to another are considered.
- The forces due to internal heat generation cancel from opposite surfaces.
- Absorbed radiation is assumed to be re-radiated immediately back to space, therefore heating or cooling effects are not considered.
- The irradiance from the Sun, the dimensions, and the mass of the satellite are known.

In future work, the following a priori models are planned to be included: Earth radiation pressure (Rodriguez-Solano et al., 2012) and yaw attitude during eclipse seasons for Block II/IIA (Bar-Sever, 1996) and Block IIR (Kouba, 2009). These effects are not included in the current study in order to test only a simple box-wing model (with nominal attitude) interacting only with the solar radiation pressure.

The acceleration produced by the physical interaction between the solar radiation and a flat surface of the satellite is formulated by Milani et al. (1987) in the following way:

$$\vec{f} = -\frac{A}{M}\frac{S_0}{c}\cos\theta\left[(1-\rho)\vec{e}_D + 2\left(\frac{\delta}{3} + \rho\cos\theta\right)\vec{e}_N\right], \tag{6}$$

with:

$$\alpha + \rho + \delta = 1, \tag{7}$$

where:

A	area of the surface
M	mass of the satellite
S_0	solar irradiance at 1 AU ($\approx 1367\ \mathrm{W/m^2}$)
c	velocity of light in vacuum
α	fraction of absorbed photons
ρ	fraction of reflected photons
δ	fraction of diffusely scattered photons
$\vec{e}_D$	direction of the Sun from the satellite
$\vec{e}_N$	normal to the satellite surface

$\cos\theta = \vec{e}_D \cdot \vec{e}_N$, valid only if $\cos\theta \geqslant 0$.

Fliegel et al., 1992 use a different notation to describe the same physical phenomenon, in particular just two optical properties are used: reflectivity (v) ranging from 0 (black) to 1 (white) and specularity (μ) ranging from 0 (diffuse) to 1 (specular). Additionally the distinction between flat and cylindrical surfaces is made. The relation between both notations is direct by writing the optical properties as: $\alpha = 1 - v$, $\rho = \mu v$ and $\delta = v(1-\mu)$, which also satisfies Eq. (7). Furthermore, Fliegel et al. (1992) mention that to a good approximation the energy absorbed by the satellite bus surfaces is instantaneously re-radiated in the form of heat. Considering that this energy is radiated back to space according to Lambert's law, one gets:

$$\vec{f} = -\frac{A}{M}\frac{S_0}{c}\cos\theta\frac{2}{3}\alpha\vec{e}_N \tag{8}$$

and by adding the instantaneous re-radiated heat to Eq. (6), it can be written in the following way:

$$\vec{f} = -\frac{A}{M}\frac{S_0}{c}\cos\theta\left[(\alpha+\delta)\left(\vec{e}_D + \frac{2}{3}\vec{e}_N\right) + 2\rho\cos\theta\vec{e}_N\right], \tag{9}$$

which is only valid for the satellite bus. This assumption is correct for materials that have zero thermal capacity and completely prevent heat transfer toward the satellite interior (Cerri et al., 2010) like multilayer insulation (MLI), a common material in spacecraft design. However, using Eq. 9 for all the surfaces of the bus means to constrain the box to be completely covered by MLI, which is not true for GPS satellites. This assumption also implies that heating or cooling effects are neglected, but specially during eclipse seasons (where the satellites are up to 55 min in shadow) these effects can be significant. The adjustable box-wing model is only appropriate for solar radiation pressure and other smaller effects (like heating or cooling effects) may lead to systematic errors in the orbits or may be aliased in the box-wing parameters, but they are out of the scope of this paper.

For the solar panels we use Eq. (6). Adhya et al. (2005) have shown that the thermal force acting on the solar panels of GPS IIR satellites is roughly 1% of the SRP force on the panels. More important, according to Adhya et al. (2005), is the thermal force acting on the satellite bus with the main force component in the D direction. Both thermal effects, on the solar panels and on the bus (D component), can be accommodated in the solar panels (SP) scaling factor (Eq. 10).

From Eqs. (6) and (9) one can get the partial derivatives of the acceleration w.r.t. the optical properties of the satellite surfaces. Using the assumption of nominal attitude the main dependency is then on the angle ϵ formed by Earth, satellite and Sun (see Fig. 1). A total of seven partial derivatives of the acceleration are obtained:

Solar panels (SP) with $\cos\theta = 1$:

$$\frac{\partial\vec{f}}{\partial(1+\rho+\frac{2}{3}\delta)} = -\frac{A_{SP}}{M}\frac{S_0}{c}\vec{e}_D. \tag{10}$$

$+X$ bus surface ($+XAD$, $+XR$) with $\cos\theta = \sin\epsilon$:

$$\frac{\partial\vec{f}}{\partial(\alpha+\delta)} = -\frac{A_{+X}}{M}\frac{S_0}{c}\sin\epsilon\left(\vec{e}_D + \frac{2}{3}\vec{e}_{+X}\right), \tag{11}$$

$$\frac{\partial\vec{f}}{\partial\rho} = -\frac{A_{+X}}{M}\frac{S_0}{c}2\sin^2\epsilon\vec{e}_{+X}. \tag{12}$$

$+Z$ bus surface ($+ZAD$, $+ZR$) with $\cos\theta = \cos\epsilon$:

$$\frac{\partial\vec{f}}{\partial(\alpha+\delta)} = -\frac{A_{+Z}}{M}\frac{S_0}{c}\cos\epsilon\left(\vec{e}_D + \frac{2}{3}\vec{e}_{+Z}\right), \tag{13}$$

$$\frac{\partial\vec{f}}{\partial\rho} = -\frac{A_{+Z}}{M}\frac{S_0}{c}2\cos^2\epsilon\vec{e}_{+Z}. \tag{14}$$

The $-Z$ partial derivatives ($-ZAD$, $-ZR$) are constructed in the same way as for $+Z$, with the appropriate sign changes. The partial derivatives are plotted in Fig. 2.

In addition to the seven parameters of the solar radiation pressure box-wing model with nominal attitude, two more parameters have been included to fit the GPS

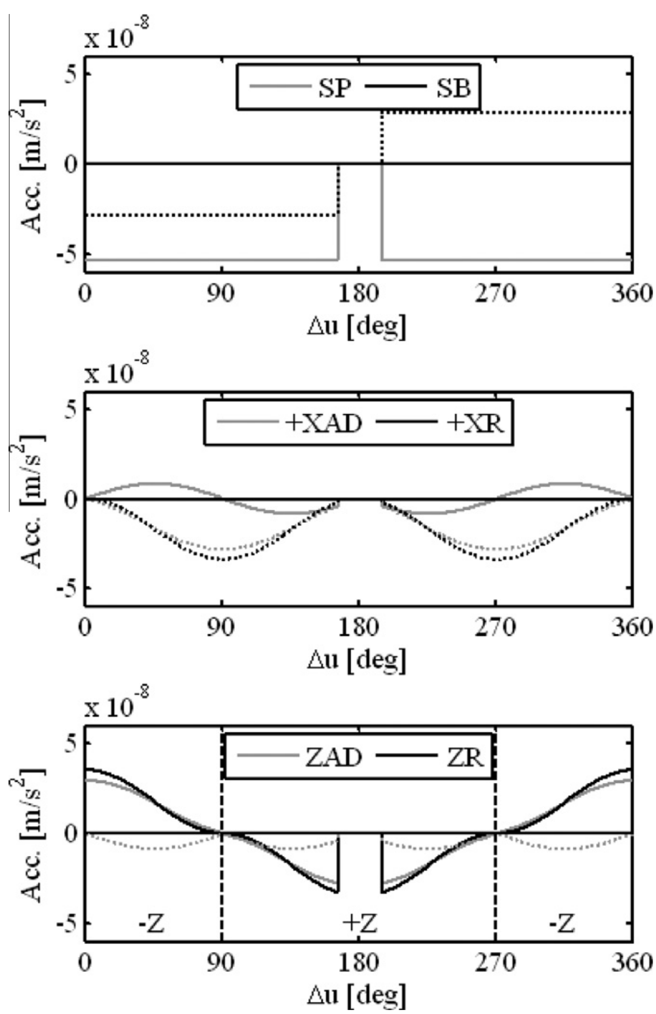

Fig. 2. Partial derivatives of the box-wing model (defined in Eqs. (10) to (15)) for a Block IIR satellite and $\beta_0 = 0°$: top in DB Sun-fixed reference frame, middle and bottom in ZX body-fixed reference frame. D and Z are shown in solid lines, B and X are shown with dotted lines. The Sun is eclipsed for $|\Delta u - 180°| < 14°$.

tracking data: (1) the Y-bias parameter ($Y0$) to compensate a constant acceleration acting on the Y-axis of the satellite, and (2) a rotation lag of the solar panels around their rotation axis.

Fliegel et al. (1992) explain the causes of the so called Y-bias, mentioning that alignment requirements of the solar panels of 0.5° to 1° are reported in the Rockwell specifications. More importantly Kuang et al. (1996) computed the misalignment angles of the solar panels w.r.t. nominal attitude using a simple SRP model, obtaining also small deviations of 1° to 2° from nominal attitude. In general, changes of the nominal attitude of the satellite will affect the SRP, producing non-modeled accelerations. Assuming that there might be small misalignment biases in the nominal rotations around X, Y and Z (Fig. 1), how can they be taken into account by our box-wing model? There are known changes in the yaw attitude (rotation around Z) of the satellites (Bar-Sever, 1996; Kouba, 2009) specially during eclipse seasons, but other smaller misalignment biases in the yaw angle are assumed to be partially absorbed by the box-wing and Y-bias parameters. A simple calculation shows that the deviation from nominal yaw attitude introduced by the yaw bias for GPS II/IIA satellites (Bar-Sever, 1996) will manifest in a constant acceleration acting along the Y-axis. A change in the rotation angle around the X-axis will lead directly to small accelerations in the Y-axis which again can be absorbed by the Y-bias.

For the rotation angle around the Y-axis we distinguish between a misalignment bias of the satellite bus and of the solar panels. The acceleration produced by a non-nominal angle in the satellite bus can be partially absorbed by the parameters of the box part of the box-wing model. For the solar panels a misalignment bias around the Y-axis causes a constant acceleration in the B direction (Fig. 1), which cannot be compensated by the box-wing model. In fact the box-wing model (seven parameters and Y-bias) adjusted to the GPS tracking data results in a much lower performance w.r.t. the purely empirical CODE model, see Fig. 7. Indeed, Springer et al. (1999) mentioned that the periodic terms in the B direction most significantly reduce orbit model deficiencies. For modeling the missing parameter of the box-wing model there are basically two possibilities: (1) a B-bias, assuming that the misalignment bias around the Y-axis is constant and does not depend on the direction of rotation of the solar panels, or (2) the misalignment bias around the Y-axis depends on the direction of rotation of the panels or more specifically on the sign of $\dot{\epsilon}$ (time derivative of Eq. (5)) which changes once per revolution. The second option has proven to be the one that fits better the GPS tracking data, with the interpretation that the solar panels follow the Sun with a small lag, therefore we have called this parameter "solar panel rotation lag" (θ_{SB}) and its partial derivative can be written (for nominal yaw attitude) as:

$$\frac{\partial \vec{f}}{\partial \theta_{SB}} = -\frac{A_{SP}}{M}\frac{S_0}{c}2\left(\frac{\delta_{SP}}{3}+\rho_{SP}\right)\mathrm{sign}(\dot{\epsilon})\vec{e}_B, \tag{15}$$

Table 1
Dimensions and optical properties of GPS Block II/IIA satellites (mass = 880 kg/975 kg, respectively).

Surface	Area $[m^2]$	α	δ	ρ
Solar panels	11.851	0.746	0.057	0.197
$+X$ bus	2.719	0.500	0.400	0.100
$+Z$ bus	2.881	0.440	0.448	0.112
$-Z$ bus	2.881	0.582	0.335	0.083

Table 2
Dimensions and optical properties of GPS Block IIR satellites (mass = 1100 kg).

Surface	Area $[m^2]$	α	δ	ρ
Solar panels	13.920	0.707	0.044	0.249
$+X$ bus	4.110	0.940	0.060	0
$+Z$ bus	4.250	0.940	0.060	0
$-Z$ bus	4.250	0.940	0.060	0

using the small angle approximation ($\sin\theta_{SB} = \theta_{SB}$ and $\cos\theta_{SB} = 1$) and with δ_{SP} and ρ_{SP} being the a priori optical properties of the solar panels (Tables 1 and 2). With this last parameter the analytical box-wing model is complete. In summary we have nine parameters:

- SP: solar panel scaling factor $(1+\rho+\frac{2}{3}\delta)$
- SB: solar panel rotation lag
- $Y0$: Y-bias acceleration
- $+XAD$: absorption plus diffusion of $+X$ bus $(\alpha+\delta)$
- $+ZAD$: absorption plus diffusion of $+Z$ bus $(\alpha+\delta)$
- $-ZAD$: absorption plus diffusion of $-Z$ bus $(\alpha+\delta)$
- $+XR$: reflection coefficient of $+X$ bus (ρ)
- $+ZR$: reflection coefficient of $+Z$ bus (ρ)
- $-ZR$: reflection coefficient of $-Z$ bus (ρ).

The constraint implied by Eq. (7) has not been imposed, i.e., $\alpha+\rho+\delta$ need not be equal to one. This allows to separate between $\alpha+\delta$ and ρ parameters and gives three extra degrees of freedom to the box-wing model.

The partial derivatives of the box-wing model (except for the Y-bias) are plotted in Fig. 2, for $\beta_0 = 0°$, when the Sun is in the orbital plane of the satellite. Note that all partial derivatives but SB are even functions over one revolution (they can be reflected w.r.t. $\Delta u = 180°$). Moreover for the satellite bus ($+X$, $+Z$ and $-Z$) there are even and odd partial derivatives over half a revolution (w.r.t. $\Delta u = 90°$ or $\Delta u = 270°$). The situation is very different, e.g. for the CODE empirical model (Eqs. (2) and (3)) where the short term (one revolution) dependency is mainly on $\cos u$ and $\sin u$. However one of the main disadvantages of the box-wing model is the high correlation between the partial derivatives (Fig. 3). In particular the absorption plus diffusion terms (AD) are highly correlated w.r.t. the solar panel scaling factor (SP). The correlation is in general lower for the reflection terms (R). The solar panel rotation lag (SB) is uncorrelated w.r.t. all other parameters, since it is the only one with an odd partial derivative over one

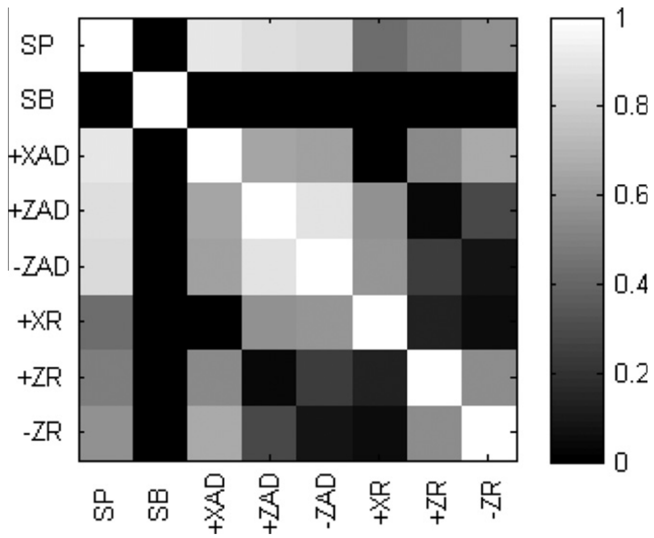

Fig. 3. Correlation between unconstrained parameters of the box-wing model for a Block IIR satellite and $\beta_0 = 0^\circ$. The Y-bias is not shown since its correlations w.r.t. all other parameters are zero.

revolution. Due to the high correlation of the absorption plus diffusion terms, additional constraints w.r.t. the a priori values (Section 5) are applied to these three terms, when the box-wing model is used to fit the GPS tracking data (Section 6).

5. A priori box-wing model

The box-wing model needs certain a priori information of the satellite. Mandatory are realistic values of mass and dimensions and helpful are the optical properties of the satellite surfaces. The main source for the a priori information are the papers of Fliegel et al. (1992) for Block II/IIA and Fliegel and Gallini (1996) for Block IIR. The dimensions presented in Tables 1 and 2 were obtained by computing a simple average of the area of all elements (flat and cylindrical) contributing to a given surface. The optical properties were weighted according to the area of the respective elements.

Applying this simple procedure to the Block IIA, the cylindrical elements of the $+X$ surface sum to 3.331 m^2 ($\alpha = 0.579$, $\delta = 0.337$, $\rho = 0.084$), additionally the flat $+X$ surface is 1.553 m^2, meaning that the +X surface would have an effective area of 4.884 m^2. In Fliegel et al. (1992) the area of each navigation antenna adapter is given as 0.181 m^2 (there are 12 antennas in total). We have computed an approximate value of 0.032 m^2, derived from pictures and dimensions of the Block II/IIA antenna array (Wübbena et al., 2007), which is more than five times smaller than 0.181 m^2. We have also considered that just 9 antennas (instead of 12) contribute to the $+X$ effective area, due to shadowing between antennas. Additionally in Fliegel et al. (1992) a plume shield is reported, which also contributes to the effective $+X$ surface (diameter 1.84 m, thickness 0.22 m). In satellite bus drawings (Bar-Sever et al., 2009) the diameter seems to be about 0.57 m. Wübbena et al. (2007) reports a diameter of 1.34 m for the navigation antenna array, but the diameter of the plume shield cannot be much larger than the one of the antenna array. As no better information was available a diameter of 0.57 m was used for the plume shield, if this value is incorrect it should have just a small impact on the scale of the box-wing parameters (Section 6). Considering 9 antennas of 0.032 m^2 and a plume shield of 0.57 m, we obtain a cylindrical surface of around 35% of 3.331 m^2, by adding 1.553 m^2 we get the value of Table 1. The rest of the dimensions provided by Fliegel et al. (1992) seem to be correct and are in accordance with other satellite drawings (e.g. Feltens, 1991). For the Block IIR satellite dimensions no conflicts were found. However, these satellites have large antenna arrays (W-sensor, low-and high-band) which are reported by Fliegel and Gallini (1996) to contribute 0.5 m^2 in the $\pm Z$ directions, therefore this value was added to the $\pm Z$ areas in Table 2.

The a priori information collected here is appropriate for the use with a box-wing model. If better a priori information is available, e.g. by constructing a 3-D model of the satellite and computing shadowing effects (Ziebart et al., 2005), this also may improve the quality of the estimated parameters and the computed orbits. This last step is, however, out of the scope of this paper, where just a simple (but capable of fitting the GPS tracking data) box-wing model accounting for SRP is tested.

6. On-orbit GPS optical properties

GPS orbits were generated based on one year (2007) of tracking data from the global IGS network (around 200 stations). Two solutions were computed, differing only in the solar radiation pressure modeling, one with the CODE empirical model and one with the adjustable box-wing model. We have computed 1-day orbits and the SRP model parameters were also estimated once per day. This was done with the Bernese GPS Software (Dach et al., 2007), where the box-wing model has been implemented in a development version, and using the processing scheme derived from the one used at CODE (Steigenberger et al.,

Table 3
Manual removal of specific satellites and doy (day of year) intervals, due to large outliers in the estimated parameters of the box-wing model. SVN = Space Vehicle Number.

Satellite	1st doy	2nd doy	Reason
PRN 14	296	296	Unknown
PRN 28	48	48	Unknown
PRN 04	39	51	Unknown
PRN 31	39	51	Unknown
PRN 15	72	72	End of life SVN 15
PRN 15	298	304	Start of life SVN 55
PRN 29	1	296	End of life SVN 29
PRN 29	360	365	Start of life SVN 57
PRN 32	93	179	Start of life SVN 23

2006; Steigenberger et al., 2011). The a priori weight for the GPS phase measurements corresponds to 1 mm at the zenith of ground stations (elevation dependent weighting was used). The year 2007 has the advantage that the number of Block IIA and IIR satellites is similar: 15 Block IIA and 15 Block IIR satellites (transition of PRN 15 and PRN 29 from II/IIA to IIR, see Table 3), in total 32 satellites.

The CODE empirical model consists of five unconstrained parameters (Eq. (2)): $D0, Y0, B0, BC$ and BS. Additionally, for most satellites (all except PRN 12, 15 (SVN 55), 29 (SVN 57) and 31, see also Table 3) an updated version of the CODE a priori model (Eq. (3), Springer et al., 1999) was applied as a priori acceleration. The CODE a priori model does not consider Earth radiation pressure or antenna thrust explicitly but these effects were partially absorbed in the estimated parameters of the model. Although models for these effects are available (Rodriguez-Solano et al., 2012), they have not been included in the box-wing model allowing for a clearer comparison w.r.t. the CODE model. Consequently, both models are only well suited for solar radiation pressure and smaller effects will be aliased into the estimated parameters. For the adjustable box-wing model its nine parameters (Section 4) were estimated with different constraints: (1) SP, SB and $Y0$ unconstrained, (2) $+XAD$, $+ZAD$ and $-ZAD$ tightly constrained to 0.01 (around 1 % of the a priori value) and (3) $+XR$, $+ZR$ and $-ZR$ loosely constrained to 0.1 (similar to the a priori value). Additionally for both SRP models, pseudo-stochastic pulses (Beutler et al., 2006) were estimated once per day in the radial, along- and cross-track directions, constrained to 10^{-6}, 10^{-6}, and 10^{-9} m/s respectively.

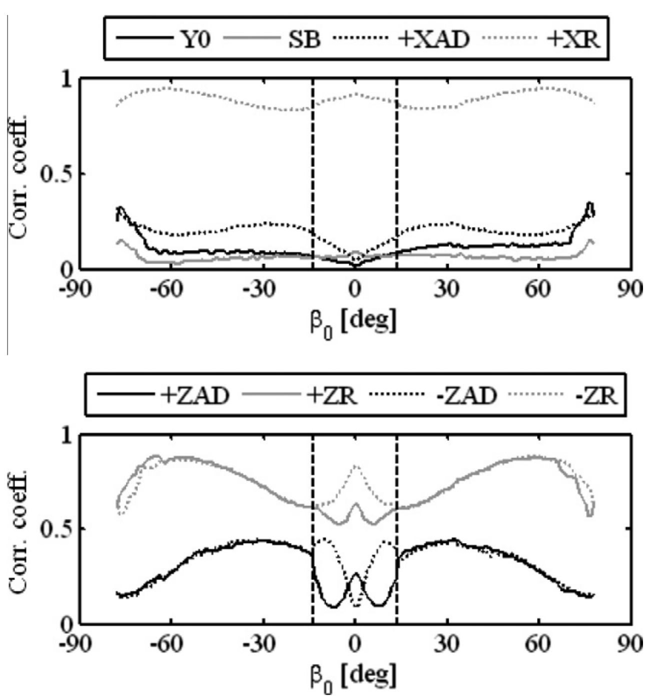

Fig. 4. Correlation of the parameters (constrained) of the box-wing model w.r.t. the solar panel parameter (SP) as a function of β_0, average over all satellites. Eclipse season for $|\beta_0| < 14^\circ$.

The constraining of the parameters of the box-wing model has as consequence that the correlation between the absorption plus diffusion terms (AD) decreases w.r.t. the other parameters. In Fig. 4 the correlation of the parameters w.r.t. the solar panel parameter (SP) is shown as a function of β_0 (note that for Fig. 3 the parameters were unconstrained), where the correlation coefficients across all satellites are averaged. The β_0 dependency is a result of the changing incidence radiation angles and exposure times of the bus surfaces as the nominal attitude of the satellite changes with β_0. Note also that the correlation coefficients, specially for the $+Z$ and $-Z$ surfaces, change when the satellite is in eclipse season ($|\beta_0| < 14^\circ$), since the Sun radiation impacts the $+Z$ surface during less time (see also Fig. 2). The high correlation between the box-wing parameters and the dependency with β_0 are issues that should be further investigated and if possible improved in the future.

Despite the above-mentioned problems, the parameters of the box-wing model (optical properties, solar panel rotation lag and Y-bias) have been successfully estimated from GPS tracking data. The results are presented in Fig. 5 (estimated parameters) and Fig. 6 (a posteriori formal errors) as a function of β_0 for most GPS satellites in 2007. The a posteriori formal errors (Fig. 6) show some outliers mostly for Block II/IIA satellites, related to satellites with less observations than others. Some satellites like PRN 29, or specific days were excluded from the presented results for both models, see Table 3, due to large outliers in the estimated parameters of the box-wing model (Fig. 5). Moreover, for the same reason, satellites performing maintenance maneuvers[4] have been excluded for the complete day where the maneuver occurred. As expected the box-wing model is only suited for well behaving satellites.

The estimated box-wing parameters are within the range of physically expected a priori values, with some scattering between days and between satellites. Ideally the parameters of the box-wing model should remain constant over time, since the optical properties should not change from day to day or a as a function of β_0. There may be, however, different scales between the parameters of different satellites, for example due to different on-orbit mass values (assumed to be the same for all satellites of the same type, see Tables 1 and 2). The increase of the scattering as a function of β_0 like, e.g. for $+ZR$ and $-ZR$ for β_0 around 60° (Fig. 5), is an indication of the higher correlation of the parameters with others and as a function of β_0 (Fig. 4), also visible in the larger a posteriori formal errors (Fig. 6). The large scattering for $|\beta_0| < 14^\circ$, in the case of parameters like $+XR$ and SB for Block IIA satellites, is most likely associated with the known yaw maneuvers performed during eclipse seasons (Bar-Sever, 1996), which were not yet included in our box-wing model.

[4] GPS satellite maneuvers detected by CODE: ftp://ftp.unibe.ch/aiub/REPRO_2008/GEN/SAT_2007.CRX, accessed on 11 August 2011.

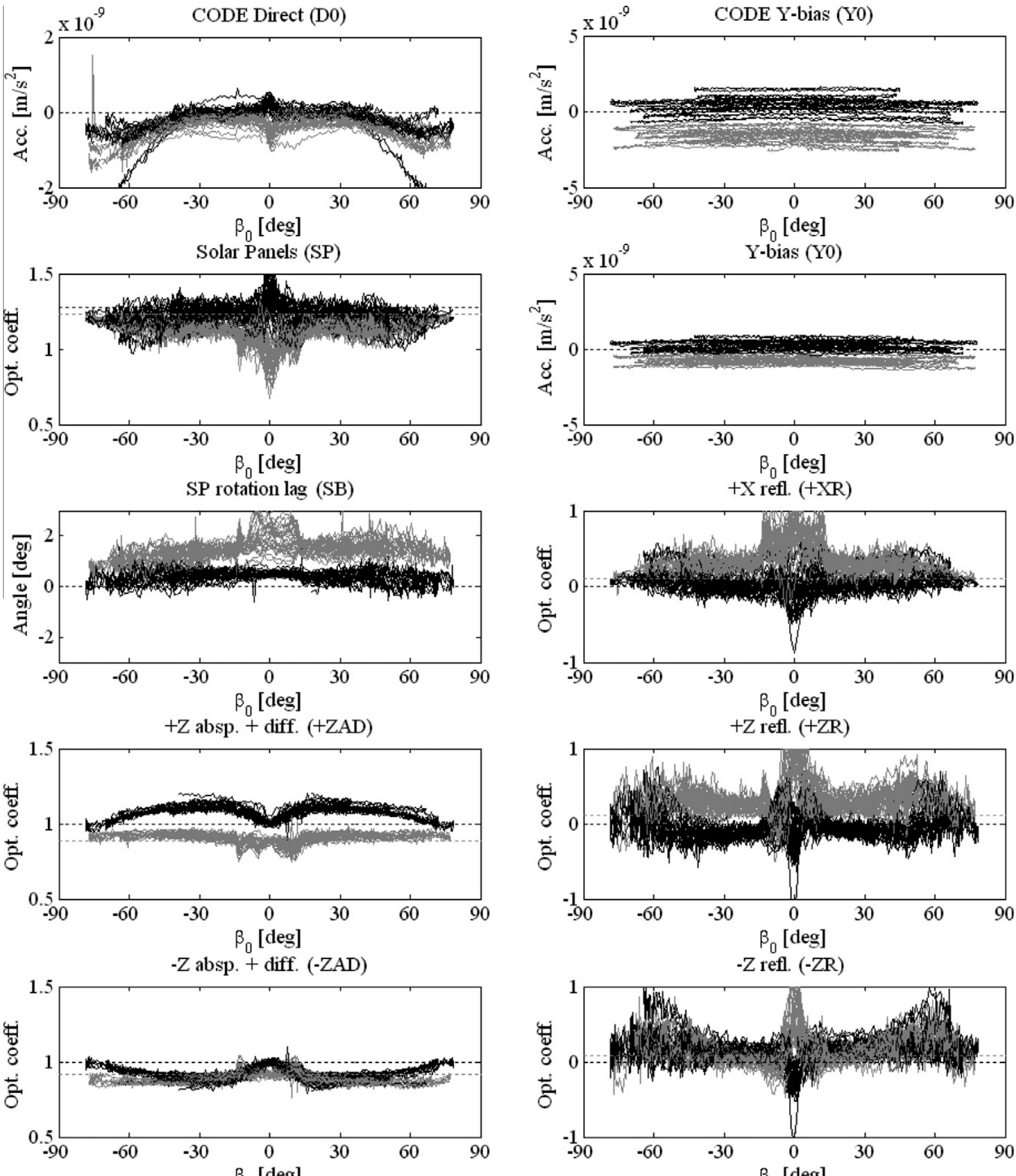

Fig. 5. Estimated parameters of the box-wing model for most GPS satellites in 2007, $+XAD$ is not shown since there is not significant variation w.r.t. the a priori value. Estimated $D0$ and $Y0$ parameters of the CODE empirical model are shown (at the top) for comparison. Block II/IIA satellites are shown in gray and Block IIR in black. A priori values are shown with dashed lines.

The estimated absorption plus diffusion parameters for the $\pm Z$ surfaces ($+ZAD$ and $-ZAD$) for the Block IIR satellites are very interesting and the results were not expected. The variation of these parameters with β_0 looks more like a systematic effect (mismodeling problem in the box-wing model), since these two parameters were tightly constrained to the a priori values. For $+XAD$ (not shown in Fig. 5) no variation with β_0 is observed, however the

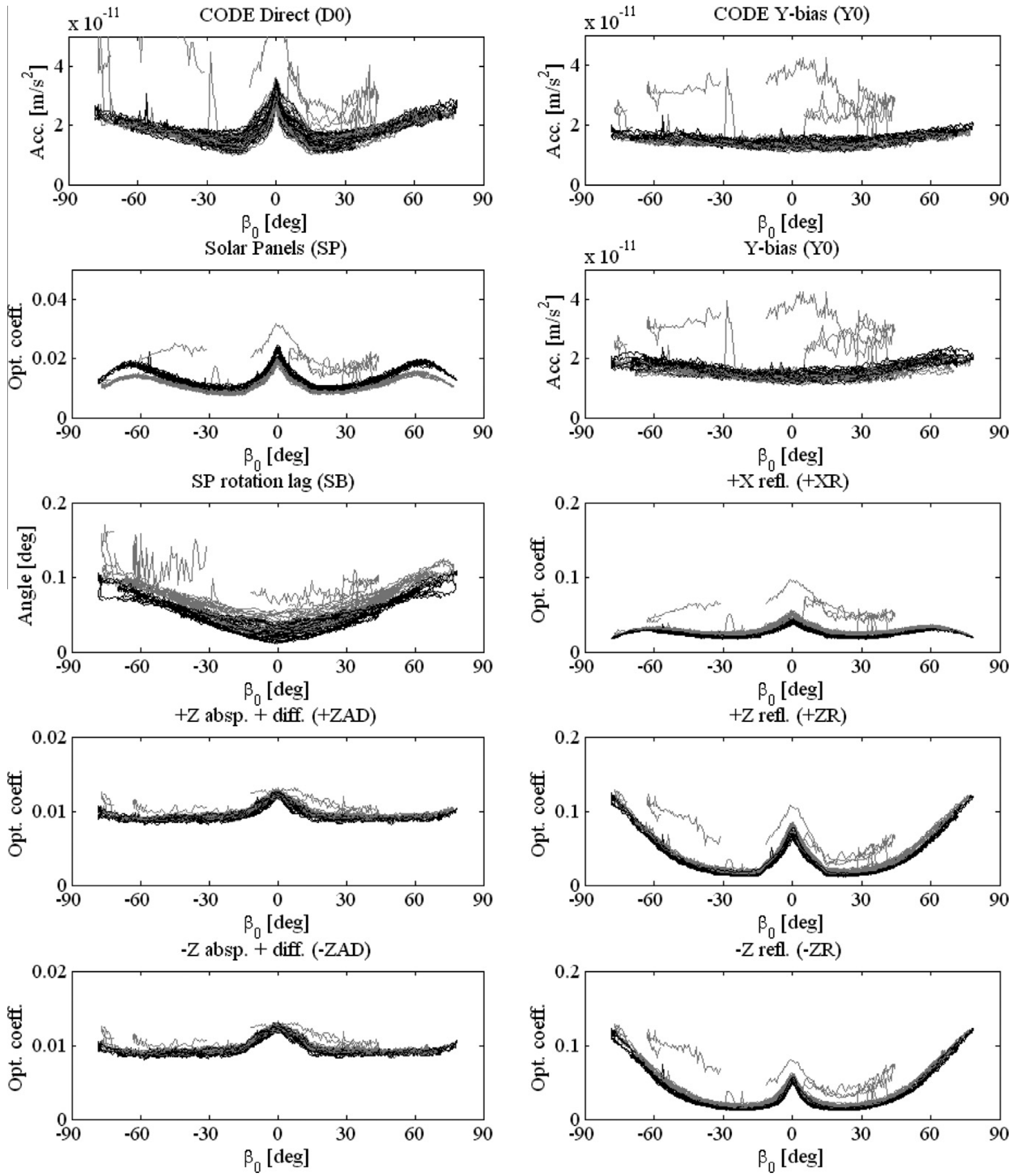

Fig. 6. A posteriori formal errors of the estimated parameters of the CODE and box-wing models (see Fig. 5). Block II/IIA satellites are shown in gray and Block IIR in black.

correlation is also lower w.r.t. the solar panel parameter (Fig. 4). A possible reason for the variation in $+ZAD$ and $-ZAD$ are heating or cooling effects (not considered in the box-wing model), since the bus of Block IIR is reported to have very high absorption coefficients (almost like a black body). Another possibility is that the optical properties of the $+Z$ and $-Z$ surfaces are not equal (Table 2) as reported by Fliegel and Gallini (1996). However, these possibilities have to be further investigated.

For comparison, the $D0$ (direct solar radiation) and $Y0$ (Y-bias) estimated parameters of the CODE empirical model are also shown in Fig. 5 and their a posteriori formal errors in Fig. 6. Firstly, note the significant reduction of the Y-bias when using the box-wing model for both satellite types, indicating that the more physical box-wing model helps to reduce the not fully understood Y-bias (see Section 3). This is consistent with the observation that the correlation of the Y-bias w.r.t. the rest of parameters (including Keplerian elements and pseudo-stochastic pulses) is significantly reduced when introducing the box-wing model. For comparing the $D0$ and SP parameters, we should first consider that an error of 0.1 (observed scatter) w.r.t. the a priori value is equivalent to 4.2×10^{-9} m/s^2 for Block IIR satellites ($1 + \rho + \frac{2}{3}\delta = 1.278$). So, in fact, the variation of the CODE empirical parameter is lower than the one of the box-wing model. We should nevertheless consider that the first one uses the CODE a priori model which is based on few years of GPS tracking data, while the box-wing model is just based on one day.

7. Quality of GPS orbits

The box-wing model not only fits well the GPS tracking data but also produces orbits with a similar precision as the ones obtained with the CODE empirical model. The metrics used here to quantify the precision of the computed orbits are not exhaustive, i.e., other metrics are also important, e.g. SLR validation, but can give a first insight into the performance of the adjustable box-wing model.

One quality measure of the orbits are the estimated pseudo-stochastic pulses, initially introduced to compensate for orbit modeling deficiencies. These pulses show a reduction (mainly in radial direction, Fig. 7) when using the box-wing instead of the CODE model. This reduction in the pulses confirms that the box-wing model results in a more physical representation of the GPS orbits than the CODE model. If the solar panel rotation lag (SB) is not estimated, i.e., when using only nominal attitude, the pulses in the radial direction show a large increase (note also the different scale in Fig. 7). Note also that the radial pulses without the SB parameter are larger for Block II/IIA than for Block IIR satellites, corresponding to the larger solar panel rotation lag observed in Fig. 5.

The precision of the computed orbits can also be assessed by comparing orbit overlap errors and prediction errors between the CODE and the box-wing model (Fig. 8). The orbit overlap errors are computed as the magnitude of the vector (radial, along- and cross-track) obtained by taking the difference between consecutive 1-day orbits at the day boundaries. For the prediction errors, the magnitude of the difference vector between predicted 7-day and estimated 1-day orbits is taken, then the RMS (over one day) is taken as prediction error. In addition, for computing

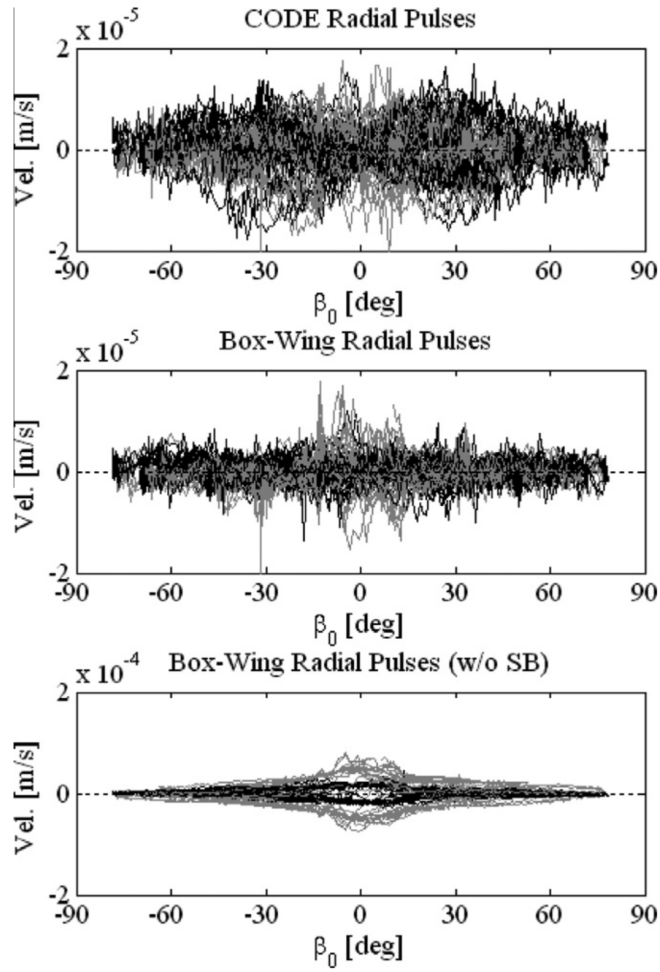

Fig. 7. Estimated pseudo-stochastic pulses in the radial direction for the CODE empirical, box-wing, and box-wing without SB models, for most GPS satellites in 2007. Block II/IIA satellites are shown in gray and Block IIR in black. Please note the different scale of the lowest figure.

the errors shown in Fig. 8, the RMS values for all satellites of the same type (Block II/IIA and IIR) are averaged as a function of the β_0 angle. Fig. 8 shows that the performance of the box-wing and CODE models is similar, specially in the orbit overlap errors. The orbit prediction errors are larger for the box-wing model during eclipse season ($|\beta_0| < 14°$) for Block II/IIA satellites. This type of satellite performs longer noon and midnight maneuvers than the Block IIR satellites, affecting the GPS phase measurements and the dynamical orbit parameters during eclipse seasons. Therefore, as the box-wing model adopts nominal yaw attitude, the effect of the not modeled yaw attitude impacts the box-wing derived orbits more than those obtained with the CODE empirical model, where the parameters in the B direction are capable of partially absorbing the effects caused by non-nominal attitude. A small reduction in the prediction errors is observed for the box-wing model around $\beta_0 = \pm 60°$. Although the prediction performance of the CODE and box-wing models is similar, it should be taken into account that the first one uses the CODE a priori model, derived from several years of data, while the

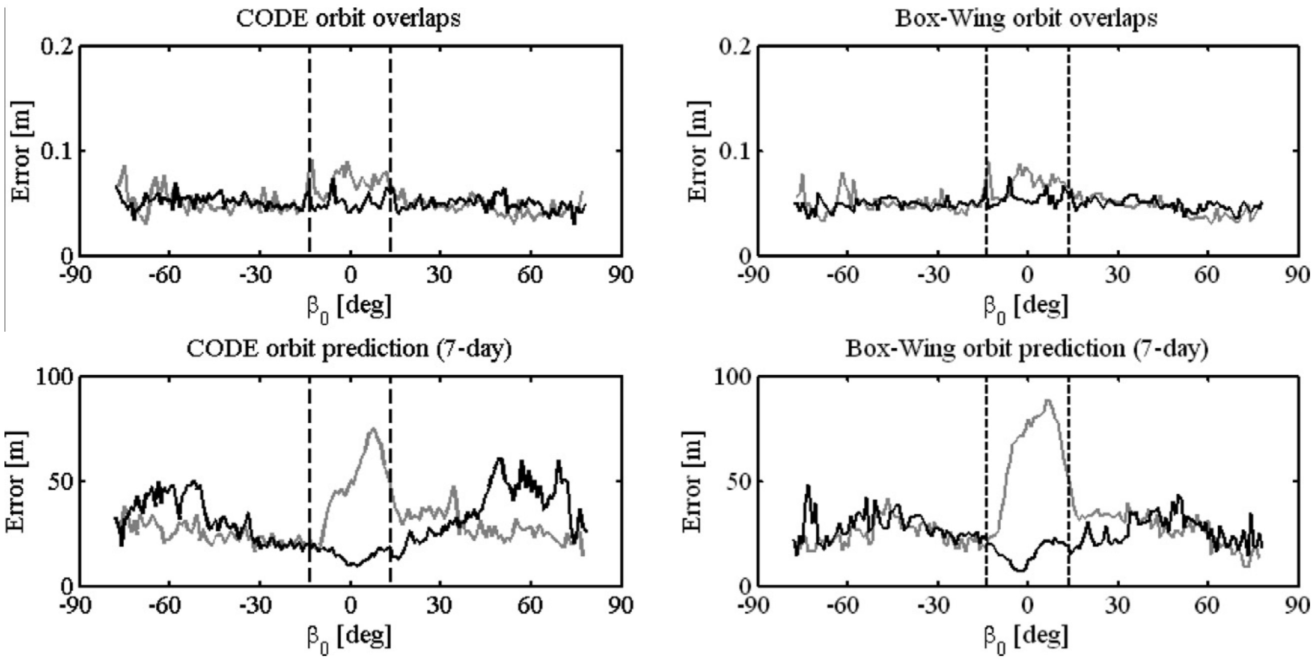

Fig. 8. Orbit overlap and prediction (after 7 days) errors for the CODE empirical (left) and the box-wing (right) models. Average over all satellites of the same type, Block II/IIA (gray) and Block IIR (black). Eclipse season for $|\beta_0| < 14^\circ$.

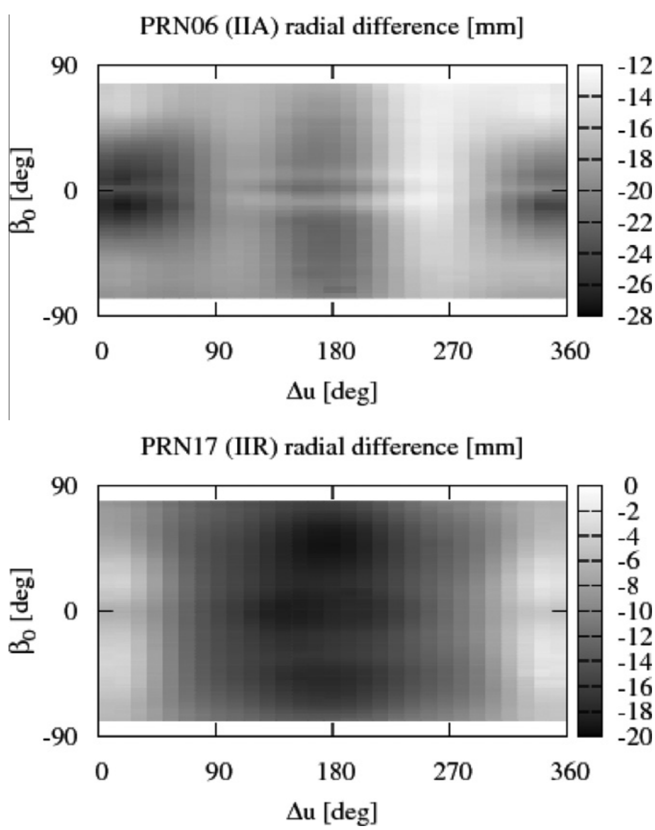

Fig. 9. Orbit differences (box-wing minus CODE empirical) in the radial direction for PRN06 (Block IIA) and PRN17 (Block IIR).

prediction with the box-wing model is based on adjusting one day of GPS tracking data, making this result indeed remarkable.

The quality of the orbits using the two models is very similar, but there are significant and systematic differences between them. Fig. 9 shows the radial differences for two specific satellites (PRN 06 and PRN 17) in a Sun-fixed reference frame. The along- and cross-track orbit differences are of smaller magnitude and therefore not shown. First of all note the negative radial bias of about 1–2 cm. This bias is in the "correct" direction, i.e., it has the potential to further reduce the bias of SLR minus GPS measurements. Recent comparisons of SLR minus GPS measurements (Bar-Sever et al., 2009) show a negative bias of 1.2–2.2 cm with associated scatters of 1.6–2.5 cm. Secondly, there are also systematic radial differences between the orbits, which are evident when plotted in a Sun-fixed reference frame, and between the block types. Similar patterns are observed when plotting the acceleration differences due to SRP in the radial direction between the box-wing and the CODE empirical model, see also Section 8. There is a β_0 dependency in the radial differences specially for Block IIA satellites, but the major dependency for both blocks is on the Δu angle (the argument of latitude of the satellite w.r.t. the argument of latitude of the Sun) where the partial derivatives (Section 4) of the CODE empirical model and the adjustable box-wing model also differ most. The acceleration caused by the solar panel rotation lag is shown in Fig. 10 for Block IIA and for a specific β_0 angle, for Block IIR (not shown) the acceleration is of smaller magnitude since the rotation lag angle is also smaller (Fig. 5). Note that the radial acceleration shows a shift at $\Delta u = 0^\circ$ and $\Delta u = 180^\circ$. A similar asymmetry can also be seen in Fig. 9 in the radial orbit differences between the box-wing and CODE models.

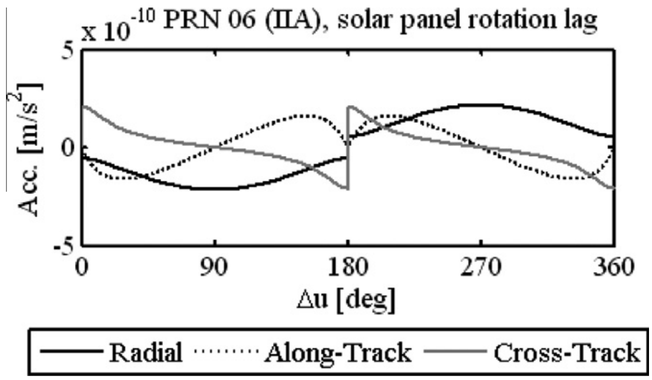

Fig. 10. Acceleration caused by the solar panel rotation lag for $\beta_0 \approx 15^\circ$ and PRN 06 (Block IIA, doy 102).

8. Reconstruction of SRP acceleration

As a final step in this study, we have reconstructed the acceleration due to SRP obtained with the box-wing model. For this purpose the partial derivatives (Section 4), the a priori dimensions (Section 5) and the estimated box-wing parameters using GPS tracking data (Section 6) were combined. The resulting acceleration is plotted in Fig. 11 for PRN 06 and PRN 17. $\beta_0 \approx 15^\circ$ was chosen since the satellites are outside of eclipse season (without yaw attitude problems) and because a small β_0 allows better comparison with other models. Specifically the T20 and T30 models (Fliegel et al., 1992; Fliegel and Gallini, 1996), the GSPM.04b model (Bar-Sever and Kuang, 2004), the Lockheed Martin model (LOCK, Marquis and Krier, 2000) for Block IIR satellites and an updated version of the CODE empirical model (Springer et al., 1999), were compared with the adjustable box-wing model (BOXW).

Fig. 11 shows large differences between the SRP models. Note that the variation of the CODE empirical model is much smaller w.r.t. the other models. The CODE empirical model fits very well the GPS tracking data but the resulting acceleration has poor physical meaning, except for the scale. This low variation of the CODE model would indicate that the features observed in the radial orbit differences (Fig. 9), correspond to features in the acceleration introduced mainly by the box-wing model.

In the case of the Block IIR satellites the other four models, two based just on a priori information (T30 and LOCK) and two based on GPS tracking data (GSPM and box-wing) behave very similar, with only small differences. For Block IIA the situation is more interesting, T20 and GSPM are similar and both differ from BOXW, in particular for $90^\circ < \Delta u < 270^\circ$, i.e., when the $+Z$ surface of the satellite bus is illuminated by the Sun (Fig. 1). This difference between the models for Block IIA is not only present for a particular β_0 angle but it is systematic. When plotting the total acceleration of the T20 and box-wing models in a Sun-fixed reference frame (β_0 vs. Δu), the T20 model shows a minimum for $90^\circ < \Delta u < 270^\circ$ while the box-wing model shows a maximum. It seems as if there would be an important difference in the construction of the

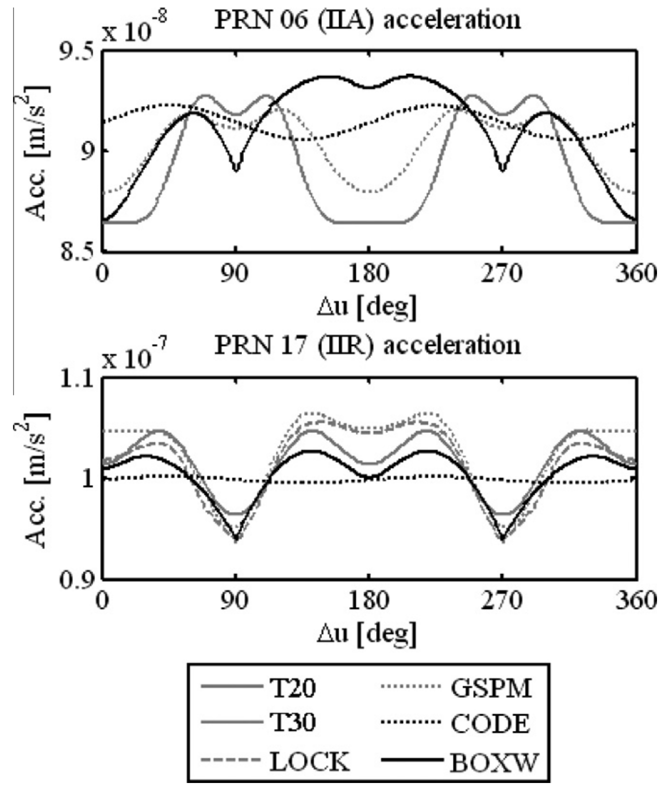

Fig. 11. Reconstructed total acceleration due to SRP for the box-wing model and comparison with existing models, for $\beta_0 \approx 15^\circ$ and PRN 06 (Block IIA, doy 102) and PRN 17 (Block IIR, doy 104).

T20 and the box-wing model. It is interesting that the T20 and GSPM models are mainly symmetric for the $+Z$ and $-Z$ surfaces, considering that the reported optical properties by Fliegel et al. (1992) are not equal for the $+Z$ and $-Z$ surfaces. The GSPM model is symmetric around $\Delta u = 180^\circ$, just like the box-wing model, but no differences can be seen when the Sun illuminates the $+Z$ or $-Z$ bus surfaces (different than for Block IIR). However, the differences in the optical properties for Block II/IIA would lead to much smaller acceleration differences between the $+Z$ and $-Z$ as the ones observed by the box-wing model. Furthermore, as Earth radiation pressure and antenna thrust were not considered in the box-wing model they may also contribute to the discrepancy between models for Block II/IIA satellites. This problem requires further investigation to decide which model is "correct", however when computing SLR–GPS residuals (Urschl et al., 2007) the T20 model has shown problems while the adjustable box-wing model could rather reduce the SLR–GPS bias (Fig. 9), favoring probably the adjustable box-wing model.

9. Conclusion

A new model for the solar radiation pressure on GPS satellites has been developed. The model simplifies the satellite to a box (satellite bus) and a wing (solar panels), and assumes an ideal yaw attitude of the satellite, but allows for a misalignment of the solar panels. The model has been derived based on the physical interaction between the solar radiation and the box-wing satellite. The model is capable of fitting the GPS tracking data by adjusting mainly the optical properties of the satellite surfaces. The misalignment of the solar panels w.r.t. their nominal orientation is modeled as a rotation lag of the solar panels around their rotation axis. In particular this last parameter of the box-wing model, not previously identified for GPS satellites, is a key factor for fitting the tracking data and obtaining highly precise GPS orbits.

The nine parameters of the box-wing model, optical properties of the surfaces, solar panels rotation lag and Y-bias, together with the satellite orbits have been estimated by fitting one year of GPS tracking data from the IGS network (around 200 global stations). The estimated parameters show a main dependency on the β_0 angle, the Sun elevation angle above the orbital plane, mainly due to correlations of box-wing parameters. However, other effects are also visible, like the not nominal yaw attitude of the Block II/IIA satellites during eclipse seasons or not modeled surface forces specially for Block IIR satellites. Smaller effects than SRP like Earth radiation pressure, antenna thrust and heating or cooling effects have not been considered in this study making the box-wing model only appropriate for solar radiation pressure.

The computed GPS orbits using the adjustable box-wing model show a similar performance as the ones generated with the CODE empirical model, when looking at overlap and prediction errors, and a better performance when looking at the radial pseudo-stochastic pulses which show a reduction. However, systematic differences between the two types of orbits are observed. This indicates that the empirical CODE model fits well the GPS tracking data but cannot totally compensate for the accelerations induced by solar radiation pressure. On the contrary, the box-wing model introduces features in the accelerations and in the orbits due to the physical modeling of solar radiation pressure.

From the estimated parameters of the box-wing model the acceleration due to solar radiation pressure was reconstructed. The acceleration was also compared to the one provided by the available analytical and empirical models. The differences between models are large, in particular for the Block II/IIA satellites. This indicates that further work should be invested in understanding the differences between the models.

This paper highlights that there are still open issues in the modeling of solar radiation pressure, in particular for GPS satellites (but also for other satellites like GLONASS). The further modeling and understanding of the non-conservative forces impacting the GNSS satellites is very important. This cannot only improve the satellite orbits but also the geodetic parameters derived from them.

GPS satellites are very good to test developments in the modeling of non-conservative forces affecting the satellites due to: (1) the high quality of GPS data provided by the IGS network and (2) the availability of a priori information concerning the satellite structure and optical properties. Improvement in the orbits of these satellites is, however, limited. Once the non-conservative forces acting on GPS satellites are well modeled, the adjustable box-wing model can be applied to satellites were the optical properties are not available or not known, like GLONASS. The orbit improvement there could be higher. Finally, reprocessing of GPS data has proven to be an important scientific tool to test different modeling approaches in a consistent way.

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