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Effects of the Earth-reflected Sunlight on the Orbit of the LAGEOS Satellite

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Summary. The geophysical satellite LAGEOS presents a secular semi-major axis decay modulated by long-periodic oscillations corresponding to an along-track acceleration of the order of $10^{-10}\,\mathrm{cm\,s^{-2}}$. We propose that a cause of these periodic perturbations is the radiation pressure due to the sunlight reflected by the Earth. If the seasonally variable albedo asymmetry between the Northern and Southern Earth hemispheres is taken into account, the resulting perturbing force is both in amplitude and in frequency signature similar to the observed unmodeled acceleration.

Key words: celestial mechanics – Earth atmosphere – artificial satellites – radiation pressure

1. Introduction

LAGEOS was launched on May 1976 with the purpose of recovering significant geophysical parameters through the determination of its orbit. Although this task has been performed with unprecedented accuracy, in the course of the orbital analysis the effort for reducing the unmodeled accelerations below the 10^{-9} cm s^{-2} level had to face serious problems (Smith and Dunn, 1980; Schutz and Tapley, 1980; Gaposchkin, 1980). The most striking unpredicted effect is a secular drift in semi-major axis of about -1 mm/d; many models have been proposed to explain this drag-like deceleration of about $3\,10^{-10}$ cm s^{-2} (see Afonso et al., 1980; Mignard, 1981; Rubincam, 1980, 1982). As the long-arc analysis has been extended, it became apparent that the semi-major axis decrease is modulated by long-periodic oscillations; the corresponding along-track accelerations have amplitudes of the order of 10^{-10} cm s^{-2}. Several mechanisms which could explain the secular decay of semi-major axis have been proposed, e.g., neutral atmospheric drag, charged particle drag or other electromagnetic dissipative effects. Even when these effects produce perturbations of an adequate order of magnitude, it seems very difficult to model quantitatively their time dependence, because of the poor knowledge of the space environment at the LAGEOS' altitude. Therefore, at the 10^{-10} cm s^{-2} level, it is reasonable to take into account other unmodeled or unproperly modeled perturbations, more suitable for a quantitative analysis even if not capable of explaining the secular part of the deceleration effect. There are different possible physical causes which could produce such small forces, but in most cases the effects on the orbital energy average out over one revolution of the satellite.

As we shall see in the following, this averaging out does not occur in the case of the force of radiation pressure due to the sunlight reflected by the Earth, expecially because of the spatial and temporal variability of the Earth's albedo. The relevance of the albedo effects is well known: as noted by Smith (1970) in a review paper on the subject: "At ... [these] heights, where the albedo perturbation is probably much larger than air drag and for orbits in complete sunlight, the albedo could be the major perturbing force of the semi-major axis". But the albedo is influenced by so many geographical and meteorological variables that it is very difficult to find a model simple enough for celestial mechanical computations and realistically describing significant features of the Earth. For instance, the albedo model used by Gaposchkin (1980; constant term plus term proportional to the squared sine of the latitude) is too simple to yield significant long-periodic perturbations. Although no albedo effect can be responsible for the strictly secular part of the semi-major axis decay, a number of long-periodic variations arise when the adopted albedo model is realistic enough.

In this paper we describe a model for the long-periodic orbital effects due to the albedo corresponding to well-known geographical and meteorological phenomena. We give explicit analytic expressions for the long-term perturbations in semi-major axis which show, both in the signature and in the order of magnitude, a significant resemblance with the observed residuals (as computed at the University of Texas and at the Goddard Space Flight Center; see Fig. 1, private communication by Smith, 1982).

This model contains very few free parameters to be fitted with the data and predicts many spectral lines with well defined amplitudes and phases; it could be checked in an unambiguous way provided other sources of perturbation with similar frequency signatures were adequately accounted for (see Sect. 4).

2. Perturbative Model

The Earth-reflected radiation pressure is an extremely complex phenomenon. First, even under very simplified assumptions on the optical properties of the Earth, the geometry of the reflected radiation light-paths is complex giving cumbersome integrals for the resulting radiation pressure force (Lochry, 1966; Levin, 1962). Second, the Earth's albedo is not uniform but shows relevant spatial and temporal variations depending both on the kind of surface (e.g., desert, forest, snow, ice or water) and on the cloud cover (see Gube, 1982). Third, the behavior of the different surface elements is not well represented by Lambert's diffusion law.

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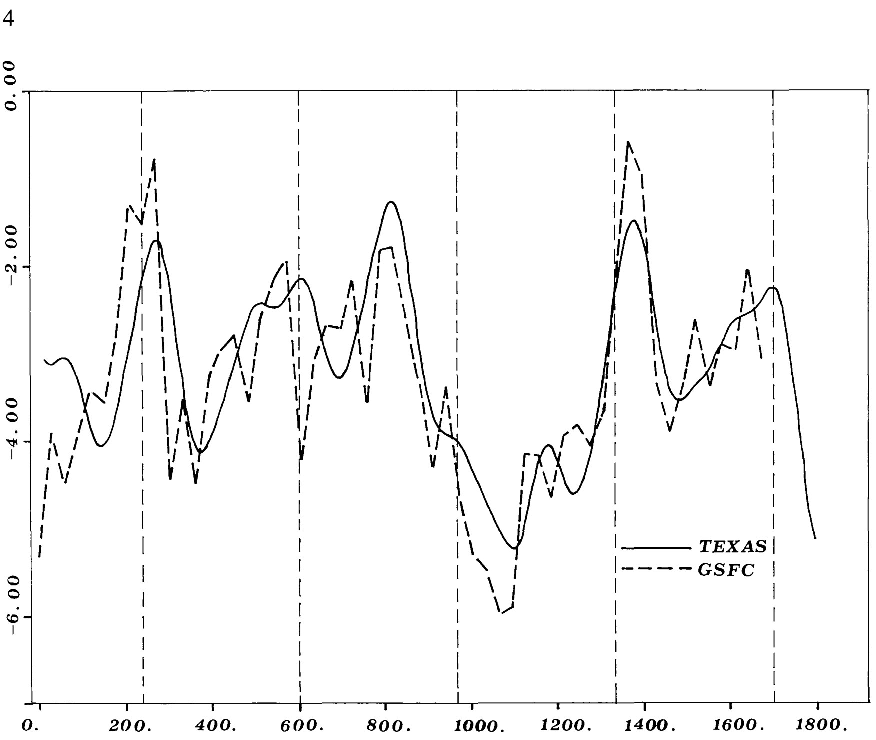

Fig. 1. LAGEOS along-track acceleration as determined by Goddard Space Flight Center – Geodynamics Branch (in monthly arcs) and by University of Texas (5-yr arc) (Smith, 1982, private communication). The horizontal scale is in Julian Days past May 7, 1976; the vertical scale unit is $10^{-10}\,\mathrm{cm/s^2}$

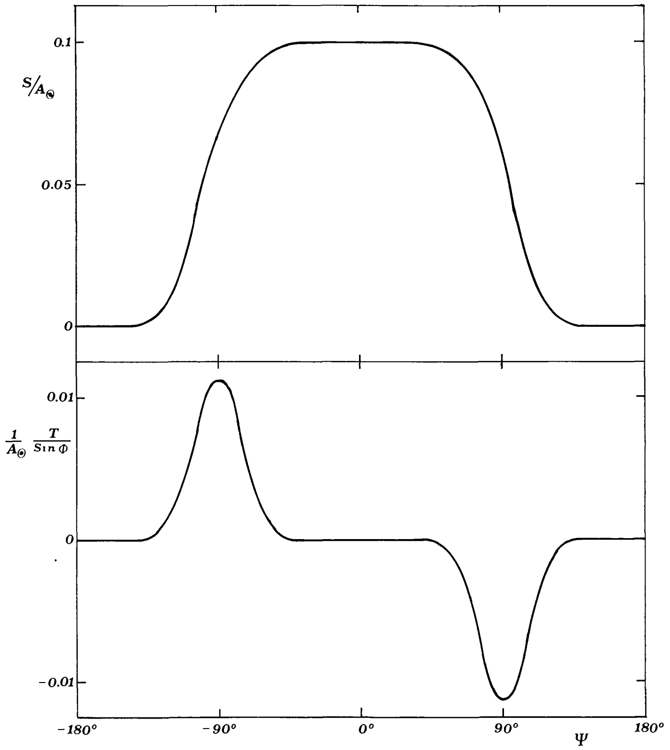

Fig. 2. Radial and pseudo-transverse acceleration vs. satellite-Earth-Sun phase angle ψ according to Levin's model applied to LAGEOS ($A_\odot$ and Φ are defined in Sect. 2)

Therefore, a mathematically tractable model must choose, among a number of effects, the ones which give rise to perturbations accumulating over long time spans (for instance, as analyzed by Sehnal, 1981, the infrared radiation gives a relevant contribution to the total radiation pressure but does not produce detectable long-term effects on the LAGEOS' orbit).

For this choice, a suitable procedure consists in decomposing the perturbing acceleration into its S, T, W components along the radial, in-plane transverse and out-of-plane directions; then the variation of these components during one revolution determines [via the planetary equations in the Gauss form, see Roy (1978, Sect. 6.7.4)] the short-periodic, long-periodic and secular effects. A qualitative idea of the behaviour of these components is given by Fig. 2 (adapted from Levin, 1962). If the albedo has no strong large scale variations, the radial component S is very roughly constant for most of the half revolution during which the satellite is over the illuminated hemisphere and is negligible in the other half. On the contrary, the acceleration component perpendicular to the plane of the terminator ("pseudo-transverse" component) gives a kick forward when the satellite enters the dark hemisphere and a kick backward when it enters the illuminated hemisphere; then the transverse component T is obtained by projecting the "pseudo-transverse" one on the satellite orbital plane. For the semi-major axis a the Gauss' equation, at the first order in the eccentricity e, is:

$$\frac{da}{dt}=\frac{2}{n}[T+e(S\sin M+T\cos M)]\,, \tag{1}$$

where M is the mean anomaly and n is the mean motion. Hence, for a satellite in a nearly circular orbit (for LAGEOS $e=0.004$), the long-term evolution of the semi-major axis is dominated by the average of the T component. This average would be zero if the optical properties of the Earth's surface were always equal at antipodal points, because the two opposite kicks would be equal; in this case the dominant effect would be demultiplied by a factor e [this indeed occurs for the albedo models used by Lautman (1977a) and Gaposchkin (1980)].

On the other hand, since the sub-satellite point crosses the terminator at opposite latitudes, there are at least two reasons why the average of T is not zero. First, the Northern and Southern hemispheres of the Earth have significantly different mean albedos (reflecting in part the different sea vs. land distribution). Second, for each hemisphere the mean albedo depends strongly on the season,

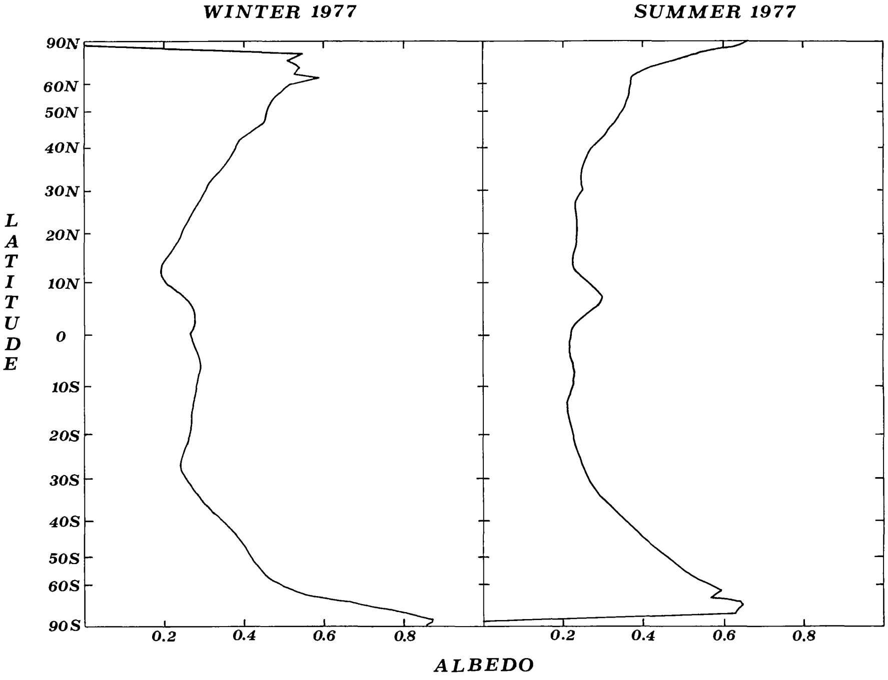

Fig. 3. Zonally and seasonally averaged albedo of the Earth's surface vs. the latitude, during winter and summer 1977 (adapted from Winston et al., 1979). Both the asymmetry between Northern and Southern hemisphere and the seasonal variations are apparent

due to changes in cloudiness, snow cover, vegetation, etc. (see Fig. 3). A simple model accounting for both these phenomena gives the average $\bar{T}$ of T over one revolution (resulting from the unbalancing of the two terminator kicks) as:

$$\bar{T}=(A\cos\phi_P\sin\bar{\lambda}_\odot+B\cos\phi_P)\sin\Phi\,, \tag{2}$$

where: Φ is the angle between the orbital plane and the terminator one ($0\leqq\Phi\leqq\pi$), and $\sin\Phi$ accounts for the projection of the "pseudo-transverse" component on the orbital plane; ϕ_P is the colatitude of the intersection between the orbital plane and the terminator plane (corresponding to the entrance of the satellite in the "day") and the factor $\cos\phi_P$ accounts for the fact that the asymmetry between the two hemispheres is more relevant as the terminator is crossed closer to the poles; $\bar{\lambda}_\odot=\lambda_\odot-\alpha$ is a seasonal phase angle defined by the longitude $\lambda_\odot$ of the sun and by a phase lag α accounting for the shift of the meteorological seasons with respect to the astronomical seasons. The amplitudes of the two effects described above are given by the constant coefficients B (for the mean "hemispheric" asymmetry) and A (for the seasonal variation).

Since the radial component S produces a small effect on the semi-major axis for low eccentricities, a very simple model is accurate enough:

$$S=\begin{cases} D\cos\psi & \text{for} \quad \psi\leqq\dfrac{\pi}{2} \\ 0 & \text{for} \quad \psi\geqq\dfrac{\pi}{2}, \end{cases} \tag{3}$$

where ψ is the Sun-Earth-satellite "phase" angle. As shown by Lautman (1977a), a more refined model including a more complex dependence of S on ψ (valid when the satellite "sees" the terminator) gives the same effect on a, provided the constant D is readjusted.

An order-of-magnitude estimate of these coefficients can be derived in the following way. The direct solar radiation pressure on LAGEOS produces an acceleration $A_\odot$ of about $3.7\,10^{-7}$ cm s^{-2}, and for $a\simeq2R_\oplus$ the maximum value of the S component (of the albedo acceleration) is about 10% of $A_\odot$ [Levin, 1962; Lautman, 1977a, Eq. (9)], so that D is about $3.7\,10^{-8}$ cm s^{-2}. For the pseudo-transverse component, the peak value is of the order of $10^{-2}A_\odot$; the integrated effect of each kick depends on the detailed assumptions [see for instance Eqs. (9) and (45) in Lautman, 1977a, and Eq. (5) in Levin, 1962], but it is approximately equivalent to a constant acceleration of $1.5\,10^{-3}A_\odot$. If we assume a 10% asymmetry between the two hemispheres and a 20% seasonal variation we have $|B|\simeq5.6\,10^{-11}$ cm s^{-2}, $|A|\simeq1.1\,10^{-10}$ cm s^{-2}, in agreement with the order of magnitude of the unmodeled along-track acceleration of LAGEOS (long-periodic components). As for the signs of A and B, if the Southern hemisphere is on the average brighter and the mean albedo increases in winter, we have $B, A>0$. All the previous assumptions (both for the order of magnitude and for the sign of A and B) are in reasonable agreement with the available data on the Earth's albedo, as derived by satellite radiation budget measurements and shown by Fig. 3 (adapted from Winston et al., 1979). In the following section we shall decompose the resulting orbital effects into Fourier components to show that our model gives the required long-periodic perturbations.

3. Long-periodic Perturbations

The satellite orbital elements $a, e, i, \Omega, \omega, v$ (true anomaly), defined as usual, allow to derive the unit vectors $\hat{h}$ (perpendicular to the orbital plane) and $\hat{r}$ (Earth's center-satellite direction) in the equatorial reference frame:

$$\hat{h}=\begin{bmatrix}\sin i\sin\Omega\\ -\sin i\cos\Omega\\ \cos i\end{bmatrix},\qquad \hat{r}=\begin{bmatrix}\cos\Omega\cos u-\sin\Omega\sin u\cos i\\ \sin\Omega\cos u+\cos\Omega\sin u\cos i\\ \sin u\sin i\end{bmatrix}, \tag{4}$$

Table 1. Fourier components of the T acceleration due to the Earth's albedo on the orbit of LAGEOS $\left[\dot{a}=\frac{2}{n}T\text{; see Eq. (10)}\right]$

Argument	Period (d)	Amplitude	Phase (sine waves)
$\lambda_\odot-\Omega$	560.2	0.90 B	$3\pi/2$
$\lambda_\odot+\Omega$	271.0	0.04 B	$3\pi/2$
$2\lambda_\odot-\Omega$	221.1	0.45 A	$\pi-\alpha$
$2\lambda_\odot+\Omega$	155.6	0.02 A	$\pi-\alpha$
Ω	1049.6	0.42 A[a]	0[a]

[a] For $\alpha=0$ [see the last term of Eq. (10)]

where $u=\omega+v$. The direction of the Sun is specified by the unit vector

$$\hat{s}=\begin{bmatrix}\cos\lambda_\odot\\ \sin\lambda_\odot\cos\varepsilon\\ \sin\lambda_\odot\sin\varepsilon\end{bmatrix}, \tag{5}$$

where ε is the obliquity of the ecliptic. The intersection line between the orbital plane and the terminator plane is defined by the vector product $\hat{s}\times\hat{h}$; hence the colatitude ϕ_P at which the orbit enters the "day" half-space is given by:

$$\cos\phi_P=\frac{\langle\hat{z},\hat{s}\times\hat{h}\rangle}{|\hat{s}\times\hat{h}|}, \tag{6}$$

where $\hat{z}$ points towards the North pole. The angle Φ between the two planes is given by

$$\sin\Phi=|\hat{s}\times\hat{h}|. \tag{7}$$

Substituting in Eq. (2) we find the following expression for the averaged T component:

$$\bar{T}=\langle\hat{z},\hat{s}\times\hat{h}\rangle\cdot(A\sin\lambda_\odot+B). \tag{8}$$

The triple product can be computed from Eqs. (4) and (5):

$$\langle\hat{z},\hat{s}\times\hat{h}\rangle=-\sin i\left[\cos^2\left(\frac{\varepsilon}{2}\right)\cos(\lambda_\odot-\Omega)+\sin^2\left(\frac{\varepsilon}{2}\right)\cos(\lambda_\odot+\Omega)\right]. \tag{9}$$

Now we can directly decompose $\bar{T}$ into Fourier components:

$$\begin{aligned}\bar{T}=&-\frac{A}{2}\sin i\cos^2\left(\frac{\varepsilon}{2}\right)\sin(2\lambda_\odot-\alpha-\Omega)\\ &-\frac{A}{2}\sin i\sin^2\left(\frac{\varepsilon}{2}\right)\sin(2\lambda_\odot-\alpha+\Omega)\\ &-B\sin i\cos^2\left(\frac{\varepsilon}{2}\right)\cos(\lambda_\odot-\Omega)-B\sin i\sin^2\left(\frac{\varepsilon}{2}\right)\cos(\lambda_\odot+\Omega)\\ &-\frac{A}{2}\sin i(1-\cos^2\alpha\sin^2\varepsilon)^{1/2}\sin(\Omega+\delta),\end{aligned} \tag{10}$$

where the phase angle of the Ω harmonic is:

$$\delta=\mathrm{arctg}\left(-\frac{\mathrm{tg}\alpha}{\cos\varepsilon}\right)\simeq-\alpha. \tag{11}$$

In order to compute the frequencies corresponding to the various terms, we neglect the eccentricity of the Sun, in such a way that $d\lambda_\odot/dt$ can be considered constant, and we recall that for LAGEOS:

$$i=110^\circ,\quad \frac{d\Omega}{dt}=0^\circ\!.343/\mathrm{d}. \tag{12}$$

Table 1 summarizes the relative amplitudes and the phases of the resulting five long-periodic components of the T acceleration. The effects on the semi-major axis are obtained simply by Eq. (1): long-periodic perturbations do appear as a direct consequence of the albedo variations included in our model. We note that the frequencies appearing in Eq. (10) are the same which characterize the solar-tidal effects on the orbit [the main terms in Table 1 have the periods of the S_1, P_1, and K_1 tides; see Smith and Dunn (1980, Table 2)]. This coincidence had to be expected because the frequencies depend only on the relation between the orbit and the Sun's position; as we shall discuss in Sect. 4, the order of magnitude of the two effects is also comparable. However, the tidal effects should not be responsible for the unmodeled perturbations because they have been included in the orbit determination (whilst the albedo effects described in this paper have been neglected).

The radial component S produces effects which, although demultiplied by a factor e, are not completely negligible because $D\gg A, B$. As shown by the Gauss equation (1), to estimate the long-periodic effects of S we have to compute the average of da/dt over one revolution:

$$\frac{\overline{da}}{dt}=\frac{2}{2\pi n}\int_0^{2\pi}eS\sin M\,dM=\frac{eD}{\pi n}\int_0^{2\pi}\cos\psi\sin M f(M)dM, \tag{13}$$

where D is the constant of Eq. (3), $\cos\psi=\langle\hat{r},\hat{s}\rangle$, and $f(M)=1$ for $\psi\leqq\frac{\pi}{2}$ and $f(M)=0$ for $\psi>\frac{\pi}{2}$. Following Lautman (1977a) we express $\cos\psi$ as

$$\cos\psi=\mathscr{A}\cos u+\mathscr{B}\sin u=\mathscr{C}\cos(u-\zeta) \tag{14}$$

with

$$\begin{aligned}&\mathscr{A}=\cos\Omega\cos\lambda_\odot+\sin\Omega\sin\lambda_\odot\cos\varepsilon,\\ &\mathscr{B}=-\sin\Omega\cos\lambda_\odot\cos i+\cos\Omega\cos i\cos\varepsilon\sin\lambda_\odot+\sin i\sin\varepsilon\sin\lambda_\odot.\end{aligned} \tag{15}$$

In this way $\zeta=\mathrm{arctg}(\mathscr{B}/\mathscr{A})$ gives the angular position on the orbital plane of the sub-satellite point's "noon" (i.e., the minimum of ψ), and $\psi=\frac{\pi}{2}$ (i.e., terminator crossing) occurs for $|u-\zeta|=\pi/2$. Therefore

$$\frac{\overline{da}}{dt}=\frac{eD}{\pi n}\int_{-\pi/2}^{+\pi/2}\mathscr{C}\cos(u-\zeta)\sin M\,d(u-\zeta). \tag{16}$$

Neglecting terms of the order of e

$$M=(u-\zeta)-(\omega-\zeta), \tag{17}$$

and we get

$$\frac{\overline{da}}{dt}=-\frac{eD}{2n}\mathscr{C}\sin(\omega-\zeta). \tag{18}$$

Since $\mathscr{C}$ was defined by (14),

$$-\mathscr{C}\sin(\omega-\zeta)=\mathscr{C}\cos\left(\omega+\frac{\pi}{2}-\zeta\right)=-\mathscr{A}\sin\omega+\mathscr{B}\cos\omega \tag{19}$$

and

$$\frac{\overline{da}}{dt}=\frac{eD}{2n}(\mathscr{B}\cos\omega-\mathscr{A}\sin\omega). \tag{20}$$

Table 2. Fourier components of the T_S acceleration due to the Earth's albedo on the orbit of LAGEOS $\left[\dot{a}=\frac{2}{n}T_S,\ \dot{e}=\frac{1}{nae}T_S;\ \text{see Eq. (21)}\right]$

Argument	Period (d)	Amplitude	Phase (sine waves)
$\lambda_\odot+\omega$	300.3	0.047 eD	0
$\lambda_\odot-\omega$	465.9	0.047 eD	0
$\lambda_\odot+\omega+\Omega$	233.5	0.0034 eD	π
$\lambda_\odot-\omega+\Omega$	322.5	0.007 eD	0
$\lambda_\odot+\omega-\Omega$	420.7	0.16 eD	π
$\lambda_\odot-\omega-\Omega$	837.9	0.079 eD	0

By substituting (15) into (20) we can compute directly the Fourier components of the equivalent along-track acceleration T_S produced by the radial component S:

$$\begin{aligned}T_S=\frac{n}{2}\frac{\overline{da}}{dt}=&\frac{eD}{8}\sin i\sin\varepsilon\,[\sin(\lambda_\odot+\omega)+\sin(\lambda_\odot-\omega)]\\&+\frac{eD}{4}\cos^2\left(\frac{\varepsilon}{2}\right)\left[-\sin^2\left(\frac{i}{2}\right)\sin(\lambda_\odot+\omega-\Omega)\right.\\&\left.+\cos^2\left(\frac{i}{2}\right)\sin(\lambda_\odot-\omega-\Omega)\right]\\&+\frac{eD}{4}\sin^2\left(\frac{\varepsilon}{2}\right)\left[\sin^2\left(\frac{i}{2}\right)\sin(\lambda_\odot-\omega+\Omega)\right.\\&\left.-\cos^2\left(\frac{i}{2}\right)\sin(\lambda_\odot+\omega+\Omega)\right].\end{aligned}\tag{21}$$

In order to compute the corresponding characteristic frequencies we recall that for LAGEOS:

$$e=0.004,\qquad \frac{d\omega}{dt}=-0\overset{\circ}{.}213/\mathrm{d}.\tag{22}$$

Table 2 summarizes the relative amplitudes and the phases of the resulting six long-periodic components of the equivalent along-track acceleration T_S; the corresponding effects on a can be obtained multiplying by a factor $2/n$.

The radial component S produces also a variation of e with the same signature. The Gauss equation for e is (neglecting terms of the order of e):

$$\frac{de}{dt}=\frac{1}{na}(S\sin M+T\cos M).\tag{23}$$

Neglecting the smaller T term the average of $\frac{de}{dt}$ over one revolution can be written as a function of the equivalent along-track acceleration T_S:

$$\frac{\overline{de}}{dt}\simeq\frac{1}{nae}T_S\tag{24}$$

and Table 2 can be used again for e. The amplitude of the resulting oscillation in eccentricity is of the order of $3\,10^{-7}$ for the main 421 ds term, in good agreement with the eccentricity residuals obtained by Smith and Dunn (1978, Fig. 2b).

4. Conclusions

As summarized in Tables 1 and 2 the model predicts several long-periodic perturbations whose amplitudes and phases depend on the choice of the four parameters A, B, D, and α. For instance, we show in Fig. 4 a plot of $\bar{T}$ plus T_S [as derived by Eqs. (10 and (21)] versus time from 1976 to 1981 (as in Fig. 1) for a set of values of the parameters ($A>0$, $B/A=0.3$, $D/A=300$, $\alpha=-60°$) chosen in order to obtain a qualitative agreement with the pattern of Fig. 1. To find reliable values of the parameters a three-step procedure can be envisaged.

First, the parameters of the model must be fitted to the unmodeled semi-major axis oscillations. A second step could be a new numerical integration of the differential equations of LAGEOS' orbit including, besides the constant "drag", an albedo

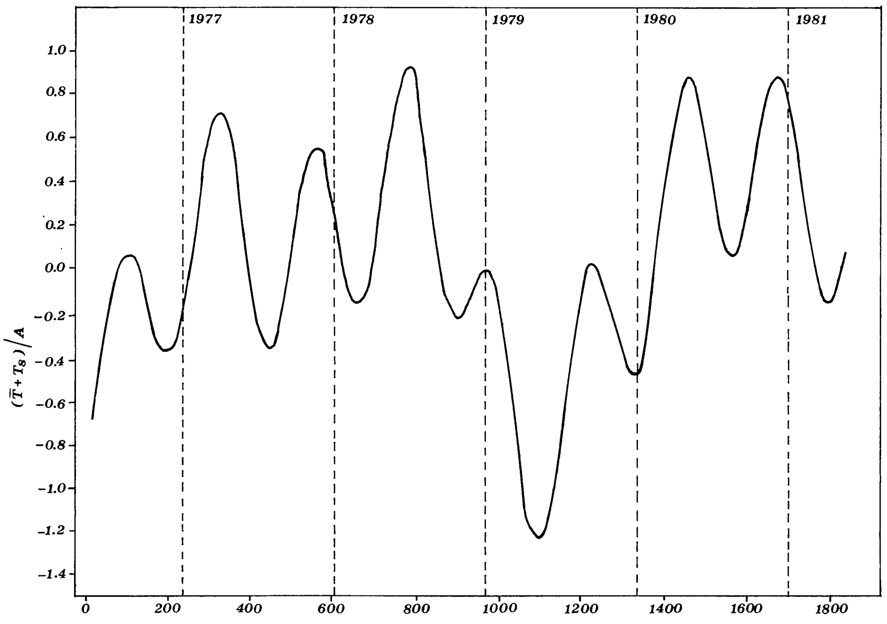

Fig. 4. LAGEOS along-track acceleration produced by the Earth-reflected sunlight. The "equivalent along-track acceleration" produced by the radial component has been included. This plot has been generated by using $B=0.3A$, $D=300A$, $\alpha=-60°$. The horizontal scale is as in Fig. 1; on the vertical axis the acceleration is expressed as a fraction of A

force term consistent with our model. Provided the numerical stability problems (arising from the steep behaviour of the terminator kicks) are adequately monitored, we expect a significant reduction of the residuals and a clear change of their spectrum. This work is presently in progress at GSFC, by implementing the albedo model in the orbit and geodetic parameter estimation program GEODYN (Smith, 1982, private communication). Third, a detailed comparison of the fitted A, B, D, and α coefficients with real albedo statistics should be tried; but this comparison is not straightforward. Although brightness data for the Earth surface are currently available (Winston et al., 1979), the complexity of the angular dependence of the reflected radiation in the various cases makes the relationship between brightness data and radiation pressure extremely hard to compute.

This fit will give satisfactory results provided that: (a) our simple model really accounts for the main effects of the Earth's reflected radiation on the orbit; (b) other perturbations with a similar signature have been appropriately modeled. This latter requirement is important, since some drag-like effect is needed to account for the average negative value of da/dt, and this raises the question of what the time variability of this drag may be. In the case of the charged-particle drag proposed by Afonso et al. (1980), Mignard (1981) and Rubincam (1982), variations could come either from changes in the satellite potential or from changes in the ion density in the plasmosphere (Bender, 1982, private communication). The former ones could be correlated with eclipses (therefore giving rise to an effect with the 560 d period), while the ion density (as well as the neutral density) is known to change with different timescales; however, the apparent lack of correlation between drag intensity and solar activity in the period 1976–1980 (see Rubincam, 1982) has still to be explained.

Another problem can arise as a consequence of the solar-tidal deformations of the Earth whose characteristic frequencies, as we noted before, are the same as in Table 1 (the main lunar-tidal terms have higher frequencies). The solar-tidal acceleration on LAGEOS is of the order of 10^{-6} cm s^{-2} (as derived via the usual frequency-independent Love number approximation), but its equivalent along-track acceleration, defined as $n/2$ times the averaged da/dt, is of the order of $5\,10^{-10}$ cm s^{-2} [this estimate can be derived as follows: the variation of orbital energy caused by tides after one revolution is given by the variation of the tidal potential after one period at a fixed position, plus its change over the displacement due to precession of the nodal line, i.e., $(2\pi/n)(d\Omega/dt)\sin i$; such perturbative terms depend obviously on the argument Ω]. If the tides are modeled with reasonable accuracy, the residual acceleration should be small, but not negligible with respect to the albedo effect. Also the modeling of eclipses could cause some problems, not only for the reasons discussed by Rubincam (1982), but also because the penumbra effects do not necessarily average out; since the eclipses are also controlled by the argument $\lambda_\odot - \Omega$, the same frequencies as before could arise.

As a conclusion, we can state that the study of the long-periodic semi-major axis perturbations of LAGEOS, first attempted in this paper to assess the effects of the albedo force term, can in general prove of great interest to determine the influence of several small perturbative effects, whose knowledge will substantially improve the attainable accuracy by the next generation of geophysical satellites.

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Note added in proof: F. Barlier and S. Leschiutta have communicated to us that, during the Symposium on Space Geodesy held at Porto Hydra (Greece) on September 1982, D. Christodoulidis and J. J. Walch reported on attempts (presently in progress) to include asymmetric albedo force model presented in this paper in numerical computations of artificial satellite orbits. As regards LAGEOS, the results are not yet conclusive: the order of magnitude of the perturbations seems to be correct, but possibly some problem arises in connection with the phase of the perturbative effects (note that our choice of $\alpha = -60°$ to derive Fig. 4 had no clear physical meaning).