Journal of Geodesy (1995) 70:140–145

Journal of Geodesy

Anisotropic reflection effect on satellite, Ajisai

A. Sengoku*, M. K. Cheng, and B. E. Schutz

Center for Space Research, Code 60605, The University of Texas at Austin, Austin, TX 78712-1085, USA

Received 31 October 1994; Accepted 1 March 1995

Summary. A model for the anisotropic reflection force acting on Ajisai is presented which includes the variable reflectivity coefficient and the force in the direction perpendicular to the incident light. This model significantly reduces the along-track orbit errors of Ajisai and smoothes the spike-like variation in the estimated drag coefficients from analysis of Ajisai laser ranging data. The model produces 17% smaller range residual RMS values in a one-year arc analysis of 1993 data, and a smaller residual RMS for a short-arc analysis in mid-year, the period from May to August, 1993.

Introduction

The Ajisai satellite is a spherical geodetic satellite launched in 1986 by the National Space Development Agency of Japan (NASDA). The primary objective for Ajisai is to determine a precise geodetic control network by satellite laser ranging (SLR) and photography (Sasaki and Hashimoto 1987). Ajisai has also contributed to recovery of geopotential field and tide models (Lerch et al. 1992). Its orbit is nearly circular with an inclination of 50 degrees and an altitude of approximately 1500 km. Ajisai is a hollow sphere covered with 1436 corner cube reflectors (CCR's) for SLR and 318 mirrors to reflect sunlight. Its spin period was 40.3 rpm at launch, but this period has been decreasing 1.65% a year (NASDA, *private communication,* 1994). The spin axis was set parallel to the Earth rotation axis so that the flashing of reflected sunlight could be seen from any ground location.

To assure greater flash luminosity from the mirrors, the diameter of Ajisai was chosen to be 2.15m, whereas the diameter of other spherical satellites is smaller, such as LAGEOS and STARLETTE which are 60 cm and 24 cm in diameter, respectively. The large area to mass ratio (mass=685kg) of Ajisai results in large accelerations due to surface forces. The study of the surface forces on Ajisai is required for precision orbit determination to support high accuracy geodetic applications, geopotential recovery and tide models. Modeling of the nongravitational forces acting on a satellite has become a major limitation in precision orbit determination.

Asymmetric reflectivity of LAGEOS was proposed by Rubincam et al.(1987a), Scharroo et al.(1991) and Ries et al.(1991) in order to account for observed rapid changes, or spikes, of along-track force during the eclipse seasons. The mechanism by which the asymmetry of LAGEOS arises is not fully understood, it may be due to uneven surface finish, which was done by hand. Asymmetric reflectivity acceleration along the spin axis of LAGEOS can explain a part of the observed along-track residuals. The anisotropic radiation effects on Ajisai are much larger than those of LAGEOS due to the large area to mass ratio and the anisotropic nature of Ajisai.

The estimation of the drag coefficient, C_d, in precision orbit determination of a satellite is used to account for the average errors in atmosphere density over the estimation interval. However, the estimated C_d is highly correlated with other unmodeled along-track orbit errors. Modeling the effect of asymmetric reflectivity is required in order to correctly interpret the estimated C_d as satellite drag information.

In this paper, a model of the anisotropic radiation pressure on Ajisai is proposed. This model is validated through orbital analysis of Ajisai using SLR data. The correction of C_d due to the anisotropic effects is presented.

* on leave from Hydrographic Department of Japan

Correspondence to: A. Sengoku

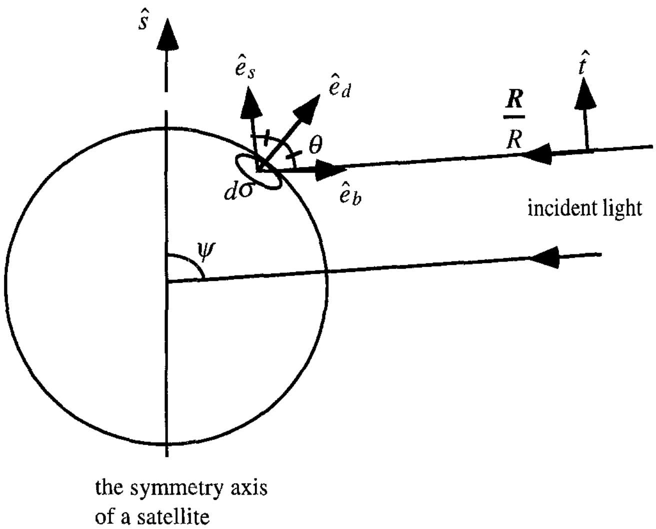


Fig. 1. Reflection of light at the surface of a satellite.

A model of the anisotropic reflection force

The radiation pressure is the vector sum of the photon thrust on all surface elements, which can be classified by the reflection property of the surface element into specular, diffuse or backward, as follows,

$$\boldsymbol{F} = \iint_{\sigma} (d\boldsymbol{F}_a + d\boldsymbol{F}_s + d\boldsymbol{F}_d + d\boldsymbol{F}_b)$$
$$= -\frac{I}{c}(A\hat{e}_b + \iint_{\sigma} (r_s f_s \hat{e}_s + \frac{2}{3} r_d f_d \hat{e}_d + r_b f_b \hat{e}_b)$$
$$g\cos\theta d\sigma). \qquad (1)$$

The subscripts a, s, d and b denote absorption, specular, diffuse and backward reflection, respectively. Backward reflection means reflection by the CCR back along the path of the incident light. In Eq.(1), I is the flux of incident light, c is the speed of light, A is the cross-sectional area, r is the reflectivity, f is the fraction of area covered, θ is the local angle of incidence (i.e., the angle between the incident light and normal of the surface element) and σ is the area of the satellite surface. $\hat{e}_b$, $\hat{e}_s$ and $\hat{e}_d$ stand for unit vectors toward backward to the incident light, specularly reflected and normal from the surface, respectively (Figure 1). Furthermore, g is 1 when the surface element is illuminated by the incident light and is 0 otherwise. The factor 2/3 results from Lambert's cosine law of diffuse reflection.

The force per unit mass, or acceleration, due to radiation pressure along the sun-satellite direction for an axially symmetric satellite is,

$$\ddot{r}_l = \kappa C_R \frac{A}{m} \frac{\boldsymbol{R}}{R^3}, \qquad (2)$$

where

$$C_R = 1 - \iint_{\sigma} (r_s f_s \hat{e}_s + \frac{2}{3} r_d f_d \hat{e}_d + r_b f_b \hat{e}_b) \cdot$$
$$\frac{\boldsymbol{R}}{R} g\cos\theta \frac{d\sigma}{A}. \qquad (3)$$

$\boldsymbol{R}$ is the heliocentric radius vector to the satellite in astronomical units (AU), $\kappa = I|_{R=1AU} / c = 4.560 \times 10^{-6} (Nm^{-2})$, m is satellite mass and C_R is the reflectivity coefficient. Note that the reflectivity coefficient is not a constant if the satellite is not spherically symmetric. If a satellite is axially symmetric, C_R is a function of ψ, where ψ is an angle between the incident light and the symmetry axis (Figure 1). Variation of acceleration due to radiation pressure along the sun-satellite direction is,

$$\Delta\ddot{r}_l = \kappa(C_R - < C_R >)\frac{A}{m}\frac{\boldsymbol{R}}{R^3}, \qquad (4)$$

where $< C_R >$ is the average reflectivity coefficient.

For axially symmetric but non-spherically symmetric satellites, the radiation pressure acceleration perpendicular to the sun-satellite direction is not zero and is given by,

$$\ddot{r}_t = \kappa C_{AR} \frac{A}{m} \frac{\hat{t}}{R^2}, \qquad (5)$$

where

$$C_{AR} = -\iint_{\sigma} (r_s f_s \hat{e}_s + \frac{2}{3} r_d f_d \hat{e}_d) \cdot \hat{t} g\cos\theta \frac{d\sigma}{A}, \qquad (6)$$

$$\hat{t} = -\frac{\boldsymbol{R} \times \hat{s}}{|\boldsymbol{R} \times \hat{s}|} \times \frac{\boldsymbol{R}}{R}. \qquad (7)$$

In this expression, C_{AR} is the anisotropic reflectivity coefficient and $\hat{s}$ is the unit vector along the symmetry axis of satellite. C_{AR} is also a function of ψ for an axially symmetric satellite. The acceleration $\ddot{r}_t$ is within a plane containing both the incident sun light and the symmetry axis. The acceleration normal to both $\boldsymbol{R}$ and $\hat{s}$ is zero due to axial symmetry.

Both $\Delta\ddot{r}_l$ and $\ddot{r}_t$ are referred in the following sections as the anisotropic reflection acceleration. The average radiation pressure in the sun-satellite direction is usually modeled in Ajisai orbit determination, whereas the anisotropic reflection effect has been neglected.

Ajisai's surface is mostly covered with four elements: CCRs, mirrors, bases and groundwork. Each CCR was made of fused silica and its surface was not coated. Theoretical computation shows that the CCR reflects light to the source if the incident angle is smaller than 16.9 degrees, or specularly if it is larger than 43.2 degrees which is the critical angle of fused silica (Hashimoto, *private communication*, 1994). Otherwise, the incident light penetrates the CCR, since the bottom side of the CCR is not coated, and is reflected at the base under the CCR. The mirrors were made of aluminum alloy and the surface was coated by silicon oxide. Reflection from the mirror surface is specular. The base, which is shown as a polygonal plate in the laser reflector assembly in Figure 2, supports the CCRs and was constructed from an alloy of aluminum. The groundwork consists of a plastic body and are depicted as the spherical body and the upper/lower cap in Figure 2. It has been assumed for this paper that the base and groundwork are diffuse since their surfaces were not

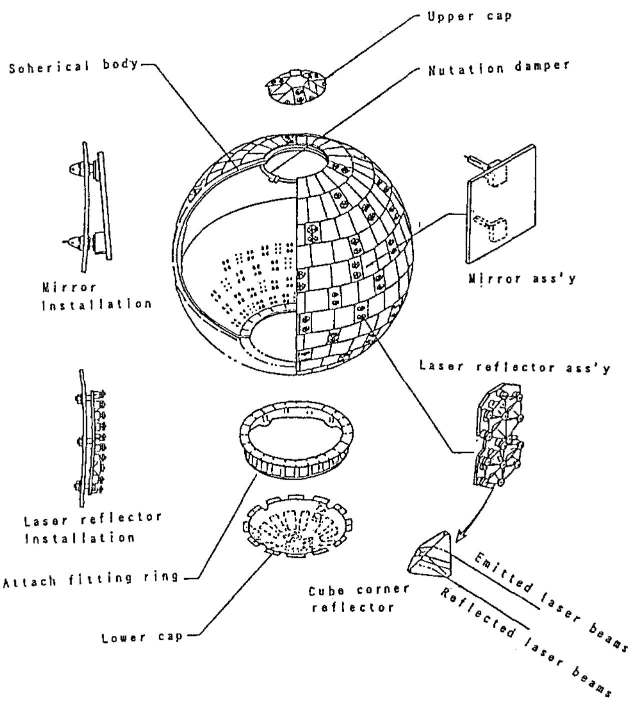


Fig. 2. Structure of Ajisai (after Sasaki and Hashimoto, 1987).

especially polished.

Ajisai consists of three parts: a main body and two polar caps. It is axially symmetric with respect to the figure axis, as shown in Figure 2. At both polar caps there are fewer mirrors than at the main body since these locations do not contribute significantly to the flashing capability. The south polar cap has 9 more mirrors than the north cap in order to optimize opportunities by ground observers in the northern hemisphere. The anisotropy of Ajisai's surface causes asymmetric recoil of photon thrust. The reflectivity and the fraction of area covered for respective surface elements are shown in Table 1 (Hashimoto, *private communication,* 1994).

The spin axis of Ajisai was aligned to the Earth spin axis at orbit insertion (Hashimoto, *private communication,* 1994). In 1988, timing observations of Ajisai's flash by the

Table 1. Fraction of area covered and reflectivity of each element

	south polar cap	main body	north polar cap
mirror	0.266 / 0.837	0.813 / 0.874	0.223 / 0.849
CCR	0.113 / 0.975	0.070 / 0.975	0.046 / 0.975
base	0.621 / 0.390	0.073 / 0.470	0.731 / 0.348
groundwork	-	0.044 / 0.100	-

Hydrographic Department of Japan (JHD) revealed that the spin axis of Ajisai was mostly parallel to the Earth spin axis with deviation of a few degrees (Kanazawa, *private communication*, 1994). Kubo (1987) showed that gravitational attraction of the Earth combined with the asymmetrical mass distribution of Ajisai causes both precession (period: 63 years) and nutation (period and amplitude of the main term: 120 days, 4320 arc seconds). Kubo's theory suggests that the amplitude of precession due to Earth's gravitational field is as small as a few degrees. Ajisai has a mechanism that damps free nutation immediately, and the symmetry axis of Ajisai coincides with the spin axis. Consequently, in this paper it is assumed that both the spin axis and symmetry axis of Ajisai are parallel to the Earth's spin axis.

The reflectivity coefficient and anisotropic reflectivity coefficient of Ajisai are obtained by numerical integration of Eqs.(3) and (6). The result is shown in Figure 3. Both coefficients are functions of declination of the incident light δ since both the symmetry axis and spin axis of Ajisai is aligned to the Earth axis. The reflectivity coefficient reaches its minimum when $\delta = \pm 90$ degrees and the maximum occurs when δ is about 30 degrees in both hemispheres. These extremes occur because the cross section of the specular component near the rim, which makes the reflectivity coefficient smaller, is minimum at these latitudes. If some part of the incident light is reflected specularly at the base or groundwork, the total reflectivity coefficient will decrease. Some part of the incident light that penetrates the CCR and is reflected at the base might be absorbed in the CCR or reflected again at the CCR to another direction. Such effects also bias the total reflectivity coefficient. The anisotropic reflectivity coefficient shows sinusoidal variations and large asymmetry with respect to the satellite equator.

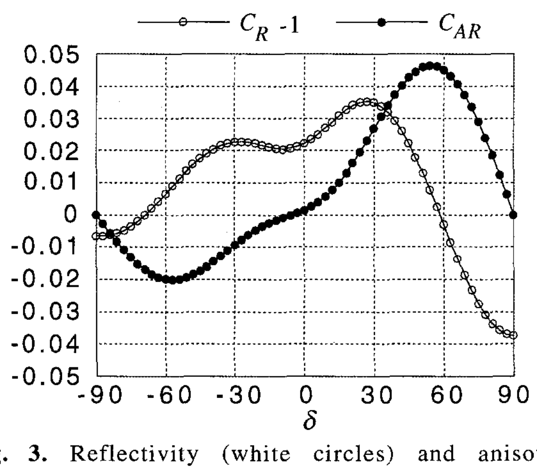


Fig. 3. Reflectivity (white circles) and anisotropic reflectivity coefficients (black circles) of Ajisai as a function of declination of the incident light δ.

Figure 4 illustrates the reflectivity and anisotropic reflectivity coefficient vs. time. The variation of the reflectivity coefficient is 1.5% and the maximum anisotropic reflectivity acceleration is 2% of the longitudinal component. These variations are not negligible, especially in the shadowing periods. It is also noted that the anisotropic reflection acceleration, which is the sum of $\Delta\ddot{r}_l$ and $\ddot{r}_t$, is large in mid-year for the period from May to August, since both coefficients attain large values. This is because in mid-year there is more illumination of the north pole of Ajisai where mirrors are sparse and the surface is less specular.

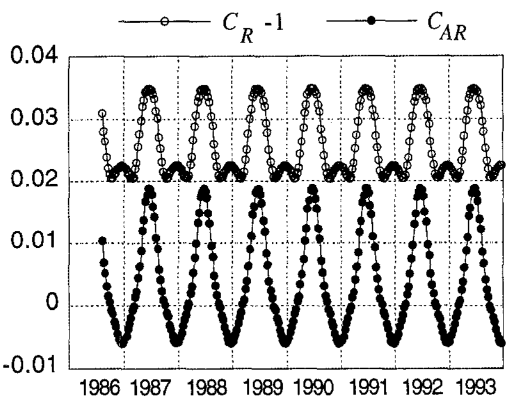


Fig. 4. Reflectivity (white circles) and anisotropic reflectivity coefficients (black circles) of Ajisai as a function of time. The year is shown along the bottom. Incident light is solar radiation only.

Along-track acceleration due to anisotropic reflection and *Cd* correction

The effect of the acceleration due to anisotropic reflection on the along-track component of the orbit becomes most significant during the shadowing periods. The averaged along-track acceleration due to anisotropic reflection can be expressed as follows:

$$< T_a > = \frac{1}{2\pi}\int_{M_2}^{M_1+2\pi}(\Delta\ddot{r}_l + \ddot{r}_t)\cdot\hat{\beta}dM, \tag{8}$$

where M is the orbit mean anomaly and $\hat{\beta}$ is the along-track unit vector. The subscripts *1* and *2* refer to the position when Ajisai enters and exits the Earth's shadow, respectively.

Neglecting terms with order of eccentricity and assuming that the acceleration is constant during one revolution (Scharroo et al. 1991), it follows that,

$$< T_a > = \frac{(\Delta\ddot{r}_l + \ddot{r}_t)\cdot(\hat{\alpha}_1 - \hat{\alpha}_2)}{2\pi}, \tag{9}$$

where $\hat{\alpha}$ is radial unit vector. During non-shadowing periods, we have $\hat{\alpha}_1 = \hat{\alpha}_2$, thus the averaged along-track acceleration is zero in this approximation. Furthermore, the effect of $\Delta\ddot{r}_l$ is always negligible for circular satellites since $\hat{\alpha}_1 - \hat{\alpha}_2$ is perpendicular to the sun-satellite direction if the earth is a sphere.

$$< T_a > = \frac{\ddot{r}_t\cdot(\hat{\alpha}_1 - \hat{\alpha}_2)}{2\pi}. \tag{10}$$

In precision orbit determination of a satellite at low altitude, the drag coefficient C_d has to be estimated to accommodate errors in the drag model. The acceleration due to air drag is typically expressed as,

$$\ddot{r}_{ad} = -\frac{C_d A\rho}{2m}v_r\mathbf{v}_r, \tag{11}$$

where ρ is atmospheric density, the $\mathbf{v}_r$ is the relative velocity vector to the atmosphere, and is essentially in the along-track direction for near circular orbit of Ajisai. Thus, the estimated C_d from analysis of Ajisai tracking data also absorbs the unmodeled along-track orbit errors, for example, due to the anisotropic reflection.

The effects of anisotropic reflection on the estimation of C_d can be computed as follows. Let T_{total}, T_{ad} and T_a express the total, air-drag, and anisotropic reflection accelerations in the along-track direction, respectively. Thus,

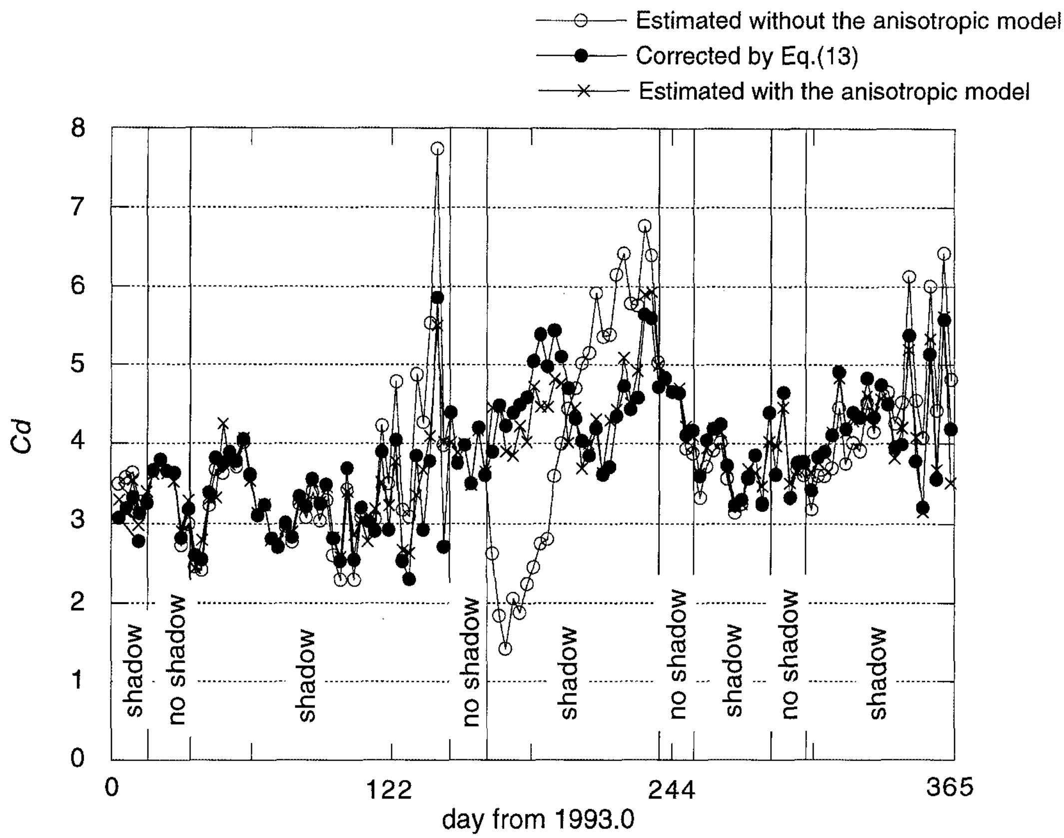
$$T_{total} = T_{ad} + T_a = -\frac{C_d^{est}A\rho v_r^2}{2m}, \tag{12}$$

where C_d^{est} is estimated C_d. From Eqs.(10), (11) and (12),

$$\begin{aligned} C_d - C_d^{est} &= \frac{2m}{A\rho v_r^2} < T_a > \\ &= \frac{m}{\pi A\rho v_r^2}\ddot{r}_t\cdot(\hat{\alpha}_1 - \hat{\alpha}_2). \end{aligned} \tag{13}$$

The model of the acceleration due to anisotropic reflection represented by Eqs.(4) and (5) is installed in the University of Texas orbit analysis system, UTOPIA, for analysis of Ajisai SLR data. The Ajisai SLR data are analyzed using a one year arc by estimating the initial position and velocity vectors, 3-day sub-arc C_d and yearly reflectivity coefficient. The JGM-3 geopotential model (Tapley et al. 1994) and the CSR background ocean tide model (Cheng et al. 1993) are used. The SLR data consist of 30-second normal points (compressed ranges) produced from Ajisai quick-look data obtained at 14 SLR stations in 1993. The RMS of the laser range residuals using the nominal force models is 102 cm for the one year arc. Adding the anisotropic model, without adjusting any parameter in the model, the residual RMS is reduced to 85cm, which is a 17% improvement.

Figure 5 shows the comparison of the 3-day C_d estimates with and without the anisotropic reflection model, and the corrected C_d obtained by using Eq.(13). Note that the large variations in mid-year are removed by modeling of the anisotropic reflection and the RMS of the C_d estimates, about the mean, decreases from 1.10 to 0.76. The C_d estimates are consistent with the results computed from Eq. (13) as shown in Figure 5. This result strongly suggests that the C_d estimates were contaminated by the unmodeled reflection effects. That is, the C_d estimates did not represent errors in the atmospheric density model used in the orbit determination, but the large variations are caused by the anisotropic acceleration. C_d correction for Ajisai is small for higher solar activity period when atmospheric density and its variation are higher, since C_d correction is inversely proportional to ρ according to Eq.(13). The maximum C_d correction in



lower solar activity period amounts to as much as 200% in 1987, while less than 50% in higher solar activity periods (assuming a nominal C_d of approximately 2). This result suggests that the along-track orbit errors in higher solar activity periods are dominated by errors in air drag model.

A short arc analysis of the Ajisai SLR data is performed. In the analysis, six 5-day arcs in July-August 1993, when solar activity was lower, are used. The estimated parameters are the Ajisai position and velocity at the epoch of each arc, a reflectivity coefficient and 2.5-day C_d. The result shows that the anisotropic reflection effects on the Ajisai orbit are not negligible, even in a short arc. In this case, the RMS of the six 5-day arcs decreases from 8.1 cm to 6.9 cm by including the anisotropic reflection model.

Discussion

At an altitude of 1500 km, the radiation pressure is by far the largest nongravitational force for Ajisai. The typical magnitude of radiation pressure acceleration of Ajisai is $2\times10^{-8}(ms^{-2})$ which is 15-400 times larger than air drag acceleration. The amplitude and direction of radiation pressure acting on Ajisai are changing according to its asymmetric surface property, which causes the acceleration to be comparable to that of air drag.

Modeling of the anisotropic reflection of Ajisai for both long arc and short arc analysis of Ajisai SLR data considerably reduces the along-track orbit errors, and smoothes away the impulsive variations in the estimated C_d. The model gives a smaller RMS for Ajisai laser ranging residuals, which provides evidence that the anisotropic reflection model represents an improved component of the nongravitational force model. The anisotropic reflection effect is not negligible even in short-arc analyses in mid-year.

The spin axis orientation is a vital factor in the studies of the anisotropic reflection effect and thermal forces. Ajisai's spin axis orientation was observed to be aligned to the Earth's polar axis in 1988. Electro-magnetic interaction with the magnetic field of the Earth might induce variations in spin axis orientation of Ajisai, though gravitational torques do not. Dynamical analysis of the 1993 Ajisai SLR data by tentatively varying the spin axis orientation shows that the smallest residual RMS is obtained in the case when the spin axis is nearly parallel to the earth's axis. This result suggests the validity of our assumption on the spin axis orientation.

The striking difference between anisotropic reflectivity models of LAGEOS and Ajisai is the orientation of the anisotropic reflectivity induced acceleration. LAGEOS models assume that the direction of the acceleration is aligned with the satellite's spin axis (Scharroo et al. 1991; Ries et al. 1991). However, in the Ajisai model, the orientation of the anisotropic reflection acceleration of Ajisai depends on the declination of the sun. The complex distribution of surface elements of Ajisai leads to a less symmetrical property of photon thrust.

Other surface forces such as thermal recoil might influence Ajisai's orbit because of Ajisai's large area to mass ratio. Thermal recoil derived by the Earth radiation (Yarkovsky effect, Rubincam 1987b, 1988) causes an along-track acceleration, which biases the C_d estimates. Solar-heating effects (Yarkovsky-Schach effect, Rubincam 1982; Slabinski 1988; Afonso et al. 1989; Scharroo et al. 1991; Ries et al. 1991) give rise to an along-track acceleration during the shadowing periods, but its amplitude is much smaller than the reflection recoil. The model given in this paper can also be applied to the radiation pressure force due to Earth's albedo. The smaller reflectivity at the poles of Ajisai might need to be taken into account. Further study of surface forces acting on Ajisai is required.

Acknowledgments. We gratefully acknowledge the kind support by Hidekazu Hashimoto of the National Space Development Agency of Japan, who kindly provided us valuable technical information of Ajisai. We express our gratitude to Dr. John C. Ries for his helpful comments. We also acknowledge the careful readings and helpful comments by the reviewers. The first author was supported by the Japanese government fellowship on space overseas research.

references

Afonso, G., F. Barlier, M. Carpino, P. Farinella, F. Mignard, M. Milani, and A. M. Nobili (1989), Orbital effects of Lageos seasons and eclipses, *Ann. Geophys. A, 7(5)*, 501.

Cheng, M. K., R. J. Eanes, C. K. Shum, B. E. Schutz, and B. D. Tapley (1993), Ocean Tide Model for Satellite Orbit Determination, *The 12th International Symposium on Earth Tides*, Beijing, China, Aug. 4-7.

Kubo, Y.(1987), The Rotation of the Geodetic Satellite AJISAI Around Its Center of Mass, *Proceedings of the twentieth symposium on Celestial Mechanics*, 52, National Astronomical Observatory, Tokyo.

Lerch, F., R. Nerem, B. Putney, T. Felstentreger, B. Sanchez, S. Klosko, G. Patel, R. Williamson, D. Chinn, J. Chan, K. Rachlin, N. Chandler, J. McCarthy, J. Marshall, S. Luthcke, D. Pavlis, J. Robbins, S. Kapoor, and E. Pavlis (1992), *NASA Tech. Memo.* 104555, NASA Goddard Space Flight Center, Greenbelt, MD.

McCarthy, D. D. (ed.), IERS Standards(1992) (1992), *IERS Tech. Note 13*, Observatoire de Paris, Paris.

NASDA (1993), *Report of Evaluation on Ajisai's Performance*, NASDA, Tukuba.

Ries, J. C., R. J. Eanes, and M. M. Watkins (1991), The along-track acceleration of Lageos, *Eos Trans. AGU, 72*, 17, Supplement p.95.

Ries, J. C., C. K. Shum, and B.D. Tapley (1993), Surface Force Modeling for Precision Orbit Determination, *Environmental Effects on Spacecraft Positioning and Trajectories, Geophysical Monograph 73, IUGG, 13*, 111.

Rubincam, D. P. (1982), On the secular decrease in the semimajor axis of Lageos's orbit, *Celest. Mech., 26*, 361.

Rubincam, D. P. (1987a), Earth anisotropic reflection and the orbit of LAGEOS, *J. Geophys. Res., 92*, 11,662.

Rubincam, D. P. (1987b), Lageos orbit decay due to infrared radiation from Earth, *J. Geophys. Res., 92*, 1287.

Rubincam, D. P. (1988), Yarkovsky thermal drag on Lageos, *J. Geophys. Res., 93,* 13805.

Sasaki, M. and H. Hashimoto (1987), Launch and Observation Program of the Experimental Geodetic Satellite of Japan, *IEEE Transactions on Geoscience and Remote Sensing, GE-25, 5*, 526.

Scharroo, R., K. F. Wakker, B. A. C. Ambrosius, and R. Noomen (1991), On the along-track acceleration of the Lageos satellite, *J. Geophys. Res., 96*, 729.

Slabinski, V. J. (1988), Lageos acceleration due to intermittent solar heating during eclipse periods, *Proc. 19th Regular Meeting of Division on Dynamical Astronomy - American Astronomical Society*, Gaithersburg, MD, July 25-26.

Tapley, B. D., M. M. Watkins, J. C. Ries,G. W. Davis, R. J. Eanes, S. R. Poole, H. J. Rim, B. E. Schutz, C. K. Shum, R. S. Nerem, F. J. Lerch, E. Pavlis, S. M. Klosko, N. K. Pavlis, and R. G. Williamson (1994), The JGM-3 Gravity Model, *XIX Generaly Assembly of the European Geophysical Society*, Grenoble, France, April 25-29.