# Zadanie 6

October 15, 2023

Niech $X_{\text{fl}} \in 2^{32}$,
wezmy $x = y = 2^{30}$.
Wtedy $\sqrt{x^2+y^2} = \sqrt{(2^{30})^2+(2^{30})^2} = \sqrt{2^{60}+2^{60}} = 2^{30}\sqrt{2} \in X_{\text{fl}}$,
jednak $2^{60} \notin X_{\text{fl}}$.
Aby temu zapobiec przekształcmy wzor (dla $x \geq y$ w razie potrzeby zamieniamy zmienne $x$ i $y$) :
$\sqrt{x^2+y^2} = \sqrt{x^2\left(1+\frac{y^2}{x^2}\right)} = |x|\sqrt{1+\left(\frac{y}{x}\right)^2}$
Skoro $x \geq y$ to $\sqrt{1+\left(\frac{y}{x}\right)^2} \leq 2$,
wic $\sqrt{2}\cdot\max(|x|,|y|) \in X_{\text{fl}}$.
Dlugosc euklidesowa: $||x_n|| = \sqrt{x_1^2+x_2^2+x_3^2+\cdots+x_n^2}$,
Optymalizujemy w nastpujcy sposób (zakladajac $x_i \geq x_{i+1}$) :

$$||x_n|| = \sqrt{x_1^2+x_2^2+\cdots+x_n^2}$$
$$= \sqrt{x_1^2\left(1+\frac{x_2^2}{x_1^2}+\frac{x_3^2}{x_1^2}+\cdots+\frac{x_n^2}{x_1^2}\right)}$$
$$= |x_1|\sqrt{\left(1+\frac{x_2^2}{x_1^2}+\frac{x_3^2}{x_1^2}+\cdots+\frac{x_n^2}{x_1^2}\right)}$$
$$= |x_1|\sqrt{n}$$
$$= \max(x_1,x_2,\ldots,x_n)\sqrt{n}.$$