The time-dependent capacitated profitable tour problem with time windows and precedence constraints

1. Problem description

The time-dependent capacitated profitable tour problem with time windows and precedence constraints is defined as follows. We consider a set of n requests $R_1, ..., R$n , where $R_i(i = 1, ..., n)$ is associated with the pickup node i and the corresponding delivery node $n + i$. Let $G = (N, A)$ be a directed graph, where $N = \{0, 1, ..., 2n + 1\}$ is the set of all nodes, and 0 and 2 n + 1 represent the origin and destination depot of the vehicle. We define the subsets $N_P = \{1, ..., n\}$ and $N_D = \{n + 1, ..., 2n\}$ as the pickup and delivery nodes, respectively. With each pickup node $i \in N_P$ a profit r_i and a load q_i are associated, and with each delivery node a load q_{n+i} is associated. For the requests, it must hold that $q_i = q_{n+i}$. There is no inventory at the depots and therefore $q_0 = q_{2n+1} = 0$. To serve the requests we have one vehicle available with limited capacity Q . A hard time window $[e_j, l_j]$ is associated with each node $j \in N_P \cup N_D$, where e_j and l_j represent the earliest and latest time, respectively, at which the service at node j may start. The service time is denoted by s_j . A vehicle needs to wait until time e_j , if it is arriving at node j before time e_j ; and arriving later than l_j is not allowed. We denote $[e_0, l_0], [e_{2n+1}, l_{2n+1}]$ as the time windows of the origin and the destination depot, respectively. Without loss of generality, we assume that $e_0 = 0$ and $s_0 = s_{2n+1} = 0$.

In this problem we minimize the total duration of the selected tour instead of the sum of the arc cost. As the duration is a piecewise linear function of the departure time, it is clear that the minimum duration of a tour can be computed by only considering the breakpoints of the ready time function.

1.0.1. Time dependent travel time function

Pan et al. (2019) characterized the piecewise linear travel time function proposed in (Ichoua et al., 2003), and its main idea is briefly presented below. A typical workday ($[e_0, l_0]$) is divided into non-overlapping time zones T, where the m-th time zone is $[w^{m-1}, w^m]$. It is assumed that the travel speed is constant within a time zone and will change only at the end of each time zone.Let $v^m_{i,j}$ be the travel speed along (i, j) during the m-th time zone. The actual time-dependent travel time along (i, j) with departure time t_0 (denoted as $\bar{\tau}_{i,j}(t_0)$) depends on the speed profile of (i, j) and d_{ij}, which can be calculated using Algorithm 1. Let the time-dependent travel time along (i, j) with departure time t be denoted as function $\tau_{i,j}(t)$. As an example in Figure 1, this function can be fully determined using the break points ($w^0_{i,j}$ to $w^9_{i,j}$) of the unique arc time zones $T_{i,j}$ and their actual travel times (e.g, $\bar{\tau}_{i,j}(w^1_{i,j})$, $\bar{\tau}_{i,j}(w^2_{i,j})$, and etc). Again the arc time zones $T_{i,j}$ is unique to the arc and we denote the m-th arc time zone for (i, j) as $T^m_{i,j}$. For ease of calculation, the slope ($\theta^m_{i,j}$) and the intersection with y-axis ($\eta^m_{i,j}$) of the m-th line segment (representing the m-th arc time zone) can be pre-calculated and used to fully represent the piecewise linear travel time function $\tau_{i,j}(t)$ as below:

$$\tau_{i,j}(t) = \sum_{m \in T_{i,j}} \left(\theta^m_{i,j} t + \eta^m_{i,j}\right) \mathbf{1}_{T^m_{i,j}}(t), \ \forall t \in [e_0, l_0], \tag{1}$$

where the indicator function $\mathbf{1}_{T^m_{i,j}}(t)$ indicates whether t belongs to $T^m_{i,j}$.

Algorithm 1 Calculation of actual travel time $\bar{\tau}_{i,j}(t_0)$)

1: Determine the time zone of t_0 as M
2: $t \leftarrow t_0$, $d \leftarrow d_{ij}$, $t' \leftarrow t + (d/v^M_{ij})$,
3: **while** $t' \geq w^M$ **do**
4: $\quad d \leftarrow d - v^M_{ij} \times (w^M - t)$
5: $\quad t \leftarrow w^M$
6: $\quad t' \leftarrow t + (d/v^{M+1}_{ij})$
7: $\quad M \leftarrow M + 1$
8: **end while**
9: **return** $t' - t_0$

Similarly, the backward travel time function (denoted as $\tau^{-1}_{i,j}(t)$), defined as the travel time required for the vehicle to definitely arrive at node j at time t along arc (i, j), is also a piecewise linear function and can be determined in a similar way as $\tau_{i,j}(t)$.

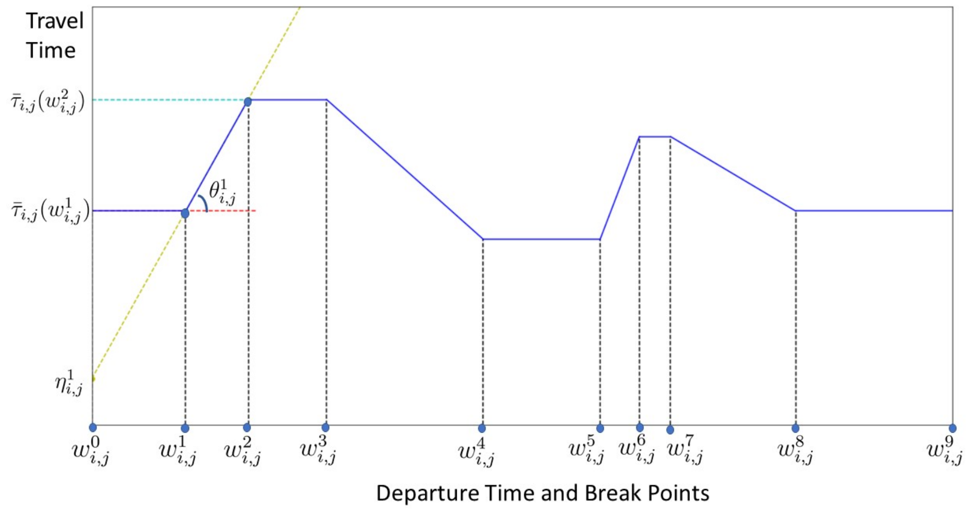

Figure 1: Example of a time-dependent travel time function.

2. TDVRPTW Test instances

The data can be retrieved from Sun et al. (2018).

References

Ichoua, S., Gendreau, M., & Potvin, J.-Y. (2003). Vehicle dispatching with time-dependent travel times. *European Journal of Operational Research*, *144*, 379–396.

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