Project 1:
Time-Dependent Vehicle Routing Problem with Time Windows

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1. Problem description

The TDVRPTW problem is defined over a directed graph $G = (V, A)$ with node set V and arc set A. Set V contains the depot 0 and the customer set $V_c = \{1, 2, ..., n\}$. Each node $i \in V$ is associated with a demand q_i and a service time s_i. Service at customer i must start within a given time window $[e_i, l_i]$. While vehicles are allowed to arrive earlier than e_i, they must wait until e_i to serve customer i, and late arrival is strictly prohibited. For the depot, e_0, s_0 and q_0 are assumed to be 0 without loss of generality, and the time windows $[e_0, l_0]$ corresponds to a typical workday. The arc set A is defined as $\{(i, j) : i, j \in V,\ i \neq j\}$ where each arc $(i, j) \in A$ is associated with an Euclidean distance $d_{i,j}$. A fleet of homogeneous vehicles K with capacity Q is available to serve the customers. Each vehicle must start from and return to the depot, and its total load cannot exceed its capacity at any node. The route's duration, defined as the arrival time at the returning depot minus the departure time at the starting depot, should not exceed the driver's maximum working hour (T_{max}) as regulated by the government and the company's policy. This is also referred to as the maximum route duration constraint.

Each vehicle has a fixed cost f if it is activated. The TDVRPTW problem aims to minimize the total cost required to serve all the customers subject to customers' time windows, service times, time-dependent travel times, vehicle capacities constraint and the maximum route duration constraint.

1.0.1. Time dependent travel time function

Pan et al. (2019) characterized the piecewise linear travel time function proposed in (Ichoua et al., 2003), and its main idea is briefly presented below. A typical workday ($[e_0, l_0]$) is divided into non-overlapping time zones T, where the m-th time zone is $[w^{m-1}, w^m]$. It is assumed that the travel speed is constant within a time zone and will change only at the end of each time zone.Let $v_{i,j}^m$ be the travel speed along (i, j) during the m-th time zone. The actual time-dependent travel time along (i, j) with departure time t_0 (denoted as $\bar{\tau}_{i,j}(t_0)$) depends on the speed profile of (i, j) and d_{ij}, which can be calculated using Algorithm 1. Let the time-dependent travel time along (i, j) with departure time t be denoted as function $\tau_{i,j}(t)$. As an example in Figure 1, this function can be fully determined using the break points ($w_{i,j}^0$ to $w_{i,j}^9$) of the unique arc time zones $T_{i,j}$ and their actual travel times (e.g, $\bar{\tau}_{i,j}(w_{i,j}^1)$, $\bar{\tau}_{i,j}(w_{i,j}^2)$, and etc). Again the arc time zones $T_{i,j}$ is unique to the arc and we denote the m-th arc time zone for (i, j) as $T_{i,j}^m$. For ease of calculation, the slope ($\theta_{i,j}^m$) and the intersection with y-axis ($\eta_{i,j}^m$) of the m-th line segment (representing the m-th arc time zone) can be pre-calculated and used to fully represent the piecewise linear travel time function $\tau_{i,j}(t)$ as below:

$$\tau_{i,j}(t) = \sum_{m \in T_{i,j}} \left(\theta_{i,j}^m t + \eta_{i,j}^m \right) \mathbf{1}_{T_{i,j}^m}(t), \ \forall t \in [e_0, l_0], \tag{1}$$

where the indicator function $\mathbf{1}_{T_{i,j}^m}(t)$ indicates whether t belongs to $T_{i,j}^m$.

Algorithm 1 Calculation of actual travel time $\bar{\tau}_{i,j}(t_0)$)

Determine the time zone of t_0 as M
$t \leftarrow t_0, d \leftarrow d_{ij}, t' \leftarrow t + (d/v_{ij}^M)$,
while $t' \geq w^M$ **do**
    $d \leftarrow d - v_{ij}^M \times (w^M - t)$
    $t \leftarrow w^M$
    $t' \leftarrow t + (d/v_{ij}^{M+1})$
    $M \leftarrow M + 1$
end while
return $t' - t_0$

Similarly, the backward travel time function (denoted as $\tau_{i,j}^{-1}(t)$), defined as the travel time required for the vehicle to definitely arrive at node j at time t along arc (i, j), is also a piecewise linear function and can be determined in a similar way as $\tau_{i,j}(t)$.

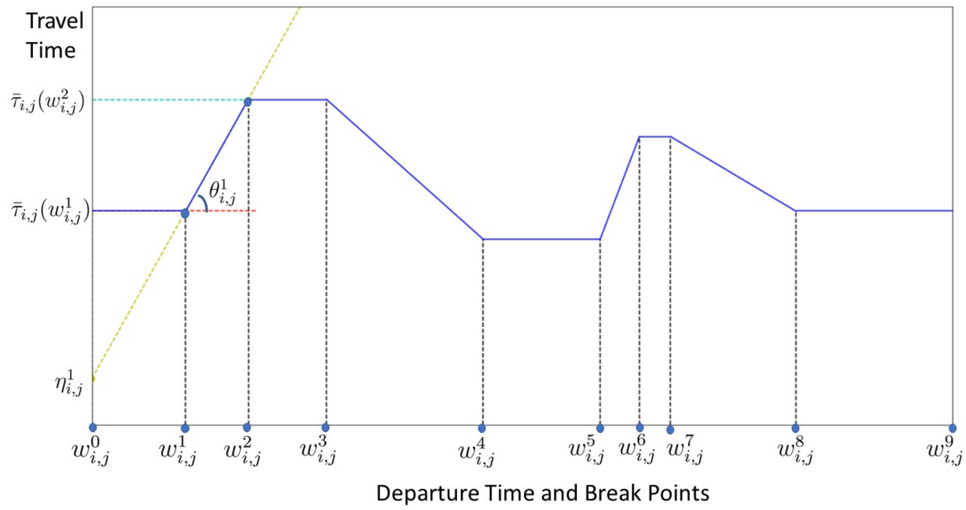

Figure 1: Example of a time-dependent travel time function.

2. TDVRPTW Test instances

This section describe the TDVRPTW test instances proposed by Pan et al. (2020) . The data set is based on the well-known Solomon's data sets with 25, 50 and 100 customers (labelled as T-25, T-50 and T-100 respectively). Solomon's data sets consist of three groups with different geographic distributions of customers: customers in the Group R instances are uniformly distributed; customers in the Group C instances are clustered together; and the customers in the Group RC instances are distributed with a mixed uniform and clustering strategy. Instances can be either of Type 1 with tight time windows or of Type 2 with wide time windows. In general, less customers can be served in a route for Type 1 instances due to narrower time windows, leading to shorter vehicle routes and a smaller solution space. To model time-dependent travel times, the workday is divided into five time zones to replicate the morning and evening peak periods from the real world. The division of the time zones are the same across all arcs as the peak and off-peak hours generally coincide for all roads. Three speed profiles are used to represent roads of different characteristics: fast links represent expressways, slow links represent central business districts and other city areas, and all other roads are considered as normal links. The assignments of arcs to the speed profiles follows the data set used in Ichoua et al. (2003). Detailed data for the time zone and the travel speed profiles for the base case is presented in Table 1 below.

As real world traffic conditions change rapidly throughout the day, it is meaningful to evaluate the performance of the algorithm on test instances with an increased number of time zones. Therefore, we derive another set of test instances (labelled as T-100-7) with 7 time zones and 100 customers from Dabia et al. (2013). The new test instances use the same speed profile (fast, normal and slow) and profile assignments to the arcs as the base case. The time zone and travel speed profiles for T-100-7 are presented in Table 2.

Table 1: Travel Speed Profile and Travel Speed for T-25, T-50 and T-100

	$Zone_1$	$Zone_2$	$Zone_3$	$Zone_4$	$Zone_5$
Period	$[0, 0.2l_0]$	$[0.2l_0, 0.3l_0]$	$[0.3l_0, 0.7l_0]$	$[0.7l_0, 0.8l_0]$	$[0.8l_0, l_0]$
Fast	1.5	1	1.67	1.17	1.33
Normal	1.17	0.67	1.33	0.83	1
Slow	1	0.33	0.67	0.5	0.83

Table 2: Travel Speed Profile and Travel Speed for T-100-7

	$Zone_1$	$Zone_2$	$Zone_3$	$Zone_4$	$Zone_5$	$Zone_6$	$Zone_7$
Period	$[0, 0.2l_0]$	$[0.2l_0, 0.3l_0]$	$[0.3l_0, 0.4l_0]$	$[0.4l_0, 0.6l_0]$	$[0.6l_0, 0.7l_0]$	$[0.7l_0, 0.8l_0]$	$[0.8l_0, l_0]$
Fast	1.5	1	1.33	1.67	1	1.17	1.33
Normal	1.17	0.67	0.83	1.33	0.83	1	1.17
Slow	1	0.33	0.5	0.67	0.5	0.67	0.83

All test instances are available online at http://www.computational-logistics.org/orlib/DM-TDVRPTW. Note that no T_max is set for the test instance and you need to determine the value of T_max to use for this project.

References

Dabia, S., Ropke, S., Van Woensel, T., & De Kok, T. (2013). Branch and price for the time-dependent vehicle routing problem with time windows. *Transportation Science*, *47*, 380–396.

Ichoua, S., Gendreau, M., & Potvin, J.-Y. (2003). Vehicle dispatching with time-dependent travel times. *European Journal of Operational Research*, *144*, 379–396.

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