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SIMULATING WAKE PATTERNS

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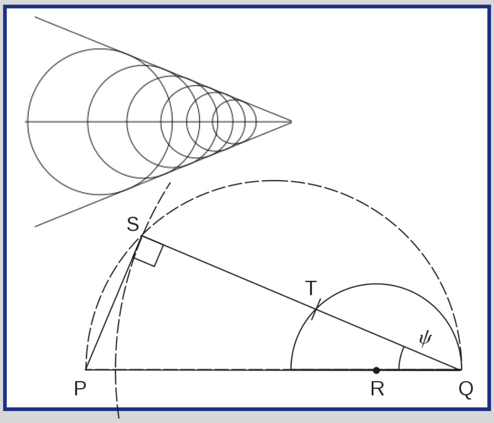
Introduction

We were struck by the fact that the wake of any slow moving object in water is completely independent of the body producing the wake. The shape of this wake was first described by Kelvin. We aimed to create numerical simulations of wake patterns in python, and compare them to Kelvin's predictions. We learned much about the physics of the wave production process by seeing which sets of assumptions in the code made in our code produced results that were reasonable.



The Kelvin Angle

A geometric Construction by Whitham (1974),seen above, shows that the angle shown above is exactly arcsin(1/3), independent of any properties of the body in the water. The only fact necessary for this deriavtion is that the phase velocity is exactly 1/2 of the goup velocity.

There's More!

SCAN ME

Kelvin's Theory

Water waves differ from sound waves because their speed is not the same for all wavelengths. Instead, it is governed by the dispersion relation:

$$\omega(k) = \sqrt{gk} \quad \Longrightarrow \quad v_{phase} = \frac{\omega}{k} = \sqrt{\frac{g\lambda}{2\pi}}$$

meaning longer waves travel faster.

As a body travels through deep water, it creates waves of many different wavelengths. Because the waves' phase velocity varies, each wavelength creates a slightly different interference pattern. When all these patterns combine, the familiar shape of a wake emerges.

Kelvin found that the angle of such wakes was independent of the vessel producing them or its speed. He observed that when the group velocity (of interference or 'beat' patterns) is exactly half the phase velocity (that of individual waves) - as is in deep water, the angle of a wake is constant at around 19.47°.

Our Python Simulation

- Initialize boat speed (v), generate range of wavelengths
- Create the result matrix (Z)
- Iterate through all wavelengths, and for each iterate through discrete timesteps (t)
- Produce a surface cosine wave, with starting point at (vt, 0) and add it to Z
- plot the points (x, y, Z[x][y])

(1)

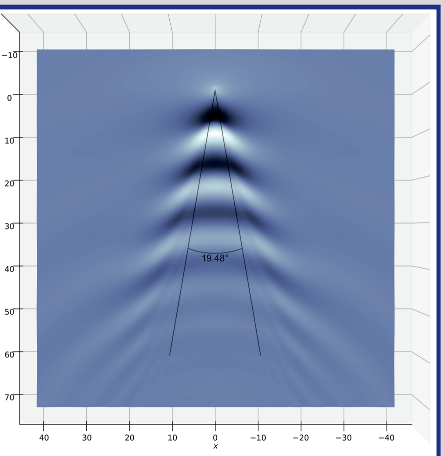


Figure 1: Simulated Kelvin wake pattern

(2)

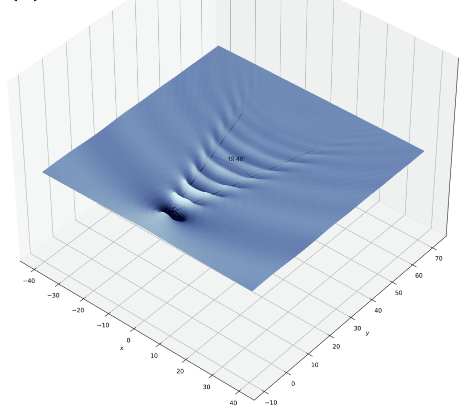


Figure 2: 3d view of the simulated Kelvin wake pattern

Results

Kelvin Regime: The predictions of Kelvin's theory are observed when the speed of the body in the water is less than the phase velocity of the fastest (i.e longest wavelength) wave produced by the body. In this case, the wake angle is independent of the boat speed and length, and observed at the predicted 19.48 degrees, as can be seen in figures [1] and [2]. Observing qualitatively, the wake is also disjoint. Moving along the outside of the wave, one encounters many peaks and throughs. This is a result of interference resulting from the difference in group velocity and phase velocity.

Mach Regime: In contrast, when the speed of the body is greater than the phase velocity of the fastest wave, the wake angle is dependent on both boat speed and wavelength. At constant velocity, wake angle increases with length, and decreases with boat speed. Moving along the edge of the wake the amplitude is constant, as can be seen in figure [3].

(3)

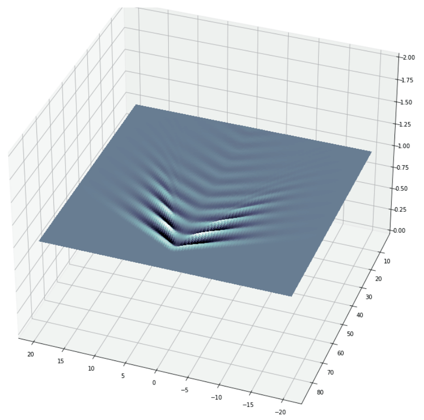


Figure 4: A 3D view of the mach angle

(3)

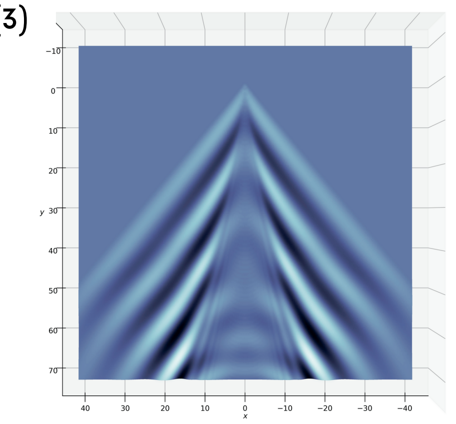


Figure 3: At large speed to hull length ratios, the mach angle becomes visible

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