Geometric Cognition and the Orthocognitum

A Unifying Field Theory for Polytopal Projection Processing, Geoepistatic Localization, and the Chronomorphic Polytopal Engine

Author: Paul Phillips, Ph.D.

Date: January 8, 2026

Abstract

The current epoch of artificial intelligence, dominated by the statistical triumphs of connectionist deep learning, faces an epistemic horizon. While Large Language Models (LLMs) and generative transformers have mastered the combinatorial syntax of human language, they remain fundamentally severed from the semantic ground of reality. They process tokens, not truths; they approximate distributions, not logical necessities. To transcend the "Second Wave" of AI and achieve the contextual adaptation, energy efficiency, and rigorous explainability demanded by the "Third Wave," we must execute a fundamental ontological shift: the **Geometric Turn**.

This white paper formally introduces the framework of **Geometric Cognition**, operationalized through the computational paradigm of **Polytopal Projection Processing (PPP)**. We posit that the fundamental unit of meaning is not the discrete symbol, nor the scalar weight, but the geometric location within a high-dimensional, topologically bounded manifold we designate as the **Orthocognitum**. Within this rigorous geometric structure, reasoning is re-conceptualized not as symbolic manipulation, but as a continuous trajectory—a **Chronomorphic** deformation—of state vectors through convex polytopes.

We introduce and define three novel formalisms essential to this paradigm: (1) **Geoepistatic Localization**, describing the non-linear, context-dependent masking and expression of semantic features based on their geometric coordinates; (2) **Epistaorthognition**, the recursive, algorithmic verification of truth-states against the topological invariants of the Orthocognitum; and (3) the **Chronomorphic Polytopal Engine (CPE)**, a computational substrate that replaces the linear Turing tape with the deformable geometry of 4-dimensional polychora. Drawing upon the neuroscience of entorhinal grid cells, the algebraic rigor of Hyperdimensional Computing (HDC), and the emerging physics of neuromorphic photonics, this paper provides an exhaustive technical and philosophical dissection of this new cognitive architecture.

1. Introduction: The Crisis of Syntax and the Geometric Turn

1.1 The Epistemic Ceiling of the Second Wave

The history of artificial intelligence can be read as a dialectic between two opposing philosophical poles: the rationalist, symbolic tradition ("Good Old-Fashioned AI" or GOFAI) and the empiricist, connectionist tradition (Neural Networks). The "First Wave" of AI excelled at explicit reasoning and logical guarantees but shattered against the brittleness of the real world. The "Second Wave," characterized by the Deep Learning revolution, solved the perception problem through massive statistical approximation. It gave us systems that can see, hear, and generate text with uncanny fluency.[1]

However, we are now witnessing the asymptotic limits of this statistical approach. Current state-of-the-art models, despite scaling to trillions of parameters, suffer from fundamental defects:

1. **Lack of Grounding:** They operate in a self-referential loop of signifiers, detached from the physical or logical constraints of the world. They hallucinate because they have no internal geometry of "truth," only a probability distribution of "likelihood."
2. **Opacity:** The decision boundary of a deep neural network is a jagged, unintelligible frontier in a millions-dimensional space. We cannot audit the reasoning of a "black box," rendering them unsuitable for high-stakes environments like defense or autonomous navigation where explainability is a safety constraint.[1]
3. **Inefficiency:** The energy cost of training and inference is approaching sustainable limits, driven by the Von Neumann bottleneck and the sheer volume of floating-point operations required to simulate simple cognitive acts.

We stand at the precipice of a "Third Wave," one that demands systems capable of **Geometric Cognition**. This is not merely a new algorithm but a new metaphysics of computation. It posits that "meaning" has a shape, "logic" has a direction, and "truth" is a bounded region in a high-dimensional space.[1]

1.2 The Geometric Hypothesis

The central thesis of this analysis is that symbolic reasoning is fundamentally a geometric operation. The brain does not process symbols as discrete tokens in a lookup table; it navigates them as locations in a conceptual space. This "Geometric Hypothesis" is grounded in the convergence of three disparate fields:

- **Neuroscience:** The discovery of grid cells in the entorhinal cortex suggests the brain uses a hexagonal coordinate system to navigate not just physical space, but abstract conceptual spaces.[2]

- **Mathematics:** The theory of **Conceptual Spaces**, pioneered by Peter Gärdenfors, provides the topological rigor for defining concepts as convex regions.[4]
- **Physics:** The emergence of **Neuromorphic Photonics** allows us to perform high-dimensional geometric transformations (Fourier transforms, matrix rotations) at the speed of light, bypassing the energy penalties of silicon logic.[1]

By synthesizing these fields, we arrive at **Polytopal Projection Processing (PPP)**: a framework where the state of a system is encoded as a dynamic 4-dimensional polytope, and computation is the rotation and projection of this shape.

1.3 Nomenclature of the New Paradigm

To rigorously discuss this framework, we must introduce a precise vocabulary that moves beyond the borrowed terms of neuroscience ("neurons," "synapses") or computer science ("tokens," "layers").

- **The Orthocognitum:** The global manifold of all valid, consistent conceptual states. It is the "world model" made geometric.
- **Geoepistatic Localization:** The mechanism by which the geometric context (location) determines the semantic expression (epistasis) of information.
- **Epistaorthognition:** The process of verifying validity; the system's ability to "know" if its current state lies within the bounds of the Orthocognitum.
- **Chronomorphic Polytopal Engine (CPE):** The hardware/software architecture that executes these dynamics, treating time as a geometric deformation of the state space.

The following sections will deconstruct each of these concepts, moving from theoretical ontology to engineering specification.

---

2. The Orthocognitum: Ontology of the Geometric Mind

2.1 Defining the Orthocognitum

The term **Orthocognitum** is a portmanteau derived from the Greek *orthos* (straight, correct, right) and the Latin *cognitum* (that which is known or conceptualized). It designates the totality of the valid, consistent semantic space available to a cognitive agent. Unlike the "latent space" of a traditional autoencoder—which is often a chaotic, sparsely populated manifold where interpolation can lead to nonsensical outputs—the Orthocognitum is a strictly structured, tessellated geometric object.

The Orthocognitum is the "Shape of the Known." It is defined by the union of all **Concept**

Polytopes that the system possesses. Following Gärdenfors' theory of Conceptual Spaces, we accept the **Convexity Constraint** as a fundamental law of this geometry.[1]

- **Axiom of Convexity:** A natural concept C is represented as a convex region within a domain. For any two points $x, y \in C$, and any scalar $\lambda \in $, the point $\lambda x + (1 - \lambda)y$ must also belong to C.

This axiom is profound. It implies that "meaning" is continuous and stable. If "Red" is a concept, and we have two shades of red, every color on the line connecting them is also "Red." The Orthocognitum, therefore, is not a cloud of points but a crystal of regions.

2.2 The Topology of Validity

The primary function of the Orthocognitum is to distinguish **Sense** from **Nonsense**. In a linguistic model, "The colorless green ideas sleep furiously" is syntactically valid but semantically void. In a probabilistic model, this sentence is assigned a non-zero probability. In the Orthocognitum, this sentence corresponds to a vector coordinate that falls into the "void" between valid polytopes—or, more accurately, tries to occupy a coordinate that violates the topological constraints of the manifold.

The Orthocognitum acts as a high-dimensional filter. It is the geometric realization of the system's "Constitution" or "Worldview."

- **The Manifold Hypothesis:** Deep learning theory relies on the Manifold Hypothesis—that real-world data lies on low-dimensional manifolds embedded in high-dimensional space.[6]
- **The Polytopal Refinement:** We refine this hypothesis. The data does not just lie on a manifold; it lies within specific **Voronoi Tessellations** on that manifold, where each cell is a convex polytope representing a distinct category or concept.[1]

Any trajectory (reasoning chain) that exits the convex hull of the Orthocognitum is, by definition, a fallacy, a hallucination, or an error. This allows for **deterministic validity checking**: is the vector inside the shape, or outside?

2.3 Grid Cells and the Metric of Meaning

How does one navigate the Orthocognitum? One needs a coordinate system. Biology provides the answer in the form of Grid Cells.
Discovered in the medial entorhinal cortex (MEC), grid cells fire at regular, hexagonal intervals, creating a tessellated map of physical space. Crucially, studies by Constantinescu et al. (2016) and Behrens et al. (2018) have demonstrated that this same mechanism is recruited to map abstract conceptual spaces.2 When humans reason about the relationships between abstract concepts (e.g., the "bird space" defined by the ratio of neck length to leg length), their brains exhibit the 6-fold symmetry characteristic of grid cell firing.
In the Orthocognitum, the **Grid Code** serves as the universal metric. It provides the **basis vectors** for the space.

- **Path Integration in Semantic Space:** Just as a rat can integrate its velocity vector to update its position in a maze, a cognitive agent can integrate a "logic vector" to update its position in the Orthocognitum.[1]
- **Vector Algebra of Thought:** If "King" is a location V_K, "Man" is a location V_M, and "Woman" is a location V_W, the reasoning "King - Man + Woman = Queen" is fundamentally a path integration task: starting at V_K, moving purely negative along the "Gender" axis, and arriving at V_Q.

The Orthocognitum is thus a navigable space. To "think" is to move. To "understand" is to know where you are relative to the boundaries of the valid polytopes.

---

3. Geoepistatic Localization: The Interaction of Context and Meaning

3.1 Defining Geoepistasis

Standard vector space models (like Word2Vec or GloVe) assume that semantic dimensions are linear and independent. A vector for "Bank" is the sum of its contexts. This leads to the problem of **polysemanticity**, where a single vector (or neuron) encodes multiple, unrelated meanings (e.g., "financial bank" vs. "river bank").[9]

To resolve this, we introduce the concept of **Geoepistatic Localization**.

- **Etymology:** Derived from *geometry* (earth/space measurement) and *epistasis* (a genetic term where the effect of one gene is dependent on the presence of 'modifier genes').[11]
- **Definition:** Geoepistatic Localization is the principle that the semantic expression of a feature vector is non-linearly dependent on its geometric localization within the Orthocognitum.

In genetics, a gene for "baldness" is epistatic to a gene for "hair color"—if you are bald, the hair color gene is silenced; it has no phenotypic expression. Similarly, in the Orthocognitum, the "context polytope" is epistatic to the "feature vector."

3.2 The Algebra of Contextual Masking

If an agent is localized within the "River Polytope" of the Orthocognitum, the semantic dimension for "Interest Rate" (associated with the financial "Bank") is geometrically masked—it is orthogonalized out of existence. The feature simply cannot be expressed in that region of the manifold.

Mathematically, we formalize this using Geometric Algebra (Clifford Algebra), specifically the Geometric Product.1

Let v be a feature vector (e.g., the token "Bank").
Let B be a bivector representing the current contextual plane (the "River" context).
The interaction is not additive ($v + B$) but geometric (vB):

$$vB = v \cdot B + v \wedge B$$

- **The Inner Product ($v \cdot B$):** Represents the **Projection**. It measures how much of the concept v aligns with the context B. If v is "Interest Rate" and B is "River," the inner product approaches zero. The meaning is suppressed.
- **The Outer Product ($v \wedge B$):** Represents the **Construction**. It creates a new, higher-dimensional object (a trivector) representing the emergent meaning of "Bank in the context of River."

This formalism allows the Orthocognitum to support **superposition without interference**. Multiple meanings can coexist in the high-dimensional space because they are separated geoepistatically. They are only "read out" when the system's trajectory enters the appropriate localization.

3.3 Polysemanticity as Geometric Packing

Deep learning researchers have observed that neural networks often exhibit "superposition," packing more features into the embedding space than there are dimensions.10 This is often viewed as a "bug" or a nuisance for interpretability.
From the perspective of Geoepistatic Localization, this is a feature. The system is utilizing the properties of high-dimensional geometry—specifically the fact that in $D=10,000$ space, there are millions of nearly orthogonal directions.1
The **Polytope Lens** framework suggests that neurons align with the vertices of high-dimensional polytopes (like the Simplex or Cross-Polytope) to maximize this packing efficiency.[15] Geoepistatic Localization posits that the network activates specific polytopes to "unlock" specific subsets of these packed features, using the geometry itself as a dynamic addressing system.

---

4. Epistaorthognition: The Logic of Validity

4.1 The Problem of Hallucination

One of the most pervasive failures of current LLMs is "hallucination"—the confident generation of false information. This occurs because LLMs are probabilistic engines; they predict the next most likely token based on statistical correlations, not logical verification. They have no internal mechanism to check if a statement is "true," only if it is "plausible."

We propose a new cognitive module to address this: **Epistaorthognition**.

- **Etymology:** *Episteme* (knowledge) + *Ortho* (correct/straight) + *Gnition* (act of knowing). Literally: "The act of knowing the correctness of one's knowledge."
- **Definition:** Epistaorthognition is the recursive, algorithmic process by which the cognitive system verifies that its current state vector and projected trajectory lie within the valid boundaries of the Orthocognitum.

4.2 The Mechanism of Verification

Epistaorthognition is not a post-hoc fact-check; it is an intrinsic geometric constraint applied at every step of the reasoning process (the Chronomorphic evolution). It functions as a **Topological Immune System**.

Step 1: The Euler Characteristic Check
The system represents its knowledge state as a polytope. For any convex 4-polytope (polychoron), the Euler Characteristic (χ) must satisfy the Poincaré-Hopf relationship:

$$N_0 - N_1 + N_2 - N_3 = 0$$

Where N_0 is vertices, N_1 is edges, N_2 is faces, and N_3 is cells.1
If a reasoning step introduces a contradiction, it structurally "tears" the polytope. The number of faces or edges will no longer balance. The Epistaorthognition module calculates χ instantly. If $\chi \neq 0$, the state is flagged as invalid (a geometric error), and the system halts or backtracks. This provides a rigorous, mathematical check on internal consistency that requires no semantic understanding of the content, only its topology.
Step 2: The Safety Polytope Projection
We can define a "Constitution" or "Safety Set" as a massive, bounding polytope P_{safe} within the Orthocognitum. This encodes inviolable rules (e.g., "Do not harm humans," "Do not violate laws of physics").
As the state vector $\Psi(t)$ evolves, Epistaorthognition continuously computes the distance function $d(\Psi, P_{safe})$.
If Ψ crosses the boundary of P_{safe}, the module applies a Projection Operator:

$$\Psi_{corrected} = \text{argmin}_{x \in P_{safe}} ||x - \Psi||$$

This "clamps" the thought back to the nearest valid surface of the Safety Polytope.1 The system is mathematically incapable of "thinking" an illegal thought, because the geometry of the Orthocognitum does not permit the vector to exist there.

4.3 Deterministic Explainability

Epistaorthognition creates a **Geometric Audit Trail**.[1] Because every valid step must adhere to these topological constraints, we can trace the reasoning trajectory. If the system makes a decision, we can visualize the path through the Orthocognitum. We can see exactly which "facet" of the concept polytope was projected to reach the conclusion. This replaces the "black box" with a transparent geometric solid.

---

5. The Chronomorphic Polytopal Engine (CPE): Architecture of the Fourth Dimension

5.1 Chronomorphism: Time as Geometry

To implement Geometric Cognition, we require a computational substrate that does not treat time as a linear sequence of clock cycles ($t_0, t_1, t_2...$), but as a geometric dimension. We call this architecture the **Chronomorphic Polytopal Engine (CPE)**.[1]

Chronomorphism (Time-Shape) posits that the evolution of a system is a continuous geometric deformation. The past, present, and future are not discrete memory addresses; they are encoded in the rotational orientation of the state polytope.

- **The Garden of Forking Paths:** How do we model the future? In a linear system, we predict a single outcome. In the CPE, we model the future as a **Polytope of Possibility**.[1]
- When the system faces a decision (a fork), it does not choose one path. It undergoes a dimensional expansion (an orthogonal rotation) that splits the trajectory into a superposition of multiple valid futures. The volume of the polytope expands to encompass the uncertainty.
- Thinking about the future is equivalent to rotating the polytope to view its "future-facing" facets.

5.2 The Polychoron Alphabet

The CPE utilizes the six convex regular 4-polytopes (polychora) as its fundamental data structures. Just as a 3D engine uses triangles, the CPE uses polychora.[1] Each has a specific cognitive function:

Polychoron	Faces/Cells	Cognitive Function in CPE
5-Cell (Simplex)	5 Tetrahedra	**Axiomatic Clustering:** Used for tightly coupled, fundamental truths where every element relates to every other. The minimal representation of a concept.

8-Cell (Tesseract)	8 Cubes	**Orthogonal Logic:** The geometry of binary choices and independent variables. Used for decision trees and "bit-like" logic in high dimensions.
16-Cell (Cross)	16 Tetrahedra	**Sparse Representation:** The dual of the Tesseract. Used for "Axis-based" forces and mutually exclusive categories (like the axes of a coordinate system).
24-Cell	24 Octahedra	**The Bridge:** A unique, self-dual object with no 3D analogue. Used to translate between dual spaces (e.g., converting a state vector into a rule vector).
120-Cell	120 Dodecahedra	**Granular State Space:** Densely connected. Used for modeling complex, chaotic systems with fine-grained transitions (e.g., fluid dynamics, social intuition).
600-Cell	600 Tetrahedra	**Continuous Approximation:** The dual of the 120-Cell. Its high density of vertices allows it to approximate continuous manifolds (curves) with high precision.

5.3 Dynamics: Double and Isoclinic Rotations

The "CPU cycles" of the CPE are 4D Rotations. In 3D, rotation occurs around an axis (line). In

4D, rotation occurs around a plane. This leaves two orthogonal dimensions free, allowing for a second, simultaneous rotation. This is the Double Rotation 1:

$$R(\alpha, \beta)$$

- **Plane A (α):** Rotates the "Physical State" (e.g., the drone's position).
- **Plane B (β):** Rotates the "Epistemic State" (e.g., the drone's confidence or goal).

This allows the CPE to process "Action" and "Thought" simultaneously and coherently in a single geometric operation.
A special case is the Isoclinic Rotation ($\alpha = \pm \beta$), where the rotation is symmetric across all dimensions. This acts as a "Global Synchronization" event—a "heave" of the entire Orthocognitum that realigns all concepts to a new context.1

---

6. Polytopal Projection Processing (PPP): The Visual Interface

6.1 Shadow Projections: The Language of Machines

While the CPE operates in 4D (or higher), we need to interface with it using standard tools. This is the role of **Polytopal Projection Processing (PPP)**. PPP treats the high-dimensional polytope as a "real" object and uses a projection operator to cast a "shadow" of it into lower dimensions (2D or 3D).[1]

This **Shadow Projection** is a dense, information-rich visual artifact.

- **Visual Encoding:** Data is not encoded in pixels, but in the *relations* between pixels—edge lengths, angles, Moire interference patterns, and textures.
- **Machine Vision Integration:** These shadows are optimized for **Vision Transformers (ViTs)**. We train standard computer vision models to "read" the geometry of the shadow. The ViT does not need to understand 4D math; it just needs to recognize that "Pattern A" (a specific shadow of a rotating 600-cell) implies "System Stability," while "Pattern B" (a distorted shadow) implies "Sensor Drift."

This allows us to leverage the massive, pre-trained power of current vision AI (the Second Wave) to interpret the precise reasoning of the CPE (the Third Wave).[1]

6.2 Hyperdimensional Computing (HDC) Algebra

Underpinning PPP is the algebra of Hyperdimensional Computing (HDC), or Vector Symbolic Architectures (VSA).1 HDC operates on vectors of $D=10,000+$.
The core operations map directly to geometric transformations in the CPE:

1. **Binding ($A \otimes B$):** This is an **Orthogonal Transformation**. It rotates the vectors

A and B into a new subspace orthogonal to both. In the CPE, this creates a new dimension of meaning (e.g., Binding "Color" and "Red" creates the specific instance "Red Color").[1]

2. **Bundling ($A + B$):** This computes the geometric **Centroid**. It finds the concept that lies "between" A and B.
3. **Permutation (Π):** A rotation that encodes sequence.

By mapping VSA operations to polytopal rotations, PPP ensures that symbolic logic (binding variables) is preserved within the continuous geometry.[17]

---

7. Neuromorphic and Photonic Implementation: The Speed of Light

7.1 The Computational Bottleneck

Simulating 4D rotations of 10,000-dimensional vectors on standard GPUs is inefficient. It involves massive Matrix-Vector Multiplications (MVM) that choke the Von Neumann bottleneck (moving data between RAM and CPU).
However, these operations are the native language of Photonics (Light).1

7.2 The Photonic Fabric

We propose implementing the CPE on **Neuromorphic Photonic** hardware, leveraging technologies from companies like **Celestial AI**, **Lightmatter**, and **LightOn**.[1]

- **Passive Rotation:** A mesh of Mach-Zehnder Interferometers (MZIs) can perform a unitary matrix rotation (the core of the CPE) on a beam of light passively. The light simply flows through the mesh, and the rotation happens at the speed of light with near-zero energy consumption.[1]
- **Optical Fourier Transforms:** A simple lens performs a Fourier transform instantly. This allows the CPE to operate in the frequency domain (Holographic Reduced Representations), where binding is a simple element-wise multiplication.[1]
- **The Memory Fabric:** Celestial AI's "Photonic Fabric" optically interconnects memory modules, creating a unified address space. This allows the entire Orthocognitum (terabytes of concept polytopes) to be stored and "rotated" against query vectors without the latency of data transfer.[1]

In this hardware, the CPE is not a simulation; it is a physical reality. The "thought" is a beam of light traversing a crystal of interferometers, refracting through the geometry of the Orthocognitum.

---

8. Strategic Applications: Calibration and Beyond

8.1 Case Study: GPS-Denied Navigation and Drift

A critical problem in defense is navigating without GPS. Inertial Measurement Units (IMUs) drift over time; errors accumulate quadratically.
The PPP Solution:

1. **Map Sensors to Rotation:** We map the IMU data (3 gyro axes, 3 accel axes) directly to the 6 rotation planes of a 4D polytope in the CPE.[1]
2. **Drift as Deformation:** As the sensor drifts, the 4D polytope "shears" away from the ideal, mathematically perfect shape defined in the Orthocognitum.
3. **Visual Calibration:** The PPP system projects the shadow of this drifting polytope. A ViT, trained on the visual signature of "perfect" vs. "drifted" shadows, detects the distortion.
4. **Concordance Correlation Coefficient (CCC):** We use a CCC loss function to train the ViT. CCC is sensitive to scaling and shifting errors—exactly the nature of drift. The ViT outputs a correction rotor that snaps the polytope back to the Orthocognitum.[1]

This converts a temporal calculus problem (integration error) into a geometric pattern matching problem, which AI excels at.

8.2 The Geometric Laboratory: Soap Films and Singularities

The CPE's ability to model 4D geometry allows it to serve as a laboratory for pure mathematics. Recent breakthroughs in the Plateau Problem (area-minimizing surfaces, like soap films) have shown that singularities can form in high dimensions.1
The CPE, specifically using the 24-Cell and Quaternion Spinor Bridge, can model these evolving surfaces. By training the PPP system to watch the "shadows" of these evolving equations, we can detect the geometric precursors to a singularity—the "tremors" in the polytope—before the singularity actually forms. This effectively creates an "AI Mathematician" capable of intuition about high-dimensional topology.1

---

9. Conclusion: Charting the Orthocognitum

The transition from Artificial Intelligence to **Geometric Cognition** is a transition from the map to the territory. By abandoning the sterile linearity of the Turing tape for the rich, high-dimensional topology of the **Orthocognitum**, we open a path to systems that do not just process data, but embody meaning.

The **Chronomorphic Polytopal Engine** provides the architecture. **Polytopal Projection Processing** provides the interface. **Geoepistatic Localization** provides the context. And **Epistaorthognition** provides the guarantee of truth.

This is a convergence of the biological (grid cells), the mathematical (polytopes), and the physical (photonics). It suggests that the "mind" is not a ghost in the machine, but a geometry in the light. We are no longer limited to predicting the next token. We are ready to navigate the shape of thought.

Table 1: Comparative Analysis of AI Paradigms

Feature	**Second Wave (Deep Learning)**	**Third Wave (Geometric Cognition / PPP)**
Data Structure	Tensors (Arrays of Scalars)	Polytopes (Vertices, Faces, Cells)
Latent Space	Unstructured, Non-Convex	**The Orthocognitum** (Tessellated, Convex)
Reasoning	Probabilistic Token Prediction	**Chronomorphic** Trajectory (Rotation)
Context	Attention Mechanism (Weights)	**Geoepistatic Localization** (Geometric Masking)
Validation	None (Hallucination Risk)	**Epistaorthognition** (Euler Characteristic Check)
Hardware	GPUs (Von Neumann)	**Photonic Fabric** (Neuromorphic/Optical)
Interpretability	Black Box (Opacity)	**Shadow Projections** (Geometric Transparency)

Table 2: The Polychoron Alphabet of the CPE

Polychoron	**Vertices**	**Cells**	**Ideal Application Domain**
5-Cell	5	5	Axiomatic logic, minimal constraint

			satisfaction.
8-Cell (Tesseract)	16	8	Binary logic, orthogonal decision trees, hypercubes.
16-Cell	8	16	Sparse coding, axis-aligned forces, resource allocation.
24-Cell	24	24	Quaternion operations, dual-space bridging, signal processing.
120-Cell	600	120	Complex system modeling, dense state spaces, fluid dynamics.
600-Cell	120	600	Continuous function approximation, high-resolution manifolds.

This white paper integrates research from the following sources:

- [1] *Geometric HDC Research Strategy*
- [1] *Polytopal Projection Processing (PPP)*
- [1] *Applying Polytopal Projection Processing*
- [1] *Chronomorphic Polytopal Engine Expansion*
- [2] *Neuroscience of Grid Cells (Constantinescu et al.)*
- [4] *Conceptual Spaces (Gärdenfors)*
- [10] *Hyperdimensional Computing (Kanerva)*
- [20] *Geometric Deep Learning (Bronstein)*

Works cited

1. Geometric HDC Research Strategy.txt
2. Organizing Conceptual Knowledge in Humans with a Grid-like Code - ResearchGate, accessed January 8, 2026, https://www.researchgate.net/publication/304031376_Organizing_Conceptual_Knowledge_in_Humans_with_a_Grid-like_Code
3. The GridCAT: A Toolbox for Automated Analysis of Human Grid Cell Codes in fMRI, accessed January 8, 2026, https://www.frontiersin.org/journals/neuroinformatics/articles/10.3389/fninf.2017.00047/full
4. Criteria for naturalness in conceptual spaces, accessed January 8, 2026, https://d-nb.info/1259104834/34
5. Convexity Is an Empirical Law in the Theory of Conceptual Spaces: Reply to Hernández-Conde | Request PDF - ResearchGate, accessed January 8, 2026, https://www.researchgate.net/publication/334019250_Convexity_Is_an_Empirical_Law_in_the_Theory_of_Conceptual_Spaces_Reply_to_Hernandez-Conde
6. DeepAtlas: a tool for effective manifold learning - bioRxiv, accessed January 8, 2026, https://www.biorxiv.org/content/10.1101/2025.08.26.672474v1.full.pdf
7. Hardness of Learning Neural Networks under the Manifold Hypothesis - NIPS papers, accessed January 8, 2026, https://proceedings.neurips.cc/paper_files/paper/2024/file/0ad6ebd11593822b8a6d5873ca9c5b0b-Paper-Conference.pdf
8. Map-Like Representations of an Abstract Conceptual Space in the Human Brain - NIH, accessed January 8, 2026, https://pmc.ncbi.nlm.nih.gov/articles/PMC7884611/
9. Interpreting Neural Networks through the Polytope Lens - LessWrong, accessed January 8, 2026, https://www.lesswrong.com/posts/eDicGjD9yte6FLSie/interpreting-neural-networks-through-the-polytope-lens
10. A Survey on Hyperdimensional Computing aka Vector Symbolic Architectures, Part I: Models and Data Transformations - arXiv, accessed January 8, 2026, https://arxiv.org/pdf/2111.06077
11. Epistasis - Wikipedia, accessed January 8, 2026, https://en.wikipedia.org/wiki/Epistasis
12. Video: Epistasis | Definition, Types & Examples - Study.com, accessed January 8, 2026, https://study.com/learn/lesson/video/epistasis-overview-examples.html
13. [PDF] On Geometric Algebra representation of Binary Spatter Codes | Semantic Scholar, accessed January 8, 2026, https://www.semanticscholar.org/paper/On-Geometric-Algebra-representation-of-Binary-Codes-Aerts-Czachor/fa34309cba9521557b4333f4547ca40afa9e3e8d
14. Hyperdimensional Computing: - Redwood Center for Theoretical Neuroscience, accessed January 8, 2026, https://redwood.berkeley.edu/wp-content/uploads/2018/01/kanerva2009hyperdimensional.pdf

15. [2211.12312] Interpreting Neural Networks through the Polytope Lens - arXiv, accessed January 8, 2026, https://arxiv.org/abs/2211.12312
16. Defining Neural Network Architecture through Polytope Structures of Dataset - arXiv, accessed January 8, 2026, https://arxiv.org/html/2402.02407v2
17. Learning Vector Symbolic Architectures | Research | Automation Technology - TU Chemnitz, accessed January 8, 2026, https://www.tu-chemnitz.de/etit/proaut/en/research/vsa.html
18. Conceptual Spaces: The Geometry of Thought | Books Gateway - MIT Press Direct, accessed January 8, 2026, https://direct.mit.edu/books/monograph/2532/Conceptual-SpacesThe-Geometry-of-Thought
19. Geometric Deep Learning: Grids, Groups, Graphs, Geodesics, and Gauges - arXiv, accessed January 8, 2026, https://arxiv.org/abs/2104.13478
20. Geometric Deep Learning - ICT Vision & Graphics Lab, accessed January 8, 2026, https://vgl.ict.usc.edu/Research/GeometricDeepLearning/GEOMETRIC%20DEEP%20LEARNING.pdf
21. Michael Bronstein - Google Scholar, accessed January 8, 2026, https://scholar.google.fr/citations?user=UU3N6-UAAAAJ&hl=fr