

## **Applications of Conceptual Spaces: The Case for Geometric Knowledge Representation**

Frank Zenker and Peter Gärdenfors (Eds.)

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**Foreword**

All chapters of this volume arise from the international conference *Conceptual Spaces at Work*, held 24-26 May 2012 at Lund University, Sweden. We gratefully acknowledge the generous sponsorship of *The Swedish Research Council* and *The Royal Swedish Academy of Letters, History and Antiquities*.

As organizers, we regret that some contributions to this meeting do not appear in the present volume. As editors, we are indebted to a number of anonymous reviewers as well as to Christi Lue and Joos Walbeek at Springer's Dordrecht office for their assistance in the publication process.

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F.Z. and P.G.

## **Editors' introduction: Conceptual spaces at work**

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**Abstract:** This introductory chapter provides a non-technical presentation of conceptual spaces as a representational framework for modeling different kinds of similarity relations in various cognitive domains. Moreover, we briefly summarize each chapter.

**Keywords:** distance measure, human conceptualization, representation, similarity, prototype

## **1. Conceptual Spaces**

### *1.1 Three kinds of cognitive representations*

Humans are extremely efficient at learning new concepts. After having been presented with only a couple of examples, we are able to abstract the general content of a new concept. A central problem for cognitive science is how this learning process and the underlying representations should be modeled. There have been two dominating approaches to these problems. The symbolic approach starts from the assumption that cognitive systems can be described as Turing machines. On this view, cognition is seen as essentially being *computation* involving symbol manipulation. The second approach is associationism, where associations between different kinds of information elements carry the main burden of representation. Connectionism is a special case of associationism that models associations using artificial neuron networks.

There are aspects of cognitive phenomena, however, for which neither symbolic representation nor associationism seems to offer appropriate modeling

tools. In particular, mechanisms of concept learning cannot be given a satisfactory treatment in any of these representational forms. Concept learning is closely tied to the notion of *similarity*, which also has turned out to be problematic to model in the symbolic and associationist approaches.

A third form of representing information that employs *geometric* structures rather than symbols or associations had been presented in the book *Conceptual Spaces: The Geometry of Thought* (Gärdenfors, 2000). Information is represented by points, vectors and regions in dimensional spaces. On the basis of these structures, similarity relations can be modelled in a natural way in terms of distances in a space.

The geometric approach to knowledge representation having received more attention over the last fifteen years, this book aims at presenting some of its areas of application and development.

### *1.2 Conceptual spaces as a representational framework*

A conceptual space consists of a number of *quality dimensions*. Examples of such dimensions are: color, pitch, temperature, weight, and the three ordinary spatial dimensions. These dimensions are closely connected to what is produced by our sensory receptors (Schiffman, 1982). However, there are also quality dimensions of an abstract, non-sensory character. In Gärdenfors (2007), for instance, the analysis has been extended to functional and action categories, and to event categories in Gärdenfors and Warglien (2012), all of which are treated in Gärdenfors (2014).

The primary function of quality dimensions is to represent various “qualities” of objects in different *domains*. The notion of a dimension should be understood literally. It is assumed that each of the quality dimensions is endowed with certain *topological* or *geometric* structures. Some quality dimensions are *integral* in the sense that one cannot fully describe an object by assign to it a value on one dimension without also giving it a value on others. For example, an object cannot be given a hue without also giving it a brightness value. Or the pitch of a sound always goes along with a particular loudness. Dimensions that are not integral are said to be *separable*, as for example the size and hue dimensions. Using this distinction, the notion of a *domain* can now be defined as a set of integral dimensions that are separable from all other dimensions. For an exact definition, see Zenker and Gärdenfors (this volume).

Conceptual spaces are particularly suited to represent different kinds of similarity relations: the closer two objects are located in a conceptual space, the more similar they are; “green,” for instance, is closer to “blue” than to “red.” If dimensions are assumed to have a metric, moreover, one can talk about *distances* in the conceptual space such that distances represent degrees of similarity between the objects represented in the space.

It is important to introduce a distinction between a *psychological* and a *scientific* interpretation of quality dimensions. The psychological interpretation generally concerns how humans structure their perceptions. Vogt’s chapter in this volume provides one model of how conceptual spaces can evolve from sensory quality dimensions. It is further assumed that these quality dimensions form the basis of word meanings, at least of the basic words that children learn first. A psychologically interesting example of a domain remains *color perception*, to which several authors in this volume refer. The scientific interpretation, in contrast, deals with how different dimensions are presented within a scientific theory, how they can give rise to empirical theories, and how to model diachronic changes as science develops (see Gärdenfors and Zenker, 2013; Zenker and Gärdenfors, 2013; this volume).

### *1.3 Properties and concepts*

Among others, the theory of conceptual spaces has been used to provide a definition of what constitutes a *natural property*. With the following criterion (Gärdenfors, 1990; 1992; 2000; 2014), the geometric characteristics of the quality dimensions are utilized to introduce a spatial structure for properties:

*Criterion P:* A natural property is a convex region in some domain.

A set is said to be *convex* if, for all points  $x$  and  $y$  in the set, all points between  $x$  and  $y$  are also in the set. Criterion P presumes, of course, that the notion of betweenness is meaningful for the relevant quality dimensions. Being a weak assumption, this demands rather little of the underlying geometric structure of a domain.

Most properties that natural languages express by simple words seem to be natural properties in the sense specified here (Gärdenfors, 2014). For instance, all

*color terms* in natural languages express natural properties with respect to the psychological representation of the three color dimensions. It is well-known that different languages carve up the color circle in different ways (Berlin and Kay, 1969), but all such carvings seemingly occur in terms of convex sets (Jäger, 2010).

Properties, as defined by criterion P, form a special case of *concepts*. More specifically, a property is based on a *single* domain, while a concept may be based on *several* domains.

The distinction between properties and concepts has been obliterated in both the symbolic and connectionist representations. In particular, both properties and concepts are represented by *predicates* in first-order languages. The predicates of a first-order language, however, correspond to several different grammatical categories in a natural language, the most important of which are adjectives, nouns and verbs. As a development of the notions in Gärdenfors (2000), Dessalles argues in his chapter that one should distinguish between concepts that are dependent on an underlying conceptual space, and predicates which are constructed “on the fly” in a particular context.

The main semantic difference between adjectives and nouns, on the one hand, is that adjectives such as “red,” “tall,” and “round” normally refer to a single domain and thus represent properties, while nouns like “dog,” “apple,” and “town” normally contain information about several domains, and thus represent concepts. Verbs, on the other hand, obtain their meaning from their role in *events*, expressing either the action being performed (“manner verbs”) or the outcome of an action (“result verbs”) (Warglien et al., 2012; Gärdenfors, 2014). In the event model proposed by Gärdenfors and Warglien (2012), for instance, actions are modelled as force vectors, or patterns thereof, and results as vectors in property domains. Another example is provided in Chella et al. (2001), who report a conceptual space describing robot actions.

Concepts are not just bundles of properties. The proposed representation for a concept also includes an account of the *correlations* between the regions of the different domains that are associated with a concept. In the “apple” concept, for instance, a very strong (positive) correlation obtains between the sweetness in the taste domain and the sugar content in the nutrition domain, while a weaker correlation holds between the color red and a sweet taste.

These considerations motivate the following definition:<sup>1</sup>

*Criterion C:* A concept is represented as a set of convex regions in a number of domains together with information about how the regions in different domains are correlated.

The kind of representation proposed in Criterion C is *prima facie* similar to *frames* (Barsalou, 1992) with slots for different *features* that have been very popular within cognitive science, linguistics, and computer science. The criterion is richer, however, since a representation based on conceptual spaces allows one to describe the structure of concepts such that objects are more or less *central* representatives of a concept. Conceptual spaces thus amount to more than a combination of extant ideas from frame theory and prototype theory, since the geometry of the domains yields predictions that are not possible in either. (For a comparison of frames with conceptual spaces, see Zenker (2014)). In his contribution to this volume, Zwarts demonstrates how existing feature analyses for particular domains can be used to construct conceptual spaces in which notions like convexity can be systematically studied.

The notion of a concept defined here displays several similarities with the *image schemas* studied in cognitive linguistics, among others by Lakoff (1987) and Langacker (1987). Although image schematic representations are often pictorial, they generally fail to specify the geometric structures of the underlying domains.

#### 1.4 Prototypes and conceptual spaces

Criterion P receives independent support from the *prototype theory* of categorization developed by Rosch and her collaborators (e.g., Rosch, 1975; 1978; Mervis and Rosch, 1981; Lakoff, 1987). The main idea in this theory is that within a category of objects, like those instantiating a property or a concept, certain members are judged to be more representative of the category than others. Robins, for instance, are taken to be more representative of the category “bird” than ravens, penguins and emus; and desk chairs are more typical instances of the category “chair” than rocking

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<sup>1</sup> A slightly more complex definition, also involving the *context* where a concept is used, is proposed in Gärdenfors (2000, ch. 4).

chairs, deck-chairs, and beanbag chairs. The most representative ones of a category are called its prototypical members. As Decock and Douven suggest in their chapter, some kinds may not have unique prototypes, which leads them to adapt Voronoi diagrams (introduced below) in order to deal with such vague concepts.

When natural properties are defined as convex regions of a conceptual space, prototype effects are indeed to be expected. In a convex region one can describe positions as being more or less central. In particular, if the space has a metric, one can calculate the “center of gravity” of a region.

One may also argue in the opposite direction and show that, if prototype theory is adopted, then the representation of properties as convex regions is to be expected, at least in metric spaces. To see this, assume that some quality dimensions of a conceptual space  $S$  are given (e.g., the dimensions of color space), and that we want to partition  $S$  into a number of categories (e.g., color categories). Starting from a set of prototypes  $p_1, \dots, p_n$  of such categories (such as the focal colors), these prototypes should thus be the central points in the categories they represent. On the additional assumption that  $S$  is a metric space, information about prototypes can now be used to generate a categorization. If we assume  $S$  to be equipped with the Euclidean metric, for instance, then—for every point  $p$  in  $S$ —we can measure the distance from  $p$  to each  $p_i$ . Moreover, by stipulating  $p$  to belong to the same category as the *closest* prototype  $p_i$ , such measurements generate a so-called *Voronoi tessellation* of the conceptual space, which is illustrated for the case of a plane in Fig. 1.

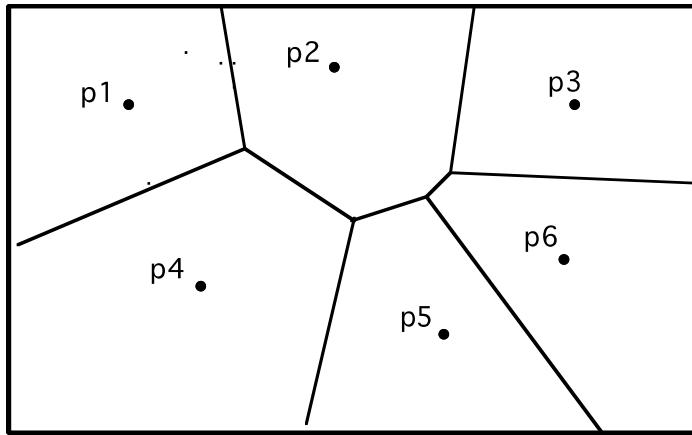


Fig. 1. Voronoi tessellation of the plane into convex sets, where p1 to p6 denote prototypes of a category, and the lines indicate category-boundaries.

A crucial property of the Voronoi partitioning of a conceptual space is that a tessellation based on a Euclidean metric always results in a partitioning of the space into *convex* regions (see Okabe, Boots, and Sugihara, 1992).

Thus, assuming that a Euclidean metric is defined on the subspace that is subject to categorization, a set of prototypes will on this method generate a unique partitioning of the subspace into convex regions. The upshot is that an intimate link obtains between prototype theory and Criterion P. Furthermore, the metric is an obvious candidate for a measure of similarity between different objects. In this way, the Voronoi tessellation provides a constructive geometric answer to how a similarity measure, together with a set of prototypes, determines a set of categories.

This concludes our brief summary of the theory of conceptual spaces. The developments since the publication of Gärdenfors (2000) have shown that geometric models of knowledge representation add significantly to our explanatory capacities, particularly to understanding cognitive processes connected with learning, concept formation, (non-monotonic) reasoning, and semantics. Such conceptual representations, moreover, prove helpful in understanding how we *communicate* about concepts and thus reach semantic agreement (Gärdenfors, 2014; Warglien and Gärdenfors, this volume). Moreover, as Scheider and Kuhn's chapter shows, for instance, such representations allow distinguishing concepts underlying data and digital information, and thus can assist people in sharing information, particularly when interoperating across computers.

## 2. Overview of chapters

To provide readers with further orientation, we now provide an overview of the contributions to this volume which—admittedly somewhat artificially—are grouped into the following overlapping categories: *Semantic spaces*, *computing meanings* and *philosophical perspectives*. These chapters either present successful applications of conceptual spaces theory in various research-areas, contrastive work, or research that seeks to further develop conceptual spaces.

### 2.1 Semantic Spaces

The theory of conceptual spaces builds on a cognitivist view of semantics. This contrasts with both extensional and intensional realistic semantics that include the referent of a linguistic expression as a meaning constituent. Conceptual knowledge is rather viewed as mental representations, modeled in conceptual spaces, which normally does without postulating a mental language (“Mentalese”). Therefore, such representations are not viewed as parts of a symbolic system with a syntactic or logical structure. They are instead treated as spatial structures that can be analyzed into their constitutive dimensions and properties, representing the semantic knowledge of an agent (Gärdenfors, 2000, 2014).

*Jean-Louis Dessalles* argues that, in addition to the structure provided by conceptual spaces, we also need the operation of *contrast*. A red face, for instance, is called “red” because it contrasts with other possible face colors, rather than being red in the prototypical sense of “red”. Contrast, as it develops during a conversation, is an essential operation that converts perceptions into the predicates expressed in communication. Thus *concepts*, which belong to a conceptual space, should be kept separate from *predicates* that are formed by a contrast operation during a conversation. According to Dessalles, predicates are dynamic representations that lack a context-independent existence. Among others, he claims that the distinction between concepts (lexical meanings) and predicates serves to avoid many of the theoretical difficulties of traditional semantics.

*Carita Paradis* deals with how human experiences of sensory stimuli such as vision, smell, taste, and touch are rendered in natural language. Next to being part of embodied cognition, sensuous cognition also deserves attention as an important source of semantic structure. To this end, she studies data from terminological

schemas for wine descriptions in wine reviews with a focus on the types of property expressions, e.g., soft, sharp, sweet, and dry, as well as object descriptors, e.g., *blueberry*, *apple* and *honey*, that are used for the different sensory experiences, and on the cross-sensory uses of these property expressions and object descriptors. In contrast to the standard view, she argues that, for instance, *sharp* in *sharp smell* does not evoke a notion of touch. When being instantiated in the sensory domain of smell, *sharp* spans closely related sensory domains. These cross-sensory uses are viewed as symptoms of synesthesia in wine-tasting, and the workings of human sensory cognition.

*Joost Zwarts* explores how to construct a conceptual space for word meanings on the basis of a system of features, his empirical basis being Dutch words for different kinds of shirts. He considers features of shirts along eight dimensions (e.g., shape, length, collar, fabric), where a few values on each dimension corresponds to these features. Based on the Hamming distance, he then defines a metric by which to compare different kinds of shirts. Investigating this space, he shows how the different kinds are related to one another, for instance, how close to another they are in the defined conceptual space. An empirical problem is that the shirts in his empirical basis cover only a small part of the logically possible combinations of features. Although clusters of shirt types can be detected, no simple mapping between the Dutch words and locations in the space is forthcoming. Therefore, the meanings of the different words, for instance, cannot be defined as a conjunction of a set of features. The sets of shirts that correspond to a given word do nonetheless cohere in terms of the underlying feature space.

*Massimo Warglien* and *Peter Gärdenfors* present a model of how meanings are negotiated, that is, the process by which agents, who each start from a different preferred conceptual representation, may converge to reach an agreement through some communication medium. They show that a model based on conceptual spaces inherits important structural elements from game-theoretic models of bargaining, particularly so if agents share overlapping negotiation regions, and thus emphasize a parallel to the Nash solution in cooperative game-theory. Should agents have disjoint solution regions, moreover, processes such as changes in the salience of dimensions, dimensional projections, and metaphorical space-transformations may be helpful in finding mutually acceptable solutions. Importantly, these processes are

not motivated by normative or rationality considerations, but presented as argumentation tools used in actual situations of conceptual disagreement.

## 2.2 Computing meanings

The second part of the volume brings up several computational issues in relation to applications of conceptual spaces. Unlike most semantic theories within linguistics, the dimensional structure of conceptual spaces is useful in generating productive computational implementations.

*Paul Vogt* uses an agent-based simulation model to investigate how conceptual spaces can evolve from a set of quality dimensions. The model builds on the Talking Heads experiment developed by Steels and others (Steels et al., 2002), where a set of agents play a large number of guessing games that incur rewards for successful communication. Vogt's model involves a 4-dimensional conceptual space that can be broken up into 3-dimensional color and 1-dimensional shape. His results show that, upon a generational turnover occurring in the set of speakers, communication evolves towards an optimal system such that the grammar represents rules for combining color with shape. The simulations also show the evolution of the conceptual spaces which underlie communication to be driven by five crucial factors: environment, embodiment, cognition, self-organization and cultural transmission.

*Simon Scheider* and *Werner Kuhn* apply conceptual spaces by way of contributing to semantic technology, and seek to improve on extant models of information sharing in human-machine-human conversations. They pursue a pragmatic approach to semantic interoperability, where conceptual spaces ground (i.e., provide the constructive basis for) *learning*—which here is an attempt at imitating conceptual content that had originated under a different perspective. Accordingly, they argue, full semantic interoperability “should be defined as [full] correspondence of conceptual perspectives of communicating agents.” The range of perspectival correspondence reaches from equivalent concepts, over (partially) comparable, to overlapping ones. For this purpose, empirical measurement-points are projected into a space whose convex regions are identified as concepts that are relevant for learning, and are illustrated through the reconstruction of cultural, scientific, and administrative land cover categories.

*Janet Aisbett, John Rickard and Greg Gibbon* explore both synergies and gaps in research on conceptual spaces. They particular work on fuzzy sets, also known as *computing with words* (CWW), which similarly attempts to address aspects of human cognition such as judgment and intuition. Outlining formal methods developed in CWW for modelling and manipulating constructs whenever membership values are imprecise, the authors describe and problematize a specific formalism of conceptual spaces based on fuzzy sets. This formalism is compared to alternative methods for aggregating property membership into concept membership. Their problem solution is a model where all constructs are fuzzy sets on a plane, and similarity of two constructs is an inverse function of the average separation between the membership functions.

*Giancarlo Guizzardi* presents a system of modal logic that distinguishes sortals from general property types, and can thus capture the semantics of objects types. His system constitutes an extension of the theory of conceptual spaces. It primarily addresses the limitations of classical (unrestricted extensional) modal logics by differentiating types that represent ascribed properties from those that carry a principle of identity (also known as sortal types). The system is exemplified, among others, by means of standard examples of “identity loss” (such as the alleged non-identity of a statue with the lump of clay constituting it), a notion he rejects. It is moreover shown how to circumvent some of the limitations that arise in representing modal (temporal) information with languages such as the Web Ontology Language (OWL) used in computer science.

*Antonio Chella* presents a cognitive architecture for a musical agent based on the architecture developed in Chella et al. (2000) for computer vision. The underlying conceptual space is constructed from the two fundamental dimensions *pitch* and *time*. The timbre of a complex tone can then be represented as a vector of the harmonic frequencies about a fundamental tone. This representation may be used to generate a higher-level conceptual space in which the distance between tones based on their consonance can be determined, and musical intervals be represented. The new conceptual space forms the semantic base for a linguistic level of musical terminology. Chella also points out the multi-faceted analogies between vision and music perception.

### 2.3. Philosophical perspectives

The theory of conceptual spaces also generates a number of philosophical issues. A fundamental question for epistemology, for instance, is how knowledge is best represented. The last part of the volume addresses some of these issues.

*Lieven Decock and Igor Douven* show conceptual spaces to be a particularly useful tool for philosophers. They discuss recent applications of the model to classical problems in metaphysics and the philosophy of language pertaining to identity such as the vagueness of terms, graded membership, and paradoxes of identity. They moreover summarize an analysis of knowledge that models the “usual suspects” (truth, belief, and justification) as a 3-dimensional space—*knowledge* being represented as one of its regions—to which additional epistemologically relevant dimensions can be added. As they show, the similarity between agents’ doxastic states and the knowledge-region can thus be exploited to account for the observation that our willingness to attribute knowledge to people is stake-sensitive. Generally, as they stress, the “investigation of many concepts must involve more than an analysis of their component parts, and must involve some construction work as well: out of the component parts, a model of the concept must be built.”

Addressing its testability, Joel Parthemore presents his unified conceptual spaces theory (UCST) and outlines various indirect ways of pitching it against empirical data by using mind-mapping software. This software is distinct from other available mind-mapping programs because it is directly based on a theory of concepts (UCST), rather than being loosely based on an underspecified theory of cognition. It also comes with a visual interface based on Voronoi tessellation. UCST offers a ‘just so’ story of how all concepts describable within this framework can be derived from three proto-conceptual entities, and thus oriented within a unified “space of spaces” along one of three dimensions that are claimed to be integral to all concepts, namely the axes of *generalization*, *alternatives*, and *abstraction*. Within this unified space, the conceptual agent is assumed to build, modify, and navigate the conceptual frameworks that serve to structure and restructure an understanding of the world around her.

*Mauri Kaipanen and Antti Hautamäki* pursue an epistemologically grounded perspectivist approach to conceptual spaces. They present a model which comprises a multi-dimensional ontospace, a full grasp of which is limited by human cognitive capabilities, as well as a lower dimensional representational space that, next to supporting particular conceptualizations of the ontospace, allows for various

alternatives thereof. Assuming that an ontospace is cognitively accessible only through the epistemic “work” of exploring alternative perspectives, they suggest that an understanding of a particular domain emerges only through having viewed it from multiple perspectives, thus abstracting further than any one given perspective. Such perspectives are said to vary individually as a function of interest, situational contexts, as well as various temporal factors; but they also remain communicable, and thus allow for interpersonally shared conceptualizations.

*Frank Zenker and Peter Gärdenfors* have in earlier work (Gärdenfors and Zenker, 2013; Zenker and Gärdenfors, 2013) shown how conceptual spaces apply to model changes of scientific frameworks (e.g., in physics), when these are treated as spatial structures, rather than as linguistic entities. In their chapter, this application is contrasted with Michael Friedman’s (2001) neo-Kantian account, which particularly seeks to render the transition from Newtonian mechanics to relativity theory as communicatively rational conceptual development. To compare different paradigms, Friedman introduces philosophical meta-paradigms as necessary elements. In contrast, Zenker and Gärdenfors argue that when theory frameworks are modeled as conceptual spaces and theories as constraints on them, then the communicative challenges said to go along with a paradigm shift become smaller. Thus, the cross-paradigmatic comparison of such frameworks that is necessary for rational scientific communication may instead proceed via the frameworks’ geometric or topological properties. This, they argue, lies closer to what scientists in fact use in their thinking and communication.

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