Algoritmos en Bioinformática

Exercises and Problems

1. **E.** Sort the following functions according to their growth:
   n, $\sqrt{n}$, $n^{1.5}$, n^2, $n \log n$, $n \log \log n$, $n \log^2 n$, $2/n$, 2^n, $2^{n/2}$, 37, $n^2 \log n$, n^3.

2. **P; Python** A classical example of a Divide and Conquer algorithm is binary search over **ordered** tables, which is essentially done according to the following pseudocode:

```
def bin_search(key, l_ints):
    m = len(l_ints)//2
    if key == l_ints[m]:
        return m
    elif key < l_ints[m]:   # left table search
        search key on l_ints up to index m-1
    else:                   # right table search
        search key on l_ints from index m+1
```

   Expand the pseudocode into a correct recursive Python function.

3. **E.** Identify a proper key operation for `bin_search`. How many key comparisons are performed at most on the table `[1, 2, 3, 4, 5, 6, 7]` in successful searches? And in unsuccessful ones?

4. **P.** In general how many unsuccessful searches will be performed at most on a table `[1, 2, 3, ..., 2**n-1]`? And successful ones?

5. **P. Fibonacci numbers and rabbit biology**. Suppose that bunnies become adults in a month and that a pair of adult rabbits, regardless of sex, produce a pair of bunnies a month. If two young shipwrecked rabbits arrive on a deserted island, how many pairs of rabbits will there be on the island after N months? (For example, after two months there would be two pairs, one of bunnies and one of adult rabbits).

6. **P; Python.** The **Fibonacci** numbers F_n are recursively defined as $F_0 = F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$ for $N \geq 2$. Write a recursive Python function `fibonacci_rec(n)` to compute them.

7. **P; Python.** Write an iterative version of `fibonacci(n)` which at each step saves the just computed Fibonacci numbers so that no recursive calls are needed.

   Hint: use first a dict for this; then, try to remove it.

8. **E; Python.** Modify our Hanoi code to write a function `hanoi_count(n_disks, a=1, b=2, c=3)` that returns the required number of single disk movements.

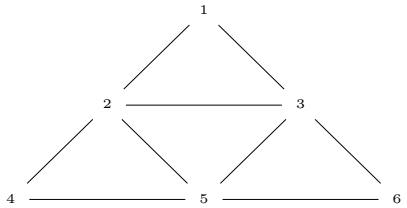
9. **P.** We have checked numerically that the number $n_{moves}(n)$ of disk moves the Hanoi algorithm makes over n disks is $2^n - 1$ but we don't have a proof on this. Try to write down one starting with the recurrence

$$n_{moves}(n) = 1 + 2 \times n_{moves}(n-1), \quad n_{moves}(1) = 1$$

   and applying induction on n.

10. **E.** Find by hand the minimum distance between the strings `bahamas` and `bananas`. Find also a longest common subsequence.

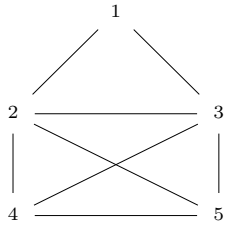
11. **E.** Write down the adjacency list of the graph below and use it to decide whether it has an Eulerian circuit or path. Use also the list to find it.



12. **E.** Given the 3–spectra
    $\mathcal{S}_1 = \{$ `ATG, TGG, TGC, GTG, GGC, GCA, GCG, CGT` $\}$
    $\mathcal{S}_2 = \{$ `ATG, GGG, GGT, GTA, GTG, TAT, TGG` $\}$
    find sequences s_i whose spectra are $\mathcal{S}_i$ writing down this problem as that of finding an adequate Hamitonian path in a suitable graph and finding it by inspection.

13. **E.** Now find sequences s_i whose spectra are $\mathcal{S}_i$ writing down a new problem of finding an adequate Eulerian path in a suitable graph and finding it from the graph's adjacency list.

14. **E.** A child's game is to try to draw the house below without lifting the pen from the sheet



Interpret this as a problem of finding an adequate **Eulerian path** in an undirected graph and find a such path starting at the lowest numbered node where it may start and "taking a walk" on the graph's adjacency list while providing adequate detail.

15. **P.** We have solved in the lectures the problem of giving optimal change of 7 maravedís with coins of 1, 3, 4 and 5 maravedís.

    Using the dynamic programming matrix, try to give a procedure to determine the coins that will enter in the optimal change.

16. **P.** The president of the United States has a leaking problem: one among his 100 person staff keeps leaking stories to the press. To find him/her out, his Chief Computing Officer tells him to give different stories to a number of different staffers and check whether the information ends up in the news until they find out (they know that he or she will leak any story he or she is given). Since coming up with stories is hard, they want to minimize the number that they use. Devise a strategy to handle stories and to find out this minimum number. (Taken from The Riddler, FiveThirtyEight.)