CW Part 01: Individual Assignment with Viva

CM 2607: Advanced Mathematics for Data Science

2024/25 Semester 01

Question 1: Application of Differentiation in AI Model Optimization

Consider a machine learning model where the loss function is given by $L(x, y) = e^{x^2+y^2} + \cos(xy)$. Your task is to:

(a) Find the partial derivatives $\frac{\partial L}{\partial x}$ and $\frac{\partial L}{\partial y}$.

(b) Write a Python function that computes the gradient vector for a given input (x, y).

(c) Use Matplotlib to visualize the loss function as a 3D surface plot over the range $x, y \in [-2, 2]$.

(d) Discuss the significance of gradient vectors in model optimization, particularly in gradient descent methods.

Question 2: Integration for Signal Processing

The amplitude of a signal $A(t)$ can be modeled by the function $A(t) = 2t^2e^{-t}$ over the interval $t \in [0, 4]$.

(a) Use the Python library SymPy to compute the definite integral $\int_0^4 2t^2e^{-t}dt$.

(b) Plot the function $A(t)$ using Matplotlib and annotate the graph with the area under the curve corresponding to the integral.

(c) Interpret the meaning of the computed integral in the context of signal energy.

Question 3: Series Convergence and Numerical Approximation

The function $f(x) = \ln(1 + x)$ can be approximated by the series:

$$f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

(a) Implement a Python script that uses the first 10 terms of this series to approximate $\ln(1 + x)$ for $x \in [-1, 1]$.

(b) Plot the approximation and the actual function $\ln(1 + x)$ on the same graph for comparison.

(c) Analyze the convergence behavior of the series, particularly focusing on the error for larger values of x.

Question 4: Fourier Transform for Data Filtering

A noisy signal $s(t) = \sin(2\pi t) + 0.5\sin(10\pi t) + 0.3\sin(50\pi t)$ is sampled over the interval $t \in [0, 1]$ with 500 sample points.

(a) Perform a Fourier Transform on this signal using NumPy and plot the frequency spectrum.

(b) Design and apply a low-pass filter to remove frequencies above 20 Hz, then plot the filtered signal.

(c) Discuss how filtering impacts the signal and its applications in real-world data science scenarios, such as noise reduction in audio processing.

Question 5: Image Processing Using Fourier and Discrete Cosine Transforms

Consider the image provided in the coursework description on Moodle. Your task is to perform the following operations using Python:

(a) Find the edges in the image using a 2D Fourier Transform. Visualize the resulting image with detected edges.

(b) Apply a Gaussian blur to the original image using a 2D Fourier Transform and compare it to a standard Gaussian blur.

(c) Apply a Discrete Cosine Transform (DCT) to the original image. Then, scale the image down to 240px $\times$ 240px using the DCT. Display the scaled image and compare it to the original.

(d) Reproduce and explain the common artifacts (ringing and blocking) that occur when an image is compressed using DCT. Provide visual examples of these artifacts.