8: Frequency, Severity & Aggregate Models

Ordinary Deductible		Franchise Deductible	
Left-censored & Shifted (LCS), Y^L	**Excess Loss, Y^P**	**Per-Payment, Y^P**	**Per-Loss, Y^L**
$Y_L=(X-d)_+=\begin{cases}0 & X\le d\\ X-d & X>d\end{cases}$ $f_{Y^L}(y)=f_X(y+d), y>0$ $F_{Y^L}(y)=F_X(y+d)$	$Y_P=\begin{cases}undefined & X\le d\\ X-d & X>d\end{cases}$ $f_{Y^P}(y)=\dfrac{f_X(y+d)}{S_X(d)}, y>0$ $F_{Y^P}(y)=\dfrac{F_X(y+d)-F_X(d)}{S_X(d)}$	$Y^P=\begin{cases}undef, & X\le d\\ X & X>d\end{cases}$ $f_{Y^P}(y)=\dfrac{f_X(y)}{S_X(d)}$ $F_{Y^P}(y)=\begin{cases}0 & 0\le y\le d\\ \dfrac{F_X(y+d)-F_X(d)}{S_X(d)} & y>d\end{cases}$	$Y^L=\begin{cases}0, & X\le d\\ X & X>d\end{cases}$ $f_{Y^L}(y)=\begin{cases}F_X(d) & ,y=0\\ f_X(y) & ,y>d\end{cases}$ $F_{Y^L}(y)=\begin{cases}F_X(d) & ,0\le y\le d\\ F_X(y) & ,y>d\end{cases}$
Expected Cost per Loss	**Expected Cost per Payment**	**Expected Cost per Payout**	**Expected Cost per Loss**
$E[Y^L]=E(X)-E(X\wedge d)$ $E[(Y^L)^k]=\int_d^\infty (x-d)^k f(x)dx$	$E[Y^P]=\dfrac{E(X)-E(X\wedge d)}{S_X(d)}$	$E[Y^P]=\dfrac{E(X)-E(X\wedge d)}{S_X(d)}-d$	$E[Y^L]=E(X)-E(X\wedge d)+d[S_X(d)]$

LLV, LEV & Policy Limits	**Limited Loss Variable**	**Policy Limits (Only applicable to Ordinary Deductibles)**
	LLV $Y=X\wedge u=\begin{cases}X & ,X<u\\ u & ,X\ge u\end{cases}$ **LEV** $E[X\wedge u]=-\int_{-\infty}^0 F(x)dx+\int_0^u S(x)dx$ $E[(X\wedge u)^k]=\int_{-\infty}^u x^k f(x)dx+u^k[1-F(u)]$ $=\sum_{x_i<u} x_i^k p(x_i)+u^k[1-F(u)]$	The payout is X if below some policy limit *u*, and *u* otherwise. N/A to franchise deductibles because nobody would buy a policy that pays nothing above the policy limit. $f_{Y^L}(y)=\begin{cases}f_X(y), & y\le u\\ 1-F_X(u), & y=u\end{cases}$ $F_{Y^L}(y)=\begin{cases}F_X(y), & y\le u\\ 1, & y>u\end{cases}$ Expected cost = E[X∧u]
Inflation	**Expected Cost, with Inflation and Ordinary Deductibles**	
	$E[Y^L]=(1+r)\left\{E(X)-E\left(X\wedge\dfrac{d}{1+r}\right)\right\}$	$E[Y^P]=(1+r)\dfrac{E[X]-E\left(X\wedge\dfrac{d}{1+r}\right)}{1-F_X\left(\dfrac{d}{1+r}\right)}$
	Expected Cost, with Inflation and Limits	
	$E[Y^L]=(1+r)\left\{E\left(X\wedge\dfrac{u}{1+r}\right)\right\}$	$E[Y^P]=(1+r)\dfrac{E\left(X\wedge\dfrac{u}{1+r}\right)}{1-F_X\left(\dfrac{u}{1+r}\right)}$
LER	$LER=\dfrac{E[X\wedge d]}{E[X]}=\dfrac{\sum\min(x_i,d)}{\sum x_i}$	

8.5 – Modifiers: Ordinary Deductible, Coinsurance, Inflation & Policy Limits

Putting deductible, coinsurance and inflation together from ground-up loss X, to construct Y^L:

$$f_{Y^L}(y)=\begin{cases}0, & X<\dfrac{d}{1+r}\\ \alpha[(1+r)X-d], & d\le X<\dfrac{u}{1+r}\\ \alpha(u-d), & X\ge\dfrac{u}{1+r}\end{cases}$$

Moments

$$E[Y^L]=\alpha(1+r)[E(X\wedge u)-E(X\wedge d)]$$

$$E[(Y^L)^2]=\alpha^2(1+r)^2\left\{E(X\wedge u^*)^2-E(X\wedge d^*)^2-2d[E(X\wedge u^*)-E(X\wedge d^*)]\right\}$$

$$Where: u^*=\frac{u}{1+r}, d=\frac{d}{1+r}$$

$$E[Y^P]=\frac{E[Y^L]}{1-F_X[d^*]}$$

8.6 – Impact of Deductibles on Claim Frequency

Back Matter: (Discrete) Compound Frequency Models

The *compound distribution* is a random sum of random variables, both discrete in this case:

$$G_S(z)=G_N[G_M(z)]$$

Thm. 6.8

If the primary distribution G_N can be written as:

$$G_N(z;\theta)=B[\theta(z-1)]$$

For some parameter θ of the primary distribution, and some function *B* that is independent of θ, then:

$$P_S(z)=B[v\theta(z-1)]=P_N(z;v\theta)$$

$$Where: v=P(X>d)$$