Modern Physics Letters A
2050213 (22 pages)

DOI: 10.1142/S0217732320502132

Gravitational Möller scattering, Lorentz violation and finite temperature

A. F. Santos*,‡ and Faqir C. Khanna†,§

*Instituto de Física, Universidade Federal de Mato Grosso, 78060-900, Cuiabá, Mato Grosso, Brazil

†Department of Physics and Astronomy, University of Victoria, 3800 Finnerty Road Victoria, BC, Canada

‡alesandroferreira@fisica.ufmt.br

§khannaf@uvic.ca; fkhanna@ualberta.ca

Received 27 March 2020
Revised 27 May 2020
Accepted 28 May 2020
Published 6 July 2020

A formal analogy between the gravitational and the electromagnetic fields leads to the notion of Gravitoelectromagnetism (GEM) to describe gravitation. A Lagrangian formulation for GEM is developed for scattering processes with gravitons as an intermediate state, in addition to photons for electromagnetic scattering. The differential cross section is calculated for gravitational Möller scattering based on GEM theory. This gravitational cross section is obtained for cases where the Lorentz symmetry is maintained or violated. The Lorentz violation is introduced with the non-minimal coupling term. In addition, using the Thermo Field Dynamics formalism, thermal corrections to the differential cross section are investigated. By comparing the electromagnetic and GEM versions, of Möller scattering, it is shown that the gravitational effect may be measured at an appropriate energy scale.

Keywords: Gravitational scattering; finite temperature; Lorentz violation.

1. Introduction

The fundamental forces are the nuclear, electromagnetic, and the gravity. However, gravity dominates over other forces on astronomical scale. Gravity is a classical theory while other forces have a quantum nature. Although the incompatibility between general relativity and quantum mechanics emerges at high energies, in lower energies an effective field theory has been constructed such that gravity and quantum mechanics work together perfectly well over a certain range of energy

‡Corresponding author.
§Professor Emeritus – Physics Department, Theoretical Physics Institute, University of Alberta, Edmonton, Alberta, Canada.

scales. For a review, see Refs. 1–5. The standard model (SM) describes all forces and their interactions except the gravitation. Although successfully confirmed, i.e., its predictions were confirmed by numerous experiments, the SM leaves unresolved a variety of issues. The SM is not a fundamental theory since gravity is missing from all types of interaction. A unified theory that combines the SM and gravity has been investigated. Many theoretical approaches seeking to unify quantum theory and general relativity. A fundamental theory would emerge at energies approaching Planck scale ($\approx 10^{19}$ GeV). There are several well-known candidates for quantum gravity models, such as string theory,[6] warped brane worlds,[7] loop quantum gravity,[8] Horava–Lifshitz gravity,[9] among others. An old idea of unification involves the gravitation and electromagnetism theories.

There are some similarities between gravity and electromagnetism, even though the two theories are different. Numerous attempts have been made to unify gravity and electromagnetism theories. Here the gravitoelectromagnetism (GEM) theory is considered. GEM is a theory of gravity that is based on a formal analogy between the gravitational and the electromagnetic phenomena. This analysis started with Faraday, who considered experiments to detect if the gravitational field is induced by electric current in a circuit.[10] Maxwell formulated a theory of gravitation considering the similarity between Newton's law and Coulomb's law[11] and Maxwell-like equations for gravity were developed by Heaviside.[12,13] Following these ideas, Weyl proposed a generalization of the Riemann geometry[14] and Kaluza and Klein developed an extension of gravity theory to five dimensions.[15] There was an effort by Einstein to develop other theories to combine with the gravitational theory.[16] However, this did not succeed. At the present time, the gravity and other forces have been considered similar. This has led to the conclusion of a unique theory of gravity that includes an effective field that provides a description of gravity. Several studies about GEM theory have been developed.[17–28] GEM is based on two assumptions: (i) there is a gravitomagnetic field connected with moving masses and (ii) the gravitational field propagates at the speed of light. Some experiments, such as LAGEOS, LAGEOS II, LAGEOS III, and the Gravity Probe B have been investigated using the GEM theory, particularly the gravitomagnetic effect.[29,30]

There are three different ways to analyze the GEM theory: (i) using the similarity between the linearized Einstein and Maxwell equations[28]; (ii) a theory based on an approach using tidal tensors[31] and (iii) the decomposition of the Weyl tensor (C_{ijkl}) into $\mathcal{B}_{ij} = \frac{1}{2}\epsilon_{ikl}C^{kl}_{0j}$ and $\mathcal{E}_{ij} = -C_{0i0j}$, the gravitomagnetic and gravitoelectric components, respectively.[32] In this paper, the Weyl tensor approach is used. The Weyl tensor is connected with the curvature tensor. In addition, the analogy between electromagnetism and general relativity is based on the correspondence $C_{\alpha\sigma\mu\nu} \leftrightarrow F_{\alpha\sigma}$, where the Weyl tensor is the free gravitational field and $F_{\alpha\sigma}$ is the electromagnetic tensor. Furthermore, the components $\mathcal{B}_{ij}$ and $\mathcal{E}_{ij}$, in a nearly flat space-time, lead to the Maxwell-like equations, i.e., the GEM equations. The GEM analogy is limited by the fact that the Maxwell field propagates on a given space-time, whereas the gravitational field itself generates the space-time. The other limitation is the fact that the GEM degrees of freedom only depend on the Weyl tensor. An important difference between two theories is that electromagnetic fields are vectors, whereas GEM fields are tensors. A Lagrangian formulation for GEM is developed.[33] A symmetric gravitoelectromagnetic tensor potential, that describes the gravitational interaction, is constructed. A fundamental difference between this formalism and the weak-field approximation of the Einstein field is that the GEM potential is considered a fundamental gravitational field in flat space-time, instead of the small perturbation, $h_{\mu\nu}$. In addition, it allows us to consider a full Lagrangian that describes gravitons, fermions, and photons and their interactions. Using this Lagrangian formalism, the differential cross section for the gravitational Bhabha scattering has been calculated.[34] Furthermore, the gravitational cross section for some processes in different gravitational theories has been investigated. For example, the interaction between gravitons and fermions in the teleparallel gravity has been studied,[35] propagators and vertices for both the Yang–Mills and gravitational fields have been analyzed,[36] graviton interactions in direct analogy to that of electromagnetism have been developed,[37] among others. Here GEM theory is used to determine the differential cross section for the gravitational Möller scattering.

Möller scattering is an electron–electron scattering with photons in the intermediate state.[38] It is similar to Bhabha and Compton scattering that deal with particles and photons as electromagnetic field. These do not include the gravity as a field. It is dominated by the electromagnetic interaction, however, the graviton exchange may also be considered to find the amplitude of the process. The corresponding effects are small but may be measurable at a very high-energy scale where the gravitational interaction is dominant. Here the electromagnetic and gravitational differential cross sections of Möller scattering are compared. In addition, thermal corrections for the differential cross section of the gravitational Möller scattering is calculated. Thermo Field Dynamics (TFD) formalism is used to introduce finite temperature effects. Studies about Möller scattering at finite temperature have been considered. For example, this scattering process with photons in the intermediate state at finite temperature in the presence of Lorentz-violating parameter has been investigated,[39] gravitational scattering with graviton in the intermediate state at finite temperature has been studied,[40] among others.

Temperature effects are introduced in two distinct, but equivalent formalism: (i) the Matsubara formalism or the imaginary time formalism,[41] that is based on a substitution of time, t, by a complex time, $i\tau$; and (ii) the real-time formalism that consist of two distinct, but equivalent, formalisms: the closed time path formalism[42] and the TFD formalism.[43–47] Here the TFD formalism will be used. This formalism depends on the doubling of the original Fock space and uses the Bogoliubov transformation. These two spaces, the original and the tilde space, are related by a mapping, called the tilde conjugation rules. While the Bogoliubov transformation consists in a rotation involving these two spaces that introduce the temperature effects. Furthermore, corrections for the cross section due to the Lorentz violation at zero and finite temperature are calculated.

The violation of the Lorentz symmetry is a possibility in theories, which attempt to unify all fundamental forces of nature. A fundamental theory would emerge at energies approaching the Planck scale ($\approx 10^{19}$ GeV). This would involve finding new physics. In order to study the consequences of the Lorentz violation an extension of the SM has been developed.[48–50] The Standard Model Extension (SME) consists of known models of the SM and general relativity plus all possible terms that violate Lorentz and CPT symmetry. The structure of the SME is one way to investigate the Lorentz and CPT violations. There is another objective to examine Lorentz-violating developments out of this broad framework. This alternative involves nonminimal coupling terms that modify the vertex interaction between fermions and gravitons, as an example see Ref. 51. Here this last option is considered. Then the usual covariant derivative will be modified by a nonminimal coupling term.

This paper is organized as follows. In Sec. 2, an introduction to effective field theory is briefly discussed. In Sec. 3, a brief introduction to the GEM Lagrangian formalism is presented. In Sec. 4, the graviton–fermions interaction is introduced. The transition amplitudes for t-channel and u-channel for Feynman diagrams are calculated. Then the differential cross section for the gravitational Möller scattering is determined for different situations: (i) the cross section is calculated at zero temperature and the Lorentz symmetries are maintained. In addition, a comparison between electromagnetic and GEM Möller scatterings is developed. (ii) In order to calculate the thermal corrections for the cross section, the TFD formalism is introduced and then the cross section at finite temperature is obtained. (iii) Corrections due to the Lorentz symmetry violation for the cross section at zero temperature are determined. (iv) Next, the cross section for the case of Lorentz violation at finite temperature is calculated. In Sec. 5, some concluding remarks are presented. Natural units, $\hbar = c = 1$, are used unless otherwise stated.

2. Gravity as an Effective Theory — An Introduction

In this section, an introduction to gravity as an effective theory is briefly presented. For a review, see Refs. 1–5. Effective field theory is a common and well-known procedure in particle physics. It is very useful to study gravity and quantum mechanics together. Is it possible? Are general relativity and quantum mechanics compatible? The answer is yes, if an effective field theory is considered.

The main point in this method consists in the separation of known physics at the working scale, low-energy scale, from unknown physics at much higher energies, Planck scale, for example. It is expected that at higher energies new degrees of freedom and new interactions come into play. Then effective field theory allows us to analyze the physics at present energies without making unwarranted assumptions about what is going on at high energies.

The gravity is defined as a fundamental theory that yields general relativity at the low-energy limit. Then general relativity is an effective field theory of a complete theory defined at a very high-energy scale. A complete gravitational Lagrangian is constructed for quantities that are invariant under the general coordinate transformations. Then

$$S = \int d^4x\sqrt{g}\left\{\Lambda + \frac{2}{\kappa^2}R + c_1R^2 + c_2R_{\mu\nu}R^{\mu\nu} + \cdots + \mathcal{L}_m\right\}. \tag{1}$$

Here the gravitational Lagrangian has been ordered in a derivative expansion where Λ is of order ∂^0, R is of order ∂^2, R^2 and $R_{\mu\nu}$ are of order ∂^4, and so on. The parameters of this Lagrangian are determined using experiments. For a review of how these parameters are constrained either locally or through cosmological observations, see Refs. 52–55. The first term, the cosmological constant, appears to be nonzero but it is so tiny that it is not relevant on ordinary scales, i.e., the Λ is extremely small on the scale of the size of the observed universe.[56,57] The second term is the Einstein action, with $\kappa^2 = 32\pi G$, where G is the gravitational constant. The terms with curvature squared yield effects that are tiny on normal scales if the coefficients c_1 and c_2 are of order unity. So, on the low-energy scale, the effect of terms with R^2 is only a small correction to Einstein theory. The general relativity is obtained for $c_1 = c_2 = 0$. However, it is very unlikely that in fact $c_1 = c_2 = 0$. Indeed there is no known reason to require that c_1 and c_2 vanish completely. Then the gravitational Lagrangian can be written as a simple energy expansion, where the contributions due to coefficients c_1 and c_2 do not influence known physics at low energies. However, at very large scales, this is not entirely true. These modifications are considered as an alternative to dark energy and therefore c_1 and c_2 do alter the behavior at low energies. How can high-energy modifications of the gravitational action have anything to do with late-time cosmological phenomenology? Some results in the literature claiming that terms responsible for late-time gravitational phenomenology might be predicted by some more fundamental theory. Then dark energy is an effective phenomenon in GR.[52–55] Using that $i\partial \sim p$, derivatives turn into factors of the momentum or energy. So the curvature term is of order p^2. The graviton energy can be arbitrarily small at low enough energies, then the p^2 term in the Lagrangian is dominant. Therefore, the higher order Lagrangians will have small effect at low energies compared to the Einstein term.

The most fundamental gravitational Lagrangian should be an infinite number of parameters such as κ, c_1 and c_2. If fundamental theory of gravity is known, these are liable to be predicted. When a classical effective field theory is treated, a more general Lagrangian has to be considered. In such a case only the effect of the Einstein Lagrangian is visible in any test of general relativity. Here, it is important to emphasize that, this result is to low-energy scale since that at high energy the contributions of the others terms are comparable to the Einstein term. In this paper, the gravity is considered as an effective field theory. In particular, the gravitational model chosen is the GEM theory that is equivalent to the Einstein theory in a certain limit, which will be discussed in Sec. 3.

In this context, gravity as an effective field theory, it is possible to develop a quantum field theory from the gravitational field. It is possible to carry this by taking the gravity in the weak-field approximation as

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \tag{2}$$

where $h_{\mu\nu}$ is a small perturbation. The indices here and in subsequent formulas are raised and lowered with the Minkowski metric. Then the Ricci tensor and Ricci scalar will lead to the Einstein equations of the form

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \equiv \frac{\kappa}{2}O_{\mu\nu\alpha\beta}h^{\alpha\beta} = \frac{\kappa^2}{4}T_{\mu\nu}, \tag{3}$$

where $T_{\mu\nu}$ is the energy–momentum tensor and

$$O^{\mu\nu}{}_{\alpha\beta} \equiv (\delta_\alpha^{(\mu}\delta_\beta^{\nu)} - \eta^{\mu\nu}\eta_{\alpha\beta})\Box - 2\delta_{(\alpha}^{(\mu}\partial^{\nu)}\partial_{\beta)} + \eta_{\alpha\beta}\partial^\mu\partial^\nu + \eta^{\mu\nu}\partial_\alpha\partial_\beta. \tag{4}$$

It is important to note, in order to obtain this result, only the Einstein term in the Lagrangian has been considered. Next a Green function is defined as

$$O_{\mu\nu}{}^{\alpha\beta}G_{\alpha\beta\gamma\delta}(x-y) = \frac{1}{2}I_{\mu\nu\gamma\delta}\delta_D^{(4)}(x-y), \tag{5}$$

where $I_{\mu\nu\gamma\delta}$ is an identity tensor.

Then Gauge transformations are used to develop the second quantization of the weak-field gravitational field. This leads to a quantization of the field. All this makes it a Quantum Mechanical theory in a Quantum Mechanical form. Further details may be found in an extensive study of Ref. 3.

In addition, considering the perturbation $h_{\mu\nu}$ is assumed to be sourced by gravitating bodies and obeys the relation $|h_{\mu\nu}| \ll 1$ so that a linear-approximation analysis may be utilized. In addition, the trace-reverse tensor, $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$ is defined.[58] Here h is the trace of $h_{\mu\nu}$. Then using these definitions and considering only linear terms, $h_{\mu\nu}$ in the Einstein equations,

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{6}$$

where $G_{\mu\nu}$ and $T_{\mu\nu}$ are the Einstein tensor and the energy–momentum tensor, respectively, become

$$\Box\bar{h}_{\mu\nu} = 16\pi G T_{\mu\nu}. \tag{7}$$

Here the harmonic gauge condition, $\bar{h}^{\mu\nu}{}_{,\nu} = 0$ has been imposed.

A different development of theory of relativity has led to a diverse model to investigate an entirely different method. This is an entirely distinct development of a theory of gravity. This theory provides an interesting way of understanding the different process through cosmological constant observations. This provides an alternate process to understand development of physics in this area. Then there is a method developed by Mashhoon.[28] This deals with classical and quantum gravity to understand gravitational induction. In addition, it is established that the Faraday Law and Lenz's law are a part of electrodynamics.

3. Lagrangian Formalism for the GEM Theory

Here a brief introduction to the GEM Lagrangian formalism is presented. More details about this formulation have been done in Ref. 33. The GEM describes the dynamics of the gravitational field in a manner similar to that of the electromagnetic field. In order to construct a Lagrangian of the GEM field, the Maxwell-like equations are considered. The GEM equations are given as

$$\partial^i \mathcal{E}^{ij} = -4\pi G \rho^j, \tag{8}$$

$$\partial^i \mathcal{B}^{ij} = 0, \tag{9}$$

$$\epsilon^{(i|kl}\partial^k \mathcal{B}^{l|j)} + \frac{\partial \mathcal{E}^{ij}}{\partial t} = -4\pi G J^{ij}, \tag{10}$$

$$\epsilon^{(i|kl}\partial^k \mathcal{E}^{l|j)} + \frac{\partial \mathcal{B}^{ij}}{\partial t} = 0, \tag{11}$$

where G is the gravitational constant, ϵ^{ikl} is the Levi-Civita symbol, ρ^j is the vector mass density and J^{ij} is the mass current density. The symbol $(i|\cdots|j)$ denotes symmetrization of the first and last indices, i.e., i and j. The quantities $\mathcal{E}^{ij}$ and $\mathcal{B}^{ij}$ are the gravitoelectric and gravitomagnetic fields, respectively. These GEM fields are constructed from the Weyl tensor and its dual, respectively, by contracting with an arbitrary time-like vector, i.e.,

$$\mathcal{E}_{ab} = C_{abcd}u^c u^d, \quad \mathcal{B}_{ab} = \frac{1}{2}\epsilon_{acd}C^{cd}{}_{be}u^e, \tag{12}$$

with u^a being a four-velocity vector field, such that $u^a u_a = -1$ and the Weyl tensor that is connected with the curvature tensor is defined as

$$\begin{aligned} C_{\alpha\sigma\mu\nu} &= R_{\alpha\sigma\mu\nu} - \frac{1}{2}(R_{\nu\alpha}g_{\mu\sigma} + R_{\mu\sigma}g_{\nu\alpha} - R_{\nu\sigma}g_{\mu\alpha} - R_{\mu\alpha}g_{\nu\sigma}) \\ &\quad + \frac{1}{6}R(g_{\nu\alpha}g_{\mu\sigma} - g_{\nu\sigma}g_{\mu\alpha}), \end{aligned} \tag{13}$$

where $R_{\alpha\sigma\mu\nu}$ is the Riemann tensor, $R_{\mu\nu}$ is the Ricci tensor and R is the Ricci scalar.

To construct the GEM Lagrangian a gravitoelectromagnetic tensor potential $\tilde{\mathcal{A}}$, with components $\mathcal{A}^{ij}$ are defined. It is a rank-2 symmetric tensor field. Using this gravitational tensor potential and the GEM counterpart of the electromagnetic scalar potential ϕ, the fields $\mathcal{E}^{ij}$ and $\mathcal{B}^{ij}$ are written as[33]

$$\mathcal{E} = -\mathrm{grad}\,\varphi - \frac{\partial \tilde{\mathcal{A}}}{\partial t}, \tag{14}$$

$$\mathcal{B} = \mathrm{curl}\,\tilde{\mathcal{A}}. \tag{15}$$

Like the Maxwell equations, the GEM equations (8)–(11) may be written as

$$\partial_\mu F^{\mu\nu\alpha} = 4\pi G \mathcal{J}^{\nu\alpha}, \tag{16}$$

$$\partial_\mu \mathcal{G}^{\mu\langle\nu\alpha\rangle} = 0, \tag{17}$$

where $\mathcal{J}^{\nu\alpha}$ is a rank-2 tensor, which depends on the mass density, ρ^i and the current density J^{ij} and $F^{\mu\nu\alpha}$ is the GEM tensor defined as

$$F^{\mu\nu\alpha} = \partial^\mu \mathcal{A}^{\nu\alpha} - \partial^\nu \mathcal{A}^{\mu\alpha}, \tag{18}$$

and $\mathcal{G}^{\mu\nu\alpha}$ is the GEM dual tensor that is written as

$$\mathcal{G}^{\mu\nu\alpha} = \frac{1}{2}\epsilon^{\mu\nu\gamma\sigma}\eta^{\alpha\beta}F_{\gamma\sigma\beta}. \tag{19}$$

The nonzero components of $F^{\mu\nu\alpha}$ are $F^{0ij} = \mathcal{E}^{ij}$ and $F^{ijk} = \epsilon^{ijl}\mathcal{B}^{lk}$, where $i, j = 1, 2, 3$.

This construction leads to the GEM Lagrangian density,

$$\mathcal{L}_G = -\frac{1}{16\pi}F_{\mu\nu\alpha}F^{\mu\nu\alpha} - G\mathcal{J}^{\nu\alpha}\mathcal{A}_{\nu\alpha}. \tag{20}$$

It is important to note that the GEM theory is only valid to describe the weak gravitational fields. In the weak field approximation, there is a relation between the GEM theory and General Relativity. In order to analyze such a connection, the field equation (16) is written as

$$\Box A^{\nu\alpha} - \partial^\nu(\partial_\mu A^{\mu\alpha}) = 4\pi G\mathcal{J}^{\nu\alpha}, \tag{21}$$

where the GEM tensor, given in Eq. (18), has been used. Using the Lorenz-like gauge condition, i.e., $\partial_\mu A^{\mu\alpha} = 0$, Eq. (21) becomes

$$\Box A^{\nu\alpha} = 4\pi G\mathcal{J}^{\nu\alpha}. \tag{22}$$

Note that, the Lorenz-like gauge condition can be used due to the gauge transformation of the GEM potentials, that is similar to the electromagnetic potential. More details about gauge transformations of GEM field are given in Ref. 59.

Therefore, comparing Eqs. (22) and (7), it is possible to observe that GEM theory is equivalent to the linearized theory of gravity, or general relativity in a weak-field approach.

Now there are important points that should be interpreted carefully. A fundamental difference is that the GEM potential, $A_{\mu\nu}$, is the fundamental gravitational field in flat space-time, instead of a small perturbation, $h_{\mu\nu}$, that is defined in the weak-field approximation. Then the tensor potential has nothing to do with the perturbation of the space-time metric. Instead, it is connected directly with the description of the gravitational field in flat space-time. In addition it should be considered so that the quantization of GEM field leads to spin-2 gravitons.

An advantage of this formalism is that it allows a consideration of a full Lagrangian, Eq. (20), that describes gravitons, fermions, photons and their interactions. The vertices for interactions between fermions, photons and gravitons have been presented.[33] All this leads to a calculation of the differential cross section of the Möller scattering in GEM field.

4. The Cross Section for Gravitational Möller Scattering

Here the cross section of the gravitational Möller scattering is calculated for four distinct cases: (i) Lorentz invariant at zero temperature; (ii) Lorentz invariant at finite temperature; (iii) Lorentz-violating at zero temperature; and (iv) Lorentz-violating at finite temperature;

Before studying the gravitational Möller scattering with Lorentz violation, let us provide more details on the derivation of the gravitational cross section at zero and finite temperature, Secs. 4.1 and 4.2, as calculated in Ref. 60. These details will be useful for the calculations with Lorentz violation and finite temperature in Secs. 4.3 and 4.4.

4.1. *Lorentz invariant cross section at zero temperature*

The Lagrangian that describes the gravitational Möller scattering is given as

$$\begin{aligned}\mathcal{L} = &-\frac{1}{16\pi}F_{\mu\nu\alpha}F^{\mu\nu\alpha} - \frac{i}{2}(\bar{\psi}\gamma^\mu\partial_\mu\psi - \partial_\mu\bar{\psi}\gamma^\mu\psi)\\ &+ m\bar{\psi}\psi - \frac{i\kappa}{4}\mathcal{A}_{\mu\nu}(\bar{\psi}\gamma^\mu\partial^\nu\psi - \partial^\mu\bar{\psi}\gamma^\nu\psi),\end{aligned} \tag{23}$$

where ψ is the fermion field with $\bar{\psi} = \psi^\dagger\gamma_0$, m is the fermion mass, γ^μ are Dirac matrices and $\kappa = \sqrt{8\pi G}$ is the coupling constant.

The main objective is to calculate the cross section for the Möller scattering using the fermion-graviton interaction, Eq. (23). There are two diagrams that contribute for this process: t-channel and u-channel diagrams (Fig. 1).

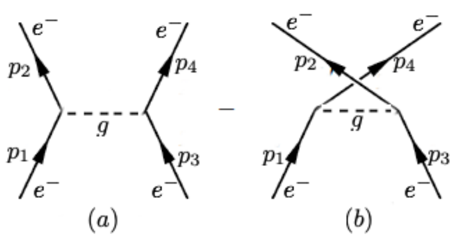

Fig. 1. The diagram (a) represent the t-channel and (b) the u-channel cross section of $e^- + e^- \to e^- + e^-$ scattering with g being a graviton.

The graviton propagator is defined as

$$D_{\mu\nu\alpha\rho}(q) = \frac{i}{2q^2}(\eta_{\mu\alpha}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\rho}) \tag{24}$$

where q is the momentum transferred. The graviton–fermions vertex[33] is

$$V^{\mu\nu} = -\frac{i\kappa}{4}(\gamma^\mu p_1^\nu + p_2^\mu\gamma^\nu). \tag{25}$$

The diagrams that define the graviton propagator and the interaction vertex are given in Fig. 2.

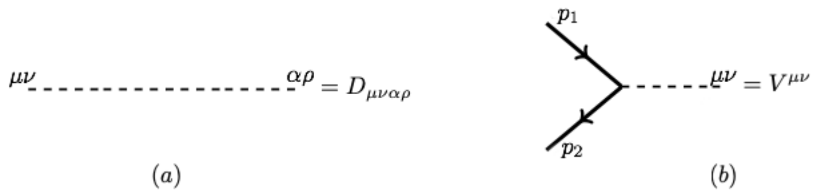

Fig. 2. The diagram (a) represent the graviton propagator and (b) the graviton–fermions interaction.

The electron–electron scattering with gravitons in the intermediate state is considered in the center of mass (CM) reference frame. This is a frame where the observer is travelling along with the CM of the system. In the CM frame, the momenta are defined as

$$p_1 = (E, \vec{p}), \quad p_2 = (E, \vec{p}), \quad p_3 = (E, -\vec{p'}) \quad \text{and} \quad p_4 = (E, -\vec{p'}), \tag{26}$$

where E and $\vec{p}$ are energy and momentum components, $|\vec{p}|^2 = |\vec{p'}|^2 = E^2$ and $\vec{p} \cdot \vec{p'} = E^2 \cos\theta$. Here, θ is the CM scattering angle.

The differential cross section is defined as

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|\mathcal{M}|^2, \tag{27}$$

where $s = 4E^2$ is the CM energy and $\mathcal{M}$ is the transition amplitude. The total transition amplitude is

$$\mathcal{M} = \mathcal{M}_t - \mathcal{M}_u, \tag{28}$$

where the relative minus sign is a consequence of the Fermi-statistics and $\mathcal{M}_t$ and $\mathcal{M}_u$ are contributions of diagrams, 1(a) and 1(b), respectively. The negative sign is due to the antisymmetry of the two electrons state. These transition amplitudes are

$$\mathcal{M}_t = \bar{u}(p_2)V^{\mu\nu}u(p_1)D_{\mu\nu\alpha\rho}(p_1 - p_2)\bar{u}(p_4)V^{\alpha\rho}u(p_3), \tag{29}$$

$$\mathcal{M}_u = \bar{u}(p_2)V^{\alpha\rho}u(p_3)D_{\mu\nu\alpha\rho}(p_3 - p_2)\bar{u}(p_4)V^{\mu\nu}u(p_1), \tag{30}$$

with u being the spinors for electrons. In order to calculate the differential cross section the relevant quantity is $|\mathcal{M}|^2$, i.e.,

$$|\mathcal{M}|^2 = |\mathcal{M}_t|^2 + |\mathcal{M}_u|^2 - 2\mathcal{M}_t\mathcal{M}_u^*. \tag{31}$$

Summing over the polarization of the electron, the total transition amplitude is

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{1}{4}\sum_{\text{spins}}(|\mathcal{M}_t|^2 + |\mathcal{M}_u|^2 - 2\mathcal{M}_t\mathcal{M}_u^*). \tag{32}$$

The calculation is carried out using

$$\bar{u}(p_2)\gamma_\alpha u(p_1)\bar{u}(p_1)\gamma^\alpha u(p_2) = \mathrm{tr}[\gamma_\alpha u(p_1)\bar{u}(p_1)\gamma^\alpha u(p_2)\bar{u}(p_2)] \tag{33}$$

and the completeness relations

$$\sum u(p_i)\bar{u}(p_i) = \not{p}_i + m \tag{34}$$

with $i = 1, 2, 3, 4$. The trace calculations involve the product of up to eight gamma matrices. Henceforth the electron mass is ignored since all the momenta are large compared with the electron mass, i.e., the ultrarelativistic limit. Thus, after some algebraic simplifications the squared transition amplitudes are

$$|\mathcal{M}_t|^2 = \frac{\kappa^4 E^4}{32}\frac{(3+\cos\theta)^3}{1-\cos\theta)^2}, \tag{35}$$

$$|\mathcal{M}_u|^2 = -\frac{\kappa^4 E^4}{32}\frac{(\cos\theta-3)^3}{(\cos\theta+1)^2}, \tag{36}$$

$$-2\mathcal{M}_t\mathcal{M}_u^* = \frac{\kappa^4 E^4}{512}\frac{(-612\cos 2\theta + \cos 4\theta + 2659)}{(1-\cos\theta)(1+\cos\theta)}. \tag{37}$$

Then the total square transition amplitude becomes

$$|\mathcal{M}|^2 = \frac{\kappa^4 E^4}{32}\left[\frac{(3+\cos\theta)^3}{(1-\cos\theta)^2} - \frac{(\cos\theta-3)^3}{(\cos\theta+1)^2} + \frac{(-612\cos 2\theta + \cos 4\theta + 2659)}{16(1-\cos\theta)(1+\cos\theta)}\right]. \tag{38}$$

Using this result, the differential cross section for graviton exchange is

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{1}{64\pi^2 s}\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2\\ &= \left(\frac{1}{256\pi^2 s}\right)\frac{\kappa^4 E^4}{32}\left[\frac{(3+\cos\theta)^3}{(1-\cos\theta)^2} - \frac{(\cos\theta-3)^3}{(\cos\theta+1)^2}\right.\\ &\quad\left. + \frac{(-612\cos 2\theta + \cos 4\theta + 2659)}{16(1-\cos\theta)(1+\cos\theta)}\right].\end{aligned} \tag{39}$$

The differential cross section for Möller scattering with graviton exchange only is different from its electromagnetic analog. The Möller scattering with photon exchange in the ultrarelativistic limit[61] is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8E^2}\left[\frac{1+\cos^2\theta/2}{\sin^4\theta/2} + \frac{1+\sin^4\theta/2}{\cos^4\theta/2} + \frac{2}{\sin^2\theta/2\cos^2\theta/2}\right], \tag{40}$$

where α is the fine-structure constant. An important note, the electromagnetic coupling constant α is dimensionless, while the gravitational coupling constant is proportional to the inverse of the energy, i.e., $\kappa \approx \sqrt{G} \approx 1/M_P$, where M_P is the Planck energy. In order to obtain the GEM coupling constant dimensionless, it should include a factor involving a characteristic energy scale of scattering process. For example, $\kappa \to \kappa' = \kappa E_c$, where E_c is an energy scale of scattering. If $E_c \lll M_P$, this implies that $\alpha \ggg \kappa'$, therefore, this confirms that the electromagnetic interaction is much stronger than the gravitational interaction. However, it is important to note that the gravitational effect would be the relevant effect at very high energy, such as the Planck scale. Here it is relevant to note that, at this very high scale of energy, some new physics, may emerge. Furthermore, nonlinear effects may arise. In order to eliminate these nonlinear effects, the limit of extremely very high energy can be avoided. In this case, our gravitational model is considered as an effective field theory, since effective field theory allows us to make predictions at intermediate energies without making unwarranted assumptions about what is going on at high energies. In addition, whatever the physics of high energy really is, it will leave residual effects at low energy.

4.2. *Cross section at finite temperature and Lorentz invariant*

Here the thermal effects for the gravitational Möller scattering are investigated. First, let us review the TFD formalism, the fundamental tools for the TFD formalism, with temperature effects are reviewed.

4.2.1. *TFD formalism*

TFD is a quantum field theory at finite temperature. This thermal theory is defined when a thermal vacuum $|0(\beta)\rangle$ is constructed. There are two basic ingredients to construct such a formalism: (a) the doubling of the degrees of freedom in a Hilbert space and (b) introduce the Bogoliubov transformation. This doubling is defined by the tilde ($\tilde{}$) conjugation rules, associating each operator in S to two operators in S_T , where the expanded space is $S_T = S \otimes \tilde{S}$, with S being the standard Hilbert space and $\tilde{S}$ the tilde or a dual Hilbert space. The Bogoliubov transformation introduces a rotation in the tilde and nontilde variables and thermal quantities. The Bogoliubov transformations are distinct for fermions and bosons.

Considering fermions with $c_p^\dagger$ and c_p being creation and annihilation operators, respectively, in the standard Hilbert space and $\tilde{c}_p^\dagger$ and $\tilde{c}_p$ being the same operators in the tilde space. For fermions, the Bogoliubov transformations are

$$c_p = \mathsf{u}(\beta) c_p(\beta) + \mathsf{v}(\beta)\tilde{c}_p^\dagger(\beta), \tag{41}$$

$$c_p^\dagger = \mathsf{u}(\beta) c_p^\dagger(\beta) + \mathsf{v}(\beta)\tilde{c}_p(\beta), \tag{42}$$

$$\tilde{c}_p = \mathsf{u}(\beta) \tilde{c}_p(\beta) - \mathsf{v}(\beta) c_p^\dagger(\beta), \tag{43}$$

$$\tilde{c}_p^\dagger = \mathsf{u}(\beta) \tilde{c}_p^\dagger(\beta) - \mathsf{v}(\beta) c_p(\beta), \tag{44}$$

where $\mathsf{u}(\beta) = \cos\theta(\beta)$ and $\mathsf{v}(\beta) = \sin\theta(\beta)$.

For bosons with $a_p^\dagger$ and a_p being creation and annihilation operators, respectively, in the standard Hilbert space and $\tilde{a}_p^\dagger$ and $\tilde{a}_p$ being similar operators in the tilde space, then the Bogoliubov transformations are

$$a_p = \mathsf{u}'(\beta) a_p(\beta) + \mathsf{v}'(\beta)\tilde{a}_p^\dagger(\beta), \tag{45}$$

$$a_p^\dagger = \mathsf{u}'(\beta) a_p^\dagger(\beta) + \mathsf{v}'(\beta)\tilde{a}_p(\beta), \tag{46}$$

$$\tilde{a}_p = \mathsf{u}'(\beta) \tilde{a}_p(\beta) + \mathsf{v}'(\beta) a_p^\dagger(\beta), \tag{47}$$

$$\tilde{a}_p^\dagger = \mathsf{u}'(\beta) \tilde{a}_p^\dagger(\beta) + \mathsf{v}'(\beta) a_p(\beta), \tag{48}$$

where $\mathsf{u}'(\beta) = \cosh\theta(\beta)$ and $\mathsf{v}'(\beta) = \sinh\theta(\beta)$.

An important characteristic of TFD formalism is that the propagator is written in two parts: one describes the flat space-time contribution and the other displays the thermal effect. As an example, consider the graviton propagator at finite temperature, which is given as

$$\langle 0(\beta)|\mathbb{T}[A_{\mu\nu}(x)A_{\rho\lambda}(y)]|0(\beta)\rangle = i\int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)}\Delta_{\mu\nu\rho\lambda}(k,\beta), \tag{49}$$

where $\mathbb{T}$ is the time ordering operator and $\Delta_{\mu\nu\rho\lambda}(k,\beta) = \Delta^{(0)}_{\mu\nu\rho\lambda}(k) + \Delta^{(\beta)}_{\mu\nu\rho\lambda}(k)$ with

$$\begin{aligned} \Delta^{(0)}_{\mu\nu\rho\lambda}(k) &= \frac{\eta_{\mu\rho}\eta_{\nu\lambda} + \eta_{\mu\lambda}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\lambda}}{2k^2}\,\tau, \\ \Delta^{(\beta)}_{\mu\nu\rho\lambda}(k) &= -\frac{2\pi i\delta(k^2)}{e^{\beta k_0}-1}\begin{pmatrix} 1 & e^{\beta k_0/2} \\ e^{\beta k_0/2} & 1 \end{pmatrix}(\eta_{\mu\rho}\eta_{\nu\lambda} + \eta_{\mu\lambda}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\lambda}), \end{aligned} \tag{50}$$

where $\Delta^{(0)}_{\mu\nu\rho\lambda}(k)$ and $\Delta^{(\beta)}_{\mu\nu\rho\lambda}(k)$ are at zero and finite temperature, respectively, and

$$\tau = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{51}$$

This formalism will be used to calculate the differential cross section at finite temperature for GEM Möller scattering.

4.2.2. *Cross section at finite temperature*

The differential cross section at finite temperature is defined as

$$\left(\frac{d\sigma}{d\Omega}\right)_\beta = \frac{1}{64\pi^2 s}\cdot\frac{1}{4}|\mathcal{M}(\beta)|^2, \tag{52}$$

where $\mathcal{M}(\beta)$ is the transition amplitude at finite temperature that is defined as

$$\mathcal{M}(\beta) = \langle f,\beta|\hat{S}^{(2)}|i,\beta\rangle, \tag{53}$$

with $\hat{S}^{(2)}$ being the second-order term of the $\hat{S}$-matrix, i.e.,

$$\hat{S}^{(2)} = -\frac{1}{2}\int d^4x\, d^4y\, \mathbb{T}[\hat{\mathcal{L}}_I(x)\hat{\mathcal{L}}_I(y)], \tag{54}$$

where $\hat{\mathcal{L}}_I(x) = \mathcal{L}_I(x) - \tilde{\mathcal{L}}_I(x)$ describes the interaction part and the thermal states are

$$\begin{aligned} |i,\beta\rangle &= c^\dagger_{p_1}(\beta) d^\dagger_{p_2}(\beta)|0(\beta)\rangle, \\ |f,\beta\rangle &= c^\dagger_{p_3}(\beta) d^\dagger_{p_4}(\beta)|0(\beta)\rangle, \end{aligned} \tag{55}$$

with $c^\dagger_{p_j}(\beta)$ and $d^\dagger_{p_j}(\beta)$ being creation operators. The total transition amplitude at finite temperature is

$$\mathcal{M}(\beta) = \mathcal{M}_t(\beta) - \mathcal{M}_u(\beta). \tag{56}$$

By taking the interaction part of the Lagrangian,

$$\hat{\mathcal{L}}_I = -\frac{i\kappa}{4}\mathcal{A}_{\mu\nu}(\bar{\psi}\gamma^\mu\partial^\nu\psi - \partial^\mu\bar{\psi}\gamma^\nu\psi) + \frac{i\kappa}{4}\mathcal{A}_{\mu\nu}(\bar{\tilde{\psi}}\gamma^\mu\partial^\nu\tilde{\psi} - \partial^\mu\bar{\tilde{\psi}}\gamma^\nu\tilde{\psi}), \tag{57}$$

the transition amplitude for the t-channel diagram becomes

$$\begin{aligned} \mathcal{M}_t(\beta) = -\frac{\kappa^2}{32}\int & d^4x\, d^4y\, d^4p\, N_p(\mathsf{u}^2 - \mathsf{v}^2)^2 e^{i(p_2-p_1)x} e^{i(p_4-p_3)y} \\ & \times \big[\bar{u}(p_2)\gamma^\mu p_1^\nu u(p_1)\bar{u}(p_4)\gamma^\alpha p_3^\rho u(p_3) + \bar{u}(p_2)\gamma^\mu p_1^\nu u(p_1)\bar{u}(p_4)p_4^\alpha\gamma^\rho u(p_3) \\ & + \bar{u}(p_2)p_2^\mu\gamma^\nu u(p_1)\bar{u}(p_4)\gamma^\alpha p_3^\rho u(p_3) + \bar{u}(p_2)p_2^\mu\gamma^\nu u(p_1)\bar{u}(p_4)p_4^\alpha\gamma^\rho u(p_3)\big] \\ & \times \langle 0(\beta)|\mathbb{T}[A_{\mu\nu}(x)A_{\alpha\rho}(y)]|0(\beta)\rangle, \end{aligned} \tag{58}$$

where the fermion field,

$$\psi(x) = \int dp\, N_p[c_p u(p)e^{-ipx} + d_p^\dagger v(p)e^{ipx}], \tag{59}$$

has been used, with c_p and d_p being annihilation operators for electrons and positrons, respectively, N_p is the normalization constant and $u(p)$ and $v(p)$ are Dirac spinors.

Using Bogoliubov transformations, the contribution due to the t-channel diagram becomes

$$\begin{aligned} \mathcal{M}_t(\beta) = -\frac{i\kappa^2 N_p}{32}\Bigg[& \bar{u}(p_2)(\gamma^\mu p_1^\nu + p_2^\mu\gamma^\nu)u(p_1)(\eta_{\mu\alpha}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\rho}) \\ & \times \tanh^2\left(\frac{\beta|k_0|}{2}\right)\bar{u}(p_4)(\gamma^\alpha p_3^\rho + p_4^\alpha\gamma^\rho)u(p_3)\Bigg] \\ & \times \left[\frac{\tau}{2k^2} - \frac{2\pi i\delta(k^2)}{e^{\beta k_0} - 1}\begin{pmatrix} 1 & e^{\beta k_0/2} \\ e^{\beta k_0/2} & 1 \end{pmatrix}\right], \end{aligned} \tag{60}$$

where $k = p_2 - p_1$. Here the definition of the graviton propagator at finite temperature, Eq. (49), is used.

In a similar way, the transition amplitude due to the u-channel diagram is given as

$$\begin{aligned}\mathcal{M}_u(\beta) = &-\frac{i\kappa^2 N_p}{32}\Big[\bar{u}(p_2)(\gamma^\alpha p_3^\rho + p_2^\alpha\gamma^\rho)u(p_3)(\eta_{\mu\alpha}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\rho})\\ &\times \tanh^2\left(\frac{\beta|q_0|}{2}\right)\bar{u}(p_4)(\gamma^\mu p_1^\nu + p_4^\mu\gamma^\nu)u(p_1)\Big]\\ &\times\left[\frac{\tau}{2q^2} - \frac{2\pi i\delta(q^2)}{e^{\beta q_0}-1}\begin{pmatrix}1 & e^{\beta q_0/2}\\ e^{\beta q_0/2} & 1\end{pmatrix}\right], \end{aligned} \tag{61}$$

where $q = p_3 - p_2$.

In order to calculate the differential cross section the relevant quantities are

$$\begin{aligned}|\mathcal{M}_t(\beta)|^2 = &\frac{\kappa^4E^4}{32}\frac{(3+\cos\theta)^3}{(1-\cos\theta)^2}[1+64\pi^2E^4(1-\cos\theta)^2\Pi_1(\beta)]\\ &\times\tanh^4\left(\frac{\beta|k_0|}{2}\right),\end{aligned} \tag{62}$$

$$\begin{aligned}\left|\mathcal{M}_u(\beta)\right|^2 = &-\frac{\kappa^4E^4}{32}\frac{(\cos\theta-3)^3}{(\cos\theta+1)^2}[1-64\pi^2E^4(1+\cos\theta)^2\Pi_2(\beta)]\\ &\times\tanh^4\left(\frac{\beta|q_0|}{2}\right),\end{aligned} \tag{63}$$

$$\begin{aligned}-2\mathcal{M}_t(\beta)\mathcal{M}_u^*(\beta) = &\frac{\kappa^4E^4}{512}\frac{(2659-612\cos 2\theta+\cos 4\theta)}{(1-\cos\theta)(1+\cos\theta)}[1-a_2\Pi_1^{1/2}(\beta)-a_1\Pi_2^{1/2}(\beta)\\ &+a_1a_2\Pi_1^{1/2}(\beta)\Pi_2^{1/2}(\beta)]\tanh^2\left(\frac{\beta|k_0|}{2}\right)\tanh^2\left(\frac{\beta|q_0|}{2}\right),\end{aligned} \tag{64}$$

where

$$\Pi_1(\beta) \equiv \frac{\delta^2(k^2)}{(e^{\beta k_0}-1)^2}\begin{pmatrix}1 & e^{\beta k_0/2}\\ e^{\beta k_0/2} & 1\end{pmatrix}^2, \tag{65}$$

$$\Pi_2(\beta) \equiv \frac{\delta^2(q^2)}{(e^{\beta q_0}-1)^2}\begin{pmatrix}1 & e^{\beta q_0/2}\\ e^{\beta q_0/2} & 1\end{pmatrix}^2. \tag{66}$$

$$a_1 = -8\pi i\tau E^2(1-\cos\theta) \quad \text{and} \quad a_2 = 8\pi i\tau E^2(1+\cos\theta). \tag{67}$$

Then the differential cross section at finite temperature, Eq. (52), for the GEM Möller scattering is

$$\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right)_\beta = &\left(\frac{1}{256\pi^2 s}\right)\frac{\kappa^4E^4}{32}\left\{\frac{(3+\cos\theta)^3}{(1-\cos\theta)^2}[1-a_1^2\Pi_1(\beta)]\tanh^4\left(\frac{\beta|k_0|}{2}\right)\right.\\ &-\frac{(\cos\theta-3)^3}{(\cos\theta+1)^2}[1+a_2^2\Pi_2(\beta)]\tanh^4\left(\frac{\beta|q_0|}{2}\right)\end{aligned}$$

$$+\frac{(2659-612\cos 2\theta+\cos 4\theta)}{16(1-\cos\theta)(1+\cos\theta)}$$
$$\times[1-a_2\Pi_1^{1/2}(\beta)-a_1\Pi_2^{1/2}(\beta)+a_1a_2\Pi_1^{1/2}(\beta)\Pi_2^{1/2}(\beta)]$$
$$\times\tanh^2\left(\frac{\beta|k_0|}{2}\right)\tanh^2\left(\frac{\beta|q_0|}{2}\right)\Bigg\}. \tag{68}$$

It is important to note that, gravitational phenomena requires measurements at high energy. This leads to high temperatures as well. Therefore, at the very high-energy limit, the thermal effects are relevant and perhaps dominant. In addition, when $T\to 0$ the cross section at zero temperature, i.e., Eq. (39), is recovered. Therefore, the cross section at zero temperature is a particular case of the result at finite temperature.

In the next sections, the Möller scattering with Lorentz-violating parameters at zero and finite temperature is studied. Similar investigation for the Bhabha scattering has been developed earlier.[62,63]

4.3. *Lorentz violation cross section at zero temperature*

The Lagrangian that describes the GEM Möller scattering with Lorentz violation is given as

$$\mathcal{L}=-\frac{1}{16\pi}F_{\mu\nu\alpha}F^{\mu\nu\alpha}-\bar{\psi}(i\gamma^\mu\overleftrightarrow{D_\mu}-m)\psi, \tag{69}$$

where ψ is the fermion field with $\bar{\psi}=\psi^\dagger\gamma_0$, m is the fermions mass, γ^μ are Dirac matrices and D_μ is the covariant derivative.

Although the structure of the SME describes all interactions of the graviton coupled to the SM fields,[50] there is an alternate procedure to introduce the Lorentz violation. This alternative involves nonminimal coupling terms that modify the vertex interaction between fermions and gravitons. The usual covariant derivative will be modified by a nonminimal coupling term, i.e.,

$$\overleftrightarrow{D_\mu}=\overleftrightarrow{\partial_\mu}-\frac{1}{2}gA_{\mu\nu}\overleftrightarrow{\partial^\nu}+\frac{i}{4}(k^{(5)})_{\mu\nu\alpha\beta\rho}\gamma^\nu F^{\alpha\beta\rho}, \tag{70}$$

where $g=\sqrt{8\pi G}$ is the gravitational coupling constant and $(k^{(5)})_{\mu\nu\alpha\beta\rho}$ is a tensor that belongs to the gravity sector of the nonminimal SME with mass dimension $d=5$.[64] Then the Lagrangian becomes

$$\mathcal{L}=-\frac{1}{16\pi}F_{\mu\nu\alpha}F^{\mu\nu\alpha}-\frac{i}{2}(\bar{\psi}\gamma^\mu\partial_\mu\psi-\partial_\mu\bar{\psi}\gamma^\mu\psi)+m\bar{\psi}\psi$$
$$-\frac{ig}{4}A_{\mu\nu}(\bar{\psi}\gamma^\mu\partial^\nu\psi-\partial^\mu\bar{\psi}\gamma^\nu\psi)+\frac{i}{4}(k^{(5)})_{\mu\nu\alpha\beta\rho}F^{\alpha\beta\lambda}\bar{\psi}\sigma^{\mu\nu}\psi, \tag{71}$$

where $\sigma^{\mu\nu}=\frac{i}{2}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)$. The interaction part of the Lagrangian is

$$\mathcal{L}_I=-\frac{ig}{4}A_{\mu\nu}(\bar{\psi}\gamma^\mu\partial^\nu\psi-\partial^\mu\bar{\psi}\gamma^\nu\psi)+\frac{i}{4}(k^{(5)})_{\mu\nu\alpha\beta\rho}F^{\alpha\beta\lambda}\bar{\psi}\sigma^{\mu\nu}\psi, \tag{72}$$

and the corresponding vertices are

$$\bullet \to -\frac{ig}{4}(\gamma^\mu p_1^\nu + p_2^\mu \gamma^\nu) \equiv V^{\mu\nu}_{(0)} \tag{73}$$

$$\circ \to \frac{i}{2}(k^{(5)})^{\mu\nu\alpha\beta\rho}\sigma_{\alpha\beta}q_\rho \equiv V^{\mu\nu}_{(1)}, \tag{74}$$

where q_ρ is the momentum transfer, which is considered as $q_\rho = \left(\sqrt{s}, 0\right)$, with s being the energy in the CM frame. It is important to note that, the first term in Eq. (72) describes the usual interaction between gravitons and fermions and the second term is a new interaction produced by the nonminimal coupling that leads to the Lorentz violation.

Our main objective is to calculate the corrections due to the Lorentz violation for the cross section. Considering the interaction Lagrangian, the transition amplitude due to the t-channel and u-channel are given, respectively, as

$$\mathcal{M}_t = \sum_{a,b} \bar{u}(p_2)V^{\mu\nu}_{(a)}u(p_1)D_{\mu\nu\alpha\rho}(p_1 - p_2)\bar{u}(p_4)V^{\alpha\rho}_{(b)}u(p_3), \tag{75}$$

$$\mathcal{M}_u = \sum_{a,b} \bar{u}(p_2)V^{\alpha\rho}_{(a)}u(p_3)D_{\mu\nu\alpha\rho}(p_3 - p_2)\bar{u}(p_4)V^{\mu\nu}_{(b)}u(p_1), \tag{76}$$

with $a, b = 0, 1$. Using the relations (33) and (34) the relevant quantity $|\mathcal{M}|^2$ becomes

$$|\mathcal{M}|^2_{LV} = |\mathcal{M}|^2_0 + |\mathcal{M}|^2_k, \tag{77}$$

where $|\mathcal{M}|^2_0$ is Lorentz invariant part, calculated in Eq. (38) and $|\mathcal{M}|^2_k$ is the Lorentz-violating part given as

$$|\mathcal{M}|^2_k = \frac{1}{64}\kappa^4 E^4 \Theta(\theta)(k^{(5)})^2, \tag{78}$$

where

$$\begin{aligned}\Theta(\theta) \approx\ & \frac{2(1+\cos\theta)^3[2(1+\cos\theta)(7+17\cos\theta)+\cos^2\theta]}{(\cos\theta-1)^3} \\ & + 8(7-23\cos\theta)\sin^8(\theta/2) + (-79-74\cos\theta+\cos 2\theta)\cot^2(\theta/2).\end{aligned} \tag{79}$$

Then the differential cross section with corrections due to Lorentz violation is

$$\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{LV}} &= \left(\frac{1}{256\pi^2 s}\right)\sum_{\mathrm{spins}}|\mathcal{M}|^2_{\mathrm{LV}} \\ &= \left(\frac{1}{256\pi^2 s}\right)\frac{\kappa^4E^4}{32}\left[\frac{(3+\cos\theta)^3}{(1-\cos\theta)^2} - \frac{(\cos\theta-3)^3}{(\cos\theta+1)^2}\right. \\ &\quad \left. + \frac{(-612\cos 2\theta + \cos 4\theta + 2659)}{16(1-\cos\theta)(1+\cos\theta)} + \frac{1}{2}\Theta(\theta)(k^{(5)})^2\right].\end{aligned} \tag{80}$$

Therefore, the Lorentz violation contribution is additive to the Lorentz invariant result. This result is important since the gravitational scattering should occur at very high energy. At this energy limit, only a small Lorentz violation is expected to emerge.

4.4. *Lorentz violation cross section at finite temperature*

In order to introduce temperature effects in the cross section of the GEM Möller scattering with Lorentz violation, consider the Lagrangian

$$\mathcal{L}_I = -\frac{ig}{4}A_{\mu\nu}(\bar{\psi}\gamma^\mu\partial^\nu\psi - \partial^\mu\bar{\psi}\gamma^\nu\psi) + \frac{i}{4}(k^{(5)})_{\mu\nu\alpha\beta\rho}F^{\alpha\beta\lambda}\bar{\psi}\sigma^{\mu\nu}\psi. \tag{81}$$

This describes the usual interaction between gravitons and fermions and the new interaction describes the nonminimal coupling between the GEM field and the bilinear fermion. Using this interaction Lagrangian the transition amplitude at finite temperature is

$$\mathcal{M}(\beta) = \mathcal{M}_0(\beta) + \mathcal{M}_k(\beta) + \mathcal{M}_{kk}(\beta) \tag{82}$$

where $\mathcal{M}_0(\beta)$ is the matrix element of the Lorentz invariant defined as

$$\begin{aligned}\mathcal{M}_0(\beta) = -\frac{1}{2}\int d^4x\, d^4y\, \langle f,\beta|\left(-\frac{ig}{4}A_{\mu\nu}(\bar{\psi}\gamma^\mu\partial^\nu\psi - \partial^\mu\bar{\psi}\gamma^\nu\psi)\right)\\ \times\left(-\frac{ig}{4}A_{\mu\nu}(\bar{\psi}\gamma^\mu\partial^\nu\psi - \partial^\mu\bar{\psi}\gamma^\nu\psi)\right)|i,\beta\rangle.\end{aligned} \tag{83}$$

The linear Lorentz violation term, i.e., $\mathcal{M}_k(\beta)$, is

$$\begin{aligned}\mathcal{M}_k(\beta) = -\int d^4x\, d^4y\, \langle f,\beta|\left(-\frac{ig}{4}A_{\mu\nu}(\bar{\psi}\gamma^\mu\partial^\nu\psi - \partial^\mu\bar{\psi}\gamma^\nu\psi)\right)\\ \times\left(\frac{i}{4}(k^{(5)})_{\mu\nu\alpha\beta\rho}F^{\alpha\beta\lambda}\bar{\psi}\sigma^{\mu\nu}\psi\right)|i,\beta\rangle,\end{aligned} \tag{84}$$

and $\mathcal{M}_{kk}(\beta)$ is the second order in the Lorentz-violating parameter. This term will be ignored since its contribution in the cross section is of the fourth order in the Lorentz-violating parameter. Hence it is very small, compared with the contribution of $\mathcal{M}_k(\beta)$ term. Note that, there are similar terms to the tilde part of the amplitude. Here, only the nontilde part is written since it describes the physical phenomenon.

Following similar steps, as in the Secs. 4.2 and 4.3, the differential cross section at finite temperature with Lorentz violation, is given as

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_\beta^{LI} + \left(\frac{d\sigma}{d\Omega}\right)_\beta^{LV}, \tag{85}$$

where $\left(\frac{d\sigma}{d\Omega}\right)_\beta^{\mathrm{LI}}$ is the differential cross section Lorentz invariant at finite temperature given in Eq. (68) and $\left(\frac{d\sigma}{d\Omega}\right)_\beta^{\mathrm{LV}}$ is the Lorentz-violating part at finite temperature that is given as

$$\left(\frac{d\sigma}{d\Omega}\right)_{\beta}^{LV} = \left(\frac{1}{256\pi^2 s}\right)\frac{\kappa^4 E^4}{64}(k^{(5)})^2\Bigg\{8(7-23\cos\theta)\sin^8(\theta/2)\left[\frac{\tau^2}{4k^4}+4\pi^2\Pi_1(\beta)\right]$$
$$\times\tanh^4(\beta k_0/2)+\frac{2(1+\cos\theta)^3[2(1+\cos\theta)(7+17\cos\theta)+\cos^2\theta]}{(\cos\theta-1)^3}$$
$$\times\left[\frac{\tau^2}{4q^4}+4\pi^2\Pi_2(\beta)\right]\tanh^4(\beta q_0/2)+\frac{\cot^2(\theta/2)}{2}$$
$$\times(-79-74\cos\theta+\cos 2\theta)\left[\frac{\tau^2}{4k^2q^2}-\frac{\pi i\tau}{k^2}\Pi_2^{1/2}(\beta)-\frac{\pi i\tau}{q^2}\Pi_1^{1/2}(\beta)\right.$$
$$\left.-4\pi^2\Pi_1^{1/2}(\beta)\Pi_2^{1/2}(\beta)\right]\tanh^2(\beta k_0/2)\tanh^2(\beta q_0/2)\Bigg\}, \quad (86)$$

where $\Pi_1(\beta)$ and $\Pi_2(\beta)$ are given in Eqs. (65) and (66). Then the results exhibit both temperature and Lorentz violation effects. In addition, the temperature effect in the gravitational Möller scattering may contribute to a new class of constraints on Lorentz violation parameters. A measurement will be possible if the experimental accuracy improves.

It is important to note that, small violation of Lorentz symmetry emerges at the Planck scale. These energies are unattainable experimentally. Lorentz violation effect has to be suppressed at lower energies, but tiny deviations may still exist, motivating sensitive tests of Lorentz invariance. Even though these effects are suppressed at observable energies, astrophysical observations can still be sensitive to new physics since tiny deviations accumulate over large distances. Typically, the strongest constraints on Lorentz violation result from astrophysical measurements.[65–67] Then the Lorentz-violating parameters become dominant well beyond the Planck scale.

5. Conclusion

The development of the Möller scattering includes the simple structure for scattering as well as for developing a structure that involves important effect of the breakdown of the interaction structure. This really leads to finding more detailed information about the effect of gravity. Einstein made an effort to obtain the role of interactions other than gravity. It required a more sophisticated physical description. And of course, use of Bhabha and Compton scattering proved to be of great help to established the overall developments of physical aspects. And overall dealing with classical and quantum gravity led to the discovery ideas like Faraday law of conduction and Lenz' law. These are just a few of the interesting developments. And even beyond that there are new developments of $f(R)$ theory of relativity that has opened a brand new field of gravity. The field to learn its importance is a major help to even the young persons entering the field will be affected and will develop new ideas and knowledge. There is no doubt that knowledge grows slowly but yields new dimensions. These are the things that keep advancement even further remains a part of the excitement.

The effective field theory is considered to be a description of gravity. In this context, the fundamental theory at very high energies leads to general relativity at low energies. Here the GEM theory is a gravitational effective field theory that is equivalent to general relativity in a weak-field approach. The gravitational electron–electron scattering, known as Möller scattering is studied using electromagnetic fields of GEM. A formal analogy between the gravitational and the electromagnetic fields lead to the notion of GEM to describe gravitation. The Weyl approach is considered to construct a Lagrangian formulation for GEM. In this formalism, a symmetric gravitoelectromagnetic tensor potential that describes the gravitational interaction is defined. The differential cross section for the GEM Möller scattering is calculated. Although the electromagnetic interaction is dominant, the graviton exchange does contribute to the amplitude of this process. The gravitational effects are small, but they may be measured at an appropriate energy scale. The GEM electron–electron scattering may provide a window for an independent study of gravitational effects at astronomical scales, where the gravitational interactions are dominant. Four different cases are considered in this study. First, the differential cross section for GEM Möller scattering at zero temperature with Lorentz invariant form is calculated. The results shown here are different from those obtained in Ref. 34, where the cross section for the gravitational Bhabha scattering is investigated. Although these scatterings are similar, the process is very different. The GEM Bhabha scattering consists in an electron–positron scattering with gravitons in the intermediate state, while the GEM Möller scattering is an electron–electron scattering with graviton in the intermediate state. The differential cross section for these processes are distinct. In the second case, thermal effects are included in the cross section. This is an interesting result since the temperature effects are relevant at astronomical scale. In the third and fourth cases, the Lorentz violation is introduced in both cases, at zero and finite temperature. If all Lorentz-violating parameters are considered, relevant effects may arise for processes at very high energies, like astrophysical processes. In addition, these results give us a reasonable estimate of the Lorentz-violating parameters at high temperatures. At present, this result is of theoretical interest and cannot provide a direct way to measure the gravitational effect. However, as stated by Donoghue,[2] gravity (here GEM) very naturally fits into the framework of an effective field theory. Then effective field theory allows us to make predictions at present energies without making unwarranted assumptions about what is going on at higher energies. In fact, it is potentially even a better effective field theory than the SM as the quantum corrections are very small and the theory shows no hint of a breakdown before the Planck scale. Therefore, constraints on the parameters of the model, such as the Lorentz-violating parameter and temperature effects, can be obtained if the precision of measurements will improve significantly. Furthermore, although high-energy astrophysics observations provide the best possibilities to detect a very small violation of Lorentz invariance, nothing has been measured so far. At present, terrestrial experiments and astrophysical observations have been used to constraint Lorentz-violating parameters. Therefore, the search for an energy scale in which this violation is dominant or observable continues.

Acknowledgments

This work by A.F.S. is supported by CNPq projects 308611/2017-9 and 430194/2018-8.

References

1. J. F. Donoghue, *AIP Conf. Proc.* **1483**, 73 (2012).
2. J. F. Donoghue, arXiv:gr-qc/9512024.
3. J. F. Donoghue, arXiv:1702.00319 [hep-th].
4. J. F. Donoghue, *Phys. Rev. D* **50**, 3874 (1994).
5. C. P. Burgess, *Living Rev. Relativ.* **7**, 5 (2004).
6. V. A. Kostelecky and S. Samuel, *Phys. Rev. D* **39**, 683 (1989).
7. C. P. Burgess, J. M. Cline, E. Filotas, J. Matias and G. D. Moore, *JHEP* **3**, 043 (2002).
8. R. Gambini and J. Pullin, *Phys. Rev. D* **59**, 124021 (1999).
9. M. Pospelov and Y. Shang, *Phys. Rev. D* **85**, 105001 (2012).
10. G. Cantor, *Phys. Educ.* **26**, 289 (1991).
11. J. C. Maxwell, *Phil. Trans. Soc. Lond.* **155**, 492 (1865).
12. O. Heaviside, *Electrician* **31**, 259 (1893).
13. O. Heaviside, *Electrician* **31**, 281 (1893).
14. H. Weyl, *Sitzungesber Deutsch. Akad. Wiss. Berli* **465** (1918); H. Weyl, *Space, Time, Matter* (Dover, 1952).
15. T. Kaluza, *Sitzungsber. Preuss. Akad. Wiss. Berlin* (*Math. Phys.*) **1921**, 966 (1921); O. Klein, *Z. Phys.* **37**, 895 (1926) [*Surveys High Energ. Phys.* 5, 241 (1986)].
16. A. Einstein, *Sitzungsber. Preuss. Akad. Wiss.* **1928**, 224 (1928).
17. H. Thirring, *Physikalische Zeitschrift* **19**, 204 (1918).
18. A. Matte, *Can. J. Math.* **5**, 1 (1953).
19. W. B. Campbell and T. A. Morgan, *Physica* **53**, 264 (1971).
20. W. B. Campbell, *Gen. Relat. Gravit.* **4**, 137 (1973).
21. W. B. Campbell and T. A. Morgan, *Am. J. Phys.* **44**, 356 (1976).
22. W. B. Campbell, J. Macek and T. A. Morgan, *Phys. Rev. D* **15**, 2156 (1977).
23. V. B. Braginsky, C. M. Caves and K. S. Thorne, *Phys. Rev. D* **15**, 2047 (1977).
24. R. T. Jantzen, P. Carini and D. Bini, *Ann. Phys.* **215**, 1 (1992).
25. A. Tartaglia and M. L. Ruggiero, *Eur. J. Phys.* **25**, 203 (2004).
26. D. Bini, C. Cherubini, C. Chicone and B. Mashhoon, *Class. Quantum Grav.* **25**, 225014 (2008).
27. Q. G. Bailey, *Phys. Rev. D* **82**, 065012 (2010).
28. B. Mashhoon, arXiv:gr-qc/0311030.
29. I. Ciufolini, *Class. Quantum Grav.* **17**, 2369 (2000).
30. C. W. F. Everitt *et al.*, *Phys. Rev. Lett.* **106**, 221101 (2011).
31. L. F. Costa and C. A. R. Herdeiro, *Phys. Rev. D* **78**, 024021 (2008).
32. R. Maartens and B. A. Bassett, *Class. Quantum Grav.* **15**, 705 (1998).

33. J. Ramos, M. de Montigny and F. C. Khanna, *Gen. Relat. Gravit.* **42**, 2403 (2010).
34. A. F. Santos and F. C. Khanna, *Class. Quant. Grav.* **34**, 205007 (2017).
35. S. C. Ulhoa, A. F. Santos and F. C. Khanna, *Gen. Relat. Gravit.* **49**, 54 (2017).
36. B. S. DeWitt, *Phys. Rev.* **162**, 1239 (1967).
37. B. R. Holstein, *Am. J. Phys.* **74**, 1002 (2006).
38. C. Möller, *Ann. Phys.* **14**, 331 (1932).
39. A. F. Santos and F. C. Khanna, *Adv. High Energ. Phys.* **2018**, 596129 (2018).
40. A. F. Santos and F. C. Khanna, *Int. J. Mod. Phys. A* **34**, 1950044 (2019).
41. T. Matsubara, *Prog. Theor. Phys.* **14**, 351 (1955).
42. J. Schwinger, *J. Math. Phys.* **2**, 407 (1961); J. Schwinger, Lecture Notes of Brandeis University Summer Institute (1960).
43. Y. Takahashi and H. Umezawa, *Coll. Phenomena* **2**, 55 (1975); *Int. J. Mod. Phys. B* **10**, 1755 (1996).
44. Y. Takahashi, H. Umezawa and H. Matsumoto, *Thermofield Dynamics and Condensed States* (North-Holland, 1982); F. C. Khanna, A. P. C. Malbouisson, J. M. C. Malboiusson and A. E. Santana, *Thermal Quantum Field Theory*: *Algebraic Aspects and Applications* (World Scientific, 2009).
45. H. Umezawa, *Advanced Field Theory*: *Micro*, *Macro and Thermal Physics* (AIP, 1993).
46. A. E. Santana and F. C. Khanna, *Phys. Lett. A* **203**, 68 (1995).
47. A. E. Santana, F. C. Khanna, H. Chu and C. Chang, *Ann. Phys.* **249**, 481 (1996).
48. V. A. Kostelecky and R. Potting, *Phys. Rev. D* **51**, 3923 (1995).
49. D. Colladay and V. A. Kostelecky, *Phys. Rev. D* **55**, 6760 (1997); *Phys. Rev. D* **58**, 116002 (1998).
50. V. A. Kostelecky, *Phys. Rev. D* **69**, 105009 (2004).
51. A. F. Santos and F. C. Khanna, *Int. J. Mod. Phys. A* **33**, 1850015 (2018).
52. A. Felice and S. Tsujikawa, *Living Rev. Relativ.* **13**, 3 (2010).
53. T. P. Sotiriou and V. Faraoni, *Rev. Mod. Phys.* **82**, 451 (2010).
54. C. P. L. Berry and J. R. Gair, *Phys. Rev. D* **83**, 104022 (2011).
55. T. Baker, E. Bellini, P. G. Ferreira, M. Lagos, J. Noller and I. Sawicki, *Phys. Rev. Lett.* **119**, 251301 (2017).
56. E. J. Copeland, M. Sami and S. Tsujikawa, *Int. J. Mod. Phys. D* **15**, 1753 (2006).
57. A. B. Antolinez, C. G. Bohmer and M. Nowakowski, *Class. Quantum Grav.* **23**, 485 (2006).
58. C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation* (Freeman, 1973).
59. J. Ramos, M. de Montigny and F. C. Khanna, *Gen. Relat. Gravit.* **50**, 83 (2018).
60. A. F. Santos and F. C. Khanna, *Int. J. Mod. Phys. A* **34**, 1950044 (2019).
61. W. Greiner and J. Reinhardt, *Quantum Electrodynamics* (Springer-Verlag, 2009).
62. A. F. Santos and F. C. Khanna, *Int. J. Mod. Phys. A* **33**, 1850015 (2018).
63. A. F. Santos and F. C. Khanna, *Mod. Phys. Lett. A* **33**, 1850061 (2018).
64. Q. G. Bailey, V. A. Kostelecky and R. Xu, *Phys. Rev. D* **91**, 022006 (2015).
65. D. Mattingly, *Living Rev. Relativ.* **8**, 5 (2005).
66. F. W. Stecker, in *Proc. of the Sixth Meeting on CPT and Lorentz Symmetry* (*CPT'13*) (2014), p. 73.
67. A. S. Friedman, R. Gerasimov, F. Kislat, D. Leon, W. Stevens, D. Tytler and B. G. Keating, arXiv:2003.00647 [astro-ph.HE].