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Gravitational Bhabha scattering

A F Santos[1] **and Faqir C Khanna**[2,3]

[1] Instituto de Física, Universidade Federal de Mato Grosso, 78060-900, Cuiabá, Mato Grosso, Brazil

[2] Department of Physics and Astronomy, University of Victoria, 3800 Finnerty Road Victoria, BC, Canada

E-mail: alesandroferreira@fisica.ufmt.br and khannaf@uvic.ca

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Abstract

Gravitoelectromagnetism (GEM) as a theory for gravity has been developed similar to the electromagnetic field theory. A weak field approximation of Einstein theory of relativity is similar to GEM. This theory has been quantized. Traditional Bhabha scattering, electron–positron scattering, is based on quantized electrodynamics theory. Usually the amplitude is written in terms of one photon exchange process. With the development of quantized GEM theory, the scattering amplitude will have an additional component based on an exchange of one graviton at the lowest order of perturbation theory. An analysis will provide the relative importance of the two amplitudes for Bhabha scattering. This will allow an analysis of the relative importance of the two amplitudes as the energy of the exchanged particles increases.

Keywords: Bhabha scattering, cross section, gravitoelectromagnetism

(Some figures may appear in colour only in the online journal)

1. Introduction

Gravitation deals with the weakest force in nature and it governs the dynamics of the large scale universe. Other forces, weak, electromagnetic and strong, are quantum in nature while the theory of gravitation is classical. No unified theory has been developed that combines these two type of forces. The Einstein theory of gravitation has been confirmed in numerous experiments dealing with various aspects of the universe. However, there are some observational data, such as the accelerated expansion of the universe, and presence of dark matter, which have not been satisfactorily explained by this theory. Alternative ideas have been suggested in attempts to solve these problems [1, 2].

[3] Professor Emeritus—Physics Department, Theoretical Physics Institute, University of Alberta Edmonton, Alberta, Canada

Numerous attempts have made to unify gravity and electromagnetism theories. These begin with Faraday in 1832. He conducted experiments in order to detect if the gravitational field is induced by electric current in a circuit [3]. Using the similarity between Newton's law and Coulomb's law, Maxwell [4] formulated a theory of gravitation. Heaviside [5, 6] developed a set of Maxwell-like equations for gravity. Generalization of Riemann geometry was proposed by Weyl [7] to unify the two theories. An extension of gravity theory to five dimensions was developed by Kaluza–Klein [8]. Here the gravitoelectromagnetism (GEM) theory [9–16] will be considered. A formal analogy between the gravitational and the electromagnetic phenomena led to the idea of GEM to describe gravitation. Such an approach is based on two assumptions. First, there is a gravitomagnetic field connected with moving masses. Second, the speed of gravitational field propagation is equal to the speed of light. GEM theory can be analyzed in three different ways: (i) using the similarity between the linearized Einstein and Maxwell equations [17], this is valid within the framework of the weak-field approximation; (ii) based on an approach using tidal tensors [18] and (iii) the decomposition of the Weyl tensor into gravitomagnetic and gravitoelectric components [19]. Although these approaches are extensively studied, there are still issues that need clarification. For example, in the case where the analogy is based on linearized Einstein theory, its limit of validity is not defined. Some studies limit the analogy to stationary configurations [20] while others claim it can be non-stationary [21]. In the approach that uses the Weyl tensor, it is known that the gravitoelectric part is regarded as a generalization of the Newtonian tidal tensor, while the gravitomagnetic part is not completely understood since there are doubts about its source. In addition it is not clear if the gravitomagnetic field vanishes in homogeneous rotating universes [22] and furthermore its Newtonian limit is not well understood. Here the Weyl tensor approach is used and a Lagrangian formulation for GEM [25] is developed. In this formalism $A_{\mu\nu}$ is a symmetric gravitoelectromagnetic tensor potential which describes the gravitational interaction. GEM allows electromagnetic scattering processes with gravitons as intermediate state in addition to the photon electromagnetic scattering. Here the GEM [25] theory is quantized to consider the interaction of gravitons with fermions.

Bhabha scattering ($e^+e^- \longrightarrow e^+e^-$) with photons in the intermediate state has a long history [26]. This scattering has been used to study quantum electrodynamics (QED) and electroweak interactions [27–29]. Experiments at colliders have observed this scattering process [30–33]. This scattering has been essential to test different theories beyond the standard model. For example, theories with extra dimensions have been investigated [34], generalized QED has been analyzed [35] and Lorentz violation theory has been studied [36, 37]. In this paper the differential cross section for Bhabha scattering in the framework of GEM is calculated. This opens a new venue to investigate the role of gravity in electromagnetic processes. Different sources of energy may be considered to analyze the gravitational Bhabha scattering. Cosmic rays are energetic, subatomic particles which produce secondary particles, when they collide with gas atoms in the interstellar medium, that are observed. The electron–positron pair may be produced in this process. This radiation is measured at a range of energies. Cosmic radiation at low energies is very common. These particles originate from sources close to the Earth such as the Sun. At high energies the production of particles decreases steeply. The highest energetic cosmic rays have energy exceeding 10^{19} eV [38–41]. Particles with such energies are produced in a distant part of the universe. These particles in transversing such large distant will interact with particles on their way to the earth. In fact these particles will produce a large number of particles in collisions. In particular by the time this shower of particles approaches the earth, the particles have reasonably large energy. The shower will have thousands of particles with high energies and can be detected on the earth.

An increase in the number of positrons relative to electrons at high energy that reach the earth has been observed [42–44]. This excess of positrons may indicate that cosmic rays come from an unknown source, such as dark matter. The scattering of e^+e^- with gravitons in the intermediate state may produce similar results. The gravitational effects in the Bhabha scattering may contribute to the understanding of these observations. Another source for investigatin gravitational effects are neutron stars. The neutron stars are objects that have high density with mass around 1–3 solar masses [45, 46]. As the gravitational field of such objects is strong, the Bhabha scattering is modified.

This paper is organized as follows. In section 2, a brief introduction to GEM is presented. In section 3, the graviton–fermions interaction is introduced. In section 4, the differential cross section for the Bhabha scattering in the GEM is calculated using different gravitational coupling constants. In section 5, some concluding remarks are made. Here $\hbar = c = 1$ are used and the metric of flat Minkowski space is given by $(-1, +1, +1, +1)$.

2. Gravitoelectromagnetism—GEM

GEM describes the dynamics of the gravitational field in a manner similar to that of the electromagnetic field. There are many interesting applications of gravitoelectromagnetism and there is a great potential for technological applications. For example, just as rotating superconductors give rise to strong magnetic fields, the so called anomalous London moment, one should also expect unusually strong gravitomagnetic and gravitoelectric fields produced by rotating superfluids [23, 24].

Here the GEM approach will be used with the Weyl tensor components (C_{ijkl}) being: $\mathcal{E}_{ij} = -C_{0i0j}$ (gravitoelectric field) and $\mathcal{B}_{ij} = \frac{1}{2}\epsilon_{ikl}C^{kl}_{0j}$ (gravitomagnetic field). These gravitoelectric and gravitomagnetic spatial tensors are, in principle, physically measurable in frames of co-moving observers [19]. An important note, the Weyl tensor is connected with the curvature tensor, it is the traceless part of the Riemann tensor, and its is defined as

$$C_{\alpha\sigma\mu\nu} = R_{\alpha\sigma\mu\nu} - \frac{1}{2}\left(R_{\nu\alpha}g_{\mu\sigma} + R_{\mu\sigma}g_{\nu\alpha} - R_{\nu\sigma}g_{\mu\alpha} - R_{\mu\alpha}g_{\nu\sigma}\right) + \frac{1}{6}R\left(g_{\nu\alpha}g_{\mu\sigma} - g_{\nu\sigma}g_{\mu\alpha}\right), \tag{1}$$

where $R_{\alpha\sigma\mu\nu}$ is the Riemann tensor, $R_{\mu\nu}$ is the Ricci tensor and R is the Ricci scalar. The Weyl tensor determines tidal accelerations due to the gravitational field, rather than absolute accelerations [11]. In addition the Weyl tensor, in terms of the energy momentum tensor, does appear to be the natural choice for the field strengths of the gravitation field.

The field equations for the components of the Weyl tensor have a structure similar to those of the Maxwell equations. The GEM equations are given as

$$\partial^i\mathcal{E}^{ij} = -4\pi G\rho^j, \tag{2}$$

$$\partial^i\mathcal{B}^{ij} = 0, \tag{3}$$

$$\epsilon^{(i|kl}\partial^k\mathcal{B}^{l|j)} + \frac{\partial\mathcal{E}^{ij}}{\partial t} = -4\pi GJ^{ij}, \tag{4}$$

$$\epsilon^{(i|kl}\partial^k\mathcal{E}^{l|j)} + \frac{\partial\mathcal{B}^{ij}}{\partial t} = 0, \tag{5}$$

where G is the gravitational constant, ϵ^{ikl} is the Levi-Civita symbol, ρ^j is the vector mass density and J^{ij} is the mass current density. The symbol $(i|\cdots|j)$ denotes symmetrization of the first and last indices, i.e. i and j.

To construct a Lagrangian formulation that describes the GEM equations, fields $\mathcal{E}^{ij}$ and $\mathcal{B}^{ij}$ are defined as (details are given in [25])

$$\mathcal{E} = -\mathrm{grad}\,\varphi - \frac{\partial\tilde{\mathcal{A}}}{\partial t}, \tag{6}$$

$$\mathcal{B} = \mathrm{curl}\,\tilde{\mathcal{A}}, \tag{7}$$

where $\tilde{\mathcal{A}}$ with components $\mathcal{A}^{\mu\nu}$ is a symmetric rank-2 tensor field, gravitoelectromagnetic tensor potential, and φ is the GEM vector counterpart of the electromagnetic scalar potential ϕ. Using the tensor $\mathcal{A}^{\mu\nu}$, a gravitoelectromagnetic tensor $F^{\mu\nu\alpha}$ is defined as

$$F^{\mu\nu\alpha} = \partial^\mu\mathcal{A}^{\nu\alpha} - \partial^\nu\mathcal{A}^{\mu\alpha}, \tag{8}$$

where $\mu,\nu,\alpha = 0,1,2,3$. Then the Maxwell-like equations are written as

$$\partial_\mu F^{\mu\nu\alpha} = 4\pi G\mathcal{J}^{\nu\alpha}, \tag{9}$$

$$\partial_\mu\mathcal{G}^{\mu\langle\nu\alpha\rangle} = 0, \tag{10}$$

where $\mathcal{J}^{\nu\alpha}$ depends on the mass density, ρ^i, and the current density J^{ij}. The non-zero components of $F^{\mu\nu\alpha}$ are $F^{0ij} = \mathcal{E}^{ij}$ and $F^{ijk} = \epsilon^{ijl}\mathcal{B}^{lk}$ where $i,j = 1,2,3$. The dual GEM tensor, $\mathcal{G}^{\mu\nu\alpha}$, is defined as

$$\mathcal{G}^{\mu\nu\alpha} = \frac{1}{2}\epsilon^{\mu\nu\gamma\sigma}\eta^{\alpha\beta}F_{\gamma\sigma\beta}. \tag{11}$$

Then the GEM lagrangian is written as

$$\mathcal{L}_G = -\frac{1}{16\pi}F_{\mu\nu\alpha}F^{\mu\nu\alpha} - G\,\mathcal{J}^{\nu\alpha}\mathcal{A}_{\nu\alpha}. \tag{12}$$

Although the symmetry properties in weak field approach of $A_{\mu\nu}$ are similar to those of $h_{\mu\nu}$ and the interactions terms in the Lagrangian look like those of the linear case, our approach is different, since the nature of $A_{\mu\nu}$ is different from $h_{\mu\nu}$. In addition, the tensor potential has nothing to do with the perturbation of the spacetime metric. Instead, it is connected directly with the description of the gravitational field in flat spacetime.

3. Graviton–fermion interaction

The interaction Lagrangian between gravitons and fermions in GEM is given as [25]

$$\begin{aligned}\mathcal{L} = &-\frac{1}{16\pi}F_{\mu\nu\alpha}F^{\mu\nu\alpha} - \frac{\mathrm{i}}{2}\left(\bar{\psi}\gamma^\mu\partial_\mu\psi - \partial_\mu\bar{\psi}\gamma^\mu\psi\right)\\ &+ m\bar{\psi}\psi - \frac{\mathrm{i}\kappa}{4}\mathcal{A}_{\mu\nu}\left(\bar{\psi}\gamma^\mu\partial^\nu\psi - \partial^\mu\bar{\psi}\gamma^\nu\psi\right),\end{aligned} \tag{13}$$

where ψ is the fermion field with $\bar{\psi} = \psi^\dagger\gamma_0$, m is the fermions mass, γ^μ are Dirac matrices that obey the spinor representation of the Clifford algebra and they anti-commute, i.e. $\{\gamma^\mu,\gamma^\nu\} = 2\eta^{\mu\nu}$, and $\kappa = \sqrt{8\pi G}$ is the coupling constant. Here the tensor potential $A_{\mu\nu}$ is the fundamental field which describes the gravitational interaction. The interaction of gravitons with fermions is given in the same way as in [47]; an essential difference being that the

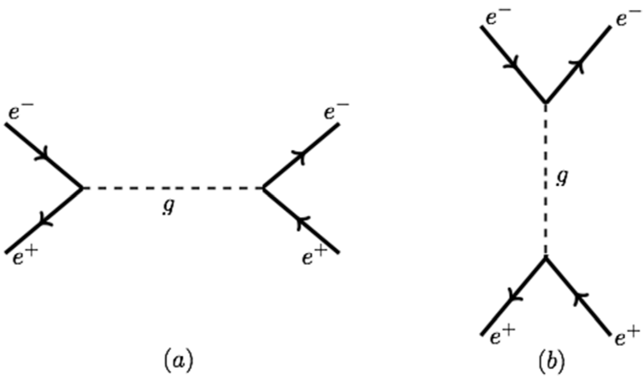


Figure 1. The diagram (a) represent the annihilation process and (b) the exchange process.

GEM potential, $A_{\mu\nu}$, is the fundamental gravitational field in flat spacetime, instead of the small perturbation, $h_{\mu\nu}$, in the weak-field approximation of the Einstein field.

Using this theory, the objective is to calculate the differential cross section for Bhabha scattering. There are two diagrams that contribute for this process: (i) the annihilation diagram, figure 1(a) and (ii) the exchange diagram, figure 1(b).

The graviton propagator is

$$\mu\nu \;\text{- - - - - - - - - - - - - - - - -}\; \alpha\rho = \frac{i}{2q^2}\left(\eta_{\mu\alpha}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\rho}\right)$$

where q is the momentum transferred. It is important to note that, the graviton propagator is expressed in terms of the tensor $A_{\mu\nu}$ and it is defined as

$$\mathrm{i}D_{\mu\nu\alpha\rho}(x-y) = \langle 0(\beta)|T\left[A_{\mu\nu}(x)A_{\alpha\rho}(y)\right]|0(\beta)\rangle, \tag{14}$$

where T is the time ordering operator and the tensor $A_{\mu\nu}(x)$ is given by

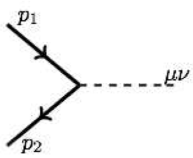
$$A_{\mu\nu}(x) = \int \frac{\mathrm{d}^3k}{\sqrt{2\omega_k(2\pi)^3}} \sum_\lambda \epsilon_{\mu\nu}(k,\lambda)\left(a_{k,\lambda}\mathrm{e}^{-\mathrm{i}k_\rho x^\rho} + a^\dagger_{k,\lambda}\mathrm{e}^{\mathrm{i}k_\rho x^\rho}\right), \tag{15}$$

with $\epsilon_{\mu\nu}(k,\lambda)$ being the polarization tensor and $a^\dagger, a$ are the creation and destruction operators, respectively. In the calculations for the graviton propagator, summation over the polarization tensor leads to

$$\sum_\lambda \epsilon_{\mu\nu}(k,\lambda)\epsilon_{\alpha\rho}(k,\lambda) = \frac{1}{2}\left(\eta_{\mu\alpha}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\rho}\right). \tag{16}$$

Then the graviton propagator is expressed in terms of the metric (Minkowski) tensor and not the tensor $A_{\mu\nu}$. This result is similar for the photon case, where the propagator is $\Delta_{\mu\nu}(k) = \frac{\mathrm{i}\eta_{\mu\nu}}{k^2}$, i.e. the photon propagator is not in terms of the vector potential A_μ, but in terms of the metric tensor.

The graviton–fermions vertex [25] is



$$= -\frac{i\kappa}{4}\left(\gamma^\mu p_1^\nu + p_2^\mu \gamma^\nu\right)$$

4. The cross section

The differential cross section for the process

$$e^- + e^+ \longrightarrow e^- + e^+ \tag{17}$$

is calculated. The particles are labeled with momentum and spin variables as $e^-(p_1, \lambda_1)$, $e^+(p_2, \lambda_2)$, $e^-(q_1, \lambda_1')$ and $e^+(q_2, \lambda_2')$ with p_1, p_2, and q_1, q_2 being the momenta of the incoming and outgoing particles, respectively. λ_1, λ_2 and λ_1', λ_2' denote helicity of the incoming and outgoing electron and positron, respectively. The helicity is the projection of the spin in the direction of motion. Here the electron–positron scattering is studied in the centre of mass (CM) frame of reference. This is a frame where the observer is travelling along with the centre of mass of the system. In the CM frame we have

$$p_1 = (E, \vec{p}), \qquad p_2 = (E, -\vec{p}), \qquad q_1 = (E, \vec{p'}) \quad \text{and} \quad q_2 = (E, -\vec{p'}), \tag{18}$$

where E and $\vec{p}$ are the coordinate components, $|\vec{p}|^2 = |\vec{p'}|^2 = E^2$ and $\vec{p} \cdot \vec{p'} = E^2 \cos\theta$.

The differential cross section is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s} |\mathcal{M}|^2, \tag{19}$$

where $s = 4E^2$ is the centre of mass energy and $\mathcal{M}$ is the transition amplitude. The total transition amplitude is

$$\mathcal{M} = \mathcal{M}_{(a)} + \mathcal{M}_{(b)}, \tag{20}$$

where $\mathcal{M}_{(a)}$ and $\mathcal{M}_{(b)}$ are contributions of diagram 1(a) and 1(b) respectively. Summing over the polarization of electron and positron leads to

$$\frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{1}{4} \sum_{\text{spins}} \left| \mathcal{M}_{(a)} + \mathcal{M}_{(b)} \right|^2. \tag{21}$$

The scattering amplitude for the diagram (a) is

$$\begin{aligned} \mathcal{M}_{(a)} = {} & \bar{v}(p_2, \lambda_2) \left[-\frac{\mathrm{i}\kappa}{4} \left(\gamma^\mu p_1^\nu + p_2^\mu \gamma^\nu \right) \right] u(p_1, \lambda_1) \\ & \times \frac{\mathrm{i}}{2(p_1 + p_2)^2} \left(g_{\mu\alpha} g_{\nu\rho} + g_{\mu\rho} g_{\nu\alpha} - g_{\mu\nu} g_{\alpha\rho} \right) \\ & \times \bar{u}(q_1, \lambda_1') \left[-\frac{\mathrm{i}\kappa}{4} \left(\gamma^\alpha q_1^\rho + q_2^\alpha \gamma^\rho \right) \right] v(q_2, \lambda_2') \end{aligned} \tag{22}$$

and for the diagram (b) is

$$\begin{aligned} \mathcal{M}_{(b)} = {} & -\bar{u}(q_1, \lambda_1') \left[-\frac{\mathrm{i}\kappa}{4} \left(\gamma^\alpha p_1^\rho + q_1^\alpha \gamma^\rho \right) \right] u(p_1, \lambda_1) \\ & \times \frac{\mathrm{i}}{2(p_1 - q_1)^2} \left(g_{\mu\alpha} g_{\nu\rho} + g_{\mu\rho} g_{\nu\alpha} - g_{\mu\nu} g_{\alpha\rho} \right) \\ & \times \bar{v}(p_2, \lambda_2) \left[-\frac{\mathrm{i}\kappa}{4} \left(\gamma^\mu p_2^\nu + q_2^\mu \gamma^\nu \right) \right] v(q_2, \lambda_2'). \end{aligned} \tag{23}$$

Here u and v are spinors for the electron and the positron, respectively. For evaluating the unpolarized cross section the relevant quantity is $|\mathcal{M}|^2 = \sum \mathcal{M}\mathcal{M}^*$, where the sum is over spins. This calculation can be accomplished using the completeness relation:

$$\sum u(p_1, \lambda_1)\bar{u}(p_1, \lambda_1) = \not{p}_1 + m \qquad \text{and}$$
$$\sum v(p_1, \lambda_1)\bar{v}(p_1, \lambda_1) = \not{p}_1 - m. \tag{24}$$

and the relation

$$\bar{v}(p_2, \lambda_2)\gamma_\alpha p_{1\rho} u(p_1, \lambda_1)\bar{u}(p_1, \lambda_1)\gamma^\alpha p_1^\rho v(p_2, \lambda_2)$$
$$= \mathrm{tr}\left[\gamma_\alpha p_{1\rho} u(p_1, \lambda_1)\bar{u}(p_1, \lambda_1)\gamma^\alpha p_1^\rho v(p_2, \lambda_2)\bar{v}(p_2, \lambda_2)\right] \tag{25}$$

the summation over both λ_1 and λ_2 is carried out. The trace calculations involve the product of up to eight gamma matrices. Henceforth the electron mass is ignored since all the momenta are large compared with the electron mass, i.e. ultra-relativistic limit. Thus, after some algebraic simplifications, using the FeynCalc package [48], the total transition amplitude is

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = -\frac{\kappa^4 E^4}{1024}\frac{(1864\cos\theta + 540\cos 2\theta + 56\cos 3\theta + 5\cos 4\theta + 1631)}{(\cos\theta - 1)^2}. \tag{26}$$

The differential cross section for $\kappa = \frac{\sqrt{8\pi G}}{c^2}$ (for dimensional analysis, $c \neq 1$ is considered) is

$$\frac{d\sigma}{d\Omega} = -\frac{G^2 E^4}{1024\, c^8 s}\frac{(1864\cos\theta + 540\cos 2\theta + 56\cos 3\theta + 5\cos 4\theta + 1631)}{(\cos\theta - 1)^2}. \tag{27}$$

Although GEM is similar to electromagnetism, the differential cross section for Bhabha scattering from graviton exchange only is very different from its electromagnetic analog. Ultra-relativistic limit of Bhabha scattering ($e^+e^- \longrightarrow e^+e^-$) with photon exchange is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2(\hbar c)^2}{64\pi^2(2E)^2}\left[\frac{1}{2}(1+\cos^2\theta) + \frac{1+\cos^4\theta/2}{\sin^4\theta/2} - \frac{2\cos^2\theta/2}{\sin^2\theta/2}\right]. \tag{28}$$

GEM and electromagnetism are described by an analogue set of equations. However, there are important differences between the two theories. For example, in GEM the gravitational field plays the role that the electromagnetic field plays in electromagnetism. The gravitational field is generated by masses while the electromagnetic field by charges. The electromagnetic fields are vectors, whereas GEM fields are tensors. Then it is expected that the differential cross section for Bhabha scattering is different for the two theories.

In the electromagnetic contribution to the Bhabha scattering the coupling constant α is a dimensionless factor. For gravity, the coupling constant κ contains a factor proportional to $\sqrt{G}$, i.e. $\kappa = \sqrt{8\pi G}$ with $\hbar = c = 1$. This coupling constant should also be dimensionless. The gravitational constant is given as $G = \frac{1}{M_P^2}$, where M_P is the Planck mass. Then it should include a factor involving a characteristic energy scale of the scattering process. To compare with electromagnetic case, the energy of the center of mass $E_{\mathrm{CM}} = 29\,\mathrm{GeV}$ is considered. Then the gravitational coupling constant becomes

$$\kappa' = \sqrt{8\pi G}\, E_{\mathrm{CM}} \approx 1,5 \times 10^{-17}, \tag{29}$$

where $M_P = 10^{19}$ GeV is used. The ratio $\frac{\alpha}{\kappa'} \approx 5 \times 10^{14}$ implies that the electromagnetic interaction is $\sim 10^{14}$ times stronger the gravitational interaction.

In the universe, at large scale, the gravitational interaction is strong. How is the coupling constant on this scale? It indicates modification in the cross section of this scattering. For this analysis, cosmic rays are considered. In the table 1 the gravitational coupling constant for different energies is given.

Table 1. E_{CM} is the energy of the center of mass, E^1_{CR} and E^2_{CR} are particle energy of the cosmic rays and E_{P} is the Planck energy.

Energy (GeV)	κ'
$E_{\mathrm{CM}} = 29$	$\approx 1,5 \times 10^{-17}$
$E^1_{\mathrm{CR}} = 10^3$	$\approx 5,0 \times 10^{-16}$
$E^2_{\mathrm{CR}} = 10^{11}$	$\approx 5,0 \times 10^{-8}$
$E_{\mathrm{P}} = 10^{19}$	$\approx 5,0$

The gravitational coupling constant increases with the energy. For example, the gravitational interaction in the energy of the center of mass is 10^{-17} smaller than the interaction in the Planck energy, i.e. $\kappa'_{\mathrm{CM}} \sim 10^{-17}\,\kappa'_{\mathrm{P}}$. At high energies Bhabha scattering would be changed due to gravitational effects.

5. Conclusion

Electron–positron scattering is analyzed using gravitoelectromagnetic (GEM) theory, a theory of gravitational field, that is similar to the electromagnetic field theory. This is based on the Weyl approach to GEM in a Lagrangian formulation. It is important to emphasize that GEM was developed in a close relationship with Maxwell's theory of electrodynamics. GEM, in a weak field approximation, has been quantized. This makes a perturbative analysis possible [25]. Bhabha scattering is a reaction process in QED. The differential cross section for Bhabha scattering in the GEM framework is calculated. The dimensionless gravitational coupling constant, κ', of the scattering process is included. A natural question is: what happens with the Bhabha scattering when the gravitational field becomes very strong? Using data from cosmic rays the coupling constant, $\kappa' = \sqrt{8\pi GE}$, is calculated for gravitation. The gravitational effects modify the Bhabha scattering results at high energies. It is interesting note that the cosmic ray measurements exhibit an excess of positrons. This has been confirmed by experiments, such as the PAMELA satellite experiment [49] and the AMS collaboration [43, 44]. There are a number of possible explanations for the positron excess. In the literature, new astrophysical sources such as dark matter and pulsars are widely suggested. One possibility asserts that these positrons may be produced via the annihilation or decay of dark-matter particles within the galactic halo [50, 51]. Another possibility considers that these particles are produced in pulsars. Pulsars are strongly magnetized, rapidly rotating neutron stars that are powered by spin down and emit electromagnetic radiation. Then high energy electrons/positrons may be generated through the cascade of electrons accelerated in the magnetosphere of pulsars [52, 53]. Although various physical origins have been explored, the positron excess is not completely understood (for a review see [54]). Then our results indicate that modifications in Bhabha scattering due to gravitational effects may be related to new physics that may be needed at very high energies. The present calculations may be interpreted as another mechanism that creates positrons in space. In addition, the success of measurements of the gravity Probe-B experiment [55], among others, that confirms the gravitomagnetic effect, indicates that the GEM contribution to the Bhabha scattering is relevant and may be measured in future experiments.

Acknowledgments

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