

Deep Learning

Week 15

Intro to Reinforcement Learning

Why you should care

Supervised learning

Given:

- objects and answers

$$(x, y)$$

- algorithm family

$$a_{\theta}(x) \rightarrow y$$

- loss function

$$L(y, a_{\theta}(x))$$

Find:

$$\theta' \leftarrow \operatorname{argmin}_{\theta} L(y, a_{\theta}(x))$$

Supervised learning

Given:

- objects and answers
- algorithm family
- loss function

(x, y)
[banner,page], ctr

$a_{\theta}(x) \rightarrow y$
linear / tree / NN

$L(y, a_{\theta}(x))$
MSE, crossentropy

Find:

$$\theta' \leftarrow \operatorname{argmin}_{\theta} L(y, a_{\theta}(x))$$

Supervised learning

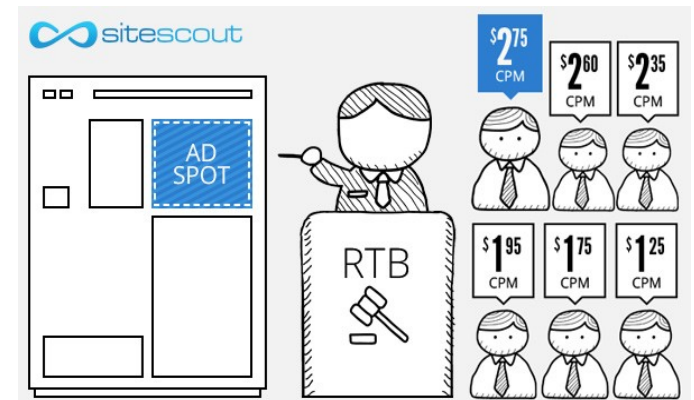
Great... except if we have no reference answers

Online Ads

Great... except if we have no reference answers

We have:

- YouTube at your disposal
- Live data stream
(banner & video features, #clicked)
- (insert your favorite ML toolkit)



We want:

- Learn to pick relevant ads

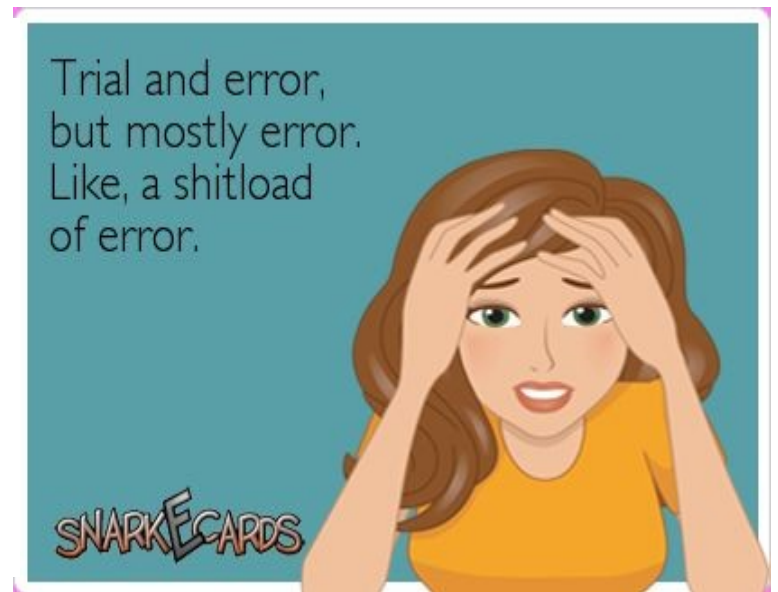
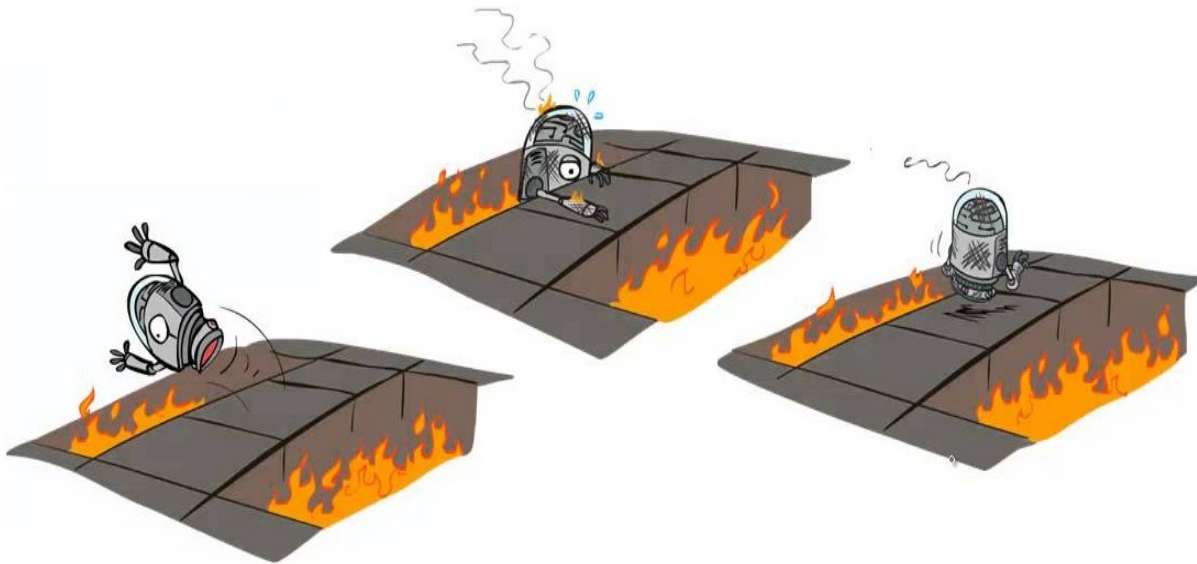


Ideas?

Duct tape approach

Common idea:

- Initialize with naïve solution
- Get data by trial and error and error and error and error
- Learn (situation) → (optimal action)
- Repeat



Trial and error,
but mostly error.
Like, a shitload
of error.

Giant Death Robot (GDR)

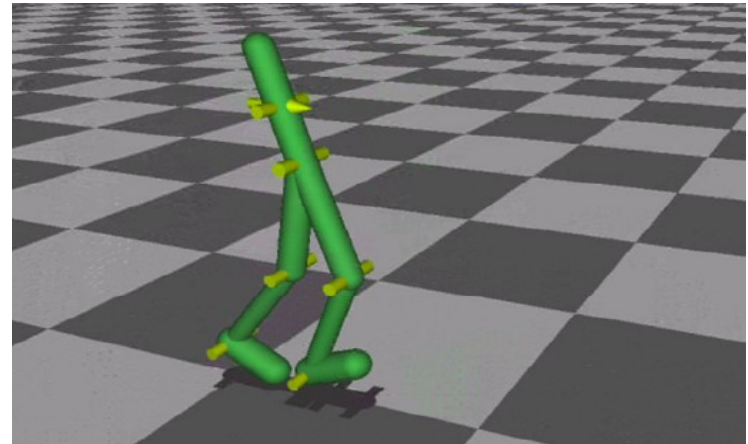
Great... except if we have no reference answers

We have:

- Evil humanoid robot
- A lot of spare parts to repair it :)

We want:

- ~~Enslave humanity~~
- Learn to walk forward

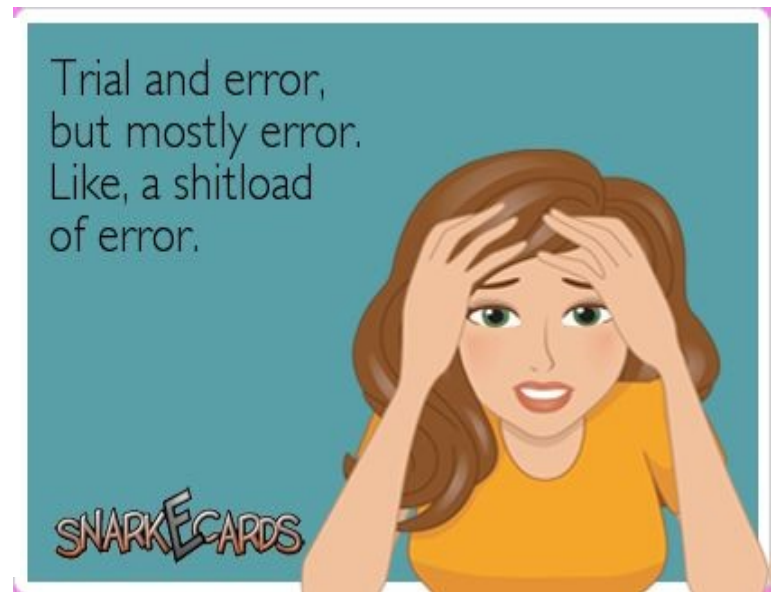
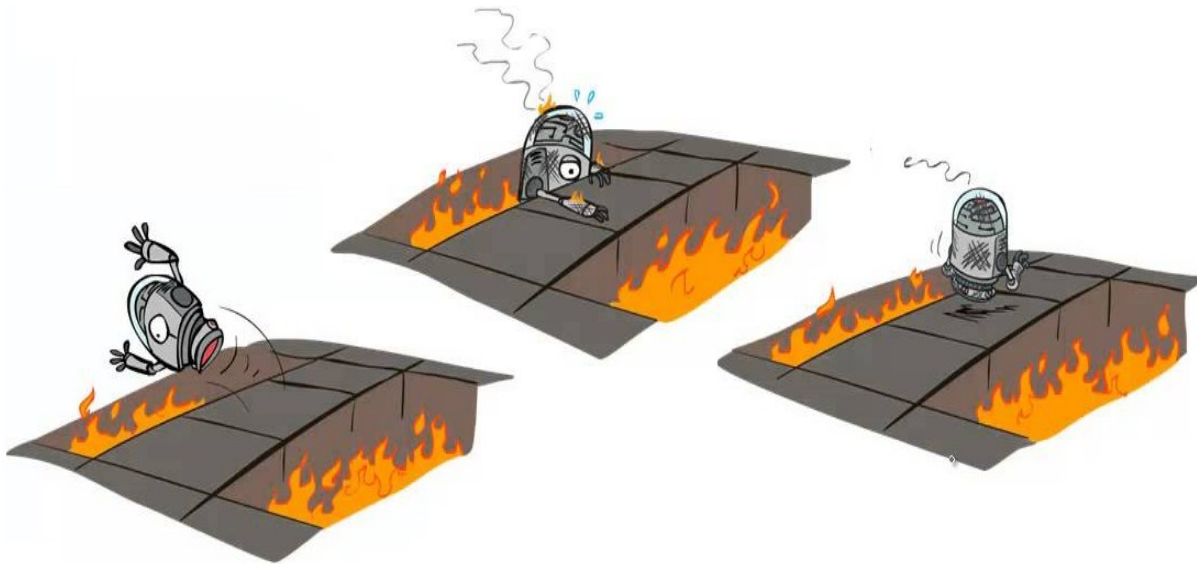


Ideas?

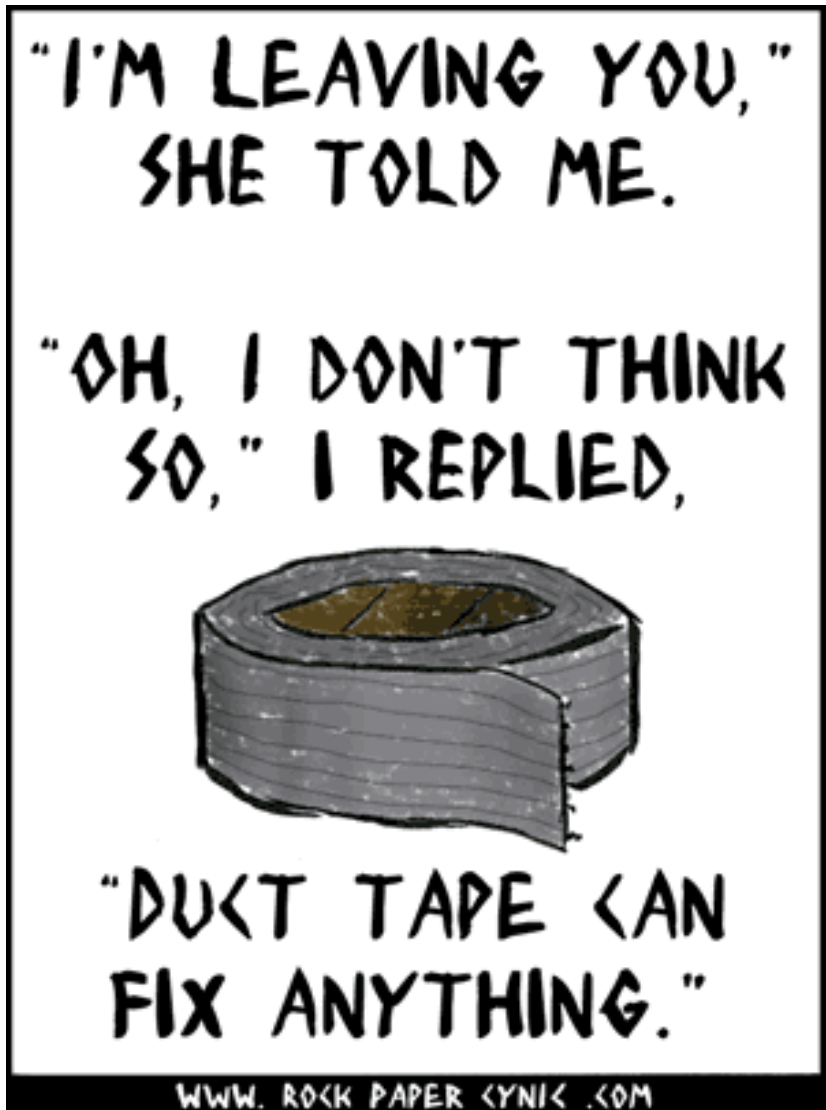
Duct tape approach (again)

Common idea:

- Initialize with naïve solution
- Get data by trial and error and error and error and error
- Learn (situation) → (optimal action)
- Repeat



Duct tape approach



Problems

Problem 1:

- What exactly does the “optimal action” mean?

Extract as much
money as you can
right now

VS

Make user happy
so that he would
visit you again

Problems

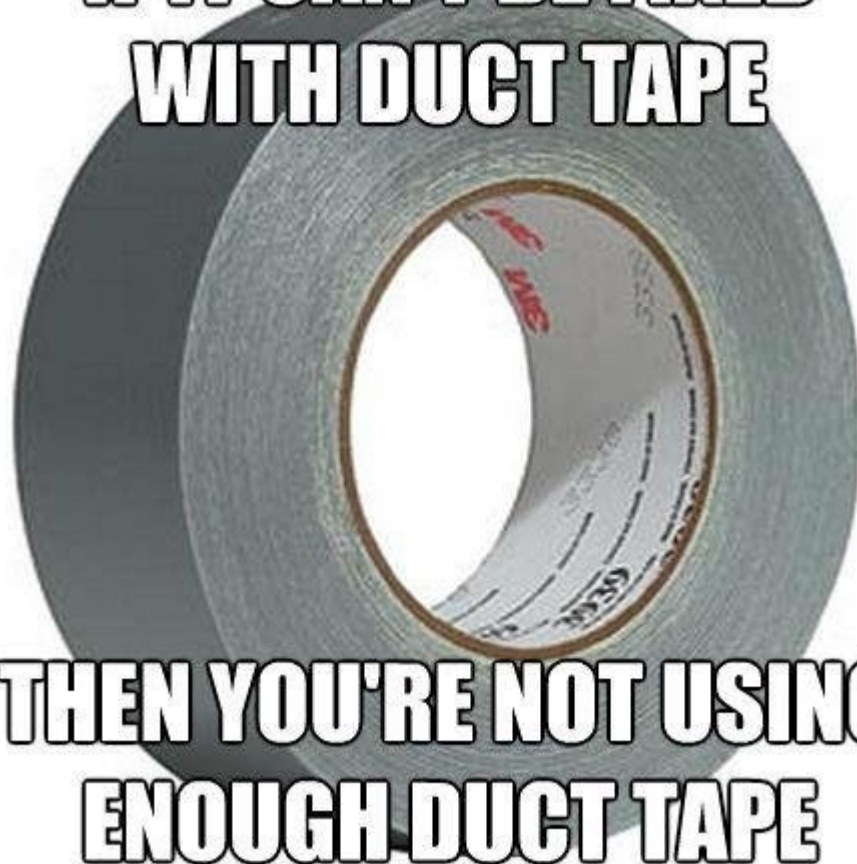
Problem 2:

- If you always follow the “current optimal” strategy, you may never discover something better.
- If you show the same banner to 100% users, you will never learn how other ads affect them.

Ideas?

Duct tape approach

**IF IT CAN'T BE FIXED
WITH DUCT TAPE**



**THEN YOU'RE NOT USING
ENOUGH DUCT TAPE**

zipmeme

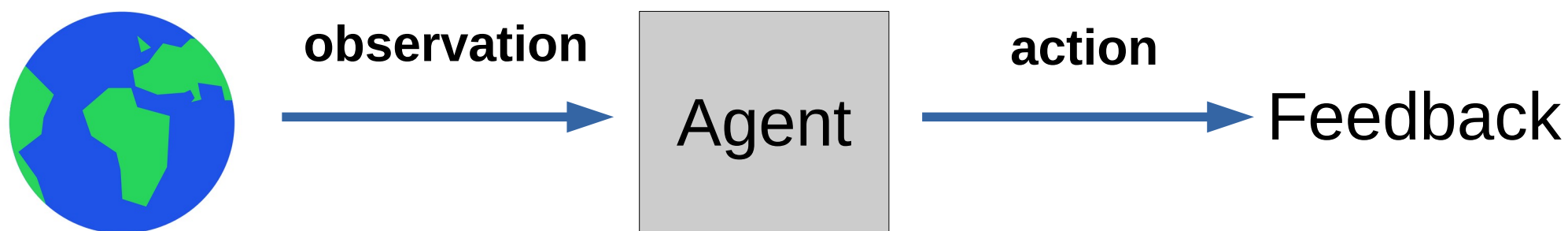
Reinforcement learning

STAND BACK



**I'M GOING TO TRY
SCIENCE**

What is: bandit



Examples:

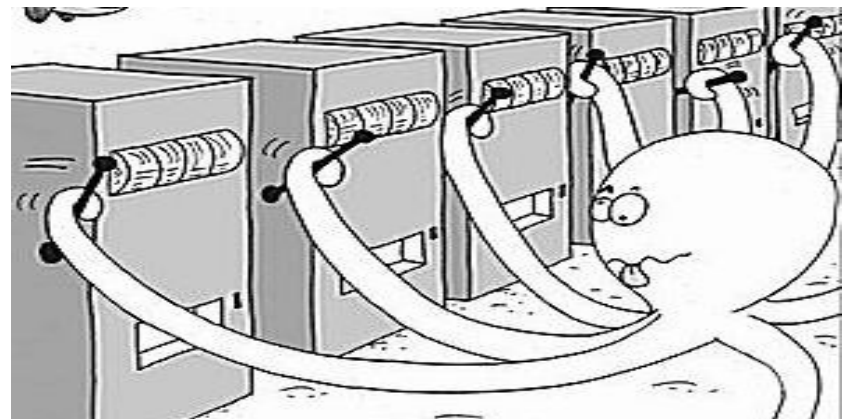
- banner ads (RTB)
- recommendations
- medical treatment

What is: bandit

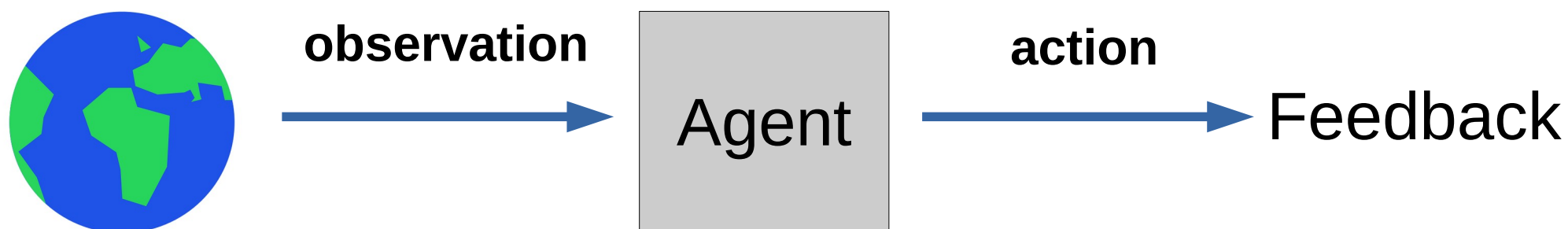


Examples:

- banner ads (RTB)
- recommendations
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What is: bandit

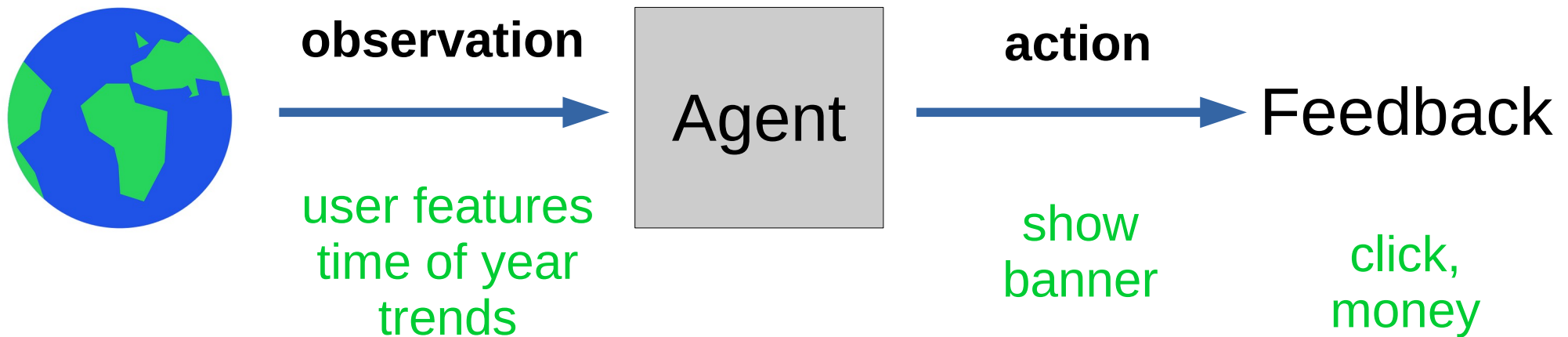


Examples:

- banner ads (RTB)
- recommendations
- medical treatment

Q: what's observation, action and feedback in the banner ads problem?

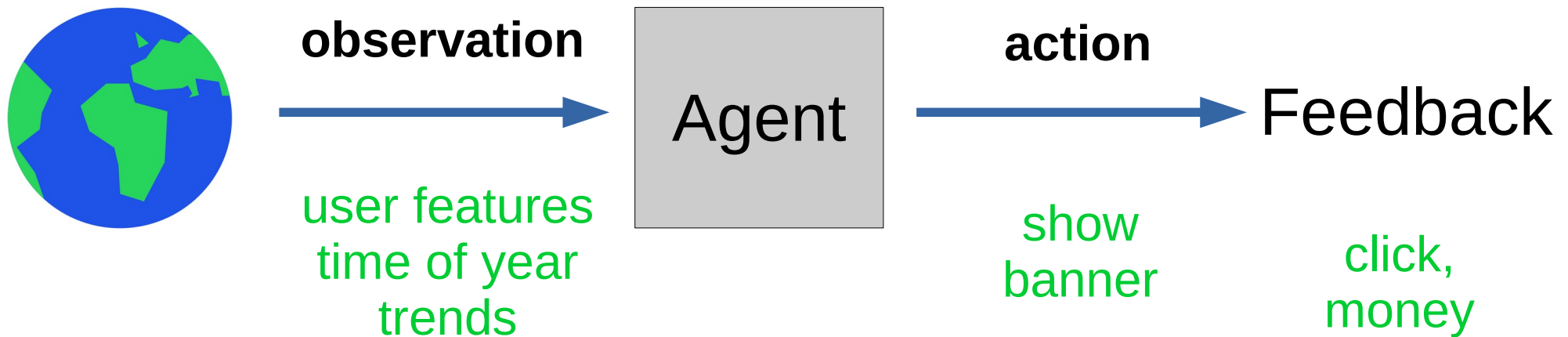
What is: bandit



Examples:

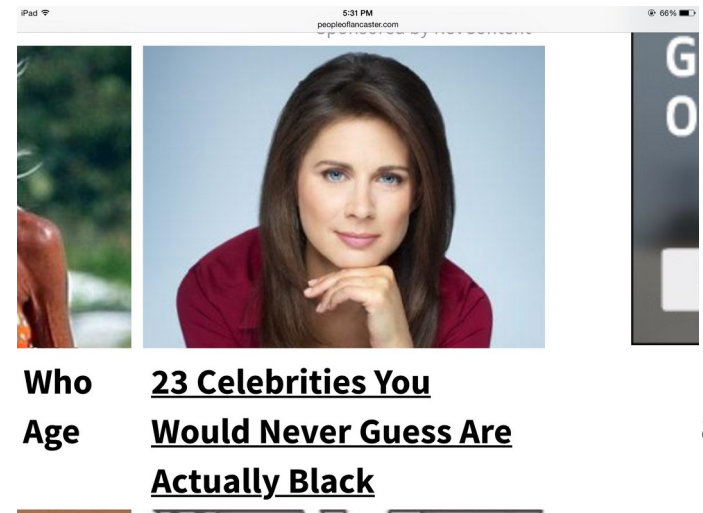
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What is: bandit

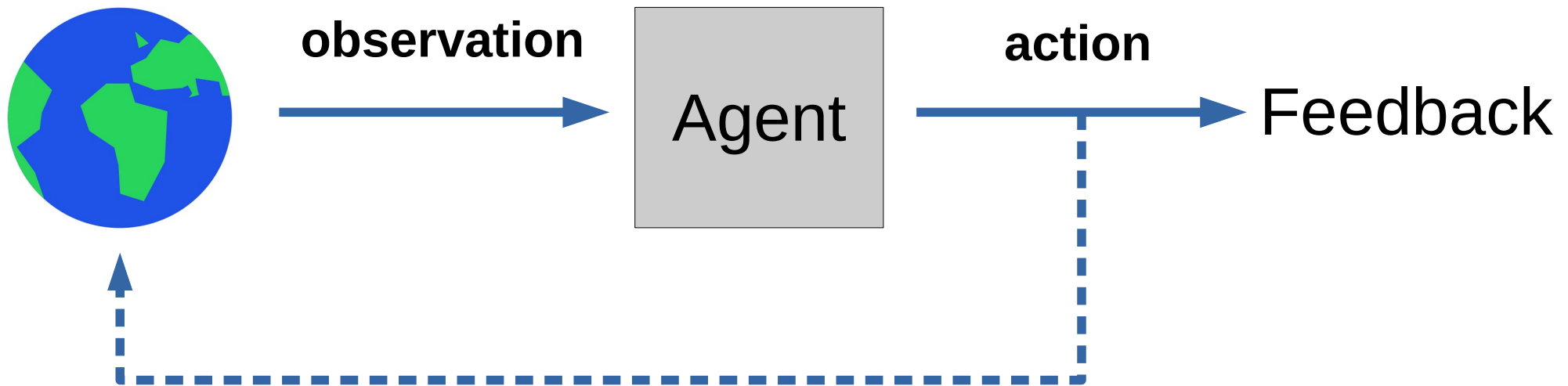


Q: You're Yandex/Google/Youtube.
There's a kind of banners that would
have great click rates: the “clickbait”.

Is it a good idea to show clickbait?

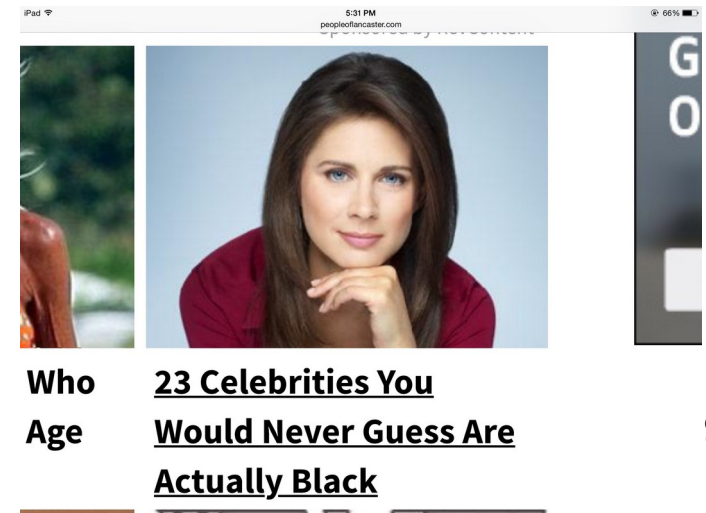


What is: bandit

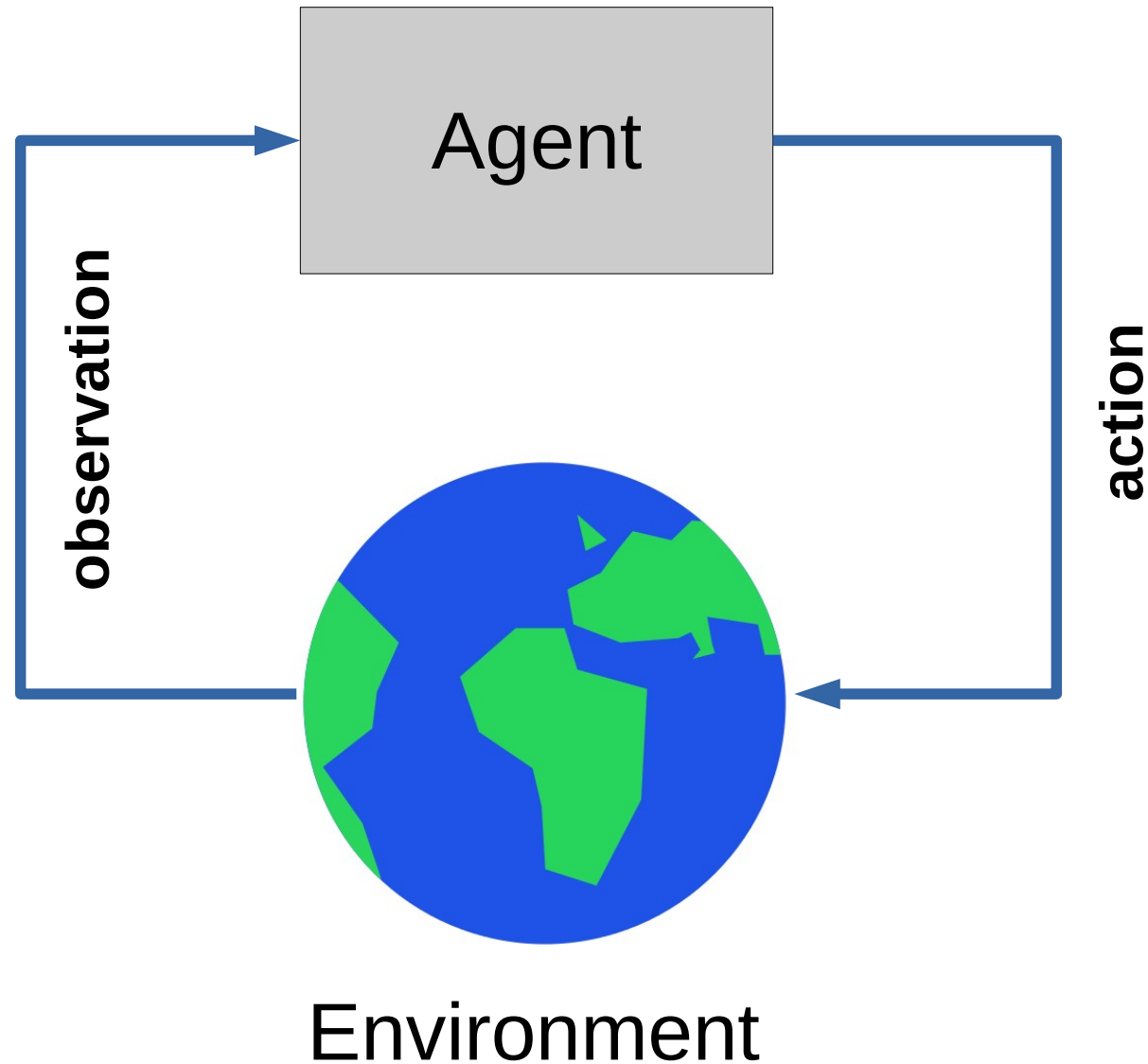


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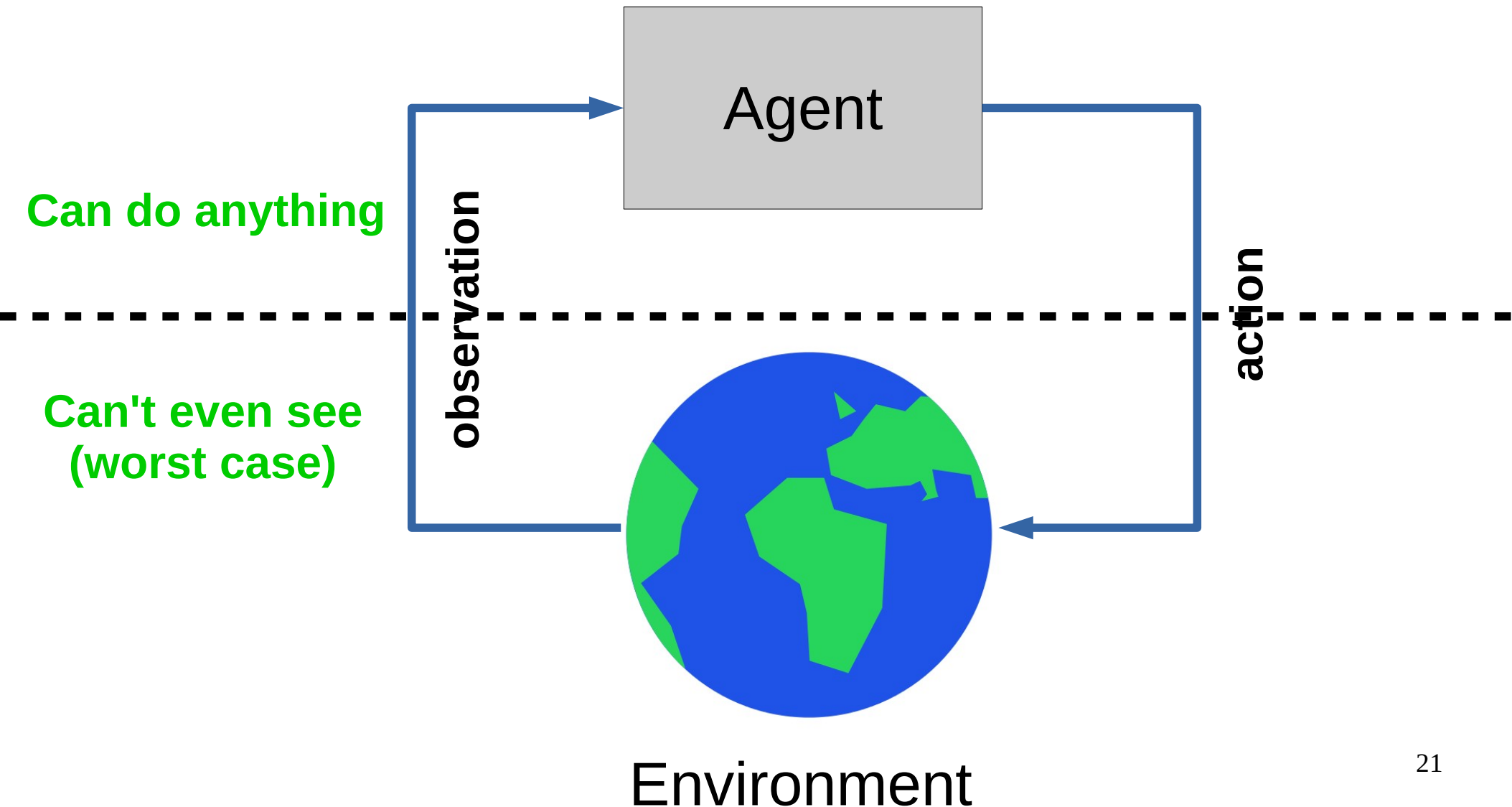
Is it a good idea to show clickbait?
No, no one will trust you after that!



What is: decision process

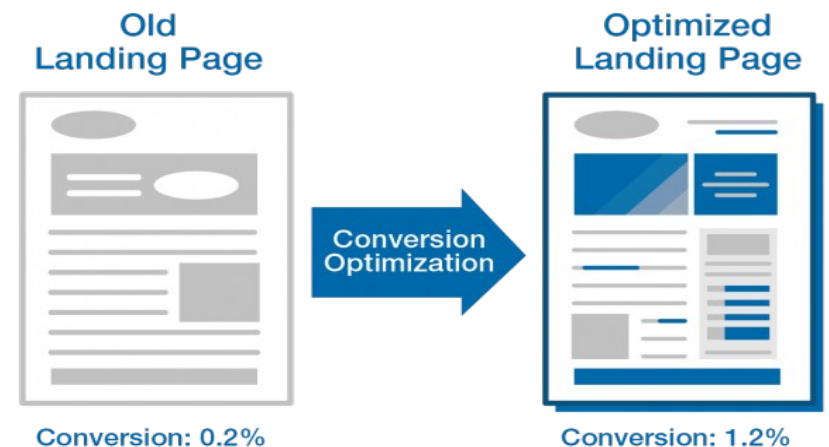
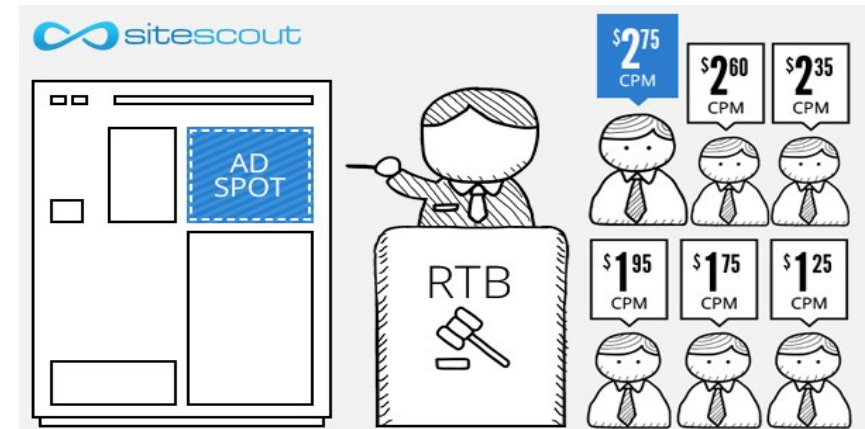


What is: decision process



Reality check: web

- **Cases:**
 - Pick ads to maximize profit
 - Design landing page to maximize user retention
 - Recommend movies to users
 - Find pages relevant to queries
- **Example**
 - Observation – user features
 - Action – show banner #i
 - Feedback – did user click?



Reality check: dynamic systems



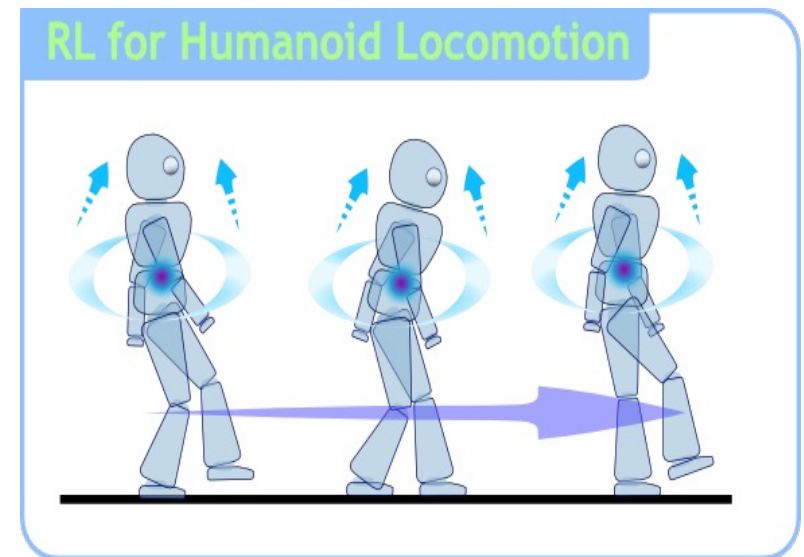
Reality check: dynamic systems

- **Cases:**

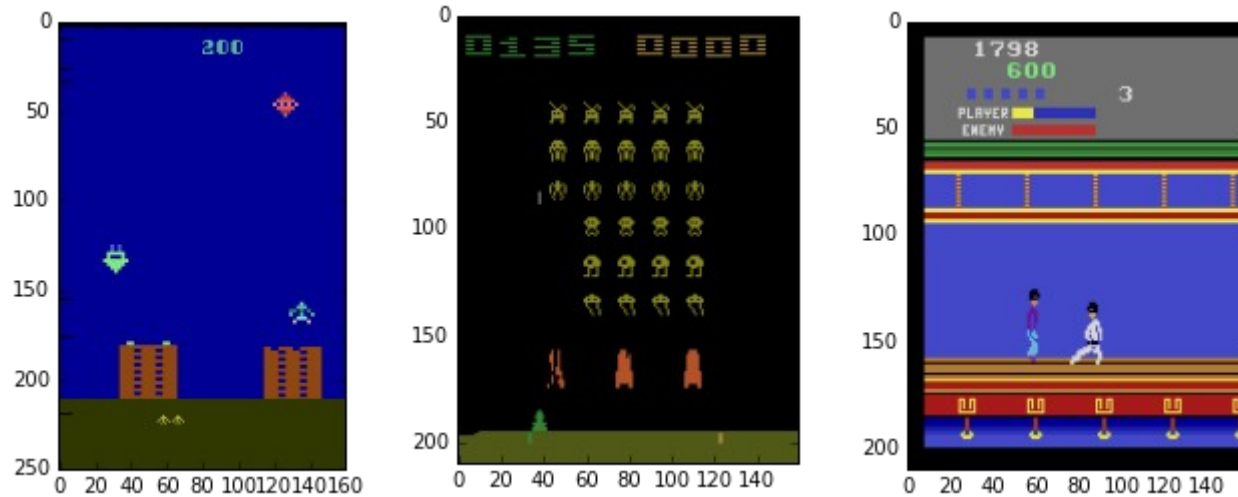
- Robots
- Self-driving vehicles
- Pilot assistant
- More robots!

- **Example**

- Observation: sensor feed
- Action: voltage sent to motors
- Feedback: how far did it move forward before falling



Reality check: videogames



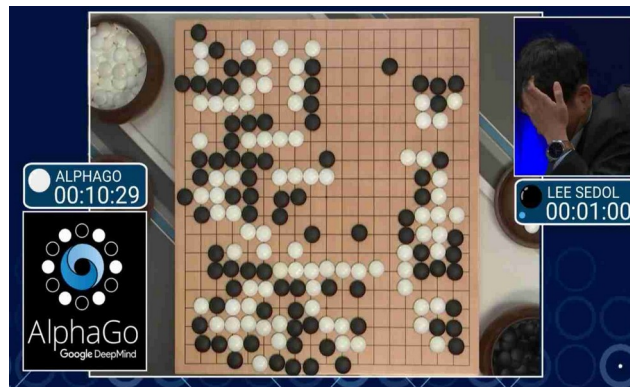
- **Q:** What are observations, actions and feedback?

Other use cases

- Personalized medical treatment



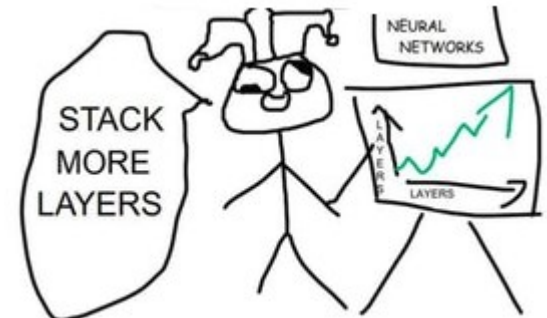
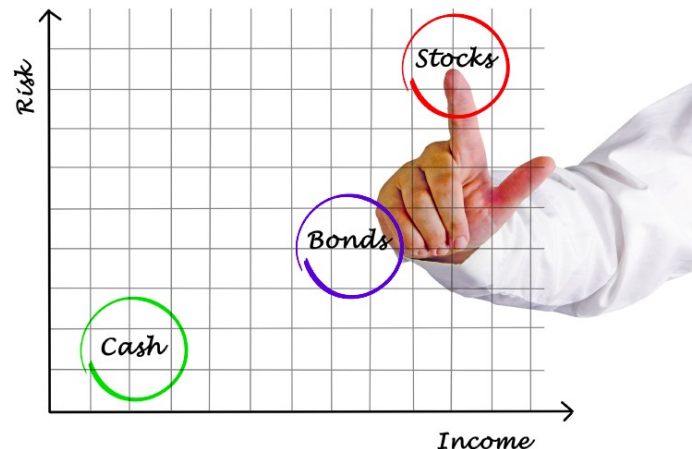
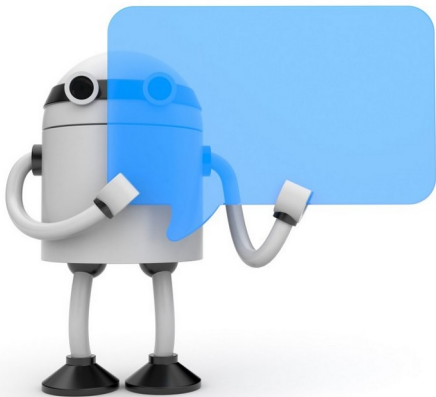
- Even more games (Go, chess, etc)



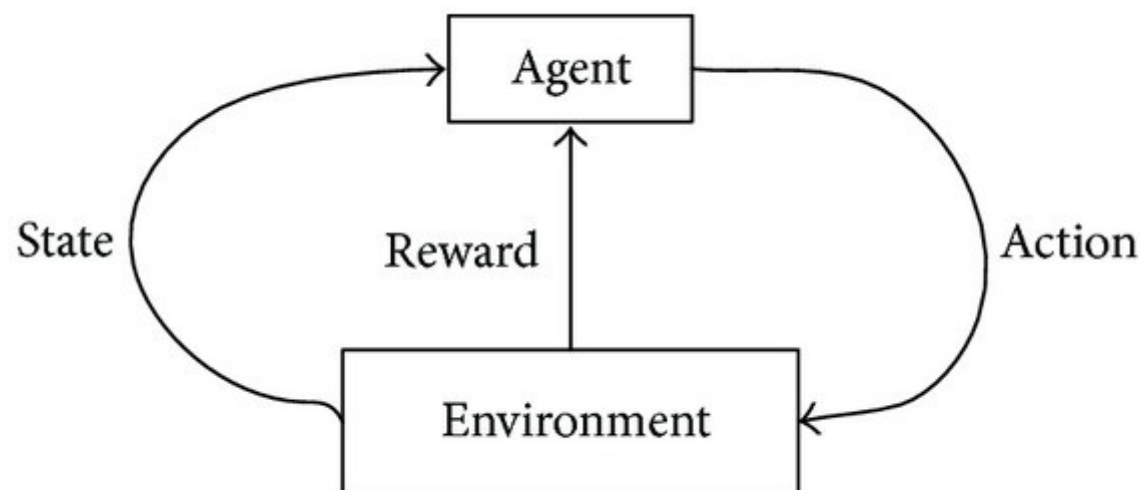
- **Q:** What are observations, actions and feedback?

Other use cases

- Conversation systems
 - learning to make user happy
- Quantitative finance
 - portfolio management
- Deep learning
 - optimizing non-differentiable loss
 - finding optimal architecture



The MDP formalism

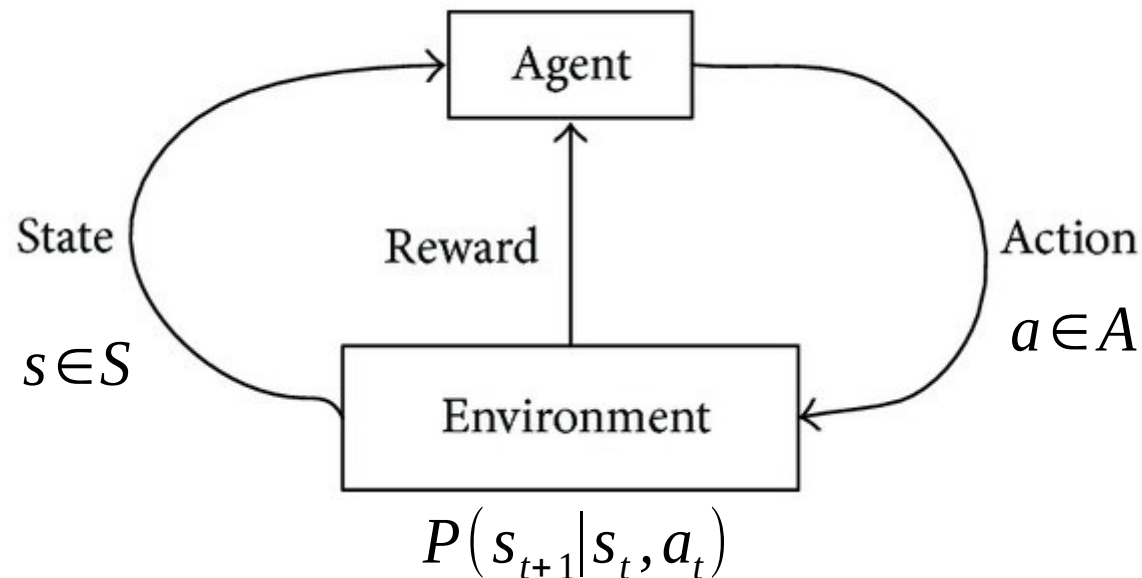


Markov Decision Process

- Environment states: $s \in S$
- Agent actions: $a \in A$
- Rewards: $r \in \mathbb{R}$

- Dynamics: $P(s_{t+1} | s_t, a_t)$

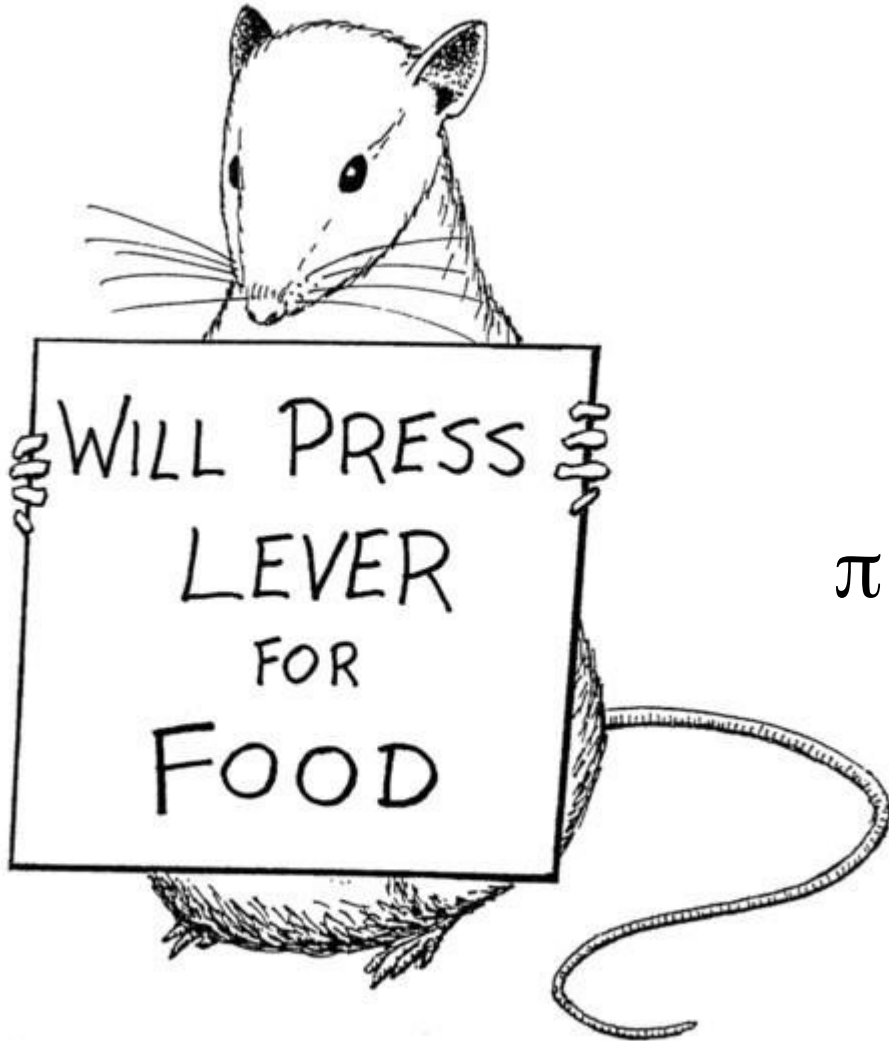
The MDP formalism



Markov Decision Process
Markov assumption

$$P(s_{t+1} | s_t, a_t, s_{t-1}, a_{t-1}) = P(s_{t+1} | s_t, a_t)$$

Total reward



Total reward for session:

$$R = \sum_t r_t$$

Agent's policy:

$$\pi(a|s) = P(\text{take action } a | \text{in state } s)$$

Problem: find policy with highest reward:

$$\pi(a|s) : E_{\pi}[R] \rightarrow \max$$

Objective

The easy way:

$Q = E_{\pi} R$ is an expected sum of rewards
that agent with policy π earns per session

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The easy way:

$Q = E_{\pi} R$ is an expected sum of rewards
that agent with policy π earns per session

The hard way:

$$\begin{array}{ccccccc} E & E & E & \dots & E & & [r_0 + r_1 + r_2 + \dots + r_T] \\ s_0 \sim p(s_0), a_0 \sim \pi(a|s_0), s_1, r_0 \sim P(s', r|s, a) & & & & s_T, r_T \sim P(s', r|s_{T-1}, a_{t-1}) & & \end{array}$$

How do we solve it?

General idea:

Play a few sessions

Update your policy

Repeat

Objective

$$J = \mathop{E}_{\substack{s \sim p(s) \\ a \sim \pi_{\theta}(s|a)}} Q(s, a) = \int_s p(s) \int_a \pi_{\theta}(a|s) Q(s, a) da ds$$

Objective

Expected reward:

$$J = E_{\substack{s \sim p(s) \\ a \sim \pi_{\theta}(s|a)}} Q(s, a)$$

We need a gradient!

Objective

$$J = E_{\substack{s \sim p(s) \\ a \sim \pi_{\theta}(s|a)}} Q(s, a) = \int_s p(s) \int_a \pi_{\theta}(a|s) Q(s, a) da ds$$

Diagram illustrating the components of the objective function J :

- Agent's policy**: Points to $\pi_{\theta}(a|s)$ in the integral.
- state visitation frequency (may depend on policy)**: Points to $p(s)$ in the integral.
- True action value**: Points to $Q(s, a)$ in the integral.

Q: how do we compute that?

Objective

$$J = E_{\substack{s \sim p(s) \\ a \sim \pi_{\theta}(s|a)}} Q(s, a) = \int_s p(s) \int_a \pi_{\theta}(a|s) Q(s, a) da ds$$

$$J \approx \frac{1}{N} \sum_{i=0}^N \sum_{s, a \in z_i} Q(s, a)$$

True action value
a.k.a. $E[R(s, a)]$

sample N sessions

Objective

$$J = E_{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}} Q(s, a) = \int_s p(s) \int_a \pi_\theta(a|s) Q(s, a) da ds$$

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True action value
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sample N sessions

Can we optimize policy now?

Objective

$$J = E_{\substack{s \sim p(s) \\ a \sim \pi_{\theta}(s|a)}} Q(s, a) = \int_s p(s) \int_a \pi_{\theta}(a|s) Q(s, a) da ds$$

parameters “sit” here

$$J \approx \frac{1}{N} \sum_{i=0}^N \sum_{s, a \in z_i} Q(s, a)$$

True action value
a.k.a. $E[R(s, a)]$

We don't know how to compute $dJ/d\theta$

Objective

$$J = E_{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}} Q(s, a) = \int_s p(s) \int_a \pi_\theta(a|s) Q(s, a) da ds$$

Wish list:

- Analytical gradient
- Easy/stable approximations

Log-derivative trick

Simple math

$$\nabla \log \pi(z) = ? ? ?$$

(try chain rule)

Log-derivative trick

Simple math

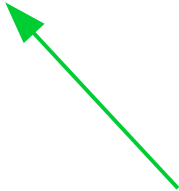
$$\nabla \log \pi(z) = \frac{1}{\pi(z)} \cdot \nabla \pi(z)$$

$$\pi \cdot \nabla \log \pi(z) = \nabla \pi(z)$$

Policy gradient

Analytical inference

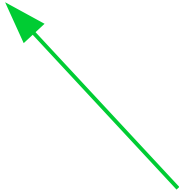
$$\nabla J = \int_s p(s) \int_a \nabla \pi_\theta(a|s) Q(s, a) da ds$$

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Policy gradient

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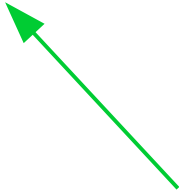
$$\nabla J = \int_s p(s) \int_a \pi_\theta(a|s) \nabla \log \pi_\theta(a|s) Q(s, a) da ds$$

Trivia: anything curious about that formula?

Policy gradient

Analytical inference

$$\nabla J = \int_s p(s) \int_a \nabla \pi_\theta(a|s) Q(s, a) da ds$$

$$\pi \cdot \nabla \log \pi(z) = \nabla \pi(z)$$


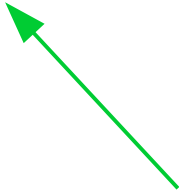
$$\nabla J = \int_s p(s) \int_a \pi_\theta(a|s) \nabla \log \pi_\theta(a|s) Q(s, a) da ds$$

that's expectation :)

Policy gradient

Analytical inference

$$\nabla J = \int_s p(s) \int_a \nabla \pi_\theta(a|s) Q(s, a) da ds$$

$$\pi \cdot \nabla \log \pi(z) = \nabla \pi(z)$$


$$\nabla J = E_{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}} \nabla \log \pi_\theta(a|s) \cdot Q(s, a)$$

Policy gradient (REINFORCE)

- Policy gradient

$$\nabla J = E_{\substack{s \sim p(s) \\ a \sim \pi_{\theta}(s|a)}} \nabla \log \pi_{\theta}(a|s) \cdot Q(s, a)$$

- Approximate with sampling

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^N \sum_{s, a \in z_i} \nabla \log \pi_{\theta}(a|s) \cdot Q(s, a)$$

REINFORCE algorithm

- Initialize NN weights $\theta_0 \leftarrow \text{random}$
- Loop:
 - Sample N sessions \mathbf{z} under current $\pi_\theta(a|s)$
 - Evaluate policy gradient

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^N \sum_{s, a \in \mathbf{z}_i} \nabla \log \pi_\theta(a|s) \cdot Q(s, a)$$

- Ascend $\theta_{i+1} \leftarrow \theta_i + \alpha \cdot \nabla J$

REINFORCE baselines

- Initialize NN weights $\theta_0 \leftarrow \text{random}$
- Loop:
 - Sample N sessions \mathbf{z} under current $\pi_\theta(a|s)$
 - Evaluate policy gradient

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^N \sum_{s, a \in \mathbf{z}_i} \nabla \log \pi_\theta(a|s) \cdot Q(s, a)$$

What is better for learning:
random action in good state
or
great action in bad state?

REINFORCE baselines

We can subtract arbitrary baseline $b(s)$

$$\nabla J = E_{\substack{s \sim p(s) \\ a \sim \pi_{\theta}(a|s)}} \nabla \log \pi_{\theta}(a|s) (Q(s, a) - b(s)) = \dots$$

$$\dots = E_{\substack{s \sim p(s) \\ a \sim \pi_{\theta}(a|s)}} \nabla \log \pi_{\theta}(a|s) Q(s, a) - E_{\substack{s \sim p(s) \\ a \sim \pi_{\theta}(a|s)}} \nabla \log \pi_{\theta}(a|s) b(s) = \dots$$

REINFORCE baselines

We can subtract arbitrary baseline $b(s)$

$$\nabla J = \mathbb{E}_{\substack{s \sim p(s) \\ a \sim \pi_\theta(a|s)}} \nabla \log \pi_\theta(a|s) (Q(s, a) - b(s)) = \dots$$

$$\dots = \mathbb{E}_{\substack{s \sim p(s) \\ a \sim \pi_\theta(a|s)}} \nabla \log \pi_\theta(a|s) Q(s, a) - \mathbb{E}_{\substack{s \sim p(s) \\ a \sim \pi_\theta(a|s)}} \nabla \log \pi_\theta(a|s) b(s) = \dots$$

Q: Can you simplify the second term?

Note that $b(s)$ does not depend on a

REINFORCE baselines

A: How to simplify the second term

$$\mathbb{E}_{\substack{s \sim p(s) \\ a \sim \pi_\theta(a|s)}} \nabla \log \pi_\theta(a|s) b(s) = \mathbb{E}_{s \sim p(s)} b(s) \mathbb{E}_{a \sim \pi_\theta(a|s)} \nabla \log \pi_\theta(a|s) =$$

$$\mathbb{E}_{s \sim p(s)} b(s) \int_a \pi_\theta(a|s) \frac{\nabla \pi_\theta(a|s)}{\pi_\theta(a|s)} da = \mathbb{E}_{s \sim p(s)} b(s) \int_a \nabla \pi_\theta(a|s) da =$$

$$\mathbb{E}_{s \sim p(s)} b(s) \nabla \int_a \pi_\theta(a|s) da = \mathbb{E}_{s \sim p(s)} b(s) \nabla 1 = 0$$

REINFORCE baselines

We can subtract arbitrary baseline $b(s)$

$$\nabla J = E_{\substack{s \sim p(s) \\ a \sim \pi_{\theta}(a|s)}} \nabla \log \pi_{\theta}(a|s) (Q(s, a) - b(s)) = \dots$$

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Gradient direction doesn't change!

$$\dots = E_{\substack{s \sim p(s) \\ a \sim \pi_{\theta}(a|s)}} \nabla \log \pi_{\theta}(a|s) Q(s, a)$$

REINFORCE baselines

- Gradient direction ∇J stays the same
- Variance may change

Gradient variance: $Var[Q(s, a) - b(s)]$
as a random variable over (s, a)

$$Var[Q(s, a)] - 2 \cdot Cov[Q(s, a), b(s)] + Var[b(s)]$$

REINFORCE baselines

- Gradient direction ∇J stays the same
- Variance may change

Gradient variance: $Var[Q(s, a) - b(s)]$
as a random variable over (s, a)

$$Var[Q(s, a)] - 2 \cdot Cov[Q(s, a), b(s)] + Var[b(s)]$$

 If $b(s)$ correlates with $Q(s, a)$, variance decreases

REINFORCE baselines

- Gradient direction ∇J stays the same
- Variance may change

Gradient variance: $Var[Q(s, a) - b(s)]$
as a random variable over (s, a)

$$Var[Q(s, a)] - 2 \cdot Cov[Q(s, a), b(s)] + Var[b(s)]$$

 Q: can you suggest any such $b(s)$?

REINFORCE baselines

- Gradient direction ∇J stays the same
- Variance may change

Gradient variance: $Var[Q(s, a) - b(s)]$
as a random variable over (s, a)

$$Var[Q(s, a)] - 2 \cdot Cov[Q(s, a), b(s)] + Var[b(s)]$$

Naive baseline: b = moving average Q
over all (s, a) , $Var[b(s)] = 0$, $Cov[Q, b] > 0$

Duct tape zone

- Superior algorithms exist
 - For harder environments google A3C, PPO, TRPO
- Regularize with entropy
 - to prevent premature convergence
- Learn on parallel sessions
 - Or super-small experience replay
- Use logsoftmax for numerical stability



THIS TIME



**Q: How is RL different
from supervised learning?**

What-what learning?

Supervised learning

- Learning to approximate reference answers
- Needs correct answers
- Model does not affect the input data

Reinforcement learning

- Learning optimal strategy by trial and error
- Needs feedback on agent's own actions
- Agent can affect it's own observations



What-what learning?

Unsupervised learning

- Learning underlying data structure
- No feedback required
- Model does not affect the input data

Reinforcement learning

- Learning optimal strategy by trial and error
- Needs feedback on agent's own actions
- Agent can affect its own observations



Reinforcement learning is easy!

1. take action



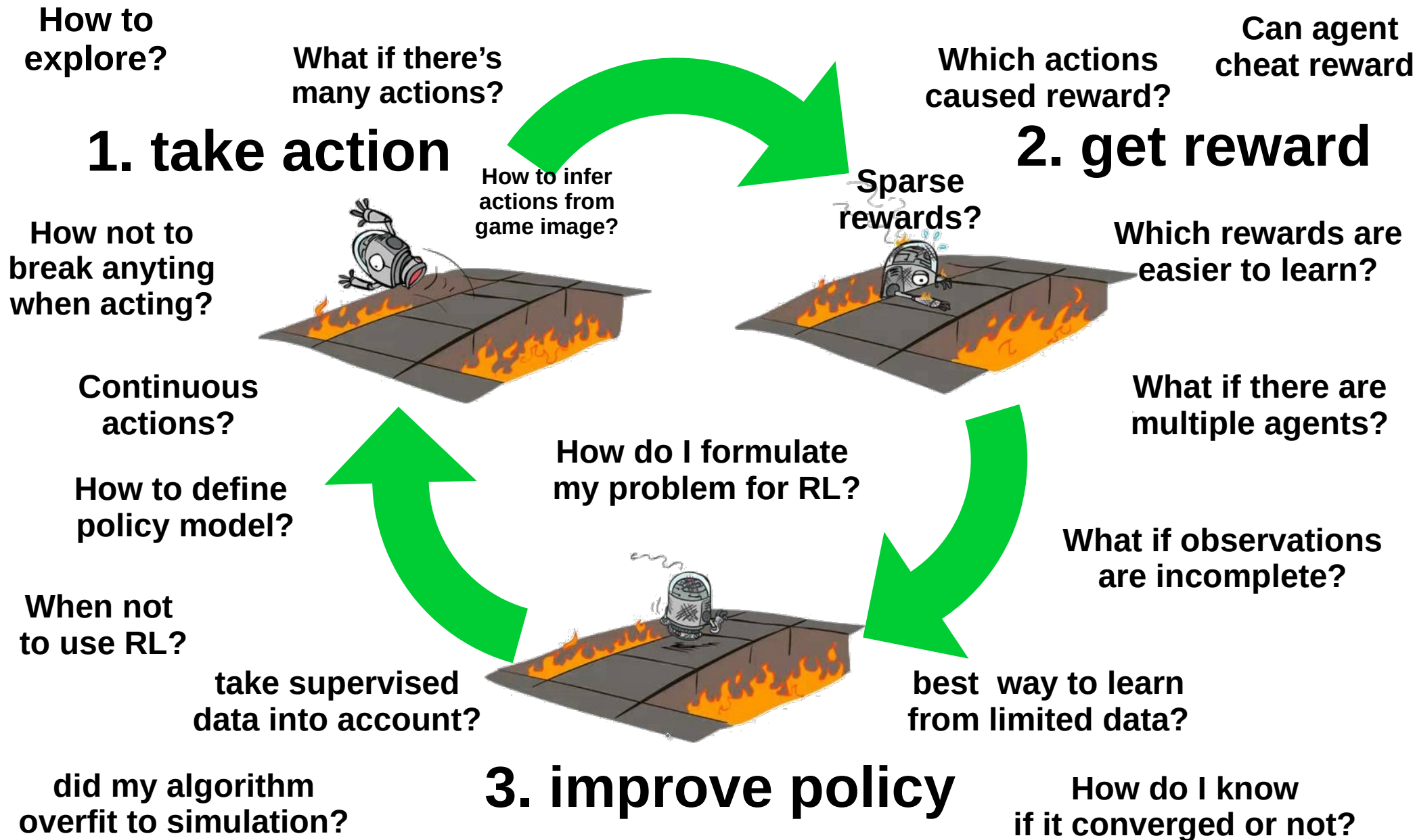
2. get reward



3. improve policy



Reinforcement learning is challenging!



Now go and implement that :)