# Testing the first law of black hole mechanics with gravitational waves

Chao-Wan-Zhen Wang<sup>1,2</sup> and Fu-Wen Shu<sup>1,2,3\*</sup>

<sup>1</sup>Department of Physics, Nanchang University, Nanchang, 330031, China

<sup>2</sup>Center for Relativistic Astrophysics and High Energy Physics, Nanchang University, Nanchang 330031, China

<sup>3</sup>GCAP-CASPER, Physics Department, Baylor University, Waco, Texas 76798-7316, USA

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GW191219\_163120 is a gravitational wave signal that is believed to have originated from a neutron star-black hole (NSBH) coalescence with an extreme mass ratio. In this work, we use data of GW191219\_163120 from LIGO and Virgo to test the first law of black hole mechanics by considering the neutron star as a perturbation to the black hole before the merger, and the remnant black hole as a stationary black hole after the merger. Our results demonstrate consistency with the first law of black hole mechanics, with an error level of about 6% at 68% credibility and 10% at 95% credibility. We also find that the higher the mass ratio of the gravitational wave source, the more consistent our results are with the first law of black hole mechanics. Overall, our study sheds light on the nature of NSBH coalescences and their implications for black hole mechanics.

#### I. INTRODUCTION

Black holes (BHs) are among the most fascinating objects in the universe, with their fruitful field for theoretical exploration of the frontiers of physics, including topics such as the singularity theorem, the cosmic censorship conjecture, and the black hole information paradox [1–3]. Over the past few decades, our understanding of BHs has improved significantly, particularly in the areas of gravitation, thermodynamics, and quantum theory. BH mechanics, in particular, plays a crucial role in the relationship between these fields and has provided us with a deeper understanding of quantum phenomena that occur in strong gravitational fields [4].

Testing BH mechanics has become increasingly important for developing a comprehensive theory of quantum gravity. The advent of gravitational wave (GW) astronomy has opened a new window on the study of BHs. In 2015, the LIGO detectors detected a GW signal from a binary BH system, GW150914, which was the first direct detection of GWs by human beings [5]. Since then, a number of GW events have been detected, providing valuable insights into BH mergers and their properties. In addition, observational testing of BH properties is made possible with the aid of various methods. For example, the black-hole area law, also known as the second law of BH mechanics, has been tested [6], along with the BH no-hair theorem [7]. Here, we present a new GW observational test on the first law of BH mechanics.

The first law of BH mechanics, also known as the Bekenstein-Smarr formula, describes an important relationship between the parameters of a BH [8–10]. It claims that a stationary axisymmetric BH will settle down to a new stationary axisymmetric BH after some infinitesimal physical process [11]. This law has important implications for our understanding of BHs and their properties. While the second law of BH mechanics

has been tested with GWs, the first law has yet to be confirmed [6].

In this letter, we propose a scheme to test the first law of BH mechanics using GWs that have been detected so far and analyze the samples from five waveform models to obtain error levels at different credible regions. Our results provide important insights into the testing of BH mechanics and its implications for the development of a comprehensive theory of quantum gravity.

In order to view the merger of a binary system as a perturbed process and test the first law of BH mechanics, it is essential to ensure that the GW event chosen has an extremely mass ratio. One such candidate is GW191219\_163120, which was detected during the second part of LIGO's and Virgo's third observing run on December 19th, 2019 at 16:31:20 UTC [12]. This event features a BH primary component with a mass of  $31.1^{+2.2}_{-2.7}$  and a neutron star secondary component with a mass of  $1.17^{+0.07}_{-0.06}$  (at 90% credible regions). After the merger, the remnant BH has a mass of  $32.2^{+2.2}_{-2.7}$ . With a mass ratio of 27, which is comparatively extreme, this event provides an opportunity to test the first law of BH mechanics, which pertains to small change.

## II. METHOD

The first law of BH mechanics is a fundamental result in the study of BHs that relates changes in the mass, angular momentum, and area of a stationary BH when it perturbs. It states that for a stationary BH in a vacuum, the variations of these quantities satisfy:

$$\delta M = \frac{\kappa}{8\pi} \delta A + \Omega \delta J,\tag{1}$$

where M, J, and A are the mass, angular momentum, and area of the BH, respectively.  $\kappa$  is the surface gravity of the BH and  $\Omega$  is its angular velocity at the event horizon. In terms of the mass, M, and the dimensionless spin magnitude,  $\chi \equiv |\vec{J}|c/(GM^2)$ , these quantities have

<sup>\*</sup> shufuwen@ncu.edu.cn

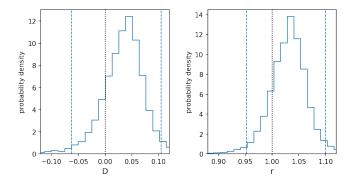


FIG. 1. The probability density distribution of difference D and the ratio r between the left and right sides of equation (1) is shown, with the blue dashed vertical lines indicating the 95% credible regions and the black dotted vertical line representing the theoretical expected value.

the following explicit form:

$$\kappa = \frac{c^4 \sqrt{1 - \chi^2}}{G^2 M (1 + \sqrt{1 - \chi^2})},$$

$$A = \frac{8\pi G^2 M^2}{c^4} \left(1 + \sqrt{1 - \chi^2}\right),$$

$$\Omega = \frac{cM\chi}{2G(r_+^2 + M^2\chi^2)},$$
(2)

where  $r_{+}$  is radius of the outer horizon and satisfies

$$r_{+}^{2} = M(1 + \sqrt{1 - \chi^{2}}). \tag{3}$$

Eq. (1) is analogous to the first law of thermodynamics, which relates changes in energy, heat, and work as  $\delta E = T \delta S - P \delta V$ . The first term on the right-hand side of the equation,  $\frac{\kappa}{8\pi} \delta A$ , represents the change in the BH's energy due to a change in its area. The second term,  $\Omega \delta J$ , represents the work done on the BH by a change in its angular momentum [8]. As a consequence, the first law of BH mechanics is an important result in the study of BHs, as it suggests that BHs possess properties similar to those of thermodynamic systems, such as temperature and entropy.

To assess the validity of the first law of BH mechanics, there are two possible methods. The first involves calculating the difference between the left and right sides of equation (1), which yields  $D = \delta M - \frac{\kappa}{8\pi}\delta A - \Omega\delta J$ . By determining the probability that the value of D approaches zero, we can evaluate the extent to which the first law holds true. Alternatively, we can define a ratio  $r = \delta M/(\frac{\kappa}{8\pi}\delta A + \Omega\delta J)$  and calculate the probability that r approaches unity. The probability distribution of difference D and ratio r can be obtained by combining the probability distribution of the following quantities:  $\delta M = m_f - m_0$ ,  $\delta A = A_f - A_0$ ,  $\delta J = J_f - J_0$ ,  $\kappa$  and  $\Omega$ . Here the subscripts "0" and "f" represent the quantities before and after the merger, respectively.

To obtain the probability distributions of  $A_{0,f}$  and  $J_{0,f}$  for BHs before and after mergers, we can combine

the posterior distributions of their masses  $m_{0,f}$  and dimensionless spin magnitudes  $\chi_{0,f}$ , as described in [13]. Similarly, the probability distributions of the BH spin magnitude  $\chi$  and orbital frequency  $\Omega$  before the merger can be obtained by combining the posterior probability distributions of the initial BH mass  $m_0$  and spin magnitude  $\chi_0$ . The probability distributions of the differences in mass  $\delta M$ , in horizon area  $\delta A$ , and in angular momentum  $\delta J$  then can be obtained by combining the probability distributions of  $m_{0,f}$ ,  $A_{0,f}$ , and  $J_{0,f}$ . We analyzed these parameters using the IMRPhenomXPHM: HighSpin waveform model to obtain the distributions of D and r, as illustrated in Fig. 1. The subplot consists of 130 bins on the left and 55 on the right. The parameters in our analysis were obtained from the LIGO and Virgo released in [14].

In order to evaluate the reliability of our GW detection data, we need to compare the actual value of r with the theoretical expected value. We can quantify the accuracy of our results using the fractional difference,  $(r_a - r_e)/r_e$ , where  $r_a$  represents the actual value of r and  $r_e$  represents the expected value of r, which is 1 [15].

By calculating this fractional difference, we can measure the level of error in our data and determine how closely it matches our theoretical expectations. This is an important step in ensuring the accuracy and validity of our results, and can help us to draw more meaningful conclusions about the nature of GWs.

Five different waveform models are considered in this letter: IMRPhenomNSBH: HighSpin, IMRPhenomNSBH: LowSpin, IMRPhenomXPHM: HighSpin, IMRPhenomXPHM: LowSpin, and SEOBNRv4PHM [16]. IMRPhenomNSBH is a tidal model for neutron star BH systems. IMRPhenomXPHM is a spin-processing IMR waveform model that takes into account subdominant modes. While SEOBNRv4PHM is a spin-precessing EOBNR waveform model that is based on SEOBNRv4HM.

#### III. RESULTS

After analyzing five waveform models we generate five violin plots, as shown in Fig. 2. As LIGO and Virgo previously reported in [12] that the posterior probability density distribution of parameters obtained with SEOBNRv4PHM and IMRPhenomXPHM exhibited good agreement overall. The green, red, and purple violin plots also demonstrated good agreement with the first law of BH mechanics. However, the blue and orange violin plots did not. We also calculated the fractional difference of r at 68% and 95% credible regions using these waveform models and present the results in Table I.

The error levels vary among different waveform models, especially the first two models. To eliminate the effect of waveform models on our results, we combine the last three waveform models, IMRPhenomXPHM: HighSpin,

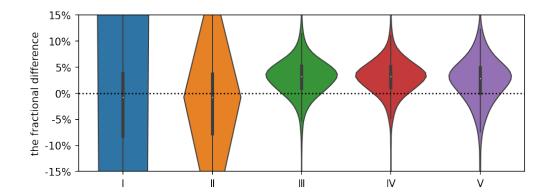


FIG. 2. Probability density distribution of the fractional difference of r. The error bars indicate the 95% credible regions as reported in Table. I. The black dotted horizontal line indicates the expect value 0. The blue/orange/green/red/purple violin plot's posterior distribution of parameters are from the waveform models named IMRPhenomNSBH: HighSpin/IMRPhenomNSBH: LowSpin/IMRPhenomXPHM: HighSpin/IMRPhenomXPHM: LowSpin/SEOBNRv4PHM, respectively.

Waveform models	68% credible region(%)	95% credible region(%)
IMRPhenomNSBH:HighSpin	(-11.8, 6.4)	(-22.6, 20.0)
IMRPhenomNSBH:LowSpin	(-11.1, 6.0)	(-20.5, 19.6)
IMRPhenomXPHM:HighSpin	(0.1, 6.1)	(-4.8, 10.0)
IMRPhenomXPHM:LowSpin	(0.1, 6.1)	(-4.3, 9.7)
SEOBNRv4PHM	(1.6, 6.2)	(-8.8, 10.5)

TABLE I. 68% and 95% credible regions of five violin plots in Fig. 2.

IMRPhenomXPHM: LowSpin, and SEOBNRv4PHM, with a weight of 1:1:2 to ensure that the weight of IMRPhenom and SEOBNR is 1:1. This is illustrated in Fig.3. The 68% credible region is (-0.10%, 6.13%), and the 95% credible region is (-4.70%, 9.94%). Our data analysis results indicate that we have tested the first law of BH mechanics with an error level of about 6% (10%) and 68% (95%) credibility.

The first law of BH mechanics, as shown in Eq. (1), pertains to small variations of M, A, and J. To test this law, we can approximate the merger of a binary system as a perturbed process and examine the fractional difference of r. We hypothesize that there should be a negative correlation between the mass ratio of GW sources and the error level. In other words, as the mass ratio of the binary increases, the fractional difference of r should decrease. To verify our hypothesis, we analyze all GW events with a mass ratio higher than 3 that have been detected thus far. Twelve events in total meet this criterion, with different mass ratios. Six of these events are from the O3a observation run of LIGO and Virgo [17], specifically GW190403\_051519, GW190412\_053044, GW190412, GW190426\_152155, GW190814, and GW190917\_114630 [18]. The remaining six events are from the O3b observation run [12], GW191113\_071753, specifically GW191219\_163120, GW200105\_162426, GW200115\_042309, GW200208\_222617, and GW200210\_092254 [14]. Our

Event	${\it Mass \ ratio}$	The fractional difference (%)
GW190412_053044	$3.07^{+1.31}_{-1.06}$	19.63
GW190412	$3.62^{+1.06}_{-1.08}$	18.08
$GW190426\_152155$	$3.66^{+4.13}_{-2.07}$	17.26
$GW200115\_042309$	$4.12^{+2.76}_{-2.64}$	26.73
$GW190403\_051519$	$4.34^{+4.33}_{-2.84}$	16.04
$GW200208\_222617$	$4.66^{+13.03}_{-3.53}$	16.71
$GW200105\_162426$	$4.74_{-1.07}^{+1.70}$	12.12
$GW190917\_114630$	$4.75^{+3.26}_{-2.13}$	21.96
GW191113_071753	$4.95^{+3.73}_{-3.51}$	13.36
$GW200210\_092254$	$8.49^{+4.52}_{-2.47}$	16.51
GW190814	$8.96^{+0.75}_{-0.62}$	8.79
GW191219_163120	$26.65^{+2.86}_{-3.29}$	6.13

TABLE II. Mass ratios and the fractional differences at 68% credible regions of twelve events in Fig. 4

analysis confirms our hypothesis, as demonstrated in Fig. 4. We show the detailed mass ratios and the fractional differences in Table II. The figure shows that, in general, the higher the mass ratio of the GW source, the smaller the fractional difference and therefore, the lower the error level.

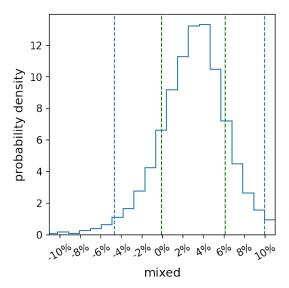


FIG. 3. The probability density distribution of the fractional difference of r is shown, where we mix the samples from the last three waveform models and use 70 bins. The green dashed vertical lines indicate the 68% credible regions, and the blue dashed vertical lines indicate the 95% credible regions. The expected value is 0.

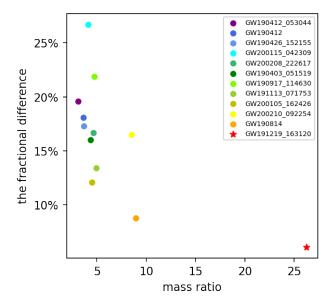


FIG. 4. 68% credible measurements of the fractional difference of different mass ratios of GW sources. Each point represents a different GW event.

## IV. CONCLUSION AND DISSCUSION

Gravitational waves provide valuable information about their sources [19]. By analyzing the data from GW

detections, we can indirectly obtain information about BH. In this letter, we consider the neutron star as a perturbation of the BH. Given that GW191219\_163120 has the highest mass ratio detected thus far, at 27, it is the most suitable event for testing the first law. Our results support the first law of BH mechanics with statistical uncertainties. Our use of GW data detected by LIGO and Virgo provides both physical evidence and an observational test of this mathematically derived law. We are the first to test this law using GWs, which adds to the growing body of knowledge about BHs and their behavior.

We find that the error level of testing the first law is related to the mass ratio of the gravitational wave sources, with higher mass ratios resulting in lower error levels and better agreement with the first law of BH mechanics. In the future, ground-based and space GW observatories will be able to further test the first law with even lower error levels and higher agreement with higher mass ratio signals.

Ground-based GW observatories, such as LIGO [20], Virgo [21], and KAGRA [22], operate in the frequency band of 10Hz-10kHz, while space GW observatories, such as LISA [23], Taiji [24–26], and TianQin [27, 28] , operate in the frequency band of 0.1mHz-1Hz. Space observatories are more sensitive to low-frequency signals than ground-based observatories and can detect GW signals emitted by a stellar BH orbiting a supermassive BH, known as Extreme Mass-Ratio Inspirals (EMRIs) and Extreme Mass-Ratio Bursts (EMRBs), with a mass ratio of at least 10<sup>4</sup> [23]. EMRI signals are continuous due to their low eccentric orbits [29], while EMRB signals are short-lived due to their high eccentric orbits [30]. LISA, for example, can detect a minimum of a few EMRIs events per year and a maximum of a few thousand EMRIs events per year [31], and two Galactic EMRBs events could be detected in LISA's two-year lifetime [32]. By analyzing events with a mass ratio of at least 10<sup>4</sup>, the error level of testing the first law of BH mechanics will be significantly lower, and the agreement with the first law will be higher.

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