Complexity in Solar Cycles

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Introduction

Complexity studies in plasma physics have provided new insights and revealed possible universalities on topics such as turbulence in laboratory plasmas, geomagnetic activity, or the physics of the solar wind. On the other hand, complex networks offer an interesting opportunity to study a wide variety of emerging phenomena from a statistical point of view. Based on these ideas, in this work we propose to build a complex network to represent the evolution of solar activity, as represented by the spatiotemporal patterns of active regions. We then performed a time series analysis on several solar patterns, obtaining information on the persistence of these series.

Complex Network

A complex networks is built based on information on the space and time evolution of active regions [1]. Complex networks are able to follow the spatiotemporal evolution of a system as well, by mapping spatial patterns to nodes and time patterns to their connections.

The degree of a node in a network corresponds to the number of connections that node has. We denote the degree of node i by k_i. For an undirected network of n nodes, the degree can be written as a function of the adjacency matrix as

$$k_i = \sum_{j=1}^{n} A_{ij}. \quad (1)$$

Fractal dimension

Fractals have their own dimension, known as a fractal dimension, which is usually a non-integer dimension that is larger than its topological dimension, D_T, and smaller than its Euclidean dimension, D_E. In particular, we calculate a box-counting fractal dimension, which is given by

$$N(\epsilon) \propto \epsilon^{-D} \quad (2)$$

Hurst exponent

The fractal dimension (D) is directly related to the Hurst exponent (H) for a statistically self-similar data set as $H = E + 1 - D$ where E is the Euclidean dimension. For a one-dimensional signal $H = 2 - D$, so from the above equation we can say that a small H has a larger fractal dimension and a rougher surface. A larger H has a smaller fractal dimension and a smoother surface. There are several ways to calculate the Hurst exponent, the most popular being based on rescaled rank (R/S) analysis.[3]

References

[1] Muñoz, V., & Flández, E. (2022). *Complex Network Study of Solar Magnetograms.* Entropy, 24(6), 753.

[2] Domínguez, M., Muñoz, V., & Valdivia, J. A. (2014), *Temporal evolution of fractality in the Earth's magnetosphere and the solar photosphere.* Journal of Geophysical Research: Space Physics, 119(5), 3585-3603.

[3] Singh, A. K., Bhargawa, A. (2017). *An early prediction of 25th solar cycle using Hurst exponent.* Astrophysics and Space Science, 362(11), 1-6.

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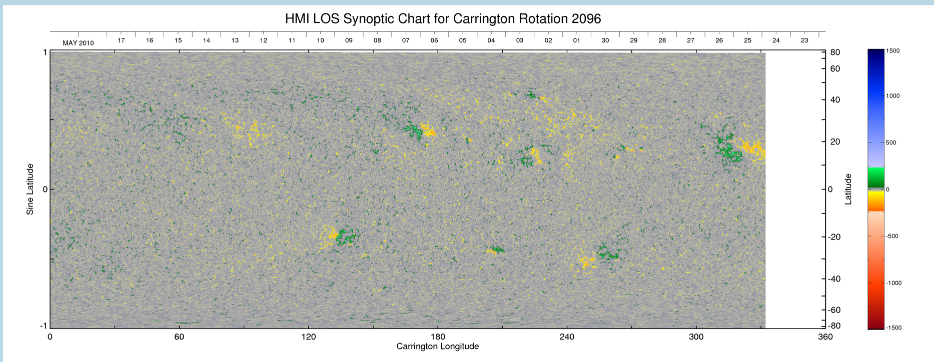
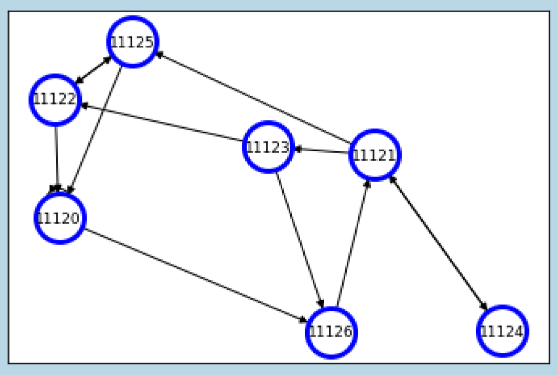
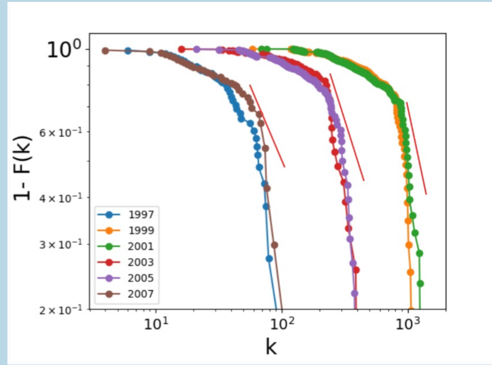
Results: Magnetograms Analisys

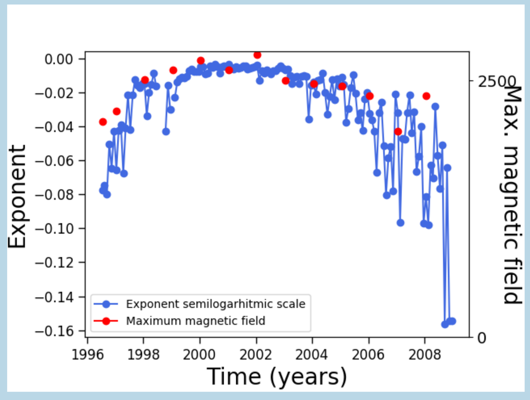
Solar magnetograms (Fig. 1) for each day in the 23rd cycle are converted to a black and white image following the same procedure described in [2]. Then, active regions are identified by means of pattern recognition algorithms. For each active region, the coordinates of its centroid are determined. These coordinates are taken as nodes of the complex network (Fig. 2). Connections between modes are given by the temporal sequence of the magnetograms. We then compute the probability density function (PDF) for the degree. Because the PDF does not describe the grade distribution, we calculate the Cumulative Distribution Function (CDF), and plot it on a semi-logarithmic scale (Fig.3).

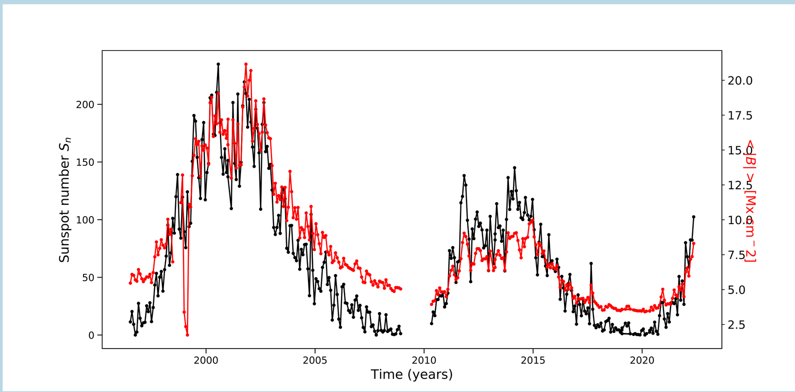


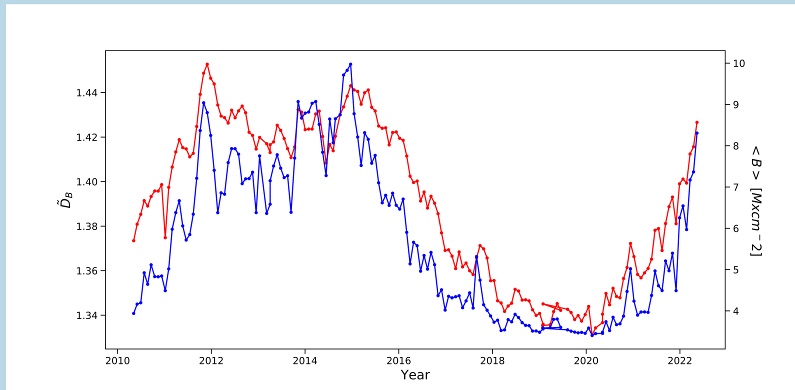




We then obtained the slope of the CDF for each one-month window (Fig.4), and plotted the slope along (blue dots) with the magnetic field strength (red dots). We find that slope correlate with solar activity. Then we can obtain the radial magnetic field by averaging over the complete magnetogram (Fig. 5, red line) which is similar to the number of sunspots (black line). Calculate the fractal dimension of Box Counting for each magnetogram of the solar cycles 24 and 25 (Fig. 6), as a way to measure the complexity of magnetic field time series and spatial patterns. We find that the fractal dimension of box counting follows the shape of the magnetic field throughout the solar cycle. These results are in agreement with the work of the Ref. [2] for SC 23.

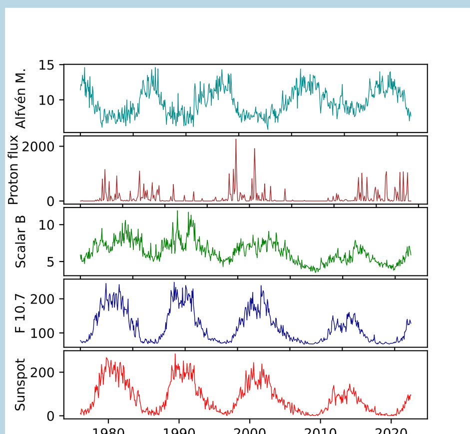


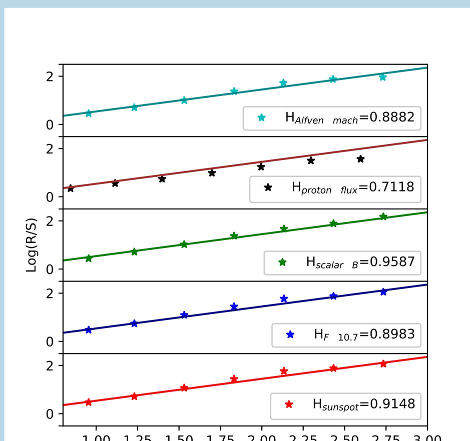


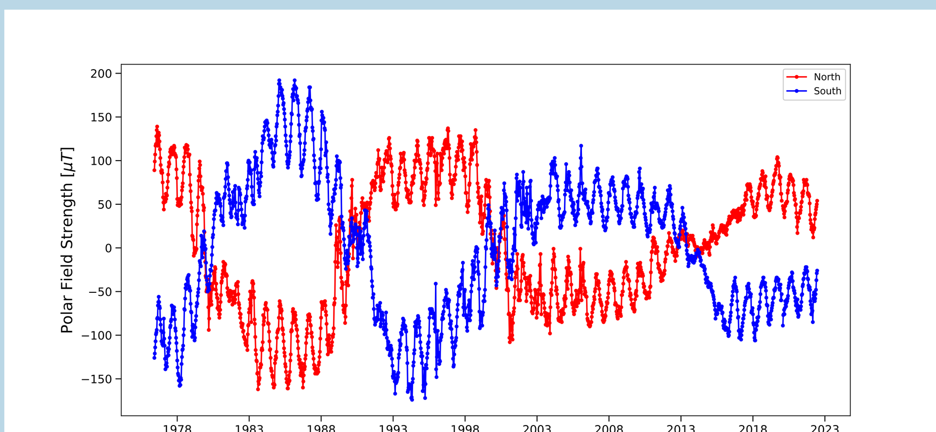


Results: Time Series Analysis

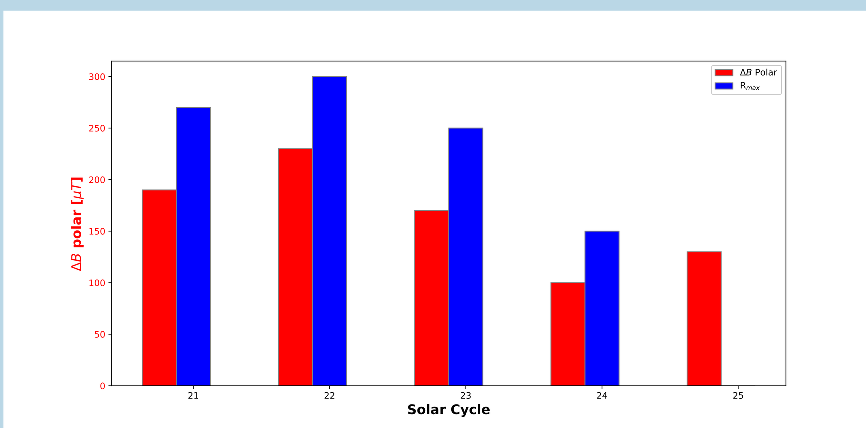
The value of the Hurst exponent (H) ranges from 0 to 1 and determines whether the given time series is completely random or has some long-term memory. Based on the value of H, a time series can be classified into three categories, namely (i) $H = 0.5$, indicative of a random series; (ii) $0 < H < 0.5$, indicating an antipersistent series and (iii) $0.5 < H < 1$ indicative of a persistent series. [3] We examined data for several solar parameters (Goddard Space Flight Center, Space Physics Data Facility (SPDF), https://omniweb.gsfc.nasa.gov/form/dx4.html) such as sunspot number, 10.7 cm radio flux, solar magnetic field, proton flux and Alfven Mach number observed for the year 1976-2016 (Fig. 7), repeating the original idea made in Ref. [3], And we obtained similar Hurst Exponent values (Fig. 8). Since we found that the most persistent series were the magnetic field series, we decided to repeat this analysis for the magnetic field at the north and south poles of the Sun (Fig 9).

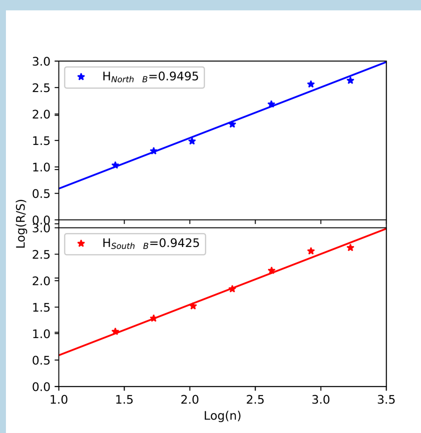






Solar physicists have identified that one of the precursors that is linked to solar activity is the strength of the magnetic field at the Sun's poles. Then for each solar minimum prior to the ascending phase of each cycle, we measure the distance between north and south polar magnetic field curves $\Delta B_{\mathbf{polar}}$, and on the other hand we record the maximum number of sunspots for each solar cycle. We found that there is a linear correlation between $\Delta B_{\mathbf{polar}}$ and the maximum number of sunspots for each cycle, i.e., when the difference between the north and south polar magnetic fields was larger, then the maximum number of sunspots was also larger (Fig. 10). Again we obtained that both the magnetic and south field series were very persistent (Fig. 11).





Conclusion: Given that the polar field series is persistent, and that this is a proxy for the strength of the subsequent solar maximum, the next step in the research is to attempt to model the magnetic field series at the poles.