

1. Introduction

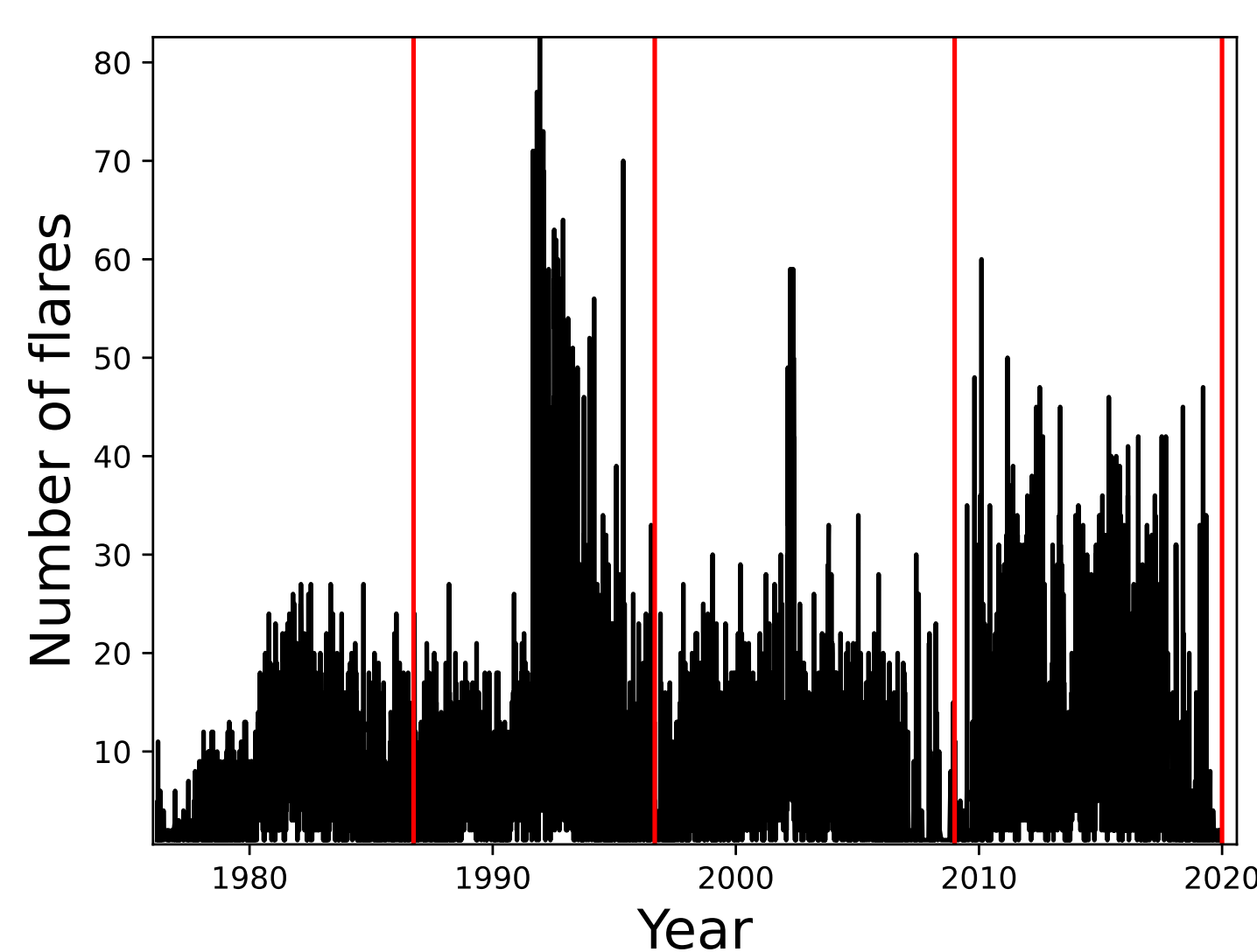
An active region (AR) is a region of the solar photosphere where the magnetic field strength is greater than that of the rest of the photosphere, and has a bipolar structure with the flow well-ordered in two islands with opposite polarity. The size distribution of the active regions shows a power law, which suggests that the emergence of magnetic flux in the solar surface occurs at all size scales. Solar flares are the manifestation of a sudden, intense, and spatially concentrated release of energy in the corona, which causes localized heating up to temperatures of $\sim 10^7$ K, and causes abundant emission of short wavelength radiation. They often occur in active regions and among the different mechanisms proposed for the origin of solar flares, it has been accepted that their appearance is due to magnetic reconnection. It has been found that the occurrence of solar flares N follows a power law distribution versus total flares energy W :

$$\frac{dN}{dW} \sim W^{-\alpha}, \quad \text{with } \alpha \sim 1.8.$$

Solar flares are related with the complexity of evolving magnetic fields in ARs. The limited predictability, nonlinearity, and self-organized critical behavior manifested by flare phenomena suggests that it should be relevant to study them from a complex systems perspective, such as complex networks approaches [1] or cellular automata based on the Lu-Hamilton model [2], approaches which can lead to new insights into the underlying complex behavior, and hint on event triggering [3]. The time series associated with solar flares have also been studied through complex networks [1], exploring the behavior of the complex network of solar flares on the solar surface and found that the complex networks of flares follow the scale-free characteristics, thus showing that flares exhibit self-organized criticality and that the complex networks can provide direct information about the behavior of solar flares. For each solar cycle (SC), we build networks of the active regions that presented solar flares.

2. Data

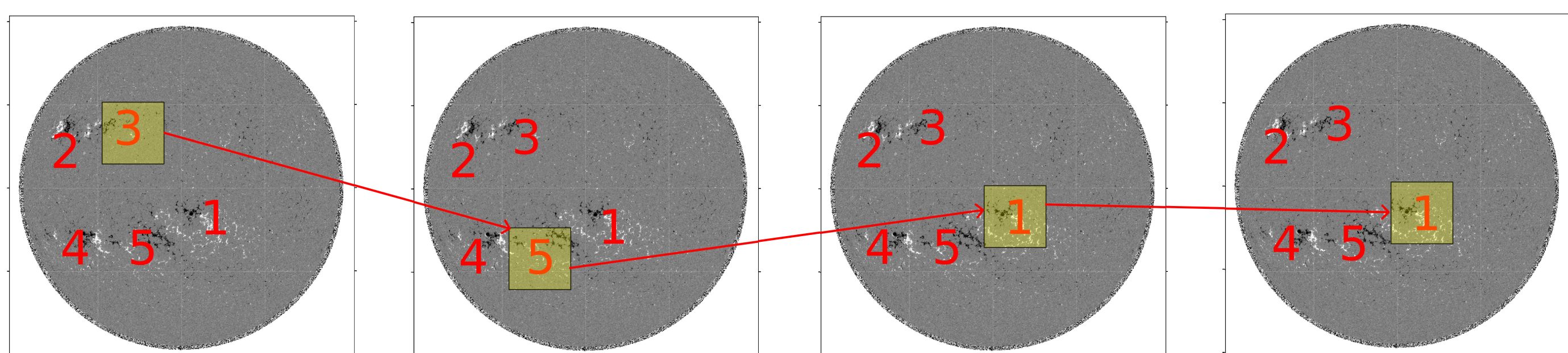
We consulted all the solar flares that occurred during SC₂₁ from March 1976 to September 1986, SC₂₂ from September 1986 to August 1996, SC₂₃ from August 1996 to December 2008, and SC₂₄ from January 2008 to December 2019. Within the total number of solar flares available in the HEK database, only some have an associated active region of origin, while several solar flares do not emerge from ARs but rather from the surface of the Sun. We have plotted the number of solar flares per day produced by active regions for each solar cycle, where it can be noticed that cycles 22 and 24 present more solar flare activity than the others.



On the other hand, the number of solar flares that do not emerge from active regions for SC₂₁ is 7701 (61% of the total solar flares emanating from the entire solar surface), SC₂₂ is 17192 (48%), SC₂₃ is 16152 (38%) and SC₂₄ is 5814 (80%). In this work we are interested in considering only solar flares from active regions, since we can associate them to zones with well-defined magnetic activity.

3. Complex network construction

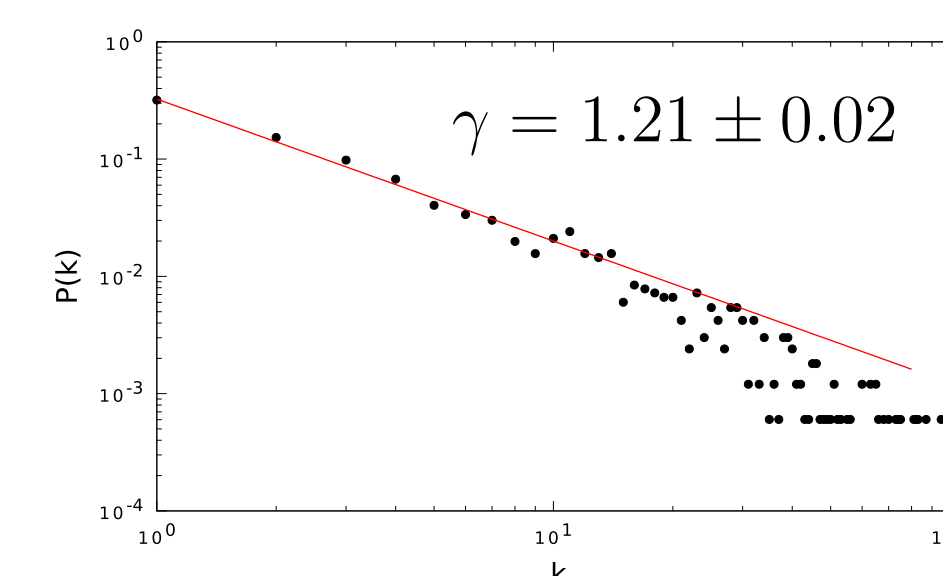
To construct the network, we consider as nodes only those active regions that emanated solar flares. Then we connect the nodes given the time sequence of occurrence of solar flares. If two consecutive solar flares arise from the same active region, then the node will have a self-connection.



This strategy follows the original idea by Abe and Suzuki for earthquake networks [4, 5], and has been successfully used to study active regions appearance on the Sun's photosphere [6].

4. Degree distribution

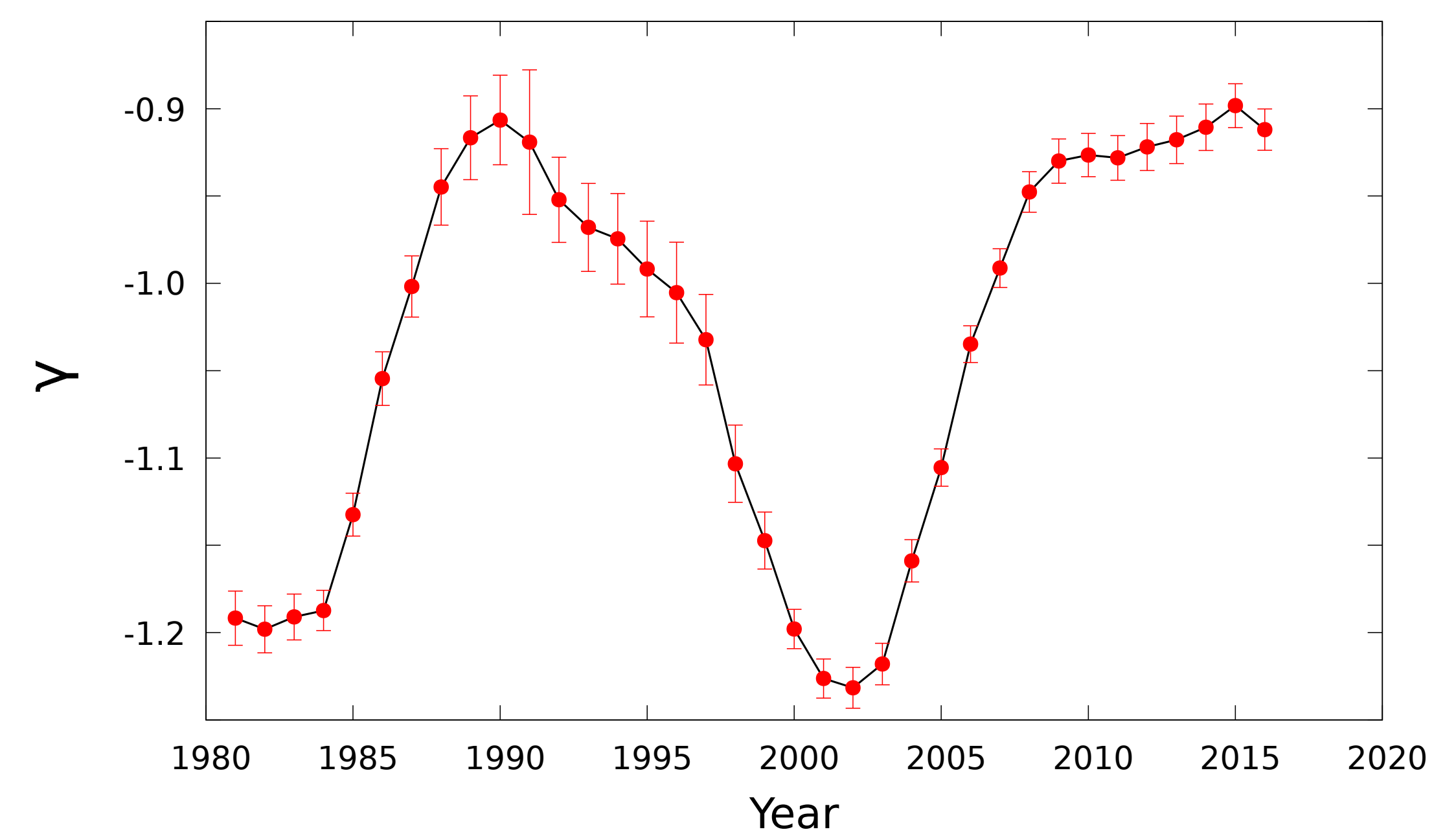
First we build a directed network, where two types of degrees can be identified for each node: the in-degree k_i^{in} , which is the number of incoming connections to a node, and the out-degree k_j^{out} . Then we determine for SC₂₁ to SC₂₄ its degree distribution $P(k)$ which in turn can provide information about the underlying process in the ARs.



In SC₂₁ to SC₂₄ a scale-free behavior can be observed, of the form $P(k) = k^{-\gamma}$, where γ is the characteristic exponent of the network, obtaining the following values: SC₂₁ = 1.21 ± 0.02 , SC₂₂ = 0.95 ± 0.04 , SC₂₃ = 1.22 ± 0.02 and SC₂₄ = 0.81 ± 0.02 . This result is consistent with other works about flares [1]. The fact that there is a scale-free distribution is consistent with the underlying process not being fully random. A type of preferential attachment may be involved, such that some active regions are more likely to yield flares, and highly connected nodes turn out to be more probable than in a random process, thus leading to a power-law behavior. Besides we notice that for these four solar cycles, we obtained that odd cycles have lower γ values than even, reminding the even-odd rule for SC.

5. Frame analysis

Then we take windows spanning 11 years shifted one year with respect to the previous one. We calculated the degree distribution for each of the networks and obtained their γ . For each window, a scale-free behaviour is found.



A striking feature of this plot is that the decay exponent does not only oscillates, but seems to exhibit a period of about two SC. The first and the second minima are separated by 22 years. This brings to mind the Sun's magnetic cycle known as the Hale cycle. There is a broad consensus that the magnetic cycle alone is caused by the inductive action of fluid motions permeating the interior of the Sun. Hale's law states that the magnetic polarity of the active regions is oppositely oriented in both hemispheres and alternates in successive sunspot cycles so that a complete magnetic solar cycle lasts approximately 22 years. These statistical properties are consistent with the Babcock-Leighton dynamo model. The Sun's global magnetic field undergoes a cyclic transition from a global poloidal field to a high-stressed toroidal field during an 11-year cycle. It explains the liquidation of the highly stressed toroidal field as a consequence of differential rotation, and the subsequent gradual decay due to meridional flows, ending in a relaxed poloidal field at the solar minimum. The most important thing is that the characteristic exponent of the distribution of the degree of the network shows a variation consistent with the Hale cycle, which is interesting result. Therefore demonstrates the enormous power of complex systems and how tools such as complex networks can deliver new information on solar physics.

Acknowledgments

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References

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