SERVO MOTOR CONTROL DESIGN REPORT

Instructions: replace the yellow highlighted text with your own words and the requested plots (that is, delete the yellow text). Note: the plots and figure used in this report should be saved using MATLAB functions or Simulink (not screen captures).

Authorship details

Author	*Don Misura Minduwara Kaluarachchi N10496262*
List others providing assistance	

Question 1	1. Working attached below 2. Working attached below

TOPIC Question 1

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* given in black ink with a circle will be the required answer for each task specified.

using $K_m = 7.09$

alpha $= 4.27$

$G(s) = \frac{K}{s(s+\alpha)}$ also given in the form $\frac{K}{s^2+\alpha s+0}$

$= \frac{7.09}{s(s+4.27)}$

$G_{closed}(s) = \frac{G(s)}{1 + G(s)\ H(s)} \times K$

$= \frac{7.09K / s^2 + 4.27s}{1 + \left[\frac{7.09}{s(s+4.27)} \times 1 \times K\right]}$

$= \frac{7.09K}{s^2 + 4.27s + 7.09K}$ ← 1

Two complex conjugate poles, therefore system is underdamped.

$s_1 = -2.135 + 1.59115i$

$s_2 = -2.135 - 1.59115i$

General form, second order system

$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

by observation $4.27 \equiv 2\zeta\omega_n$

$2.135 = \zeta\omega_n$

Settling time $(T_s) = \frac{4}{\zeta\omega_n} = \frac{4}{2.135} = 1.874$ seconds ← 2

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QUT TOPIC

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Using $K=1$, $2\zeta\omega_n = 4.27$

$$\zeta\omega_n = 2.135$$

$$\zeta = \frac{2.135}{\omega_n}$$

where $\omega_n = \sqrt{8}$ $\quad \zeta = \frac{2.135}{\sqrt{8}}$

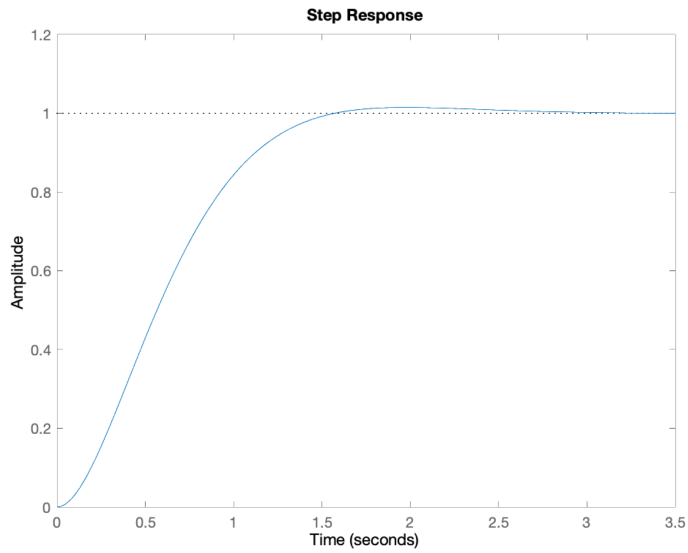
$$= \frac{2.135}{\sqrt{7.09}}$$

$$\zeta = 0.8011$$ underdamped

$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100$$

$$\%OS = 1.477 \leftarrow (2)$$

3. Plot attached below



4. Comment: Using the in-built MATLAB function "Stepinfo" (results shown below) we can see that the %OS calculated in the analytical method and the matlab model are the same therefore the plot matches the results.

```
    RiseTime: 0.9293
SettlingTime: 1.4161
 SettlingMin: 0.9081
 SettlingMax: 1.0148
   Overshoot: 1.4762
  Undershoot: 0
        Peak: 1.0148
    PeakTime: 1.9844
```

Question 2

1. Working attached below
2. Working attached below

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where $K = 7.09$
$\alpha = 4.27$

For 5% overshoot, open loop $\rightarrow G(s) = \dfrac{K}{s(s+\alpha)}$

Closed loop system,

(same algebra as Q1)

$$G_{cl}(s) = \frac{G(s)}{1 + G(s)H(s)} \times K \Rightarrow \frac{7.09K}{s^2 + 4.27s + 7.09K} \leftarrow (1)$$

to determine a value that results in a 5% OS

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}$$

$$\zeta = \frac{-\ln(5/100)}{\sqrt{\pi^2 + \ln^2(5/100)}} = \underline{\underline{+0.69}}$$ damping ratio

$(1) \rightarrow 4.27 = 2\zeta\omega_n$ from part 1

$\dfrac{4.27}{2 \times 0.69} = \omega_n$ where $\omega_n = \sqrt{\gamma}$

$(\omega_n)^2 = \gamma$

$s^2 + \beta s + \gamma$

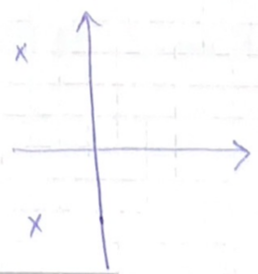
$s^2 + 4.27s + 7.09 \times 1.35$

$\gamma = \left(\dfrac{4.27}{2 \times 0.69}\right)^2$ and $\gamma = 7.09K$

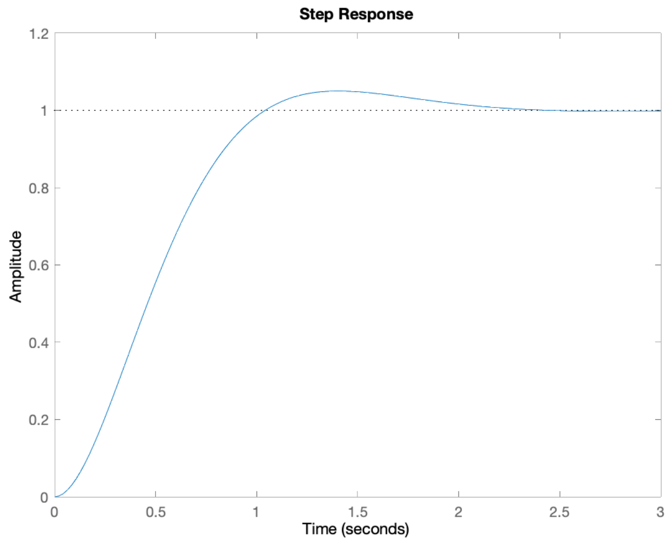
$\left(\dfrac{4.27}{2 \times 0.69}\right)^2 = \gamma = 7.09K$

$\underline{\underline{K = 1.35}} \leftarrow (2)$

closed loop poles
$$\begin{cases} s_1 = -2.135 + 2.24i \\ s_2 = -2.135 - 2.24i \end{cases} \leftarrow (2)$$



3. Plot attached below

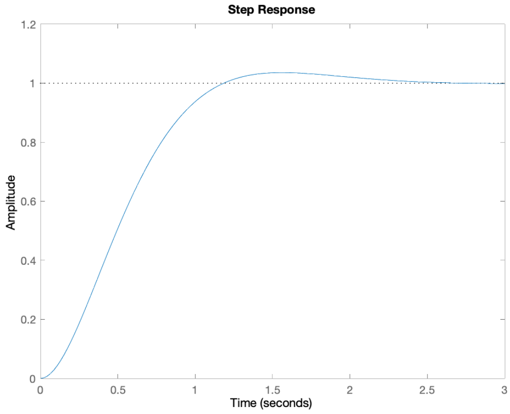
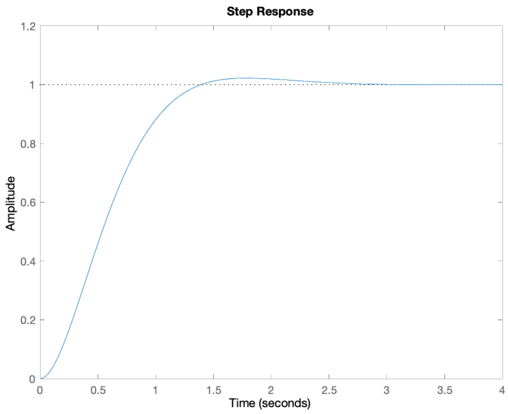
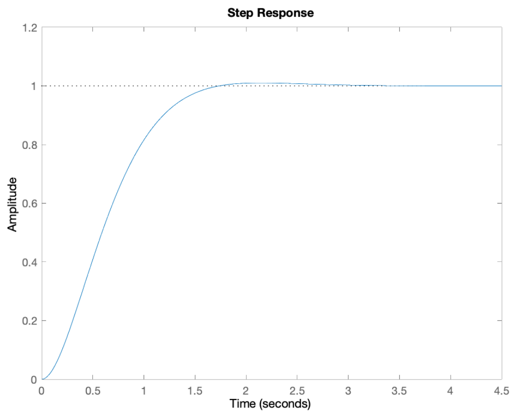


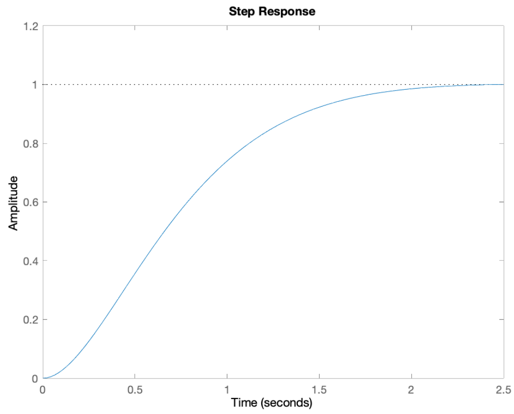
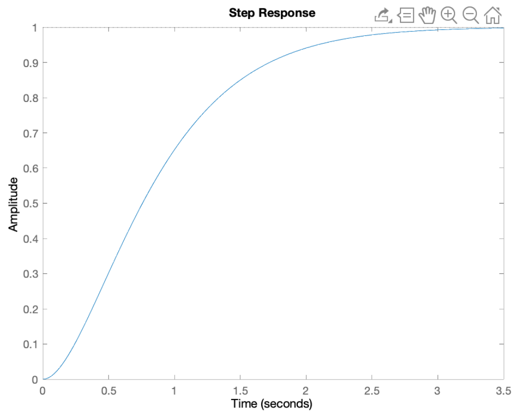
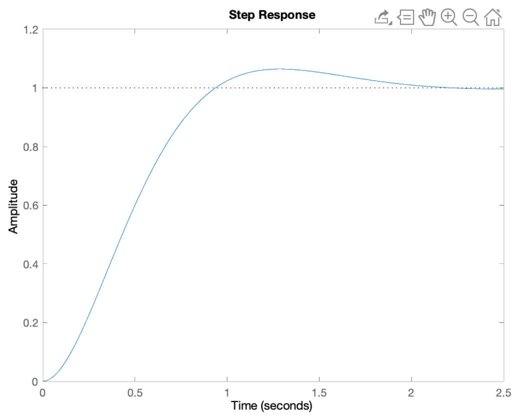
4. Comment: Using the in-built MATLAB function "Stepinfo" we can see that the matlab gives a 5% overshoot and the analytic method was used to calculate for 5% overshoot. Attached below is a screenshot when the step info function is used. This shows that plot matches the analytical results.

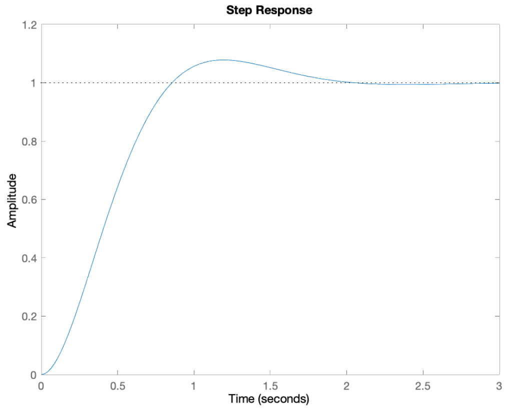
```
    RiseTime: 0.6778
SettlingTime: 1.9379
 SettlingMin: 0.9005
 SettlingMax: 1.0500
   Overshoot: 5.0005
  Undershoot: 0
        Peak: 1.0500
    PeakTime: 1.4020
```

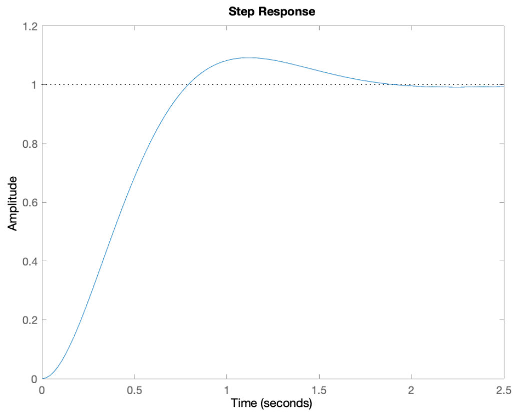
Question 3

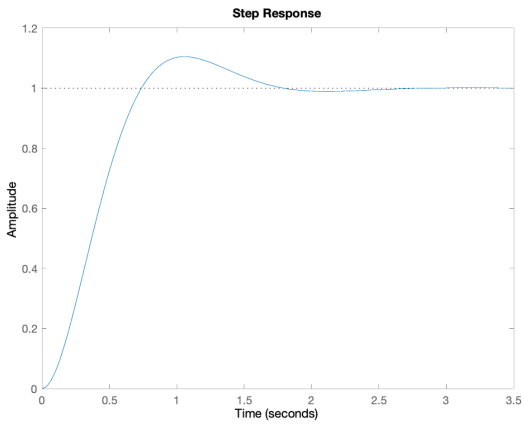
1. Table attached

K value	Closed loop poles and Plots
0.9K	 -2.1350 + 2.0140i -2.1350 - 2.0140i
0.8K	 -2.1350 + 1.7604i -2.1350 - 1.7604i
0.7K	 -2.1350 + 1.4635i -2.1350 - 1.4635i

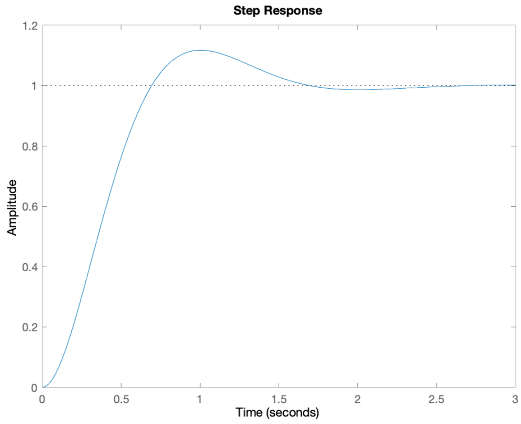
	0.6K	 -2.1350 + 1.0884i -2.1350 - 1.0884i
	0.5K	 -2.1350 + 0.4770i -2.1350 - 0.4770i
	1.1K	 -2.1350 + 2.4434i -2.1350 - 2.4434i

<table>
<tr><td></td><td>1.2K</td><td>

-2.1350 + 2.6320i
-2.1350 - 2.6320i</td></tr>
<tr><td></td><td>1.3K</td><td>

-2.1350 + 2.8080i
-2.1350 - 2.8080i</td></tr>
<tr><td></td><td>1.4K</td><td>

-2.1350 + 2.9735i
-2.1350 - 2.9735i</td></tr>
</table>

	1.5K	 -2.1350 + 3.1303i -2.1350 - 3.1303i

2. Working attached below. Effect of increasing and decreasing gain changes the location of the poles along the y axis which is what you can expect from a root locus. The branches of the root loci would pass through the axis that the closed loop poles are on. It can be observed that there is a constant settling time along the axis.

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$K = 1.35$

(1) K values, $(0.9 * K)$ $\rightarrow \dfrac{7.09 \times K}{s^2 + 4.27s + 7.09K}$

$\rightarrow s^2 + 4.27s + (7.09 \times 1.35 \times 0.9)$

$\rightarrow s_1 = -2.135 + 2.01i$

$s_2 = -2.135 - 2.01i$

(2) K values, $(0.8 * K)$ $\rightarrow s^2 + 4.27s + (7.09 \times 1.35 \times 0.8)$

$\rightarrow s_1 = -2.135 + 1.76i$

$s_2 = -2.135 - 1.76i$

(3) K values, $(0.7 * K)$ $\rightarrow s^2 + 4.27s + (7.09 \times 1.35 \times 0.7)$

$s_1 = -2.35 + 1.46i$

$s_2 = -2.35 - 1.46i$

(4) K values $(0.6 * K)$ $\rightarrow s^2 + 4.27s + (7.09 \times 1.35 \times 0.6)$

$s_1 = -2.135 + 1.09i$

$s_2 = -2.135 - 1.09i$

(5) K value $(0.5 * K)$ $\rightarrow s^2 + 4.27s + (7.09 \times 1.35 \times 0.5)$

$s_1 = -2.135 + 0.48i$

$s_2 = -2.135 - 0.48i$

(6) K value $(1.1 \times K)$ $\rightarrow s^2 + 4.27s + (7.09 \times 1.35 \times 1.1)$

$s_1 = -2.135 + 2.44i$

$s_2 = -2.135 - 2.44i$

(7) K values $(1.2 * K)$ $\rightarrow s^2 + 4.27s + (7.09 \times 1.35 \times 1.2)$

$s_1 = -2.135 + 2.63i$

$s_2 = -2.135 - 2.63i$

(8) K value $(1.3 * K)$ $\rightarrow s^2 + 4.27s + (7.09 \times 1.35 \times 1.3)$

$s_1 = -2.135 + 2.81i$

$s_2 = -2.135 - 2.81i$

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(9) K values (1.4 × K) $\longrightarrow$ $s^2 + 4.27s + (7.09 \times 1.35 \times 1.4)$

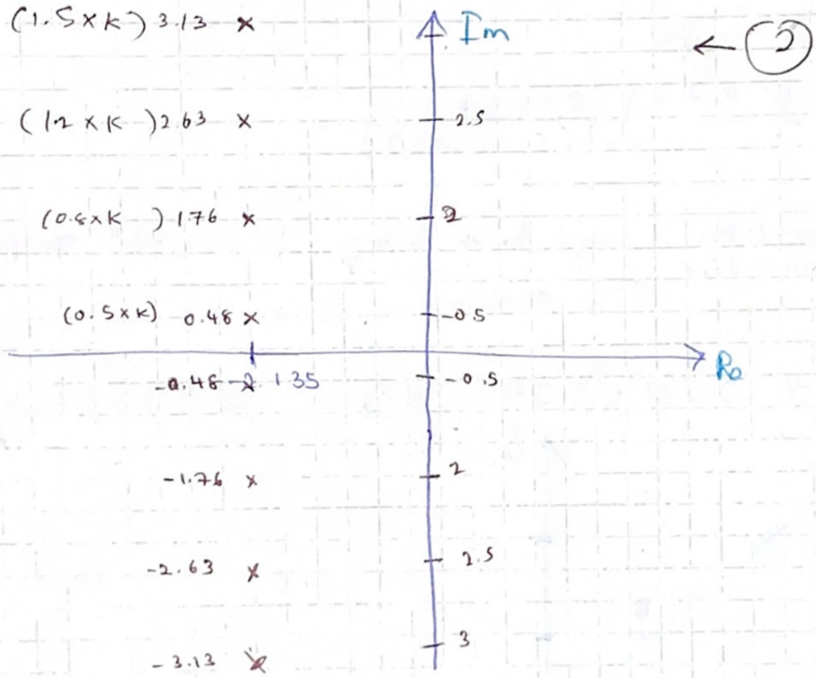
$s_1 = -2.135 + 2.97i$

$s_2 = -2.135 - 2.97i$

(10) K values (1.5 × K) $\longrightarrow$ $s^2 + 4.27s + (7.09 \times 1.35 \times 1.5)$

$s_1 = -2.135 + 3.13i$

$s_2 = -2.135 - 3.13i$



Question 4	1. Working attached below

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introducing ~~dyam~~ dynamic compensation → has half the time in Q2

for an open loop response :

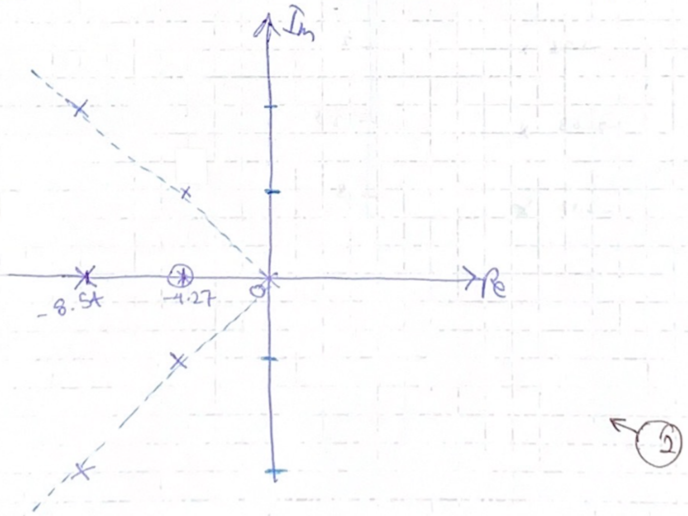
$$T = \frac{4}{\sigma} = \frac{4}{\zeta \omega_n}$$ to get half the settling time you need 2σ

new OL response

$$\frac{K \times 7.09\,(s+4.27)}{s\,(s+4.27)(s+2\times 4.27)}$$

we want the changes to be along the constant damping ratio line (↖) direction

$$s = (-2.135 + 2.24i) \times 2 = -4.270 + 4.48i$$ ①



use the eq^n $K = \frac{1}{|G|}\Bigg|_{s=-4.270+4.48i}$

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2. Working attached below

[recheck math]

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$$K = 1 \Big/ \left| \frac{7.09(\cancel{-4.270 + 4.48i} + 4.27)}{(-4.27+4.48i)(\cancel{-4.270+4.48i}+4.27)} \right|$$

$(-4.27 + 4.48i + 8.54)$

$$K = \frac{1}{\left| 7.09 / (-4.27+4.48i)(-4.27+4.48i)(-4.27+4.48i+8.54) \right|}$$

$$K = \qquad 33.44$$

$$= 5.40 \quad (3\ s.f)$$

$$7.09 \times K_c = K$$

$$K_c = \qquad \underline{\underline{4.72}}$$

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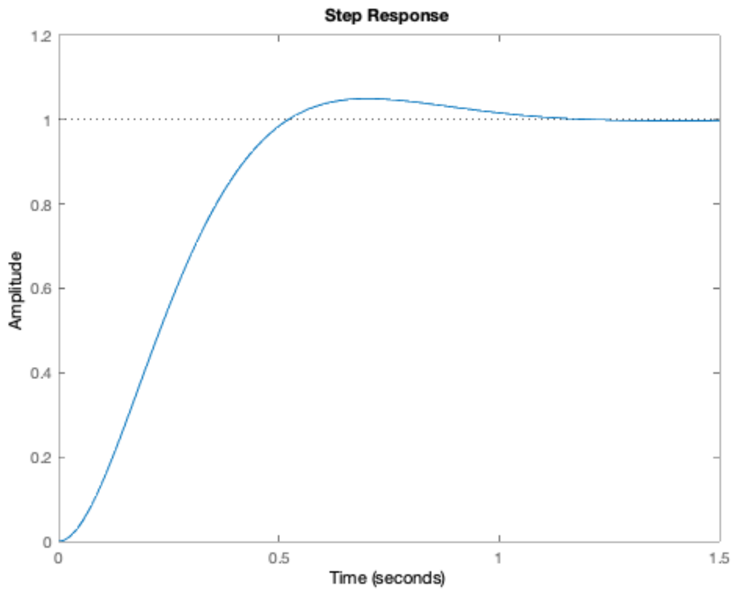
3. Plot attached below:




```
   RiseTime: 0.3389
SettlingTime: 0.9689
 SettlingMin: 0.9005
 SettlingMax: 1.0500
   Overshoot: 5.0005
  Undershoot: 0
        Peak: 1.0500
    PeakTime: 0.7010
```

4. Comment: The overshoot condition (under 5%) is satisfied, the system is stable, and the settling time is halved which is as expected therefore plots match results. It can be observed from the plot that the system settles at 1.

Question 5	1. Working attached below

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$$G(s) = \frac{K_m}{s(s+\alpha)} \times K$$

open loop response

$K_m = 7.09$

$\alpha = 4.27$

$s^2 + s\alpha + 0$

Stretching upon + possibilites of K_m & α values

$K_m \rightarrow 0.9\%,\ 1.1\%$

$\alpha \rightarrow 0.9\%,\ 1.1\%$

Case 1 $\begin{cases} K_m \rightarrow 0.9 \\ \alpha \rightarrow 0.9 \end{cases}$

convert / write in terms of a closed loop system

$$G_{cl}(s) = \frac{G(s)}{1 + G(s)H(s)} \times K$$

$$= \frac{(7.09 \times 0.9) / [s^2 + (4.27 \times 0.9)s]}{1 + \left[\frac{7.09 \times 0.9}{s(s+4.27)} \times K\right]}$$

$$= \frac{(7.09 \times 0.9) \times K}{s^2 + (4.27 \times 0.9)s + (7.09 \times 0.9)K}$$

$(4.27 \times 0.9) = 2\zeta\omega_n$

$\frac{4.27 \times 0.9}{2 \times 0.69} = \omega_n = \sqrt{\sigma}$

$$\left(\frac{4.27 \times 0.9}{2 \times 0.69}\right)^2 = 7.09 \times 0.9 \times K$$

$K = 1.21$ — 1

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the algebra is repeated for the rest of the cases.

Case 2; $K_m \rightarrow 0.9 \times K_m$ $K = \dfrac{\left(\frac{4.27 \times 1.1}{2 \times 0.69}\right)^2}{7.09 \times 0.9}$

$\alpha \rightarrow 1.1 \times \alpha$

$= 1.815$

Case 4; $K_m \longrightarrow 1.1 \times K_m$ $K = \dfrac{\left(\frac{4.27 \times 1.1}{2 \times 0.69}\right)^2}{7.09 \times 1.1}$

$\alpha \longrightarrow 1.1 \times \alpha$

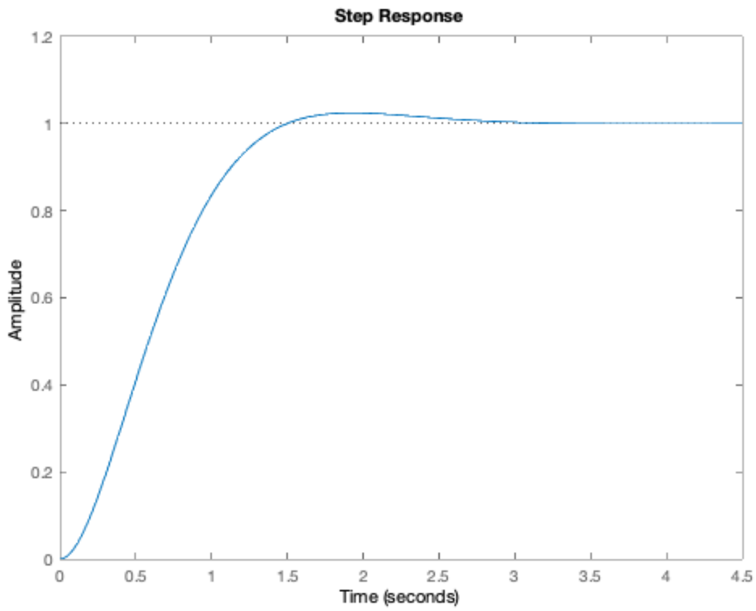
$K = 1.485$

Case 3; $K_m \longrightarrow 1.1 \times K_m$ $K = \dfrac{\left(\frac{4.27 \times 1.1}{2 \times 0.69}\right)^2}{7.09 \times 0.69}$ ← ①

$\alpha \longrightarrow 0.9 \times \alpha$ $K = 0.994$

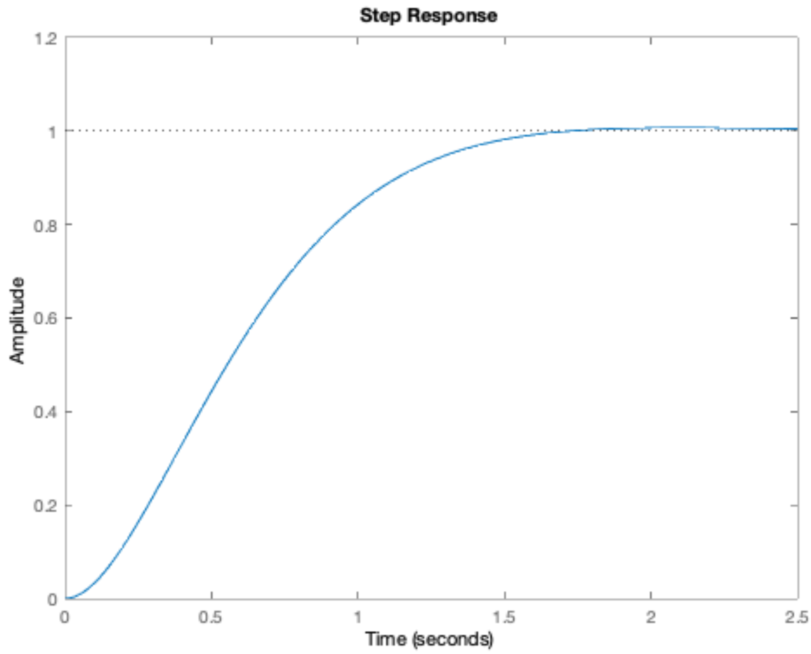
2. Justification: We use the Case 3 gain which is 0.994 so that when it is used in any system it has the lowest gain therefore allowing for an acceptable %overshoot.

3. Plot attached below




```
    RiseTime: 0.9264
SettlingTime: 2.2218
 SettlingMin: 0.9091
 SettlingMax: 1.0245
   Overshoot: 2.4520
  Undershoot: 0
        Peak: 1.0245
    PeakTime: 1.9413
```

4. Plot attached below




```
     RiseTime: 0.9482
 SettlingTime: 1.4829
  SettlingMin: 0.9013
  SettlingMax: 1.0072
    Overshoot: 0.7203
   Undershoot: 0
         Peak: 1.0072
     PeakTime: 2.0982
```

5. Comment: The highest overshoot can be observed by system 3 with an overshoot of 4.99 with the lowest overshoot coming from system 2 with an overshoot of 0.03%. System 1 which has the highest settling time while system 4 has the lowest settling time. For any unknown value of Km and alpha the overshoot will not be higher than the expected 5% therefore the system will remain robust.

Question 6	1. Working attached below

TOPIC: Question 6

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$$G_{cl}(s) = \frac{G(s)}{1+G(s)\,H(s)} \times K$$

open loop response

$$G(s) = \frac{K_m}{s(s+\alpha)} \times K$$

$$= \frac{7.09K / s^2 + 4.27s}{1+\left[\frac{7.09}{s(s+4.27)} \times 0.9 \times K \times H(s)\right]}$$

$$= \frac{7.09K}{(s^2+4.27s) + s(s+4.27)\left(\frac{7.09K}{s^2+4.27s}\right) \times H(s)}$$

$$= \frac{7.09K}{s(s+4.27) + (7.09K) \times H(s)}$$

Solving for when the H(s) is

$H(s) = 0.9$

$$= \frac{7.09K}{s^2+4.27s+(7.09\times 0.9)K}$$

$4.27 = 2\zeta\omega_n$

$$\frac{4.27}{0.69\times 2} = \omega_n$$

$$\left(\frac{4.27}{0.69\times 2}\right)^2 = 7.09\times 0.9\times K$$

$K = 1.5$ ← (1)

$H(s) = 1.1$

$$= \frac{7.09K}{s^2+4.27s+(7.09\times 1.1)K}$$

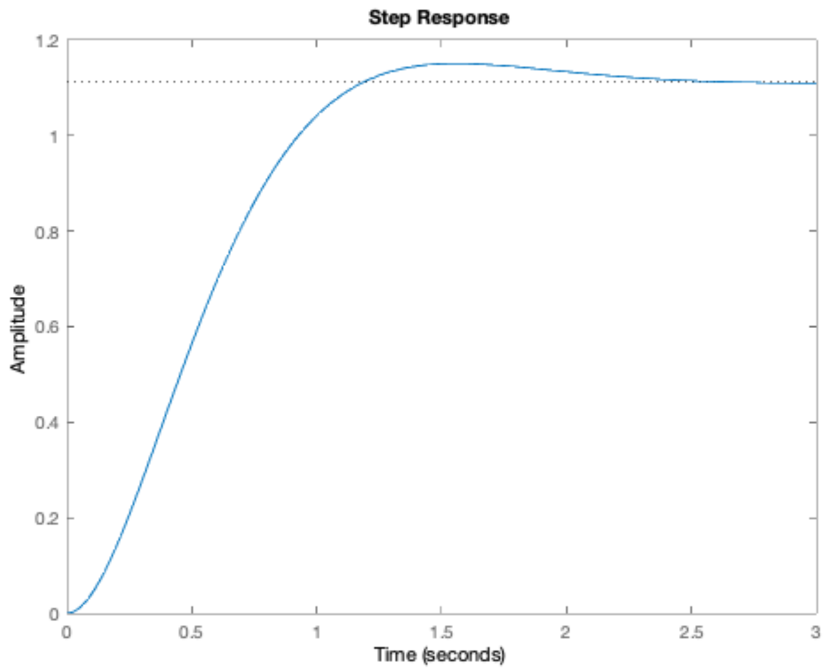
$4.27 = 2\zeta\omega_n$

$$\left(\frac{4.27}{0.69\times 2}\right)^2 = 7.09\times 1.1\times K$$

$k = 1.23$ ← (1)

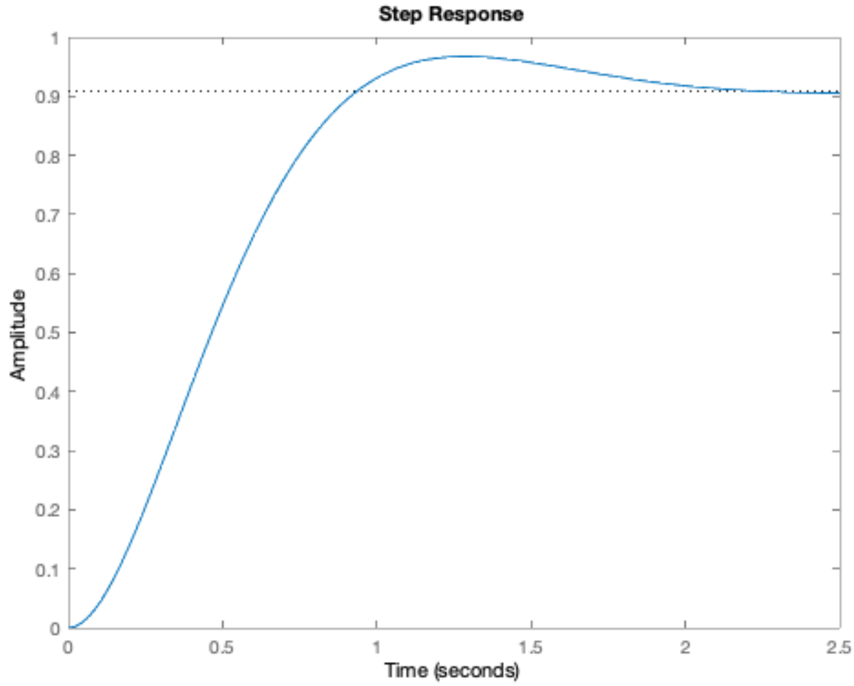
2. Comment: Both plots show that the overshoot is under 5% therefore the design is robust. The lower gain is used to ensure that the system remains under the 5% overshoot.

3. Plot attached below (H = 0.9)




```
    RiseTime: 0.7538
SettlingTime: 2.0070
 SettlingMin: 1.0006
 SettlingMax: 1.1509
   Overshoot: 3.5773
  Undershoot: 0
        Peak: 1.1509
    PeakTime: 1.5530
```

4. Plot attached below (H= 1.1)




```
    RiseTime: 0.3389
SettlingTime: 0.9689
 SettlingMin: 0.9005
 SettlingMax: 1.0500
   Overshoot: 5.0005
  Undershoot: 0
        Peak: 1.0500
    PeakTime: 0.7010
```

5. Comment: Both plots which show the unknown range of gains have systems that have an overshoot that is under 5% this will ensure that the system is still robust whether it is at the lower or higher end of the unknown values. The lower gain is chosen to ensure that the system operates under the overshoot. In terms of steady state properties, the system with a lower gain takes almost twice the time to settle and the values that it settles at is also different with one system settling at 1.1 and the other at 0.9.

Question 7	1. Working attached below

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$$G_o(s) = \frac{V_p(s)}{V_m(s)} = \frac{K_m}{s(s+\alpha)(s+\beta)}$$

open loop response.

$s^2(s^2 + \beta s + \alpha s + \alpha\beta)$

s

Closed loop response

$$G_{cl}(s) = \frac{G_o(s)}{1 + G_o(s) \times H(s)} \times K$$

$$= \frac{K \times K_m / s(s+\alpha)(s+\beta)}{1 + \left[\frac{K_m}{s(s+\alpha)(s+\beta)} \times H(s) \times K\right]}$$

$$= \frac{K \times K_m}{s(s+\alpha)(s+\beta) + \left[\cancel{s(s+\alpha)(s+\beta)} \times \frac{K_m}{\cancel{s(s+\alpha)(s+\beta)}} \times H(s) \times K\right]}$$

$$= \frac{K \times K_m}{s(s+\alpha)(s+\beta) + K_m \times H(s) \times K}$$

(a) If $\beta = 10\alpha$ (the fast pole)

$$= \frac{K \times K_m}{s(s+\alpha)(s+\beta) + K_m \times H(s) \times K}$$

when $K_m = 7.09$
$\alpha = 4.27$
$H(s) = 1$
$K = 1.35$

$$= \frac{1.35 \times 7.09}{s(s+4.27)(s+10 \times 4.27) + 7.09 \times 1 \times 1.35}$$

a. When 10*alpha

QUT TOPIC Question 7 contd

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$s(s^2 + 42.7s + 4.27s + 10 \times 4.27 \times 4.27)$

$s(s^2 + 46.97s + 182.329)$

$s^3 + 46.97s^2 + 182.329s$

$$= \frac{9.715}{s^3 + 46.97s^2 + 182.329s + 9.5715}$$

Solving the eq^2 the poles are at

$S_1 = -42.71$

$S_2 = -0.05$

$S_3 = -4.21$

due to the prescense of a fast pole S_1 and S_3 can be ignored (5x rule)

dominant pole => -0.05

$$G(s) = \frac{9.715}{(s + 0.05)}$$

Settling time = $\frac{4}{\text{pole}}$

$\approx$ 80 seconds

No %OS as it's a first order system.

b. When 2 * alpha

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Name:	Student No.	Ckd:	Date:

(b) $\dfrac{1.35 \times 7.09}{s(s+4.27)(s+2\times4.27) + 7.09 \times 1 \times 1.35}$

$s(s^2+2\times4.27s) + 4.27s + 4.27\times2\times4.27 + 7.09\times1\times1.35$

$s(s^2+12.81s+36.47) + 9.5715$

$s^3 + 12.81s^2 + 36.47s + 9.5715$

$= \dfrac{9.5715}{s^3+12.81s^2+36.47s+9.5715}$

$s_1 = -8.781$

$s_2 = -0.292$

$s_3 = -3.738$

by using the 5X rule

the $s_2 = 0.292$ is the dominant pole.

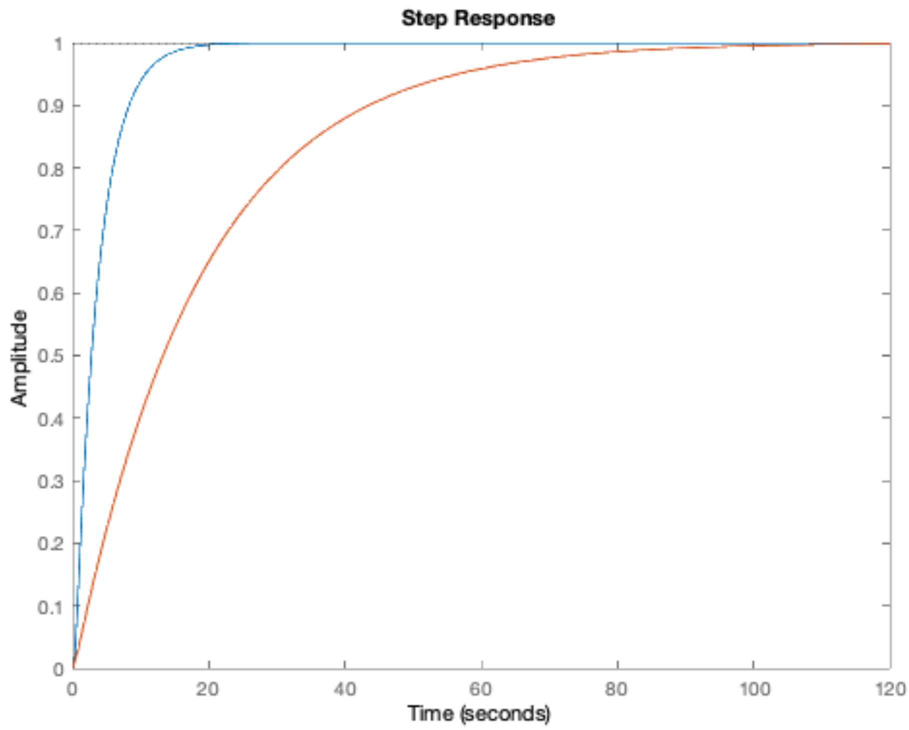
Setting time $= \dfrac{4}{pole}$

$= 13.70$ seconds.

No %OS, it's a first order system.

Science and Engineering Faculty

	2. Plot attached below




```
   RiseTime: 41.2748
SettlingTime: 73.7543
 SettlingMin: 0.9032
 SettlingMax: 0.9993
   Overshoot: 0
  Undershoot: 0
        Peak: 0.9993
    PeakTime: 137.5493
```

```
   RiseTime: 7.5720
SettlingTime: 13.8081
 SettlingMin: 0.9004
 SettlingMax: 0.9996
   Overshoot: 0
  Undershoot: 0
        Peak: 0.9996
    PeakTime: 27.0818
```

3. Comment: The new open loop response was used. The closed loop response was then used to calculate the location of the closed loop poles when there was a fast pole and when there wasn't.
   With the use of the 5x rule we can find which is the dominant pole. By observation of the pole location we can see that the system with the fast pole where it is closer to zero so it has a higher settling time as seen above. System 1 has a settling time of 73 seconds while the system 2 has a settling time of 13 seconds. Both systems end up settling at the same value even though they have a difference in their settling time.

Appendix A: Approved Extension Letter

Reference number: 211026-001121

Subject
EN01 10496262 Kaluarachchi, Don Misura Minduwara - EGB345 Assignment Extension (EXT) [Faculty of Engineering]

Response By Email (Brianna) (01/11/2021 03.30 PM)

Dear Don Misura Minduwara,

Your request for an Assignment Extension has been **approved**. The details of your extension are provided below:

Unit: EGB345 - Control and Dynamic Systems
Assignment Title: Servo Motor Control Design Task
Original Submission Due Date: 26 October 2021
Revised Approved Submission Due Date: 2 November 2021

Please submit your assignment using the normal submission process as outlined in your unit's Blackboard site.

You are required to attach a copy of this email when submitting your assignment as it is confirmation of your approved extension.

If you do not submit your assignment by the extended due date your work will not be marked and you will receive a grade of 1 or 0% against the assessment item.

Book an individual session with a success coach or specialist educator (study skills, language and writing support, STEM skills) to get back on track with your studies. Find out more about other the academic support available at qut.to/academicsupport.

If you wish to discuss this further, please quote your reference number: 211026-001121.

Kind Regards,
Brianna

Faculty of Engineering | Queensland University of Technology

O Block Podium, Level 3, Gardens Point Campus
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CRICOS No 00213J

Remember to include your full name, student number and course on all correspondence with us and to check your QUT emails regularly.

For 24/7 assistance please go to ASK QUT