EGB345 Portfolio Task

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Statement: I declare that this(Portfolio answers) is my own work.

Introduction and Review of Laplace Transforms.

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Task 1:

$$R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i\,d\tau = V_{dc}\,u(t)$$

→ needs to be converted to laplace domain

$$R\,I(s) + L\,s\,I(s) + \frac{1}{C}\frac{I(s)}{s} = V_{dc}(s)\frac{1}{s}$$

$$I(s)\left[R + Ls + \frac{1}{Cs}\right] = V_{dc}(s)\frac{1}{s}$$

$$I(s) = \frac{V_{dc}(s)/s}{R + Ls + 1/Cs}$$

$$= \cancel{\frac{V(s)}{sR + s^2L + 1/C}}$$

$$= \frac{V_{dc}}{s^2L + sR + 1/C}$$

Tutorial 1 : Numerical Sol^n

$$I(s) = \frac{\left(\frac{V_{dc}}{L}\right) \times L}{\left(s^2 + \frac{sR}{L} + \frac{1}{LC}\right) \times L}$$

multiply by L

$$= \frac{V_{dc}}{s^2L + sR + 1/C}$$

∴ task 1 is correct.

Task 2 : Determine the inverse transfer function

$$\frac{6(s+5)(s+3)}{s(s+1)(s+2)}$$

$$= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

Using the cover up method explained in tutorial

$$A = \frac{6(0+5)(0+3)}{(0+1)(0+2)} = \underline{\underline{45}}$$

$\boxed{s=0}$

$$B = \frac{6(-1+5)(-1+3)}{(-1)(-1+2)} = \frac{6(4)(2)}{-1} = \underline{\underline{-48}}$$

$\boxed{s=-1}$

$$C = \frac{6(-2+5)(-2+3)}{-2(-2+1)} = \frac{6(3)(1)}{-2(-1)} = \underline{\underline{9}}$$

$\boxed{s=-2}$

$$\hookrightarrow \frac{45}{s} - \frac{48}{s+1} + \frac{9}{s+2}$$

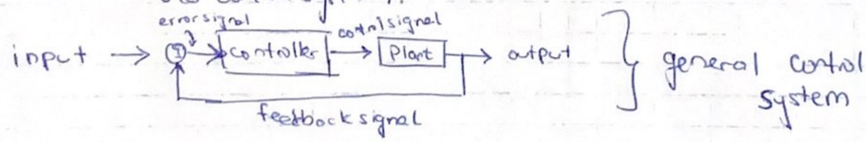
→ use laploce transform table.

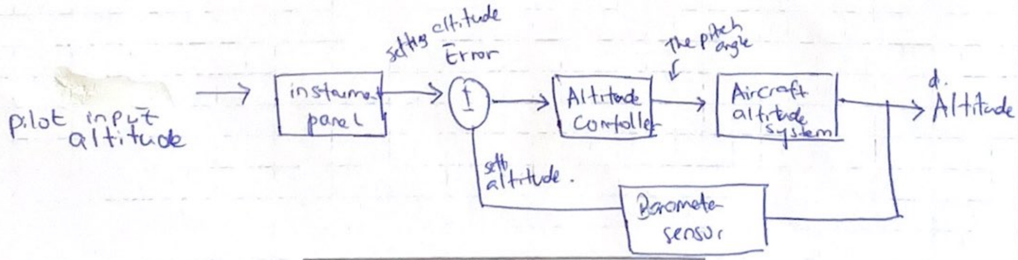
$$= u(t)\left(45 - 48e^{-t} + 9e^{-2t}\right)$$

Numerical sol^{n}

$$\left(45 - 48e^{-t} + 9e^{-2t}\right)u(t)$$

Task 3 : Draw functional digram for the aircraft altitude hold control sytem.





Electrical transfer function

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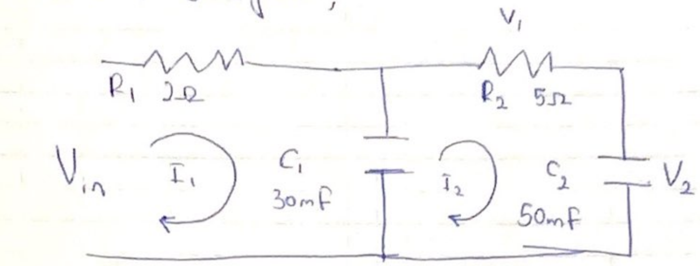
Circuit diagram,



$R \rightarrow R$

$C \rightarrow \frac{1}{sC}$

$L \rightarrow sL$

KVL loop 1: $\Sigma V_{in} = \Sigma V_{out}$

$$V_{in} = I_1 R_1 + C_1 (I_1 - I_2)$$

$$V_{in} = I_1 R_1 + \frac{1}{sC}(I_1 - I_2)$$

$$V_{in} = \left(R_1 + \frac{1}{sC_1}\right) I_1 - \left(\frac{1}{sC_1}\right) I_2 \quad (1)$$

← Keeping in i_1 and i_2 form

KVL loop 2: $\Sigma V_{in} = \Sigma V_{out}$

$$0 = R_2 I_2 + C_2 I_2 - C_1 (I_1 - I_2)$$

lapl tra

$$0 = R_2 I_2 + I_2 \times \frac{1}{sC_2} - \frac{1}{sC_1} I_1 + \frac{1}{sC_1} I_2$$

$$0 = -\left(\frac{1}{sC_1}\right) I_1 + \left(R_2 + \frac{1}{sC_1} + \frac{1}{sC_2}\right) I_2 \quad (2)$$

$$0 = \left(-\frac{1}{sC_1}\right) I_1 + \left(R_2 + \frac{2}{sC_2}\right) I_2 \quad (2)$$

$$\begin{bmatrix} R_1 + \frac{1}{sC_1} & -\frac{1}{sC_1} \\ -\frac{1}{sC_1} & R_2 + \frac{1}{sC_2} + \frac{1}{sC_1} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{in} \\ 0 \end{bmatrix}$$

← I_2 using cramers

← $V_2 = I_2 \times \frac{1}{sC_2}$

← V_{in} ~~out~~ sub in I_2

← $V_1 = I_2 R_2$

← $\frac{V_2}{V_{in}}$ subs V_{in} for $\frac{5}{s}$ and inv LT.

~~se~~

Solve for I_2 using cramers.

$$I_2 = \frac{\det\left(\begin{bmatrix} R_1 + \frac{1}{sC_1} & V_{in} \\ -\frac{1}{sC_1} & 0 \end{bmatrix}\right)}{\det\left(\begin{bmatrix} R_1 + \frac{1}{sC_1} & -\frac{1}{sC_1} \\ -\frac{1}{sC_1} & R_2 + \frac{1}{sC_2} + \frac{1}{sC_1} \end{bmatrix}\right)}$$

$$I_2 = \left(V_{in} \times -\frac{1}{sC_1}\right) \Big/ \left(\left(R_1 + \frac{1}{sC_1}\right)\left(R_2 + \frac{1}{sC_1} + \frac{1}{sC_2}\right) - \frac{1}{s^2 C_1^2}\right)$$

$$I_2 = \frac{-V_{in}/sC_1}{-\frac{R_1}{sC_1} - R_1 R_2 - \frac{R_1}{sC_2} - \cancelto{0}{\frac{1}{s^2 C_1^2}} - \frac{R_2}{sC_1} - \frac{1}{s^2 C_1 C_2} + \cancelto{0}{\frac{1}{s^2 C_1^2}}}$$

$$I_2 = \frac{-V_{in}}{-sC_1\left(\frac{R}{sC_1} + R_1 R_2 + \frac{R_1}{\cancel{s}C_2} + R_2/sC_1 + 1/s^2 C_1 C_2\right)}$$

$$I_2 = \frac{V_{in}}{\left(R_1 + R_1 R_2 sC_1 + \frac{R_1 C_1}{C_2} + R_2 + \frac{1}{sC_2}\right)}$$

$\frac{\times s}{\times s}$

$$I_2 = \frac{V_{in} \times S}{SR_1 + S^2 R_1 R_2 C_1 + \frac{SR_1 C_1}{R_2} + SR_2 + \frac{1}{C_2}}$$

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$$\Sigma \times {}^{1}/_{R_1 R_2 C_1} \longrightarrow \frac{(V_{in} \times s)/R_1 R_2 C_1}{\frac{sR_1}{R_1R_2C_1} + \frac{s^2 R_1 R_2 C_1}{R_1 R_2 C_1} + \frac{sR_1C_1}{sR_1R_2} + \frac{sR_2}{R_1R_2C_1} + \frac{1}{C_2 \times R_1 R_2 C_1}}$$

using eq^n $V = IR$ where $I = \frac{V}{R}$

use $I_2 = \frac{V_{out}}{R} \leftarrow {}^{1}/_{sC_2}$; $I_2 = \frac{V_{out}}{{}^{1}/_{sC_2}}$ $\sim V_2$ or V_1

using I_2 in original eq^n

$$\frac{V_{in} \times s}{s^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_1}\right)s + \frac{1}{R_1R_2C_1C_2}} = \frac{V_{out}}{{}^{1}/_{sC_2}}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{{}^{1}/_{\cancel{s}C_2} \times {}^{\cancel{s}}/_{R_1 R_2 C_1}}{s^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2}\right)s + \frac{1}{R_1R_2C_1C_2}} \rightsquigarrow {}^{1}/_{C_1C_2R_1R_2}$$

substitue values into the eq^n

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{{}^{1}/_{50\times10^{-3} \times 30\times10^{-3} \times 5 \times 2}}{s^2 + \left({}^{1}/_{2 \times 30\times10^{-3}} + {}^{1}/_{5 \times 30 \times 10^{-3}} + {}^{1}/_{5 \times 50\times10^{-3}}\right)s + \frac{1}{5 \times 2 \times 30\times10^{-3} \times 50\times10^{-3}}}$$

using calculator.

$$= \frac{66.6}{s^2 + 27.3s + 66.6}$$

given answer $\hookrightarrow \frac{V_2(s)}{V_{in}(s)} = \frac{66.6}{s^2 + 27.3s + 66.6}$

$\therefore$ answer is correct.

Task 2, $t=0$ $V=5V$

how long (t)? to reach 2V, $V=2V$

Since the V_{in} can be modelled as a step input (lecture 2)

$$V_{in}(t) = 5\,u(t)$$

Laplace table $u(t) \leftrightarrow \frac{1}{s}$

$$V_{in}(S) = \frac{5}{S}$$

Using above eq^n and eq^n in task 1

$$\frac{V_2(s)}{V_{in}(s)} = \frac{66.6}{s^2 + 27.3s + 66.6} \longrightarrow V_2(S) = \frac{66.5 \times}{s^2+27.3s+66.6}\frac{5}{s}$$

$$\xrightarrow{\text{Simplify}} V_2(S) = \frac{333.3}{S(S+2.71)(S+24.59)}$$

simplify further using partial fraction.

$$V_2(S) = \frac{333.3}{S(S+2.71)(S+24.59)} = \frac{A}{S} + \frac{B}{S+2.71} + \frac{C}{S+24.59}$$

Using cover up method.

$S=0$, $A = \frac{333.3}{2.71 \times 24.59} = 5.00$ //

$S=-2.71$, $B = \frac{333.3}{-2.71 \times (-2.71 + 24.59)} = -5.62$ //

$S=-24.59$, $C = \frac{333.3}{(-24.59)(-24.59+2.71)}$

$= 0.619$

≈ 0.62 //

resulting in the partial fraction

$$\frac{333.3}{s(s+2.71)(s+24.59)} = \frac{5}{s} + \frac{-5.62}{s+2.71} + \frac{0.62}{s+24.59}$$

Use laplace table and inverse laplace transform

$$\frac{5}{s} \Leftrightarrow 5u(t) \quad , \quad \frac{-5.62}{s+2.71} \Leftrightarrow -5.62u(t)e^{-2.71t}$$

$$\frac{0.62}{s+24.59} \Leftrightarrow 0.62\,u(t)\,e^{-24.59t}$$

$$V_2(t) = (5 - 5.62e^{-2.71t} + 0.62e^{-24.59t})\,u(t)$$

when the output voltage $= 2$,

$$2 = (5 - 5.62e^{-2.71t} + 0.62e^{-24.59t})\,u(t)$$

by having a look at the eq^n we can deduce that $e^{-24.59t}$ can be ignored as it will reach zero

[as james mount did in tut 2*]

$$-5 = -5.62e^{-2.71t} + 0.62e^{-24.59t}$$

$$-5 = -5.62e^{-2.71t}$$

$$-2.71t = \ln\left(\frac{-5}{-5.62}\right)$$

$$t = \frac{1}{-2.71} \times \ln\left(\frac{5}{5.62}\right) \Rightarrow t = 0.23 \text{ seconds}$$

answer Task 2, 0.23 seconds

$\therefore$ task 2 is correct.

Mechanical transfer function

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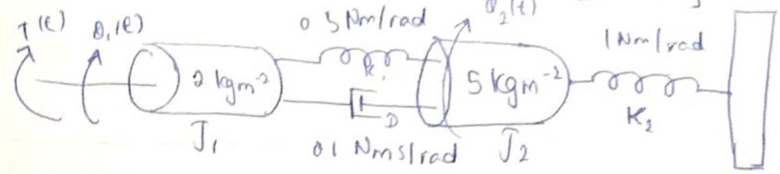
Task 1 Find the transfer function $\left[\frac{\theta_1(s)}{T(s)}\right]$



from left to right

$K_1 \leftarrow$ first spring

$K_2 \leftarrow$ second spring

$D \leftarrow$ damper

→ Two degrees of freedom

→ Two eq^ns

For main inertia 1

$$T(t) = T_i(t) + T_{spr}(t) + T_v(t)$$

$$T(t) = J_1 \times \frac{d^2\theta(t)}{dt^2} + K_1\left(\theta_1(t) - \theta_2(t)\right) + D \times \left(\frac{d\theta_1(t)}{dt} - \frac{d\theta_2(t)}{dt}\right)$$

↓ Laplace transform

$$T(s) = J_1 s^2 \theta_1(s) + K_1\left(\theta_1(s) - \theta_2(s)\right) + Ds\left(\theta_1(s) - \theta_2(s)\right)$$

Rearranging the eq^n so we can use cramers rule properly.

$$T(s) = (J_1 s^2 + Ds + K_1)\theta_1(s) - (Ds + K_1)\theta_2(s) \quad \text{eq}^n \text{ (1)}$$

For main inertia 2

$$T_{spr}(t) + T_v(t) + T_{spr2}(t) + T_{in2}(t) = 0$$

$$K_1\left(\theta_2(t) - \theta_1(t)\right) + D\left(\frac{d\theta_2(t)}{dt} - \frac{d\ \ (t)}{dt}\right) + K_2\theta_2(t) + J_2 \times \frac{d^2\theta_2(t)}{dt^2}$$

$\left[\frac{d^2\theta}{dt^2} \rightarrow s^2\theta(s)\right]$

↓ Laplace transform

$$K_1\left(\theta_2(s) - \theta_1(s)\right) + D\left(\theta_2(s) - \ (s)\right) + K_2\theta_2(s) + J_2 s^2 \theta_2(s)$$

Rearranging the eq^n same as eq^n (1)

$$-(Ds + K)\theta_1(s) + (J_2 s^2 + Ds + K_1 + K_2)\theta_2(s) = 0 \quad \text{(2)}$$

Cramer rule, $V_1 = \det\left(\begin{bmatrix} RV_1 & B \\ RV_2 & D \end{bmatrix}\right) / \det\begin{pmatrix} A & B \\ C & D \end{pmatrix}$

$$\begin{bmatrix} J_1 s^2 + Ds + k_1 & -(Ds+k) \\ -(Ds+k_1) & (J_2 s^2 + Ds + k_1 + k_2) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} T(s) \\ 0 \end{bmatrix}$$

the diagonal is same therefore equations have been used properly.

$$\theta_1 = \frac{\det\begin{pmatrix} T(s) & -(Ds+k) \\ 0 & J_2 s^2 + Ds + k_1 + k_2 \end{pmatrix}}{\det\begin{pmatrix} J_1 s^2 + Ds + k_1 & -(Ds+k) \\ -(Ds+k_1) & J_2 s^2 + Ds + k_1 + k_2 \end{pmatrix}}$$

$$\frac{\theta_1}{T(s)} = \frac{J_2 s^2 + Ds + k_1 + k_2}{J_1 J_2 s^4 + (J_1 D + J_2 D)s^3 + (J_1 k_1 + J_1 k_2 + J_2 k_1)s^2 + D k_2 s + k_1 k_2}$$

→ rearranged to find the transfer function.

→ sub in the values.

$J_1 = 2 \quad k_1 = 0.5 \quad D = 0.1$
$J_2 = 5 \quad k_2 = 1$

$$\frac{\theta_1(s)}{T(s)} = \frac{5s^2 + 0.1s + 1.5}{10s^4 + 0.7s^3 + 5.5s^2 + 0.15 + 0.5} \times \frac{1/10}{1/10}$$

$$\frac{\theta_1(s)}{T(s)} = \frac{0.5s^2 + 0.01s + 0.15}{s^4 + 0.07s^3 + 0.55s^2 + 0.01s + 0.05} \quad -(1)$$

∴ same as given answer.

Task 2: : System response

Step input of 1Nm, use laplace transform

$T(t) = u(t)$

$T(s) = \frac{1}{s}$

into eqn from task 1

$$\theta(s) = \text{eq}^{n}\ ①\ \times \frac{1}{s}$$

$$= \frac{0.5s^2 + 0.01s + 0.15}{s^5 + 0.07s^4 + 0.55s^3 + 0.015s^2 + 0.05s}$$

∴ answer is same as given answer.

Electromechanical transfer function.

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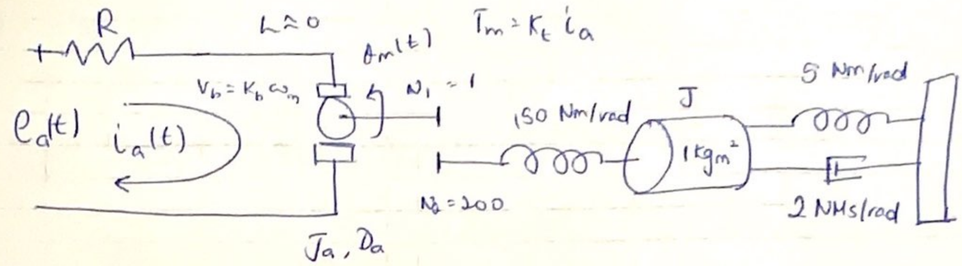
Find the transfer function for the Series Elastic Actuator



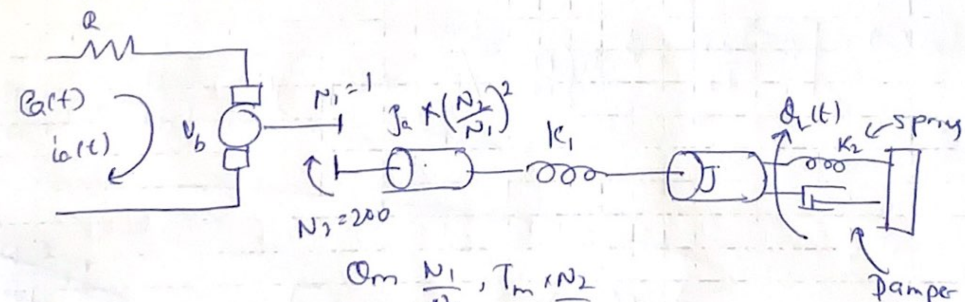
→ given that $J_{\text{acting on } J_2} = J_2 \times \left(\frac{N_2}{N_1}\right)^2$

→ other eq^ns.

Angular displacement: $\frac{\theta_1}{\theta_2} = \frac{N_2}{N_1} \longrightarrow \theta_2 = \theta_1 \times \frac{N_1}{N_2}$

Torque : $\frac{T_2}{T_1} = \frac{N_2}{N_1} \longrightarrow T_2 = T_1 \times \frac{N_2}{N_1}$

To do the question systems are broken down as explored in the tutorial.



$\theta_m \frac{N_1}{N_2}$, $T_m \times \frac{N_2}{N_1}$

Use KVL,

$$e_a(t) = R \times i_2(t) + V_b$$

$$T_m = K_t \times i_2$$

$$i_2 = \frac{T_m}{K_t}$$

Substitute the eq^n in KVL

$$e_a(t) = \frac{R}{K_t} \times T_m + K_b \cdot \omega_m$$

$$V_b = K_b \times \omega_m$$

Back emf

Laplace transform and sub in ω_m

$$\omega_m = s \cdot \theta_m$$

$$E_a(s) = \frac{R}{K_t} T_m + s K_b \theta_m(s) \quad -1$$

Summing the forces on the first mass.

$$\frac{N_2}{N_1} T_m = \left(\frac{N_2}{N_1}\right)^2 J_a s^2 \theta_m \frac{N_1}{N_2} + \left(\theta_m \frac{N_1}{N_2} - \theta_2\right) K_1$$

make T_m the subject the formulae.

$$T_m = \left(K_1 \left(\frac{N_1}{N_2}\right)^2 + s J_a\right) \theta_m - \left(K_1 \times \frac{N_1}{N_2}\right) \quad -2$$

subs 2 in T_m of eq^n 1

$$E_a(s) = \frac{R}{K_t}\left[s^2 J_a + K_1 \left(\frac{N_1}{N_2}\right)^2 \theta_m - \left(K_1 \frac{N_1}{N_2}\right)\theta_2 \right] + s K_b \theta_m(s)$$

3

Summing torques

Nod 2: $s^2 J\theta_2 + k_1\theta_L + k_2\theta_2 + sD\theta_L - k_1\theta_m \frac{N_1}{N_2} = 0$

$$k_1\theta_m \frac{N_2}{N_2} = s^2 J\theta_2 + sD\theta_L + k_2\theta_2 + k_1\theta_2$$

make θ_m subject

$$\theta_m = \frac{s^2 J\theta_2 N_2}{k_1 N_1} + \frac{sD\theta_2 N_2}{k_1 N_1} + \frac{k_2\theta_2 N_2}{k_1 N_1} + \frac{\theta_2 N_2}{N_1}$$

eq^n ④

Rearanging eq^n ③

$$E_a(s) = \frac{s^2 J_a R\theta_m}{k_t} + \frac{R_1 k_1 \left(\frac{N_1}{N_2}\right)^2 \theta_m}{k_t} - \frac{Rk_1N_1\theta_2}{k_t N_2} + sk_b\theta_m$$

$$E_a(s) = \left[\frac{s^2 J_a R}{k_t} + R_1k_1\left(\frac{N_1}{N_2}\right)^2 / k_t + sk_b\right)\theta_m - \frac{Rk_1N_1\theta_2}{k_2 N_2}$$

Subs eq^n ④ in ③ then substitute

$$E_a(s) = \left[\left(\frac{s^2 J_a R}{k_t} + R_1k_1\left(\frac{N_1}{N_2}\right)^2 / k_t + sk_b\right)\left(\frac{s^2 J\theta_2 N_2}{k_1N_1} + \frac{sD\theta_2 N_2}{k_1 N_1} + \frac{k_2\theta_2 N_2}{k_1 N_1} + \frac{\theta_2 N_2}{N_1}\right) - \frac{Rk_1N_1\theta_2}{k_t N_2}\right]\theta_L(s)$$

$$\frac{E_a(s)}{\theta_L(s)} = \text{eq}^n \text{ given.}$$

$$\frac{\theta_L(s)}{E_a(s)} = \frac{1}{\text{eq}^n \text{ above}}$$

Use values given in question

$$\frac{Q_L(s)}{E_a(s)} = 1/\left(1.8\times10^{-3}s^2 + 9/8 + 0.3s\right)\left(4/3 s^2 + 8/3 s + \frac{620}{3}\right) - 225$$

use calculator

$$\frac{Q_L(s)}{E_a(s)} = 1/\frac{3}{1250}s^4 + \frac{253}{625}s^3 + \frac{334}{125}s^2 + 65s + \frac{15}{2}$$

multiply above eq^{n} by $\frac{3}{1250}$

$$\frac{Q_L(s) \times 3/1250}{E_a(s) \times 3/1250}$$

$$\frac{Q_L(s)}{E_a(s)} = \frac{416.67}{s^4 + 168.67s^3 + 1113.3s^2 + 270.833.33s + 3125}$$

$\therefore$ task done as answer is same as numerical sol^{n}.

Time Response

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→ seperate the system into parts

↳ get motor electrical, motor mechanical, tachometer, op amp

Use KVL for motor electric circuit

$$Ea(s) = RIa(s) + V_b$$

$$T_m = R_t \, i_a \quad \text{for motor torque}$$

$$i_a = T_m / R_t$$

$$Ea(s) = Ra \frac{T_m}{K_t} + k_b \omega_m \quad \text{— eq}^n \ 1$$

For motor ~~el~~ mechanical circuit

$$T_m(s) = J_a s^2 \theta_m(s) + J_L s^2 \theta_m(s)$$

$$T_m(s) = \theta_m(s) \left[J_a s^2 + J_L s^2 \right] \quad \text{— eq}^n \ 2$$

into eq^n 1 $\leftarrow$ $\omega_m = s\,\theta_m(s)$

$$Ea(s) = \theta_m(s) \left[\frac{Ra}{Kt} (J_a s^2 + J_L s^2) + s k_b \right]$$

Rearrange for transfer f.

$$\frac{\theta_m(s)}{Ea(s)} = \frac{1}{s^2 \frac{Ra}{K_2}(Ja + J_L) + sK_b} \quad \text{— eq}^n \ 4.$$

Rearrange the eq^{n}

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K / R_a(J_a + J_L)}{s^2 + s\left(\frac{K_b \cancel{K_t}}{R_a(J_a + J_L)}\right)}$$

Use given value.

For Torque stall,

$V_b = 0$

$$E_a(s) = \frac{R_a \times T_{stall}}{K_t}$$

$$\frac{T_{stall}}{E_a} = \frac{K_t}{R_a}$$

$$\frac{K_t}{R_a} = 1.9$$

For motor speed no load

$E_a(s) = V_b$

$K_b \, \omega_{load} = E_a(s)$

$$K_b = \frac{E_a(s)}{\omega_{load}} = \frac{42}{1000}$$

$K_b = 0.042$

get the tf for tachometer

$V_m(t) = K_a \theta_m(t)$

$V_m(s) = K_a s \theta_m(s)$

laplac transform

$$\frac{V_m(s)}{\theta_m(s)} = K_a s \longleftarrow eq^{n}\ 5.$$

get the eq^n $\frac{V_a(s)}{V_m(s)}$

in a non inverting amp

use KCL

$i_{in} = i_{out}$

$$\frac{V_m - 0}{R_1} = \frac{V_a - V_m}{R_2}$$

$$V_m \times \frac{R_2}{R_1} = V_a - V_m$$

$$\left(\frac{R_2}{R_1} + 1\right)V_m = V_a$$

$$\frac{V_a(s)}{V_m(s)} = \left(\frac{Z_2}{Z_1} + 1\right) - eq^{n}\ 6$$

derive ~~the~~ voltage divider for low pass circuit

$$V_0(s) = V_a(s) \times \frac{R_0}{R_0 + SL} \longrightarrow \frac{V_0(s)}{V_a(s)} = \frac{R_0}{R_0 + SL} \leftarrow \text{eq}^n\ 7$$

combine eq^ns together.

$$\frac{\theta_m(s)}{E_a(s)} \times \frac{V_a(s)}{V_m(s)} \times \frac{V_0(s)}{V_a(s)} \times \frac{V_m(s)}{\theta_m(s)} = \frac{V_0(s)}{E_a(s)}$$

$$\frac{V_0(s)}{E_a(s)} = \frac{1}{S\left[S\left(\frac{R_a}{K_t}(J_a + J_L)\right) + K_b\right]} \times K_b S \times \left(\frac{R_2}{R_1} + 1\right) \times \left(\frac{R}{SL + R}\right)$$

rearrange eq^n

$$\frac{V_0(s)}{E_a(s)} = R\left(1 + \frac{R_2}{R_1}\right) K_a \Big/ (R + SL)\left(\frac{R_a}{K_t}(J_a + J_L) S + K_b\right)$$

Substitute value in eq^ns

$R_1 = 1000\ \Omega$, $R_a = 1000$, $L = 0.75$
$R_2 = 2000\ \Omega$, $K_a = 2$, $J_a = 8 \times 10^{-6}$
$K_b = 0.042$, $J_L = 5$
$\frac{R_a}{K_t} = \frac{42}{80}$

$$\frac{V_0(s)}{E_a(s)} = \frac{1000 \times \left[1 + \frac{2000}{1000}\right] \times 2}{(1000 + 0.75s)\left(\frac{42}{82} \times \left(8 \times 10^{-6} + 5\right) s + 0.042\right)}$$

$$= 6000 \Big/ (0.75s + 1000)(2.625s + 0.0220s)$$

Rearrange the equation

$$\frac{V_0(s)}{E_a(s)} = \frac{3048}{s^2 + 1333.3136s + 21.3328}$$

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$G(s) = \omega_n^2 / s^2 + 2\xi\omega_n s + \omega_n^2$ } general Second order system

→ Finding Natural frequency and Damping Ratio

$\omega_n^2 = 21.3328 \quad \omega_n = 4.6187$

$2\xi\omega_n = 1333.316$

$\xi = 1333.316 / 2 \times 4.6187 = \underline{\underline{144.3375}}$

We can determine that this is an Overdamped system

Since the 2nd order form is invalid we can approximate settling time for underdamped system

$T_s = 4 / \xi\omega_n$

$= 4 / 144.3375 \times 4.6187$

$= \underline{\underline{6\ ms}}$ (2nd order)

same answer

1st Order approximation

↳ $\frac{V_o}{E_a(s)} = \frac{3048}{(s+0.016)(s+1333.3)}$

$\sigma_1 = -0.016 \quad \sigma_2 = -1333.3$

intermidiate answer.

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general form for over-damped system from lecture.

$$C(t) = k_1 e^{\delta_1 t} + k_2 e^{\delta_2 t}$$

$$c(t) = k_1 e^{-0.016t} + k_2 e^{-1333.3t}$$

Same as in test 2 we can assume $e^{-1333.3t}$ as zero

$$\frac{V_0(s)}{E_a(s)} = 3048 / s+0.016$$

$$G(s) = a / s+a \qquad a = 0.016$$

Calc. settling time, $T_s = 4/a$

$= 4/0.016$

$= 250$ second, ← 1st order approximat.

∴ given answer.