QUT Centre for Robotics

EGB339 - Part 2: Robotic Arms

Lecture 1: Rigid Body Motions

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About Part 2: Robotic Arms

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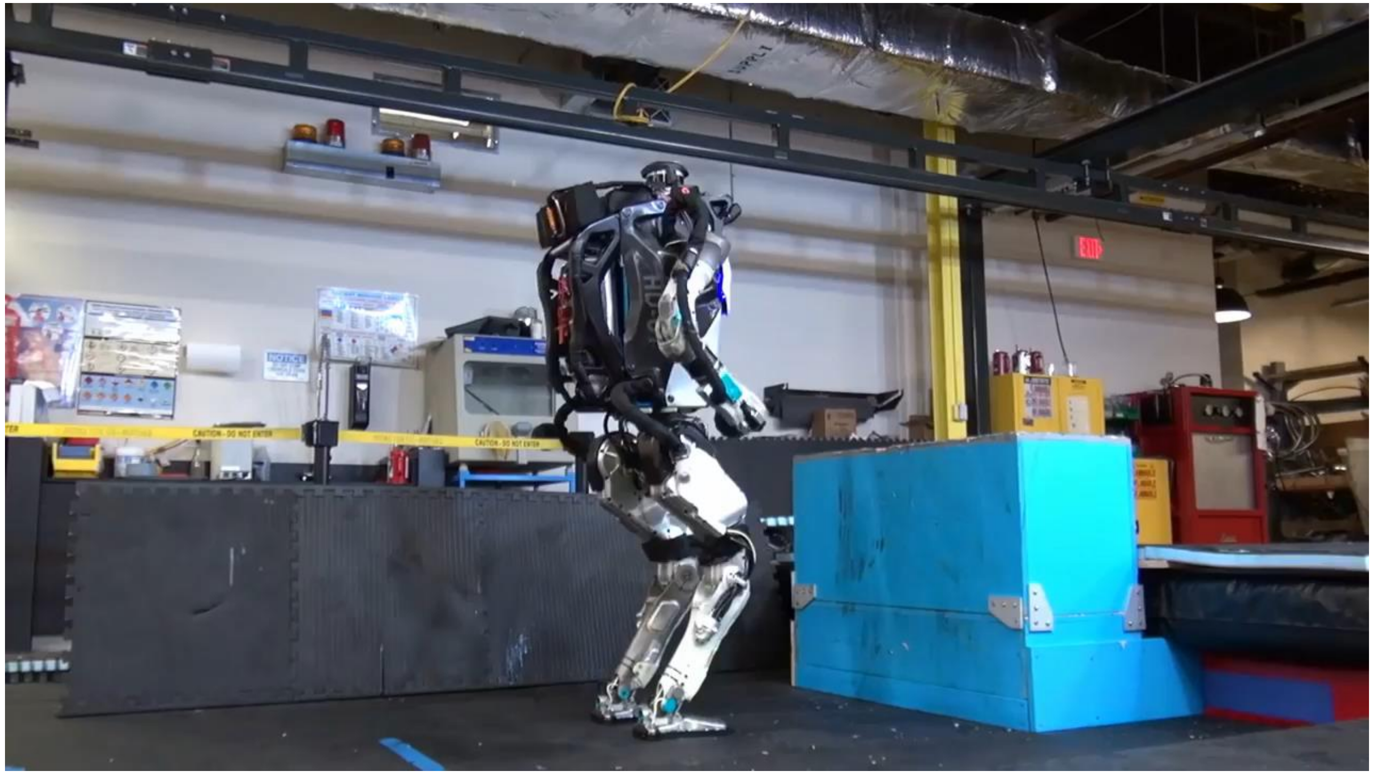
What you may expect to learn



https://www.youtube.com/watch?v=fRj34o4hN4I

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What you actually learn*

*we are aiming to provide this same experience through a simulation environment

But even just with this, you will be able to handle

https://www.youtube.com/watch?v=8_lfxPI5ObM

Outline

- Topics covered in this series of lectures
  - Rigid Body Motions (week 8)
  - Forward Kinematics (week 9)
  - Inverse Kinematics (week 10)
  - Velocity Kinematics (week 11)
  - Path and Trajectory Planning (week 12)
  - Revision (week 13)

- Topics not covered in this series of lectures
  - Dynamics
  - Control
  - Hardware
  - (Artificial) Intelligence
  - …

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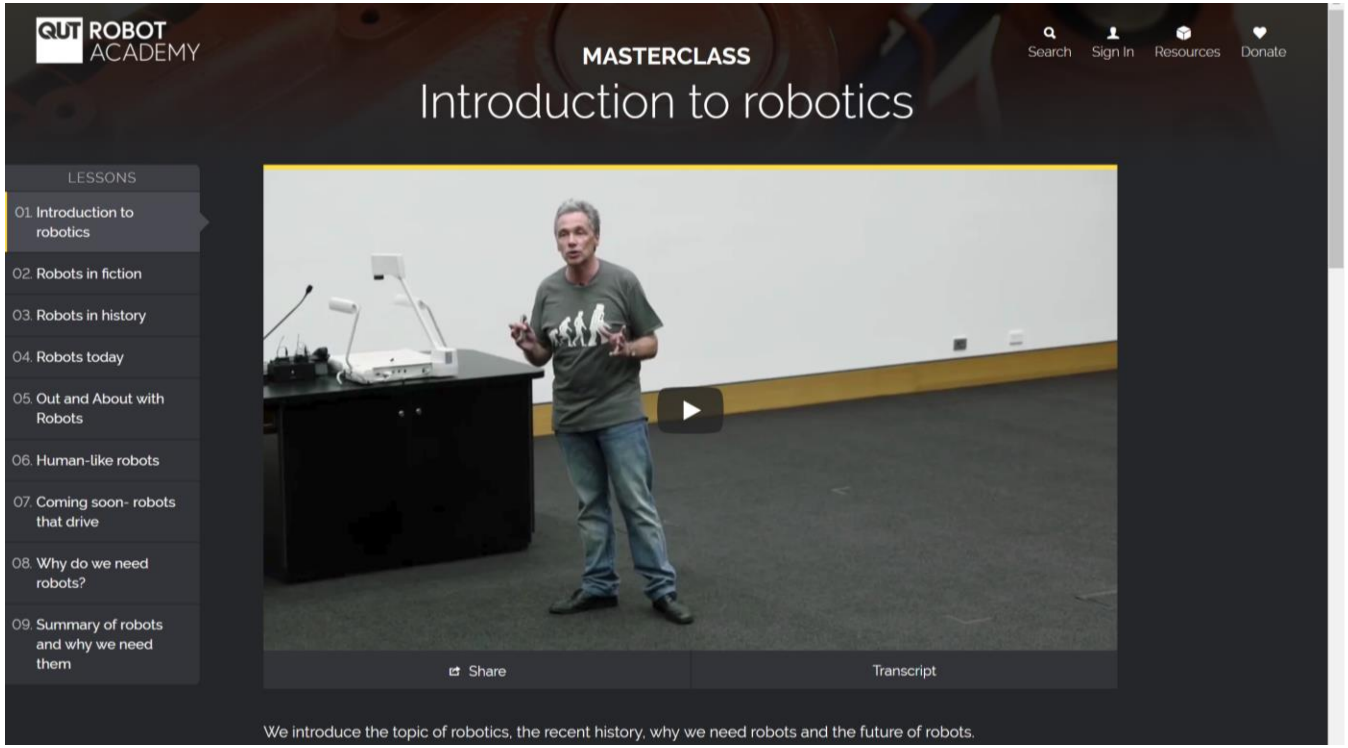
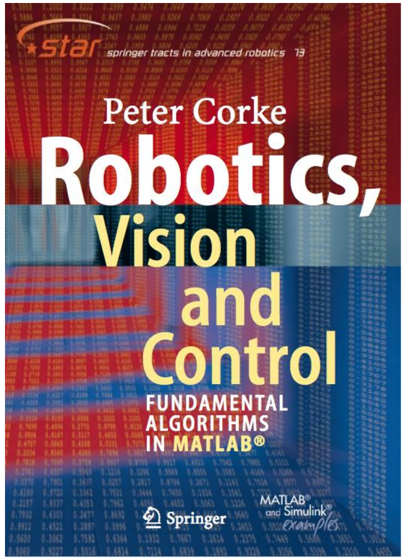
Online Resources

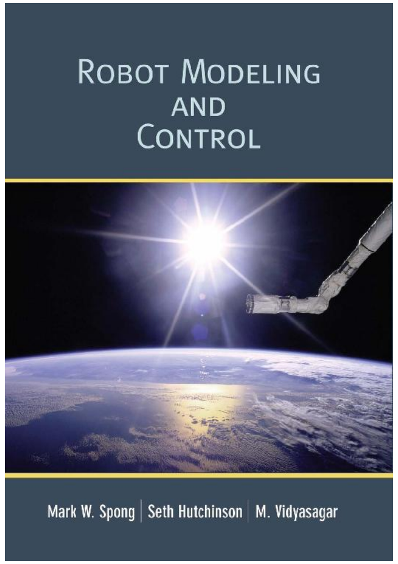
- QUT Robot Academy (by Peter Corke)
  - https://robotacademy.net.au/



Additional Resources

- Robotics, Vision and Control (Ed2)
  - By Peter Corke
  - Electronic resources in the library
  - Hard copies in the library
  - For sale in the bookshop
  - http://petercorke.com/RVC





- Robot Modeling and Control
  - By Mark W Spong; Seth Hutchinson; M Vidyasagar
  - Hard copies should be in the library
  - For sale in the bookshop

- Lectures: MilfordRobotics Youtube Channel Theory Playlist (http://bit.ly/2azZacj)
- Tutorials: Theoretical Problems and Solutions by James Mount - Youtube Playlist (http://bit.ly/2aeZys3)

Assessment

- Part 2: Robotic Arms (50%)
  - 20% prac exam Week 13
  - 30% theory exam in the final exam period
    - Timed Online Assessment,
    - Given the time limit, you’d better really master the contents than rely on the open-book
    - If you want to pass the exam, attend the tutorials!!!

Rigid Body Motions

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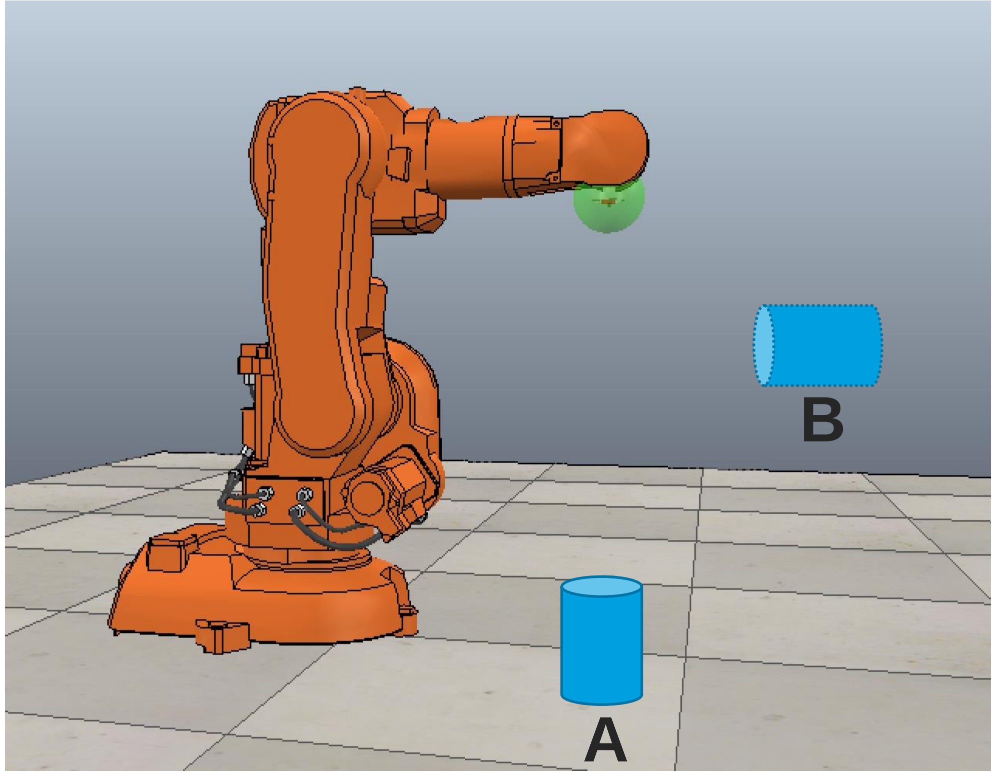
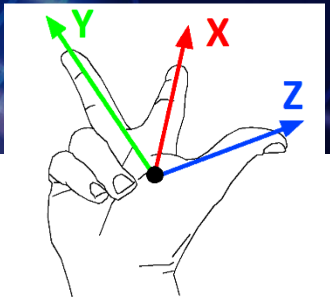
Assuming you have watched these online videos

- QUT Robot Academy (by Prof Peter Corke)
  - 2D Geometry
    - https://robotacademy.net.au/masterclass/2d-geometry/
  - 3D Geometry
    - https://robotacademy.net.au/masterclass/3d-geometry/

Motivating Problem

- Imagine one of your arms is replaced by a robotic arm, and you’re blindfolded (you don’t have sensors to detect the object in front of you).
- You are supposed to move an object from A to B.
- You want somebody to tell you where the object is and where to move it.
- How can the pose (position and orientation) of A and B be described to you?

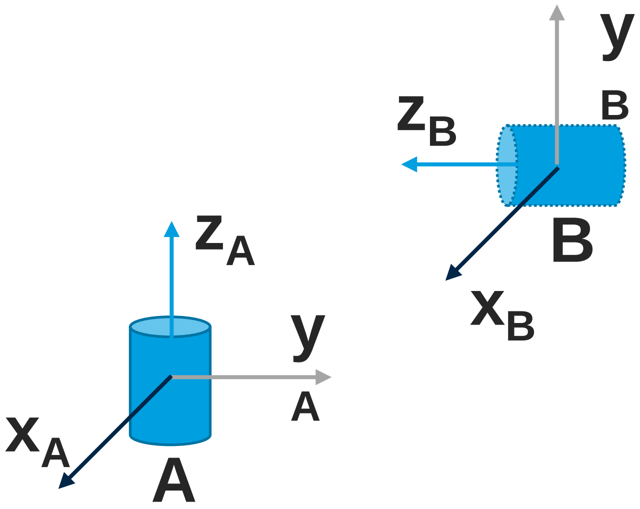




The Right-Hand Rule

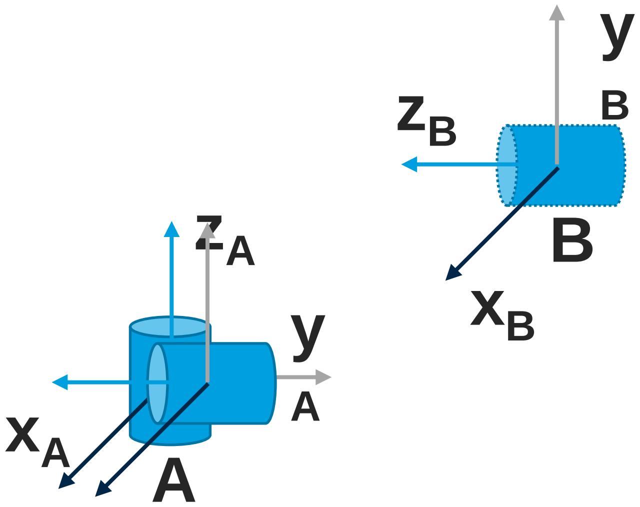
Rigid Body Motions

- Rigid Body
  - In physics, a rigid body is a solid body in which deformation is zero or so small it can be neglected. (Wikipedia)
  - In kinematics, a rigid body is a coordinate frame.



Rigid Body Motions

- The pose of B relative to A can be described with two components:
  - A rotation matrix R specifying the orientation
  - And a translation vector t specifying the position



2D Rotation Matrix

- 2D rotation

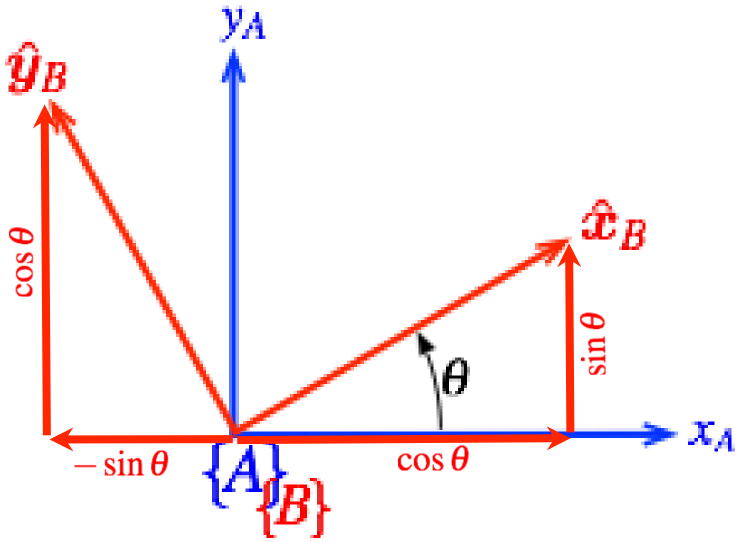
$$\hat{x}_B = \cos\theta\,\hat{x}_A + \sin\theta\,\hat{y}_A$$
$$\hat{y}_B = -\sin\theta\,\hat{x}_A + \cos\theta\,\hat{y}_A$$

- 2D rotation matrix

new x-axis

new y-axis

$$\begin{pmatrix} {}^{A}x \\ {}^{A}y \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}}_{{}^{A}R_{B}} \begin{pmatrix} {}^{B}x \\ {}^{B}y \end{pmatrix}$$



Watch https://robotacademy.net.au/masterclass/2d-geometry/?lesson=75 for the derivation

Rotation from frame {B} to frame {A}

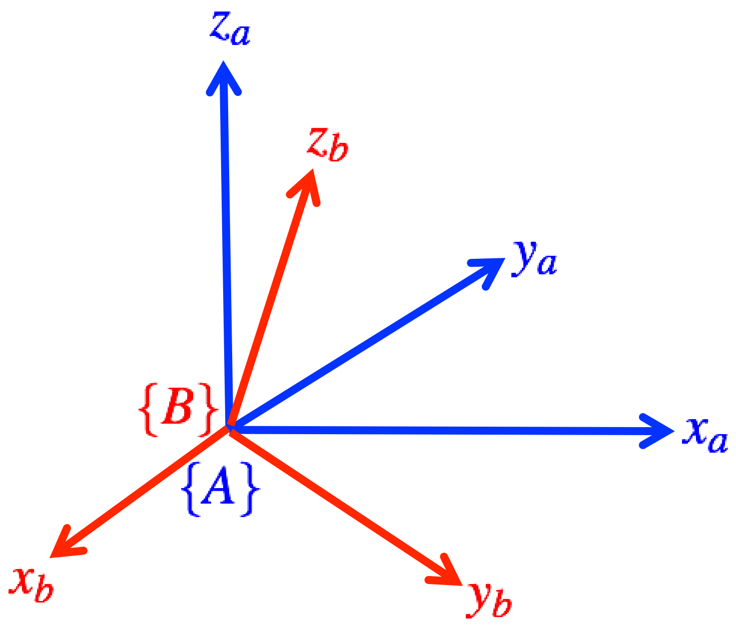
Rotates vectors from {B} to {A}

Rotation angle from {A} to {B},

3D Rotation Matrix

- 3D rotation matrix

$$\begin{pmatrix} {}^{A}x \\ {}^{A}y \\ {}^{A}z \end{pmatrix} = \underbrace{\begin{array}{c} \begin{array}{ccc} \text{new x-axis} & \text{new y-axis} & \text{new z-axis} \end{array} \\ \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \end{array}}_{{}^{A}R_{B}} \begin{pmatrix} {}^{B}x \\ {}^{B}y \\ {}^{B}z \end{pmatrix}$$



Rotation from {B} to {A}

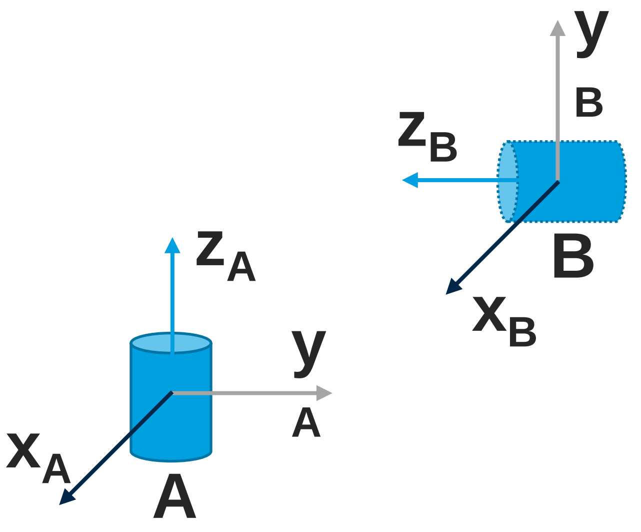
Rotation Angle from {A} to {B},

Rotates vectors from {B} to {A}

Rotation Matrix - Example

- Determine the rotation matrix from {B} to {A}

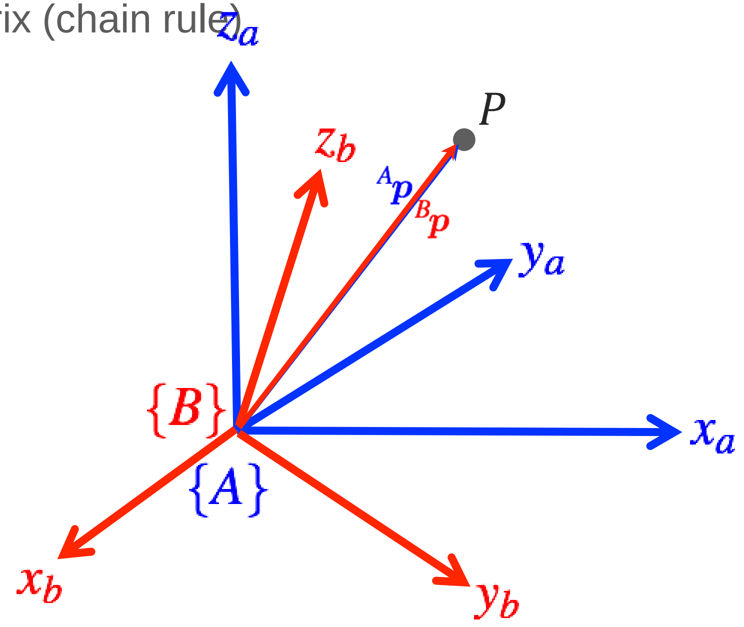
$$\begin{pmatrix} {}^{A}x \\ {}^{A}y \\ {}^{A}z \end{pmatrix} = \begin{pmatrix} \boxed{} & \boxed{} & \boxed{} \end{pmatrix} \begin{pmatrix} {}^{B}x \\ {}^{B}y \\ {}^{B}z \end{pmatrix}$$



Positions in Rotated Frames

- Positions of a point in rotated frames can be related by a rotation matrix (chain rule)

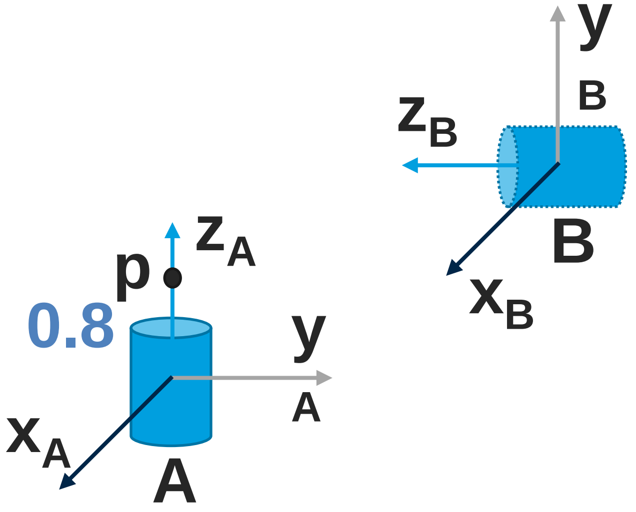
$$ {}^{A}p = {}^{A}R_{B}\,{}^{B}p $$



Only works for frames sharing the same origin (no translation involved).

Positions in Rotated Frames - Example

- Suppose P is a 3D point.
- {A} and {B} share the same origin.
- Write down by inspection



- Validate

$$ {}^{A}R_{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad {}^{A}p = \begin{pmatrix} \boxed{} \end{pmatrix} \quad {}^{B}p = \begin{pmatrix} \boxed{} \end{pmatrix} $$

$$ {}^{A}p = {}^{A}R_{B}\, {}^{B}p \qquad \begin{pmatrix} 0 \\ 0 \\ 0.8 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0.8 \\ 0 \end{pmatrix} $$

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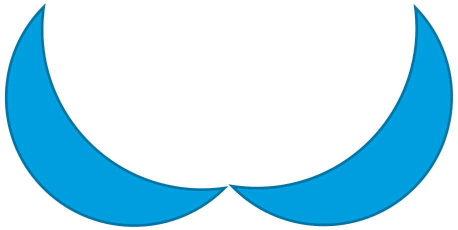
Properties of A Rotation Matrix

$$\boxed{R^T R = I \text{ or } R^T = R^{-1}}$$

- Orthogonal Matrix
  - Because the length of a vector remains the same after rotation. $(Rp)^T Rp = p^T R^T Rp = p^T p$

- Special Orthogonal Matrix $\det(R) = 1$
  - Otherwise it is a rotary reflection.

Rotation
$\det(R) = 1$



Rotary reflection
$\det(R) = -1$

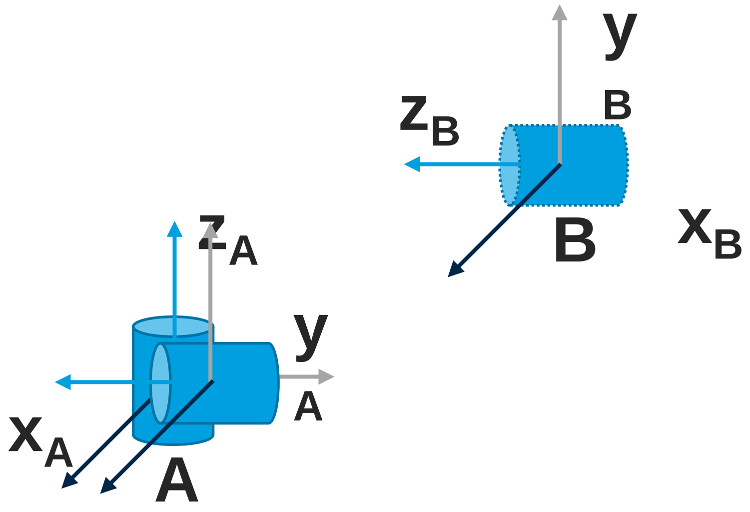
Parameterisation of A Rotation

- How many parameters at least are required to parameterise a 3D rotation?
-
-
-
-
- …

Very useful in practice; you are strongly recommended to learn them via the online videos!

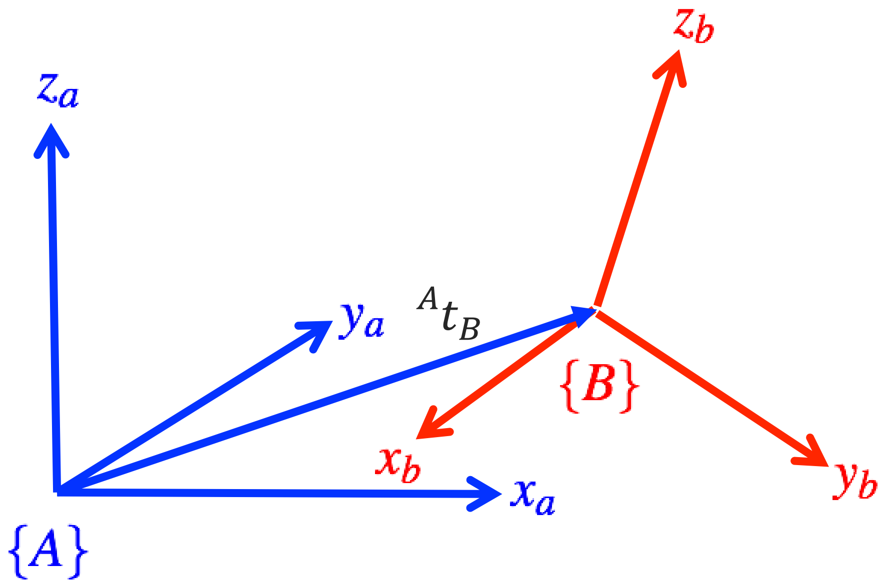
Recall: Rigid Body Motions

- The pose of B relative to A can be described with two components:
  - A rotation matrix R specifying the orientation
  - And a translation vector t specifying the position



Translation Vector

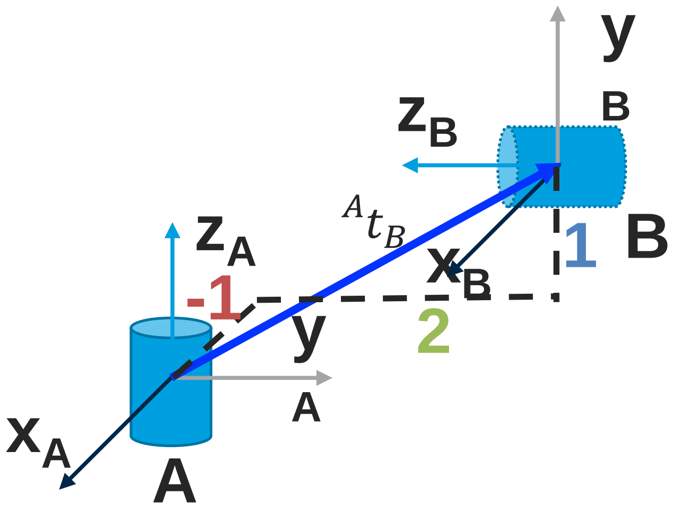
- Translation vector ${}^{A}t_{B}$ is a vector pointing from the origin of {A} to the origin of {B} evaluated in {A}.



Translation Vector - Example

- Determine the translation vector from {B} to {A}

$$ {}^{A}t_{B} = \left(\boxed{} \right) $$



Homogeneous Transformation

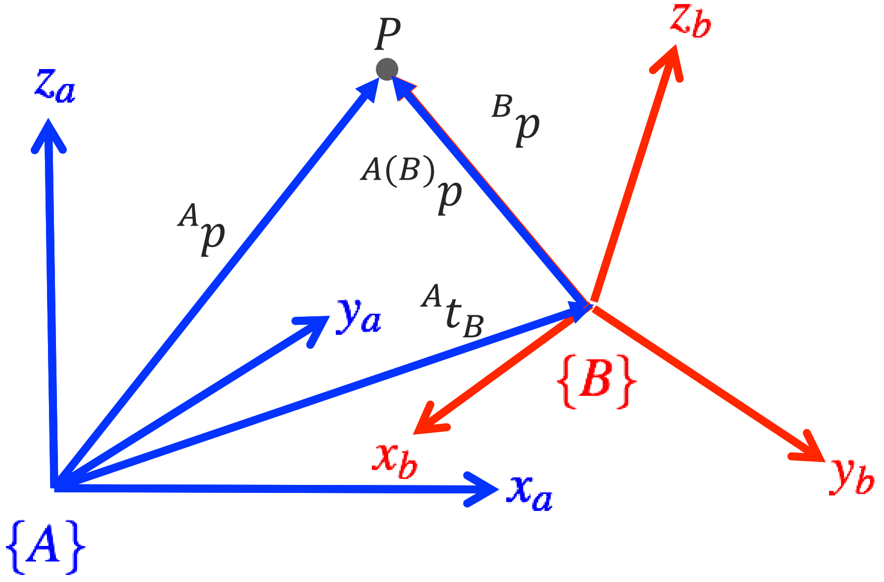
- Suppose P is a 3D point.

$$^{A}p = {}^{A}t_{B} + {}^{A(B)}p$$

$$^{A(B)}p = {}^{A}R_{B}\,{}^{B}p$$

$$^{A}p = {}^{A}t_{B} + {}^{A}R_{B}\,{}^{B}p$$

$$\begin{pmatrix} ^{A}p \\ 1 \end{pmatrix} = \begin{pmatrix} & ^{A}R_{B} & & ^{A}t_{B} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ^{B}p \\ 1 \end{pmatrix}$$



Homogeneous Transformation

$${}^A\tilde{p} \qquad {}^A T_B \qquad {}^B\tilde{p}$$

Homogeneo us Vector — Homogeneous Transformation — Homogeneous Vector

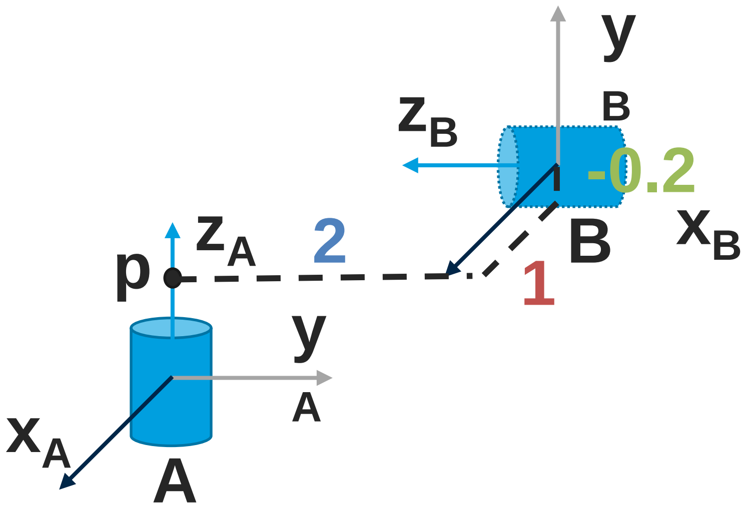
$$\begin{pmatrix} {}^A p \\ 1 \end{pmatrix} = \begin{pmatrix} & {}^A R_B & & {}^A t_B \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} {}^B p \\ 1 \end{pmatrix}$$

$${}^A\tilde{p} = {}^A T_B \, {}^B\tilde{p}$$

Homogeneous Transformation - Example

- Suppose P is a 3D point.
- Given ${}^{A}T_{B}$ and ${}^{B}\tilde{p}$, find ${}^{A}\tilde{p}$

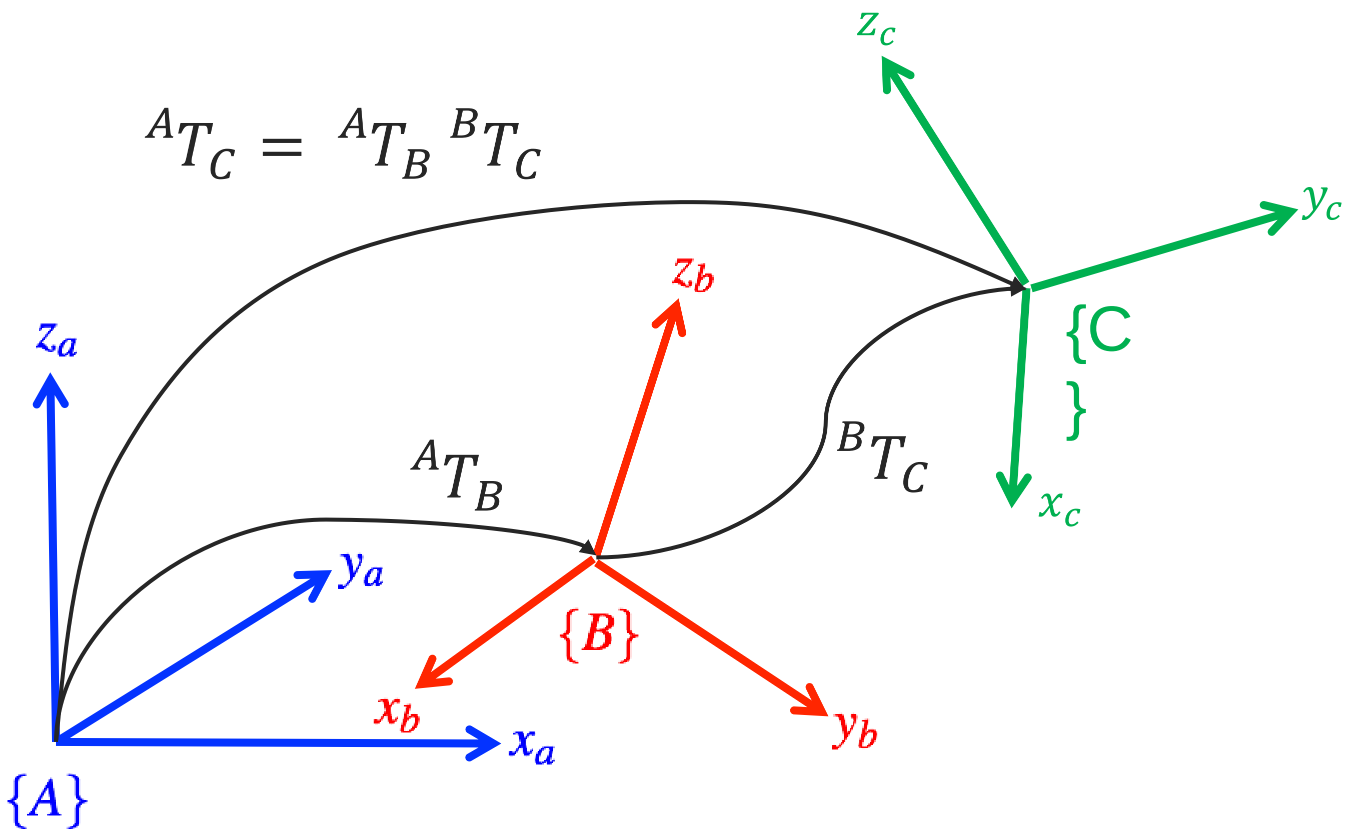
$$ {}^{A}T_{B} = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$



- Validate

$$ {}^{A}\tilde{p} = \begin{pmatrix} \boxed{} \end{pmatrix} \qquad {}^{B}\tilde{p} = \begin{pmatrix} 1 \\ -0.2 \\ 2 \\ 1 \end{pmatrix} $$

Chain Rule



Inverse of Homogeneous Transformation

$$T = \begin{pmatrix} & R & & t \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad T^{-1} = \begin{pmatrix} & R^T & & -R^T t \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$ {}^{A}T_{C} = {}^{A}T_{B}\, {}^{B}T_{C} \qquad {}^{\mathrm{A}}p = {}^{A}T_{B}\, {}^{B}p$$

$$ {}^{A}T_{B}^{-1}\, {}^{A}T_{C} = {}^{B}T_{C} \qquad {}^{A}T_{B}^{-1}\, {}^{\mathrm{A}}p = {}^{B}p$$

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Summary

- A rigid body can be represented by a coordinate frame

- Rigid body motions have two components
  - A rotational component (rotation matrix)
  - And a translational component (translation vector)

- Rigid body motions can be represented by homogeneous transformations

- Homogeneous transformations conform to chain rules and are invertible

Motivating Problem - Revisited

- Imagine one of your arms is replaced by a robotic arm, and you are blindfolded (you don't have sensors to detect the object in front of you).
- You are supposed to move the object from A to B.
- You want somebody to tell you where the object is and where to move it.
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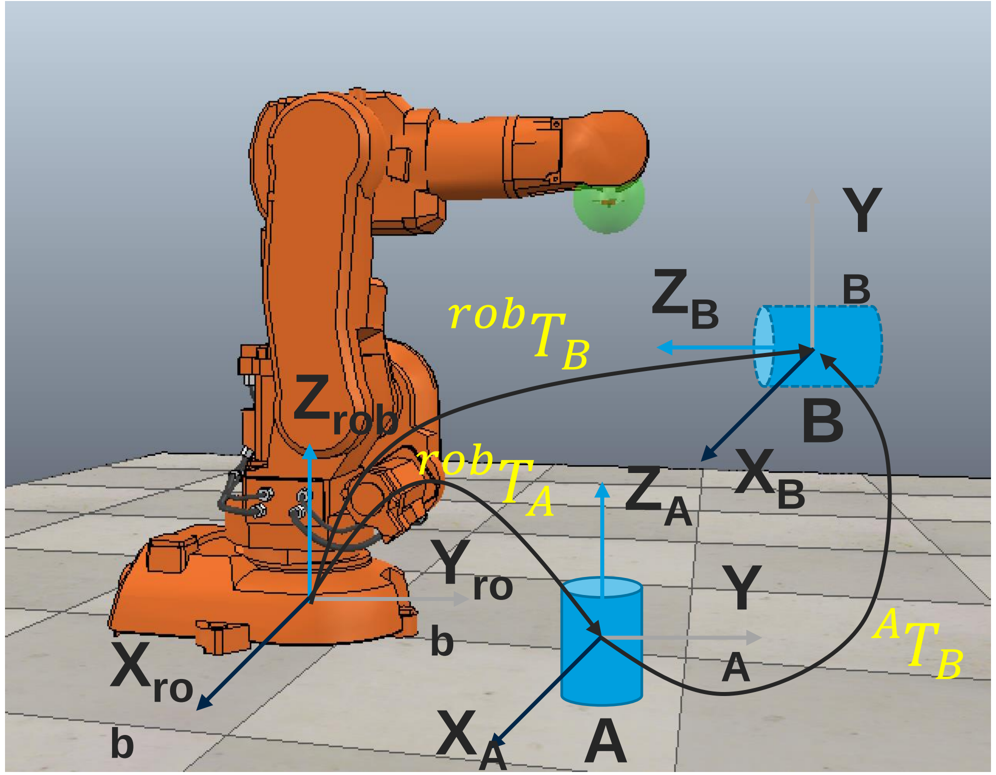

Final Remarks

- Acknowledgements
  - Some material of the slides was developed by the previous lecturers of EGB339 - Introduction to Robotics (Michael Milford, Peter Corke, and Leo Wui)

- Feedback
  - Please email any feedback to c.lehnert@qut.edu.au
  - I will try to incorporate the feedback into the next lecture.