Centre for Robotics

Part 2: Robotic Arms

Lecture 2: Forward Kinematics

Chris Lehnert (Lecturer)

Outline

- Topics covered in this series of lectures
  - Rigid Body Motions (week 8)
  - Forward Kinematics (week 9)
  - Inverse Kinematics (week 10)
  - Velocity Kinematics (week 11)
  - Path and Trajectory Planning (week 12)
  - Revision (week 13)

- Topics not covered in this series of lectures
  - Dynamics
  - Control
  - Hardware
  - (Artificial) Intelligence
  - …

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Watch these online videos

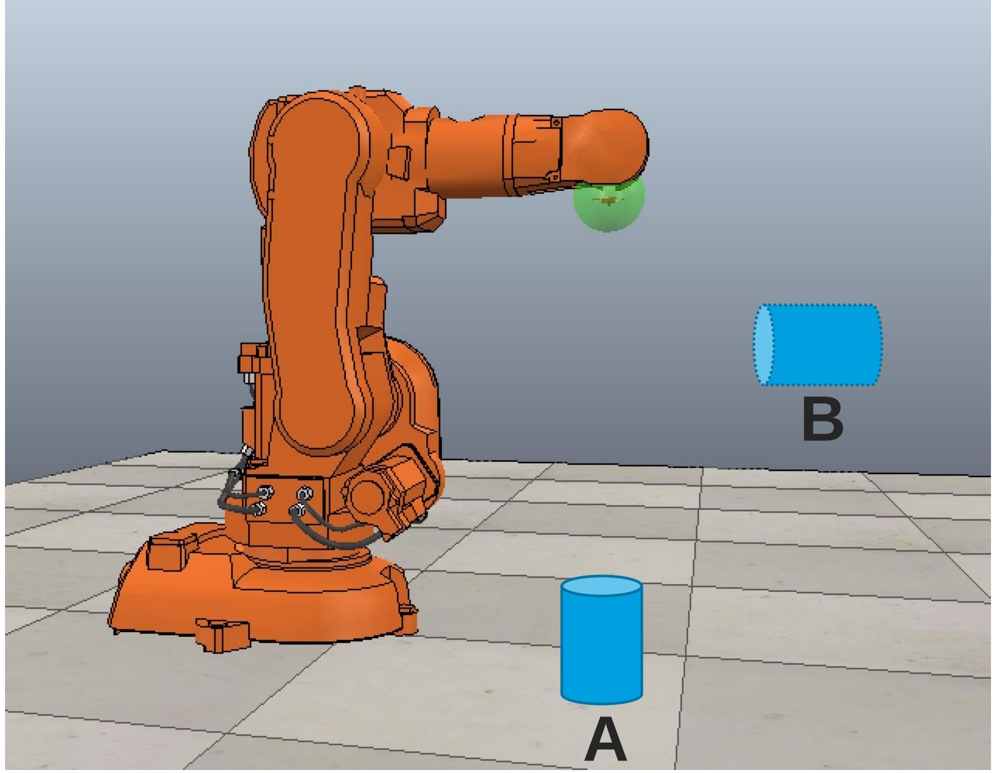
- QUT Robot Academy (by Prof Peter Corke)
  - Robotic arms and forward kinematics
    - https://robotacademy.net.au/masterclass/robotic-arms-and-forward-kinematics/

Recap of Week 8

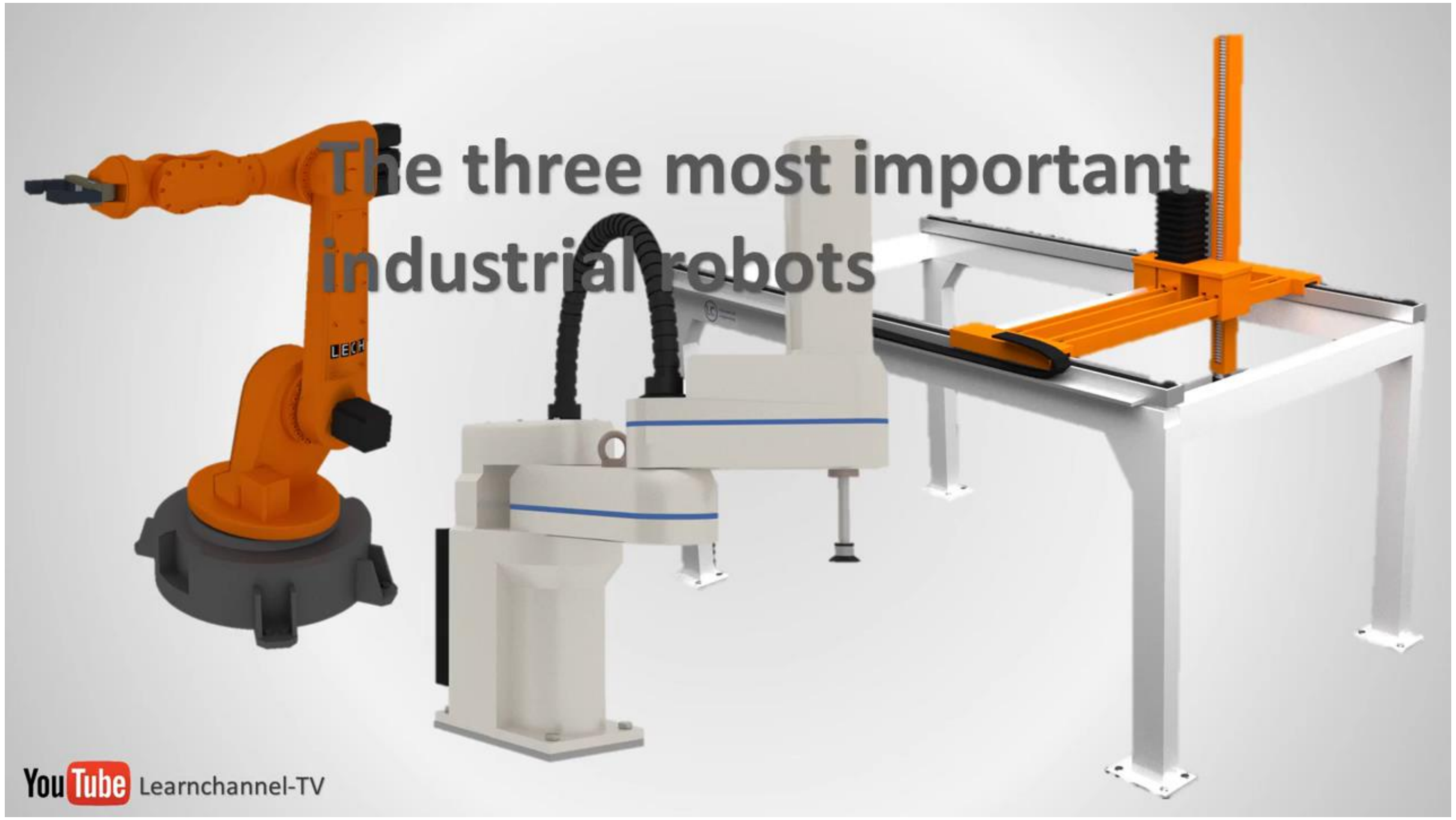
- A rigid body can be represented by a coordinate frame
- Rigid body motions have two components
  - A rotational component (rotation matrix)
  - And a translational component (translation vector)
- Rigid body motions can be represented by homogeneous transformations
- Homogeneous transformations conform to chain rules and are invertible

Motivating Problem

- Imagine one of your arms is replaced by a robotic arm. You are supposed to move an object from A to B.
- But first you want to know where your current “hand” is.
- What you already know are the geometric parameters of your arm (fixed) and the angle of each joint (variable).
- How can you calculate the pose of your “hand”?



Three Typical Robotic Arms



https://www.youtube.com/watch?v=FORcPhBaa50

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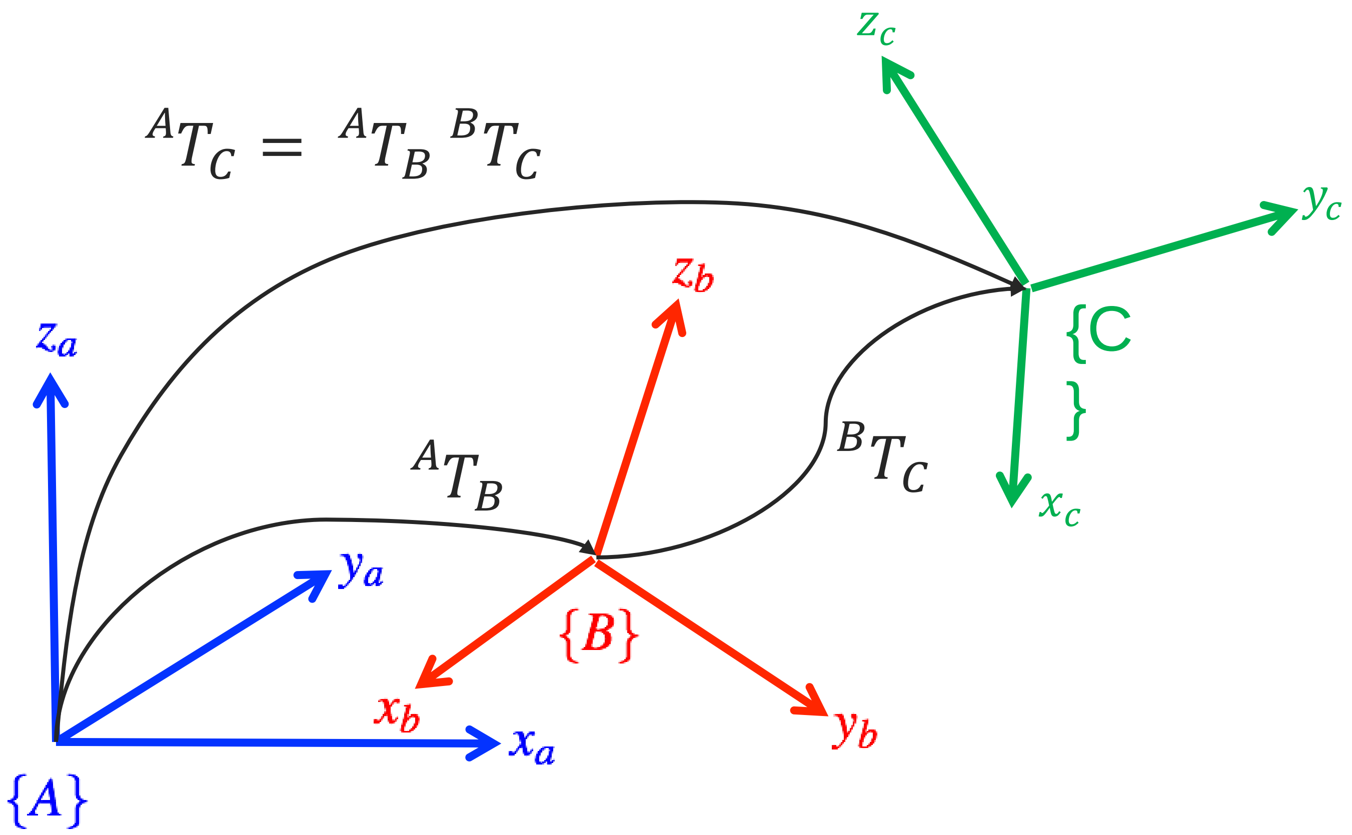
Key concepts in the video

- Joint types
  - Revolute
  - Prismatic
- Degree of Freedom (DoF)
  - The number of independent motions that can be achieved
- Tool Centre Point (TCP)
  - A point fixed on the end-effector
- Workspace
  - The set of positions that can be reached by the TCP

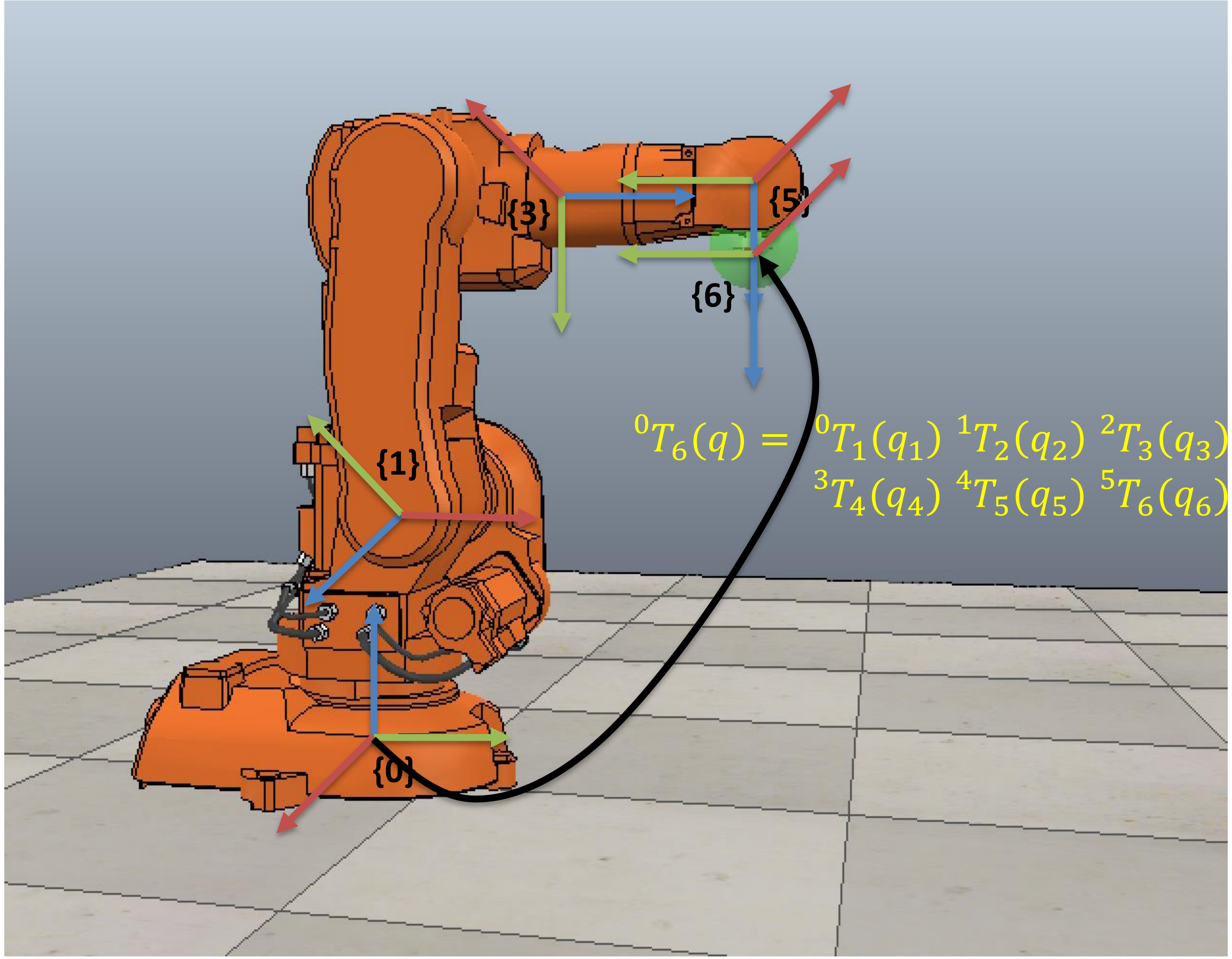
Forward Kinematics

- Forward kinematics is the process of finding the position of the TCP/the pose of the tool given the joint variables.
- Robot example in V-REP

Recall: Chain Rule



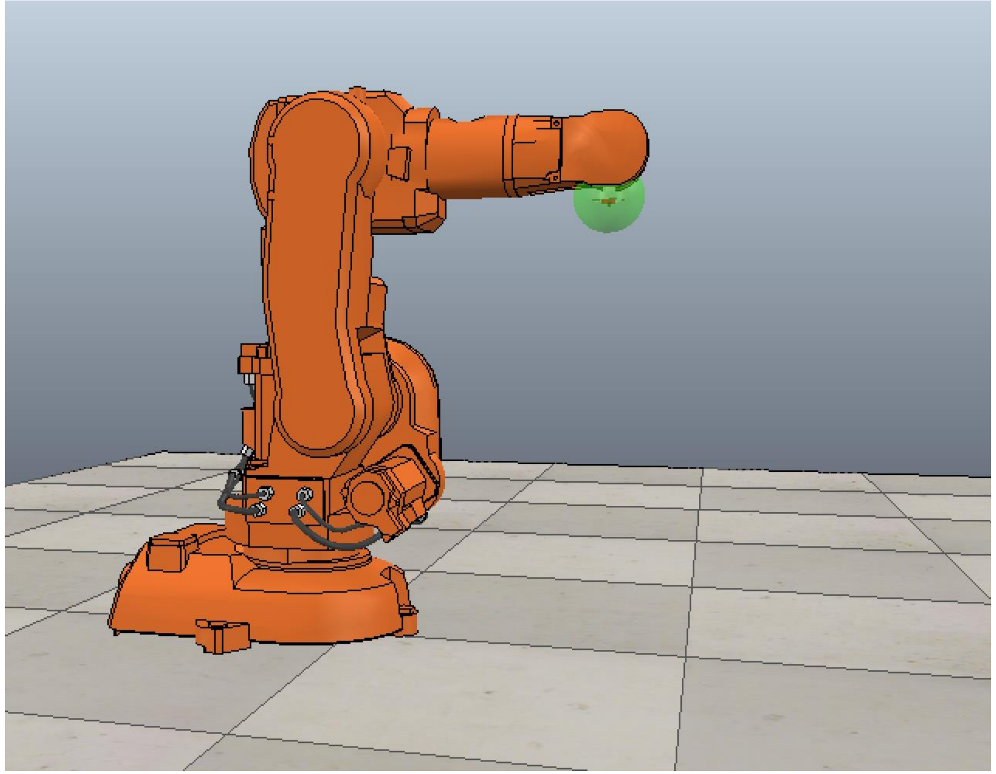
Kinematic Chain

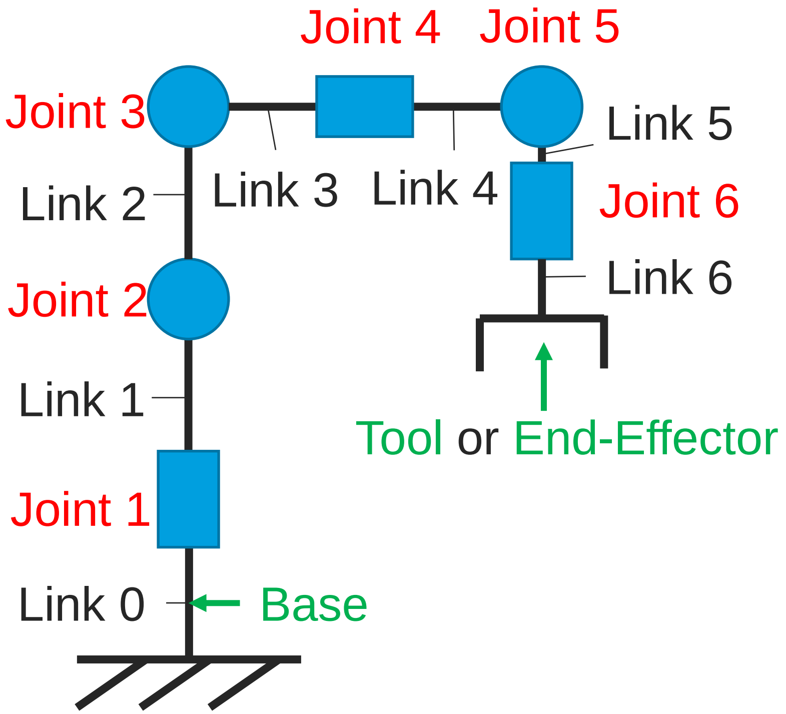


Kinematic Modelling of A Robotic Arm

1. Find the joints and links
2. Attach the frames to the links (each link is a rigid body, so we can use a frame to represent it)
3. Parameterise the homogeneous transformations between consecutive frames
4. Calculate the homogeneous transformation from the first frame to the last frame using the chain rule

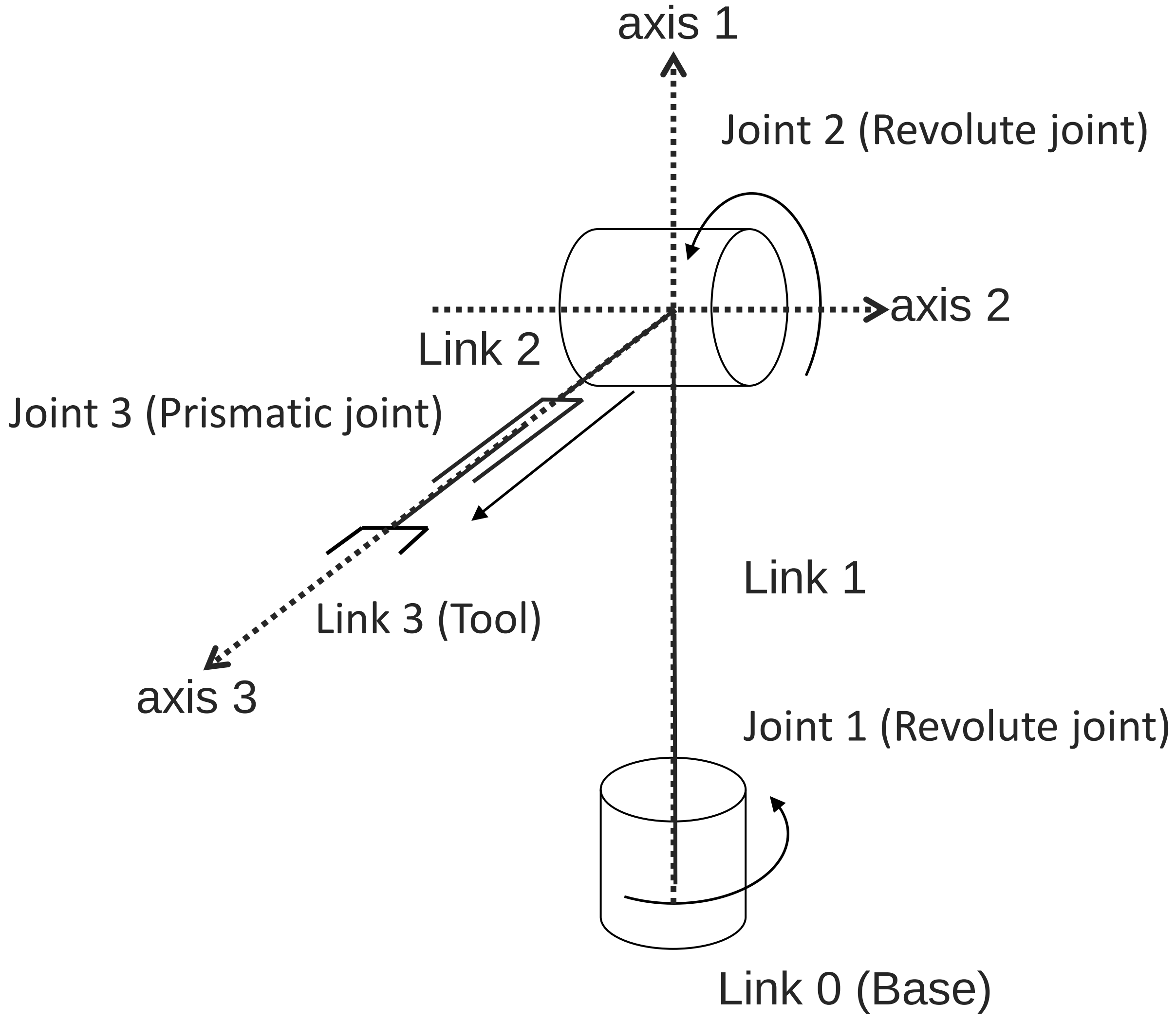
1. Find the links and joints





Example

1. Find the joints and links



2. Attach the frames to the links

- Theoretically, frames can be located anywhere, as long as they are rigidly attached to the links

- If so, there will be 6 parameters needed to parameterise the homogeneous transformation between consecutive frames

- Roboticists have found a way to use just 4 parameters to parameterise the homogeneous transformations by using a set of rules to assign the frames, which is called the Denavit-Hartenberg (D-H) Convention

    - Axis x_i **intersects** z_{i-1}
    - Axis x_i is **perpendicular** to axis z_{i-1}
    - Link frame may not be placed directly on the link

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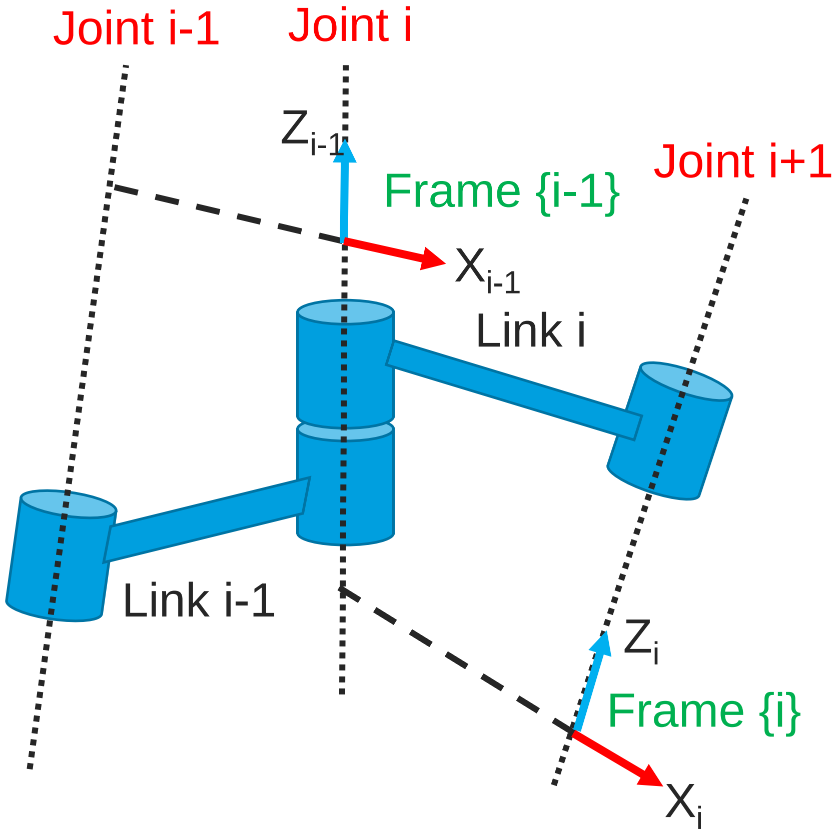
Denavit-Hartenberg (D-H) Convention

Rule 1: Z_i of Frame {i} is axis of actuation of joint i+1

- Axis of revolution of revolute joint
- Axis of translation of prismatic joint

Rule 2: X_i of Frame {i} is axis along the perpendicular pointing from axis Z_{i-1} to axis Z_i

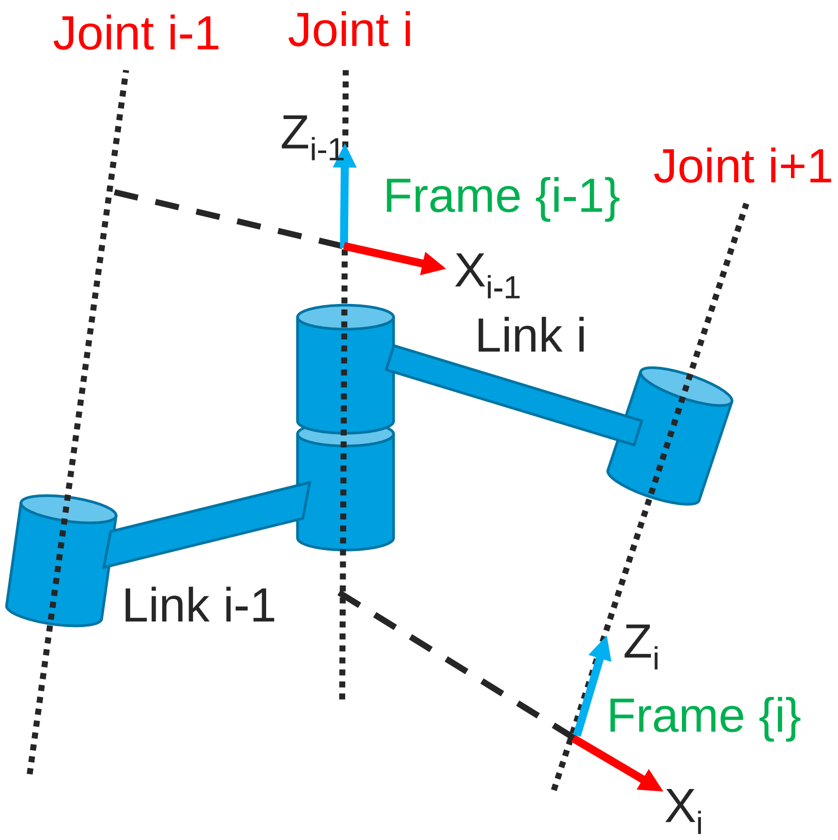
Rule 3: Derive Y_i from X_i and Z_i (right-hand rule)



Denavit-Hartenberg (D-H) Convention

Rule 2: X_i of Frame {i} is axis along the perpendicular pointing from axis Z_{i-1} to axis Z_i

- Case 1: axis Z_{i-1} and axis Z_i are not co-planar
  - There is only one line possible for X_i, which is the shortest line from axis Z_{i-1} to axis Z_i
  - O_i (origin) is at intersection of axis Z_i and the perpendicular



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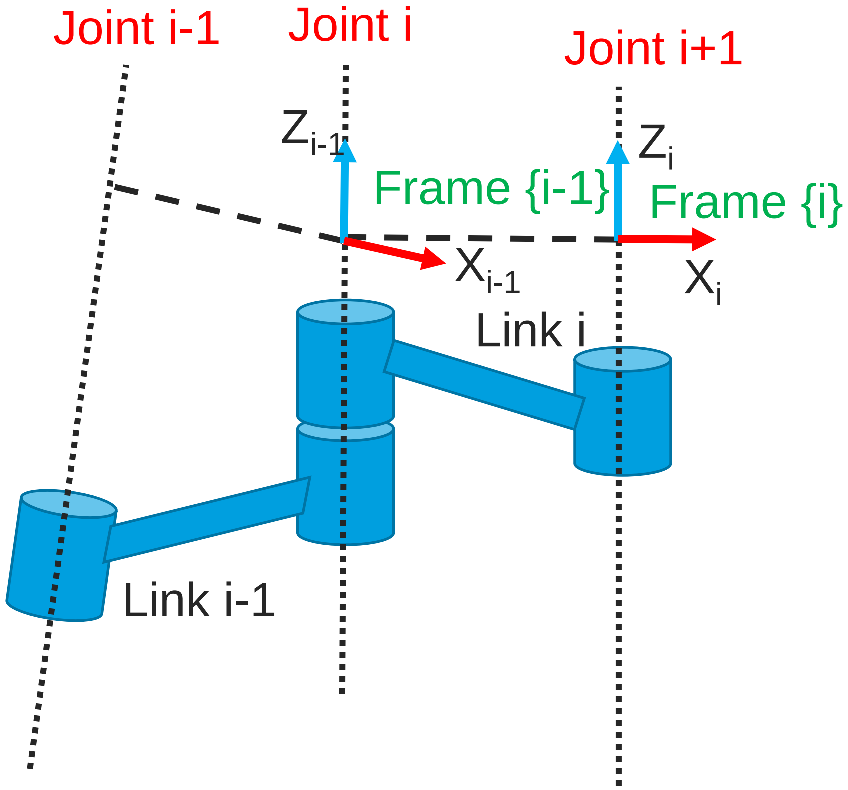
Denavit-Hartenberg (D-H) Convention

Rule 2: X_i of Frame {i} is axis along the perpendicular pointing from axis Z_{i-1} to axis Z_i

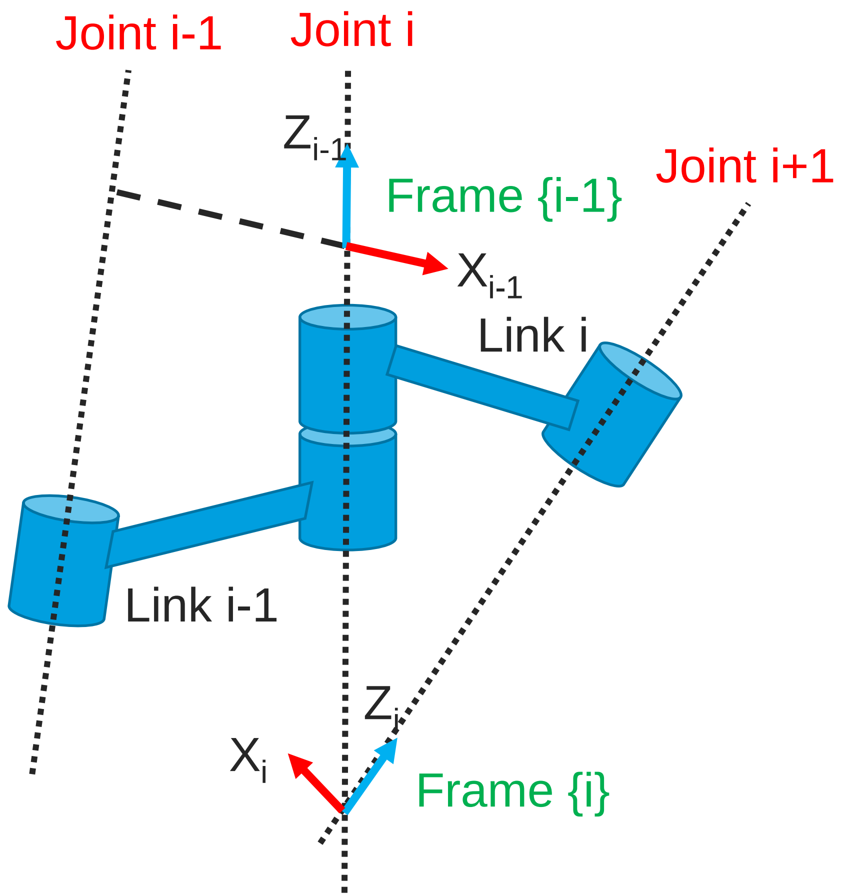
- Case 1: axis Z_{i-1} and axis Z_i are not co-planar
- Case 2: axis Z_{i-1} and axis Z_i are co-planar and parallel
  - There are an infinite number of possibilities for X_i to point from axis Z_{i-1} to axis Z_i
  - Usually (but not always) easiest to choose an X_i that passes through O_{i-1} (origin of {i-1})



Denavit-Hartenberg (D-H) Convention

Rule 2: X_i of Frame {i} is axis along the perpendicular pointing from axis Z_{i-1} to axis Z_i

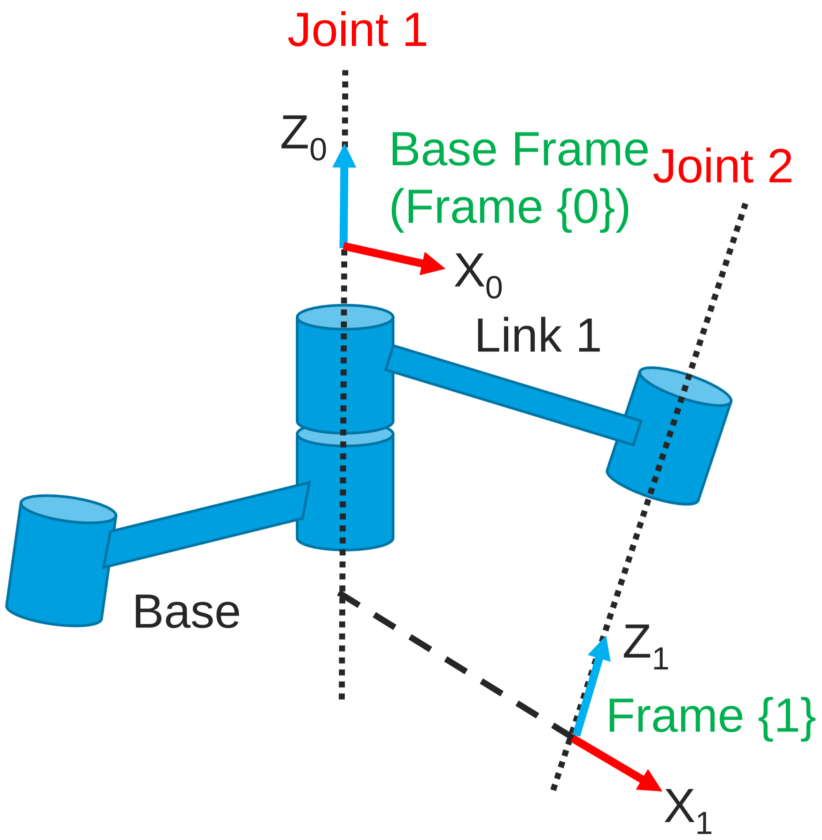
- Case 1: axis Z_{i-1} and axis Z_i are not co-planar
- Case 2: axis Z_{i-1} and axis Z_i are co-planar and parallel
- Case 3: axis Z_{i-1} and axis Z_i are co-planar and intersect
  - X_i is normal to the plane of axis Z_{i-1} and axis Z_i
  - Positive direction of X_i is arbitrary
  - Can use right-hand rule, i.e., make (Z_{i-1}, Z_i, X_i) right-handed
  - O_i naturally sits at intersection



Denavit-Hartenberg (D-H) Convention

Rule 4: Base Frame (Frame {0})

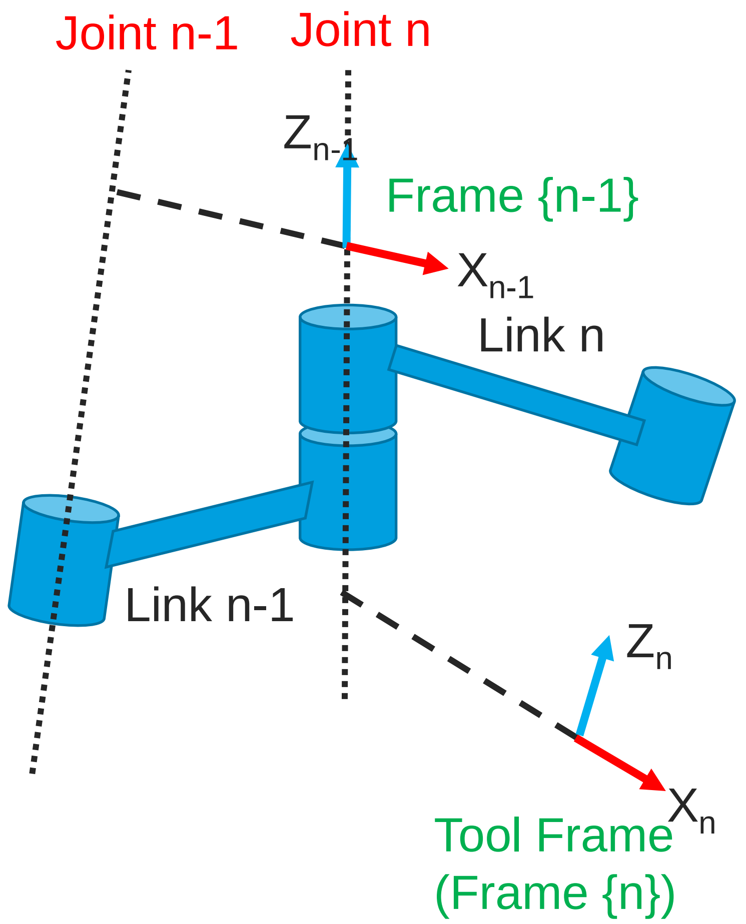
- Z_0 of Frame {0} is axis of actuation of joint 1
- X_0 of Frame {0} is set as convenient since Joint 0 does not exist
- Y_0 is derived from X_0 and Z_0 (right-hand rule)



Denavit-Hartenberg (D-H) Convention

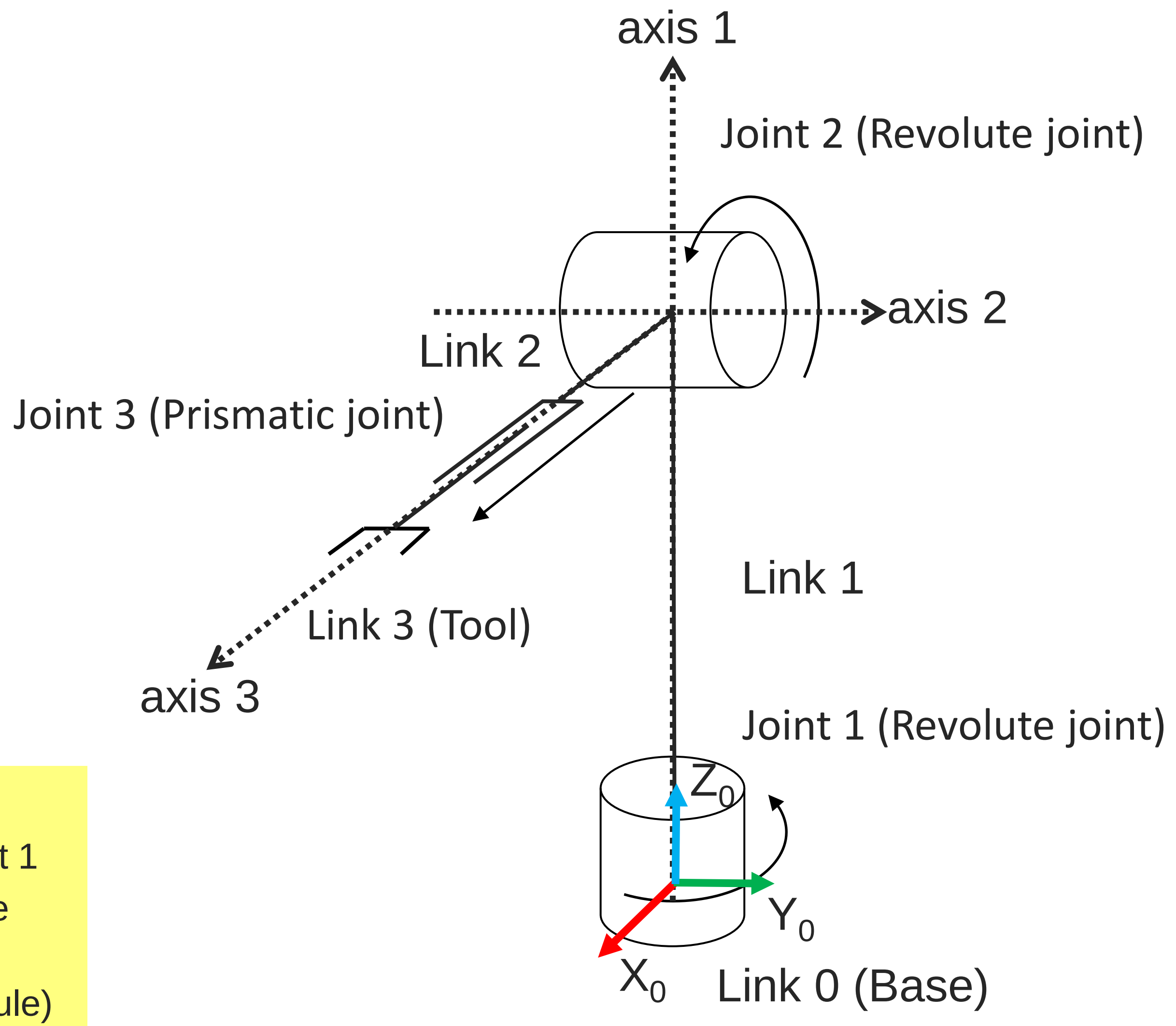
Rule 5: Tool Frame (Frame {n})

- Z_n of Frame {n} is set as convenient since Joint n+1 does not exist
- Z_n is usually (but not always) set as the approach direction of the tool
- X_n is set according to Rule 2
- Y_n is derived from X_n and Z_n (right-hand rule)



Example

2. Attach frames to links



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Example

2. Attach frames to links

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- Axis of revolution of revolute joint
- Axis of translation of prismatic joint

Rule 2: X_i of Frame {i} is axis along the perpendicular pointing from axis Z_{i-1} to axis Z_i

- Case 3: axis Z_{i-1} and axis Z_i are co-planar and intersect
  - X_i is normal to the plane of axis Z_{i-1} and axis Z_i
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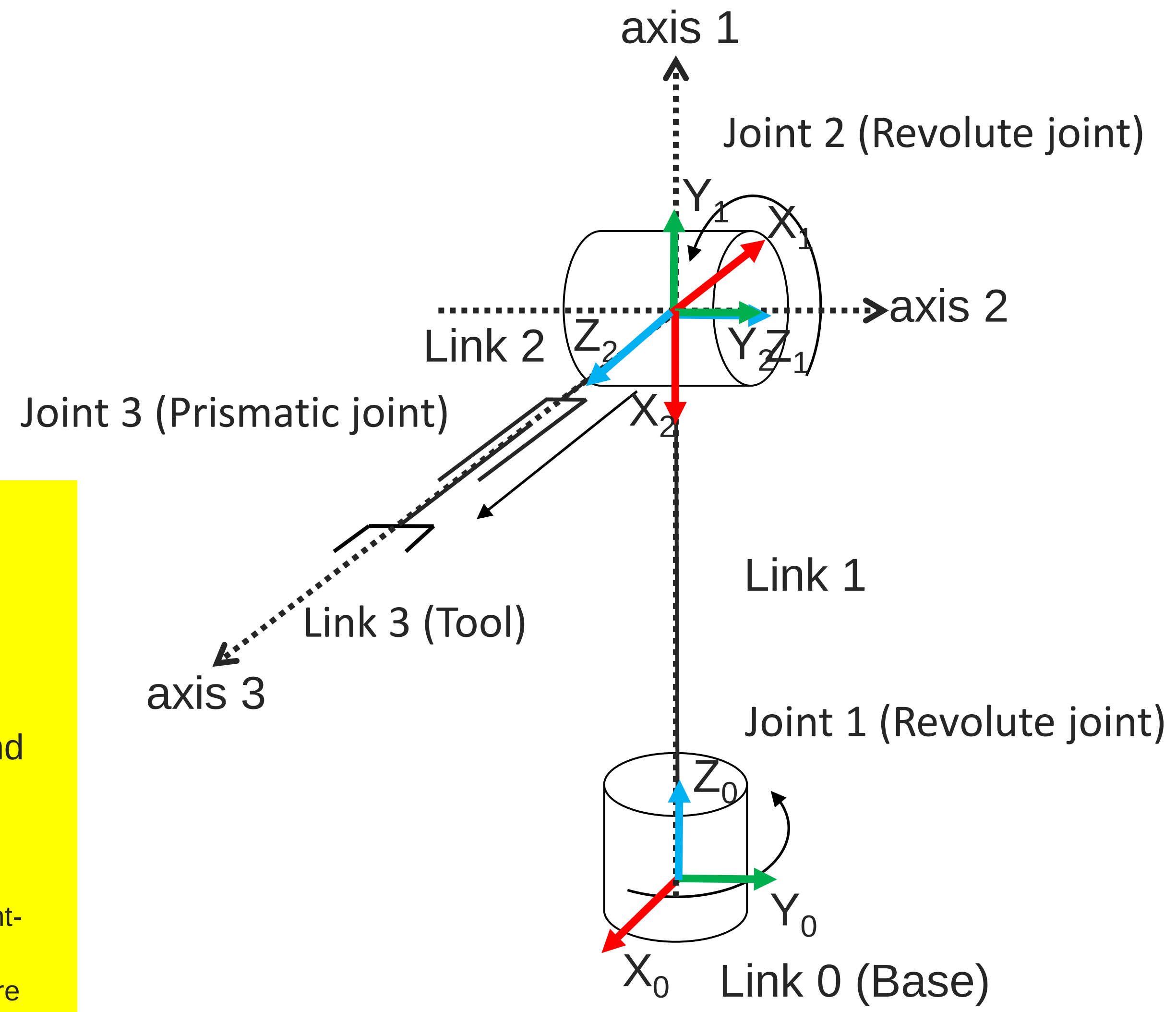
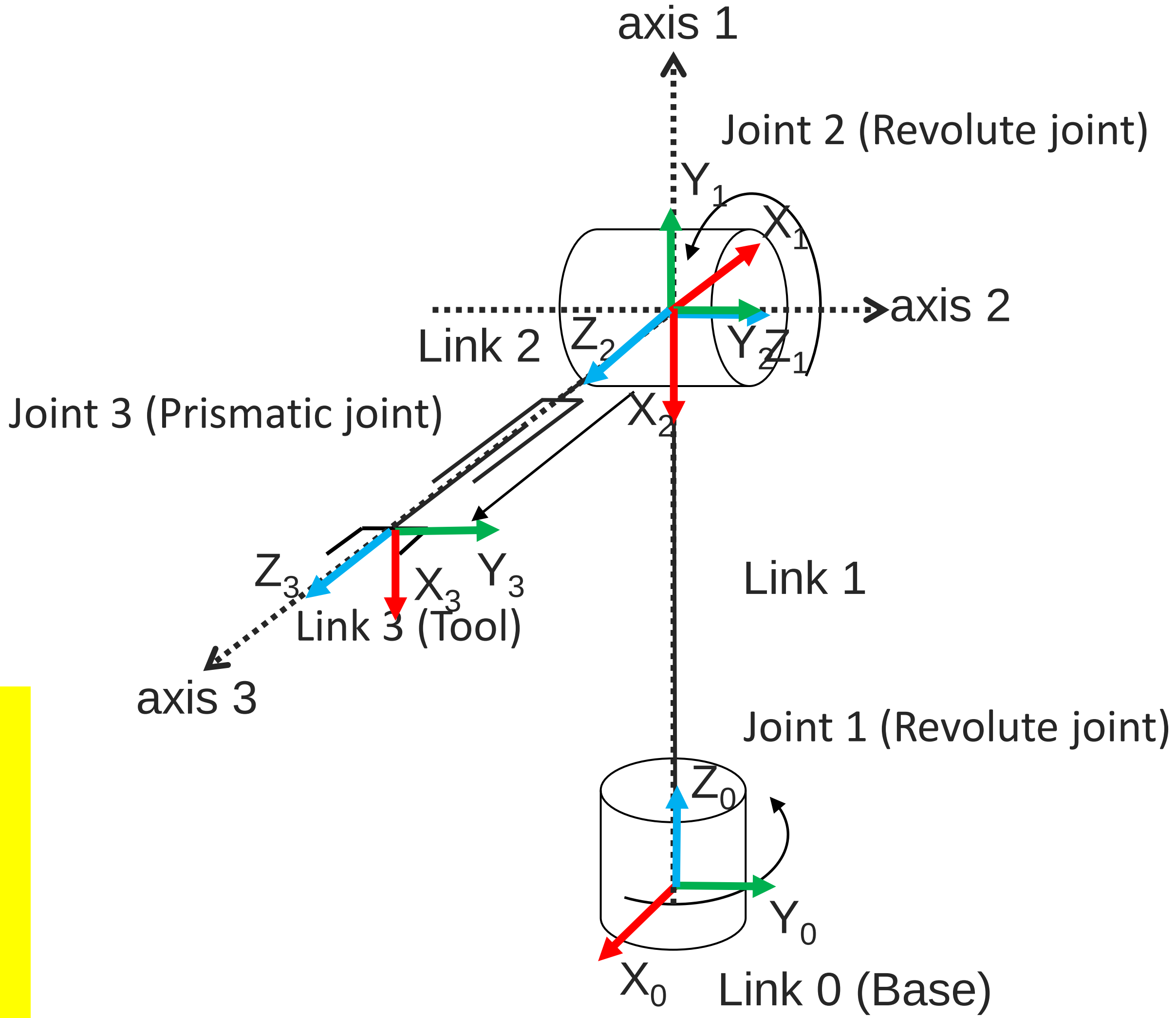
  - Can use right-hand rule, i.e., make (Z_{i-1}, Z_i, X_i) right-handed
  - O_i naturally sits at intersection but can be anywhere on axis Z_i

Rule 3: Derive Y_i from X_i and Z_i (right-hand rule)



Example

2. Attach frames to links



Rule 5: Tool Frame (Frame {n})

- Z_n of Frame {n} is set as convenient since Joint n+1 does not exist
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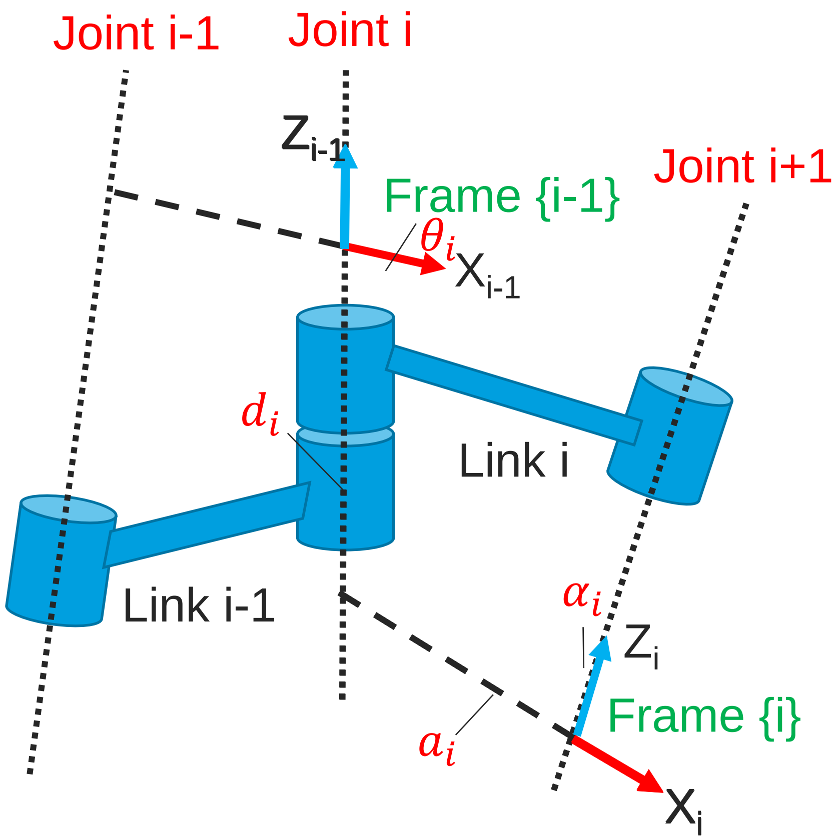
- X_n is set according to Rule 2
- Y_n is derived from X_n and Z_n (right-hand rule)

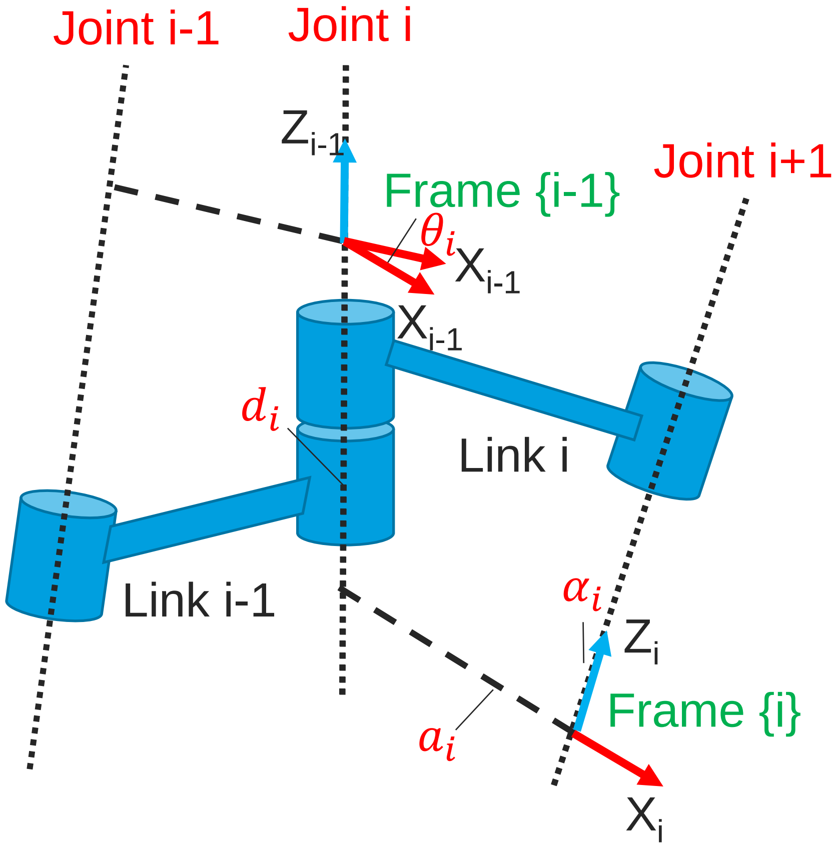
3. Parameterisation: D-H parameters

- θ_i: Joint angle
  - Angle from X_{i-1} to X_i measured about Z_{i-1}
- d_i: Link offset
  - Distance from X_{i-1} to X_i measured along Z_{i-1}
- α_i: Link twist
  - Angle from Z_{i-1} to Z_i measured about X_i
- a_i: Link length
  - Distance from Z_{i-1} to Z_i measured along X_i



3. Parameterisation: D-H parameters

$$ {}^{i-1}T_i \;=\; R_Z(\theta_i)\,T_Z(d_i)\,T_X(a_i)\,R_X(\alpha_i) $$



Some textbooks denote this matrix as A_i

3. Parameterisation: D-H parameters

$$ {}^{i-1}T_i = R_Z(\theta_i)\,T_Z(d_i)\,T_X(a_i)\,R_X(\alpha_i) $$

$$ = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

3. Parameterisation: D-H parameters

- q_i: Joint variable of joint i
- For a revolute joint

$$^{i-1}T_i\ (q_i) = R_Z(\theta_i + q_i)T_Z(d_i)R_X(\alpha_i)T_X(a_i)$$

- For a prismatic joint

$$^{i-1}T_i\ (q_i) = R_Z(\theta_i)T_Z(d_i + q_i)R_X(\alpha_i)T_X(a_i)$$

- When adding joint variables, make sure the joint variables items equal to zeros.
  - E.g., if in the given diagram q_i is at 90°, add (q_i-90°) instead of q_i
  - The advantage of this notation is, if the zero position changes, you can simply change the values subtracted from the joint variables.

$$^{i-1}T_i\ (q_i) = R_Z(\theta_i + (q_i - 90°))T_Z(d_i)R_X(\alpha_i)T_X(a_i)$$

Example

3. Find D-H parameters

i	θ_i	d_i	α_i	a_i
1				
2				
3				

- θ_i: Joint angle
  - Angle from X_{i-1} to X_i measured about Z_{i-1}
- d_i: Link offset
  - Distance from X_{i-1} to X_i measured along Z_{i-1}
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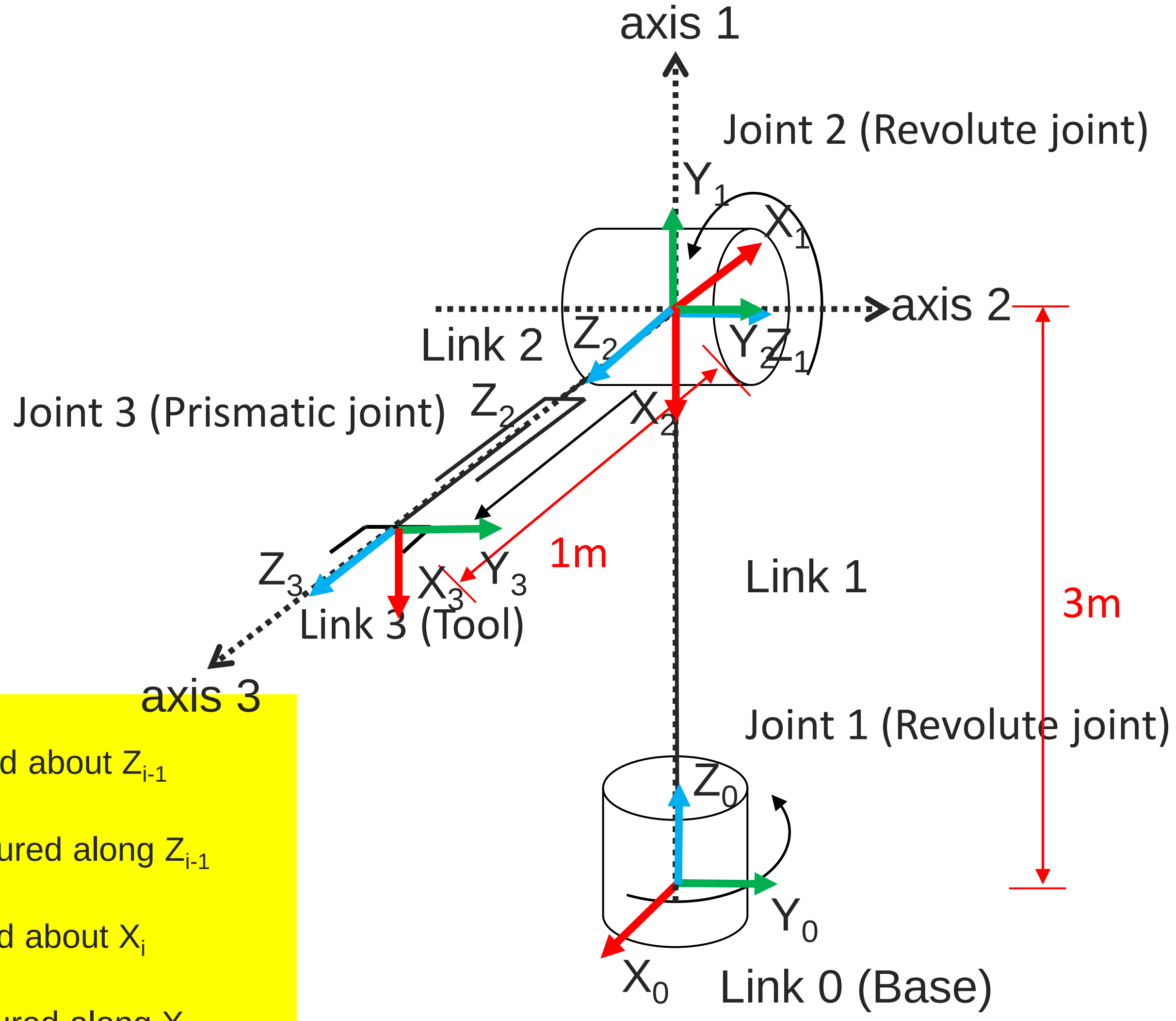

4. Calculate the forward kinematics using the chain rule

$$ {}^{0}T_n(\boldsymbol{q}) = {}^{0}T_1(q_1)\, {}^{1}T_2(q_2) \cdots {}^{n-1}T_n(q_n) $$

where $\boldsymbol{q} = \{q_1, q_2, \cdots, q_n\}$ is a set of joint variables and is called a robot configuration.

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Example

4. Calculate the

i	θ_i	d_i	α_i	a_i
1	$180^\circ + q_1$	3m	90°	0
2	$-90^\circ + q_2$	0	90°	0
3	0°	$1\text{m} + q_3$	0°	0

$$\Rightarrow \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ {}^{0}T_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{1}T_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{2}T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note shorthand: $C_i = \cos(\theta_i)$, $S_i = \sin(\theta_i)$, $\theta_i \neq q_i$

Example

4. Calculate the full kinematics

$$ {}^{0}T_{3} = {}^{0}T_{1}\ {}^{1}T_{2}\ {}^{2}T_{3} = \begin{bmatrix} c_1 c_2 & s_1 & c_1 s_2 & d_3 c_1 s_2 \\ s_1 c_2 & -c_1 & s_1 s_2 & d_3 s_1 s_2 \\ s_2 & 0 & -c_2 & 3 - d_3 c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}T_{1} = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{1}T_{2} = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{2}T_{3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

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$$

Given $q_1 = 0, q_2 = 0, q_3 = 0$
we have $\theta_1 = 180°, \theta_2 = -90°, d_3 = 1m$
and thus

$$
{}^{0}T_{3} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

i	θ_i	d_i	α_i	a_i
1	180°+q_1	3m	90°	0
2	-90°+q_2	0	90°	0
3	0°	1m+q_3	0°	0

Example

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i	θ_i	d_i	α_i	a_i
1	$180^\circ + q_1$	3m	90°	0
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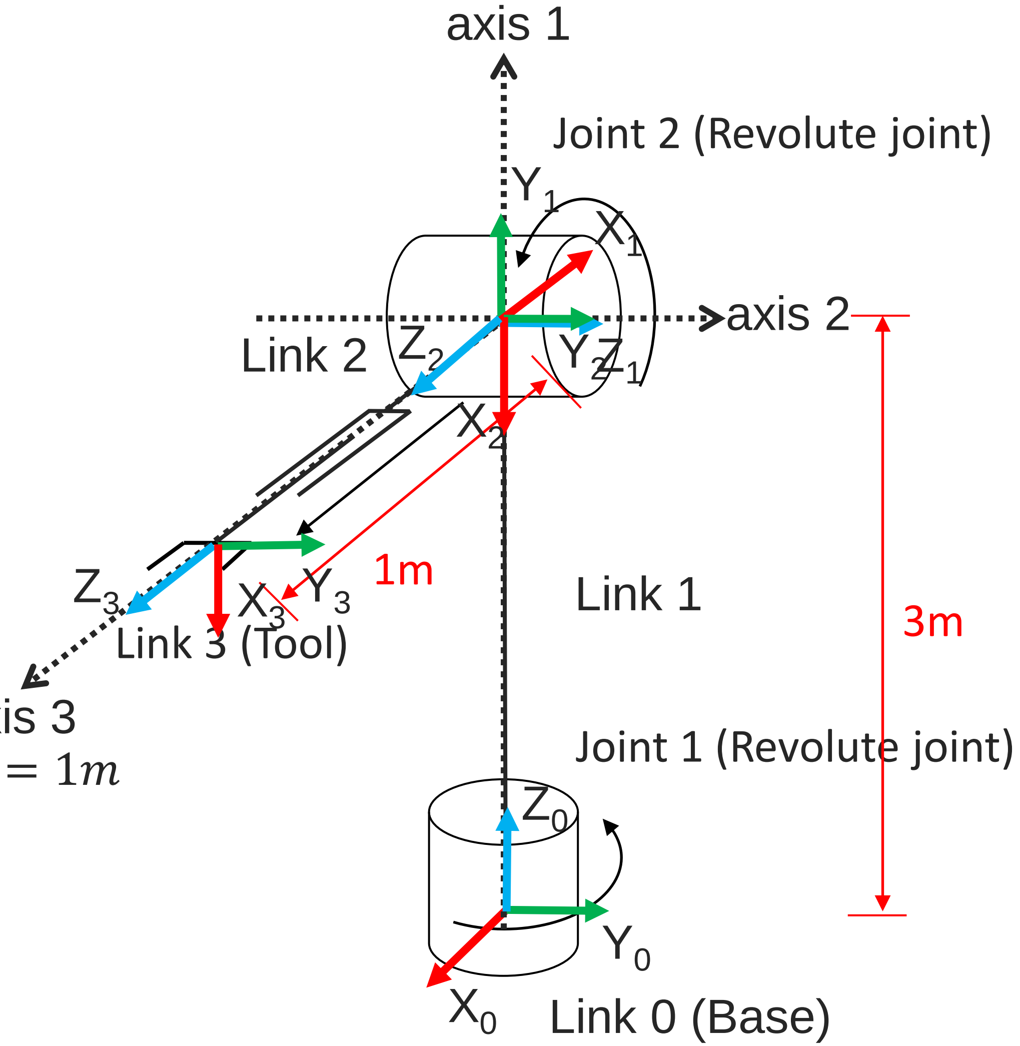
$$^0T_3 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

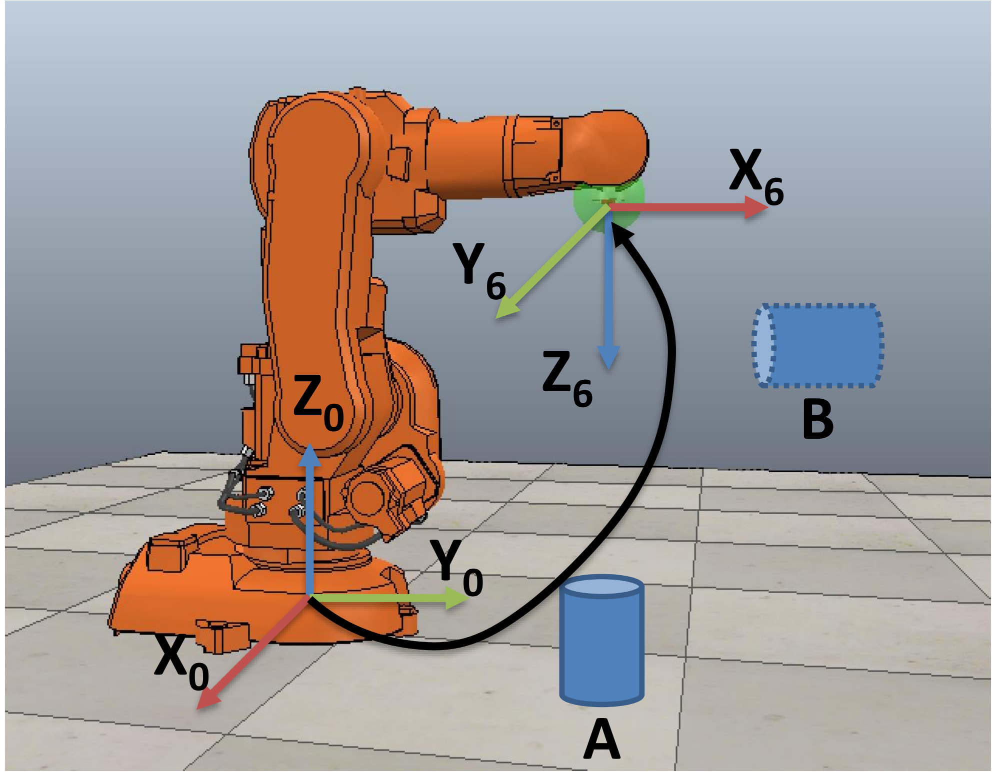


Summary

- A robotic arm can be modelled by a kinematic chain.
- The kinematics can be parameterised by the Denavit-Hartenberg (D-H) convention.
- Four D-H parameters are used to parameterise a homogeneous transformation between two consecutive frames.
- Forward kinematics is able to calculate the pose of the tool with respect to the base given a set of joint variables.

Motivating Problem - Revisit

- Imagine one of your arms is replaced by a robotic arm. You are supposed to move an object from A to B.
- But first you want to know where your current "hand" is.
- What you already know are the geometric parameters of your arm (fixed) and the angle of each joint (variable).
  - D-H parameters
- How can you calculate the pose of you "hand"?
  - Forward Kinematics



$$^{0}T_6 = fkine(q_1, q_2, q_3, q_4, q_5, q_6)$$

Final Remarks

- Acknowledgements
  - Some material of the slides was developed by the previous lecturers of EGB339 - Introduction to Robotics (Michael Milford, Peter Corke, and Leo Wu)