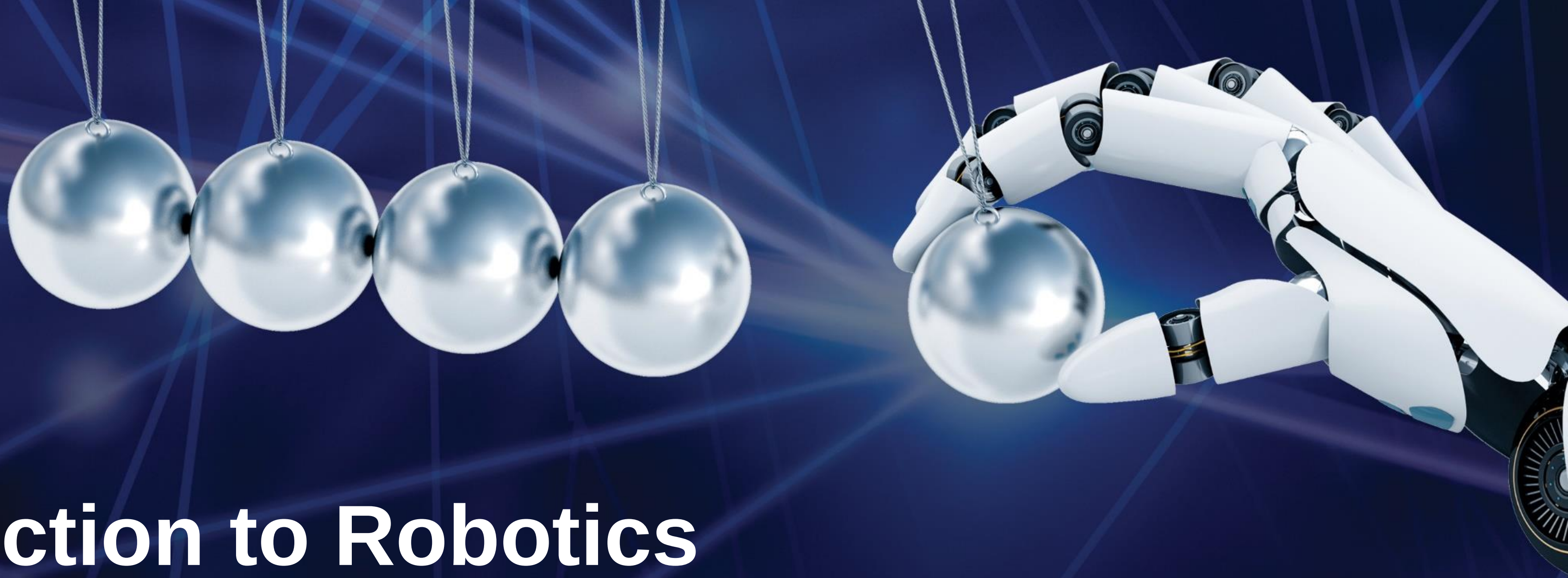
Centre for Robotics



EGB339 Introduction to Robotics
Part 2: Robotic Arms

Lecture 4: Velocity Kinematics

Chris Lehnert (Lecturer)

Outline

- Topics covered in this series of lectures
  - Rigid Body Motions (week 8)
  - Forward Kinematics (week 9)
  - Inverse Kinematics (week 10)
  - Velocity Kinematics (week 11)
  - Path and Trajectory Planning (week 12)
  - Revision (week 13)

- Topics not covered in this series of lectures
  - Dynamics
  - Control
  - Hardware
  - (Artificial) Intelligence
  - …

Watch these online videos

- QUT Robot Academy (by Prof Peter Corke)
  - Velocity Kinematics in 2D
    - https://robotacademy.net.au/masterclass/velocity-kinematics-in-2d/
  - Velocity Kinematics in 3D
    - https://robotacademy.net.au/masterclass/velocity-kinematics-in-3d/

Review of Week 10

- Given the pose of a target, we can calculate the joint variables that enable the robot to reach the target by inverse kinematics.

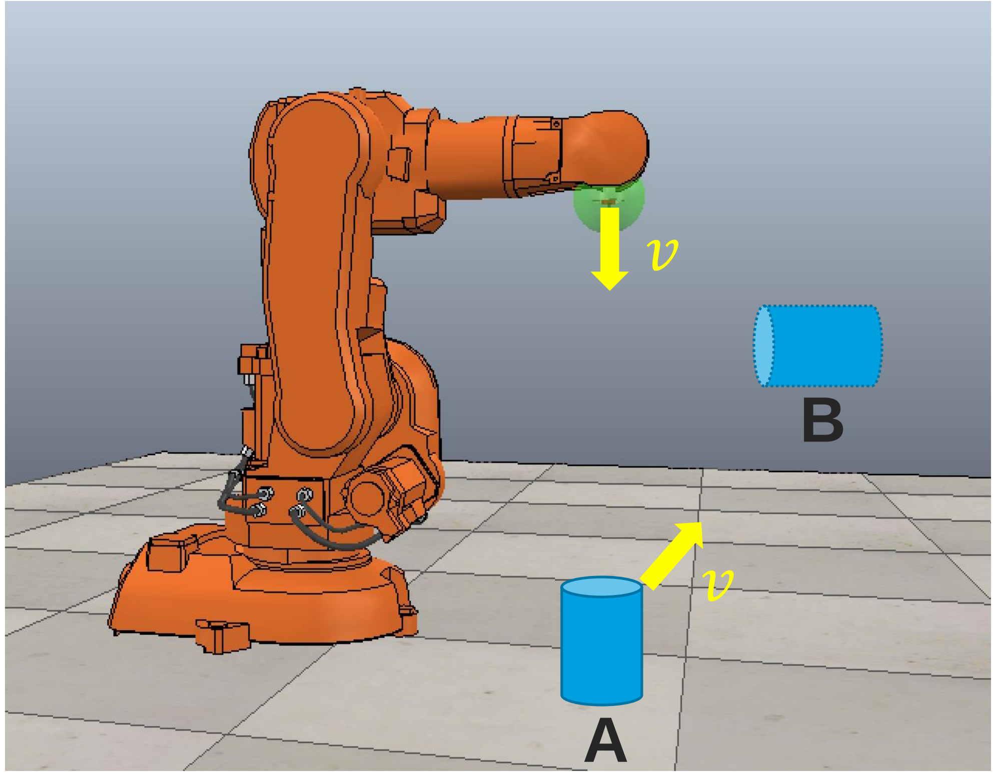
- There are three methods to solve the inverse kinematics.
  - Geometric method
  - Algebraic method
  - Numeric method

- A single pose of the target can correspond to multiple configurations of the robots.

- Singularity could occur when a non-redundant robot has an infinite number of inverse kinematics solutions.

Motivating Problem

- Imagine one of your arms is replaced by a robotic arm. You are supposed to move an object from A to B.
- Now you know where the object is in front of you (homogeneous transformation).
- You know where your “hand” is with respect to your “body” (forward kinematics).
- You also know how to move your “hand” to reach the object (inverse kinematics).
- Would you be able to move the object at a certain speed?



Motion in 3D

https://robotacademy.net.au/masterclass/velocity-kinematics-in-3d/?lesson=340

Tool Velocity

- Recall that the homogeneous transformation has the form

$$T = \begin{bmatrix} & R & & t \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

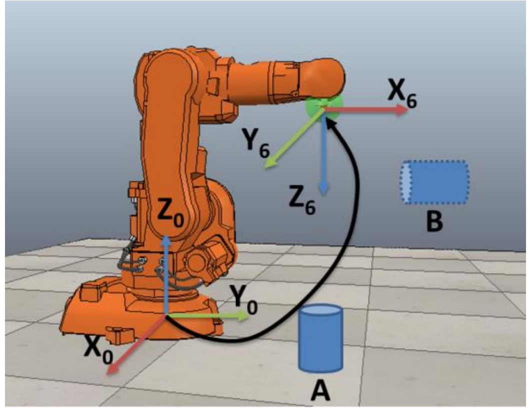
- The velocity of a tool consists of two components:
  - The rotational velocity ($\dot{R}$, or ω, based on the parameterization of the rotation matrix)
  - The translational velocity ($\dot{t}$, or v, simply the velocity in the 3D Cartesian space)

- For starters, we will only investigate the translational velocity.

Forward Kinematics and Inverse Kinematics

Forward kinematics

$$f_{\theta,d,\alpha,a} : q \rightarrow p$$



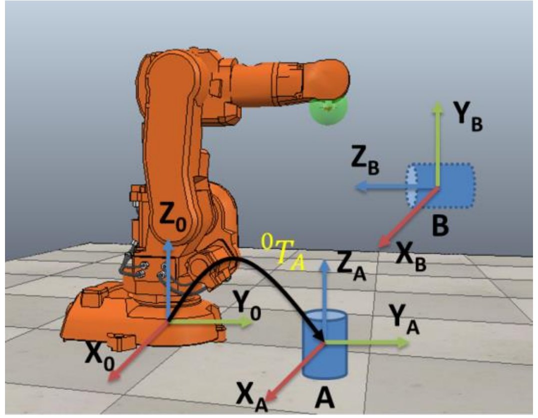
${}^{0}T_6 = fkine(q_1, q_2, q_3, q_4, q_5, q_6)$

Joint Space

$$q = [q_1, q_2, \ldots q_n]$$

Task Space

$$p = [R, t]$$



$q_A = ikine({}^{0}T_A)$

Inverse kinematics

$$f^{-1}_{\theta,d,\alpha,a} : p \rightarrow q$$

Forward and Inverse Velocity Kinematics

Forward velocity kinematics

$f_{\theta,d,\alpha,a,q} : \dot{q} \to \dot{p}$

Joint Velocity

$\dot{q} = [\dot{q}_1, \dot{q}_2, \ldots \dot{q}_n]$

Tool Velocity

$\dot{p} = [\dot{R}, \dot{t}]$

Inverse velocity kinematics

$f^{-1}_{\theta,d,\alpha,a,q} : \dot{p} \to \dot{q}$

The velocity kinematics function depends not only on the D-H parameters but also on the robot configuration q.

One dimensional example

Forward kinematics:

$$x = f_L(\theta) = 2L\cos(\theta)$$

Inverse kinematics:

$$\theta = f_L^{-1}(x) = \arccos(x/2L)$$

$\dot{x} = dx/dt$

J

Forward velocity kinematics:

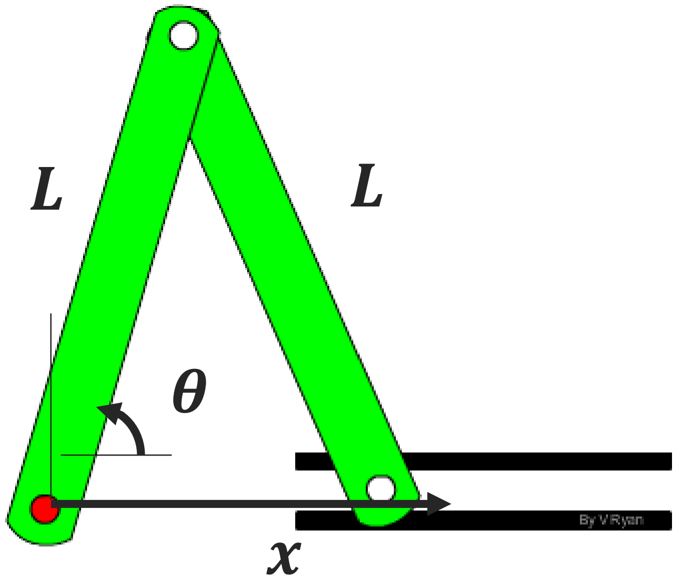
$$\dot{x} = f_L'(\theta)\dot{\theta} = \boxed{-2L\sin(\theta)}\dot{\theta}$$

J^{-1}

Inverse velocity kinematics:

$$\dot{\theta} = (f_L'(\theta))^{-1}\dot{x} = \boxed{-\frac{1}{2L\sin(\theta)}}\dot{x}$$

$\dot{\theta} = d\theta/dt$



The Jacobian

- The Jacobian is a matrix that is a function of joint configuration, that linearly relates joint velocity to tool velocity.

- Full Jacobian:

$$\dot{p} = \begin{bmatrix} \omega \\ v \end{bmatrix} = J(q)\dot{q} = \begin{bmatrix} J_\omega(q) \\ J_v(q) \end{bmatrix} \dot{q}$$

- Translational velocity Jacobian:

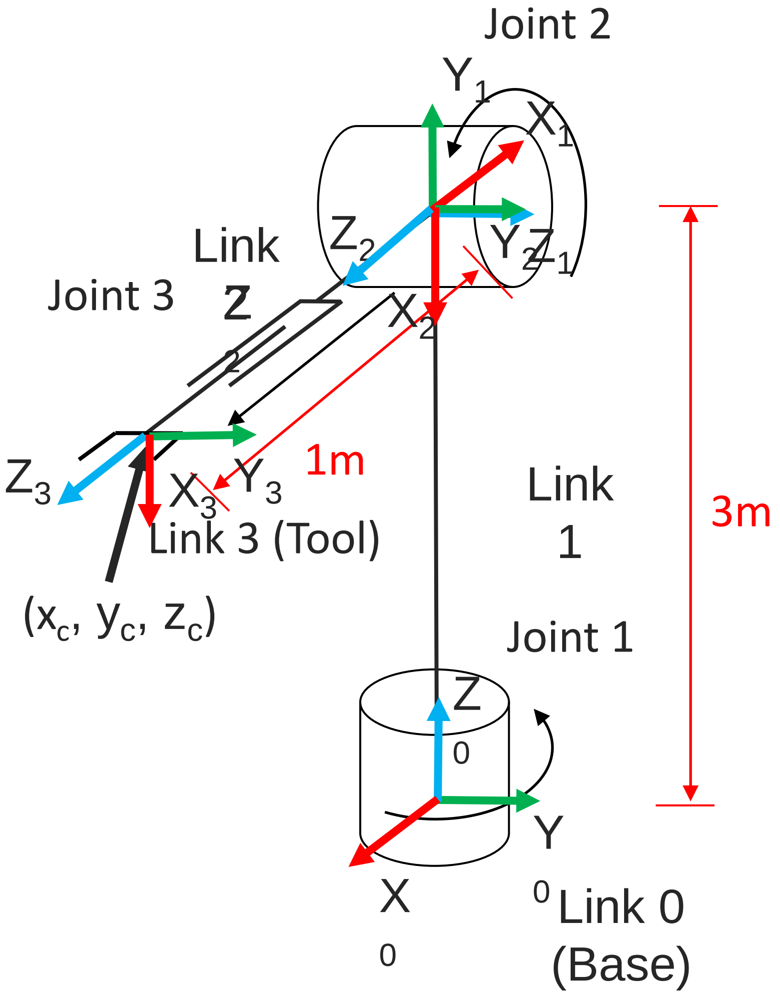
$$v = J_v(q)\dot{q}$$

The Jacobian - Example

$$v = J_v(q)\dot{q}$$

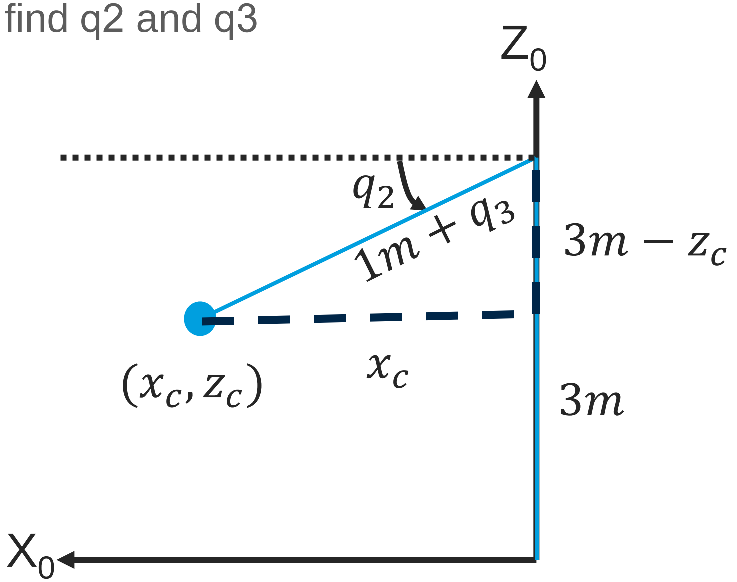
- Translational velocity Jacobian:

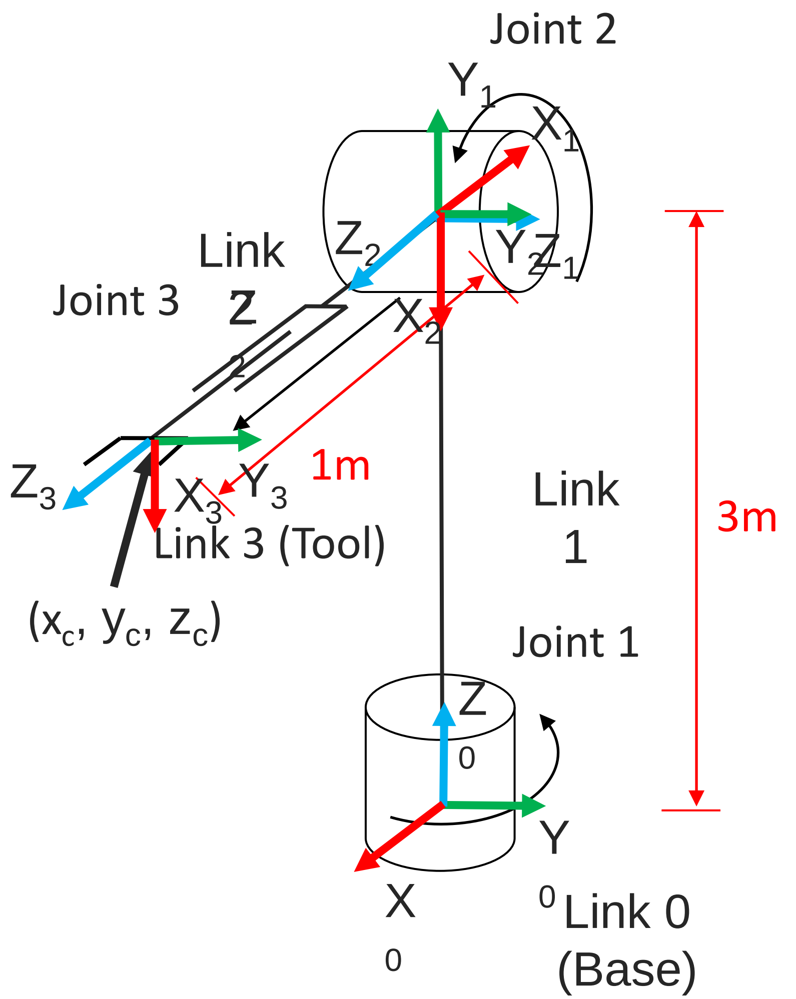
$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{z}_c \end{bmatrix} = \begin{bmatrix} \frac{\partial x_c}{\partial q_1} & \frac{\partial x_c}{\partial q_2} & \frac{\partial x_c}{\partial q_3} \\ \frac{\partial y_c}{\partial q_1} & \frac{\partial y_c}{\partial q_2} & \frac{\partial y_c}{\partial q_3} \\ \frac{\partial z_c}{\partial q_1} & \frac{\partial z_c}{\partial q_2} & \frac{\partial z_c}{\partial q_3} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$$



The Jacobian - Example

- Translational velocity Jacobian:
  - Lock joint 1 at the initial position (q_1=0) and only consider the planar motion of joint 2 and 3 (only investigate 2D problem for ease of demo)
  - I.e., given ($\dot{x}c$, $\dot{z}c$), find $\dot{q}2$ and $\dot{q}3$





The Jacobian - Example

- Translational velocity Jacobian:
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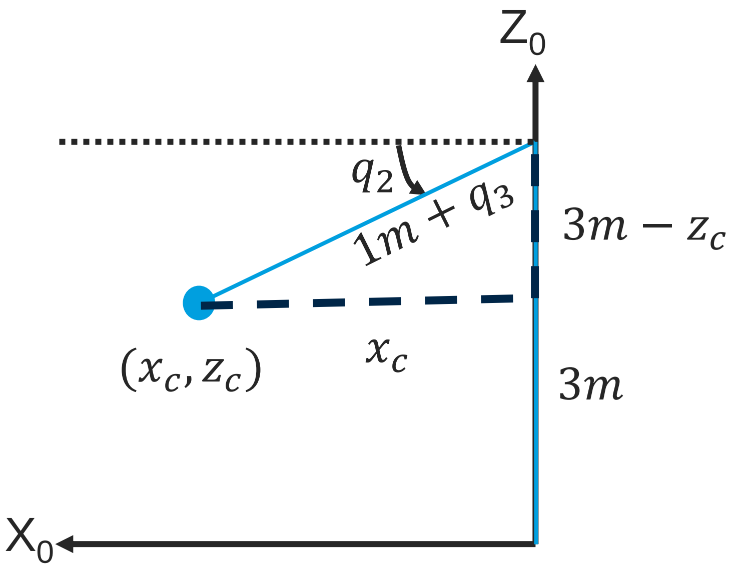


You can also work this out geometrically.

i	θ_i	d_i	α_i	a_i
1	$180^\circ+q_1$	3m	90°	0
2	$-90^\circ+q_2$	0	90°	0
3	0°	$1m+q_3$	0°	0

$$
{}^0T_3 = \begin{bmatrix} c_1c_2 & s_1 & c_1s_2 & d_3c_1s_2 \\ s_1c_2 & -c_1 & s_1s_2 & d_3s_1s_2 \\ s_2 & 0 & -c_2 & 3m - d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\begin{cases} d_3c_1s_2 = x_c \\ d_3s_1s_2 = y_c \\ 3m - d_3c_2 = z_c \end{cases}
$$

$$\Downarrow$$

$$
\begin{cases} x_c = -(1m + q_3)\sin(-90^\circ + q_2) \\ \quad\; = (1m + q_3)\cos(q_2) \\ z_c = 3m - (1m + q_3)\cos(-90^\circ + q_2) \\ \quad\; = 3m - (1m + q_3)\sin(q_2) \end{cases}
$$

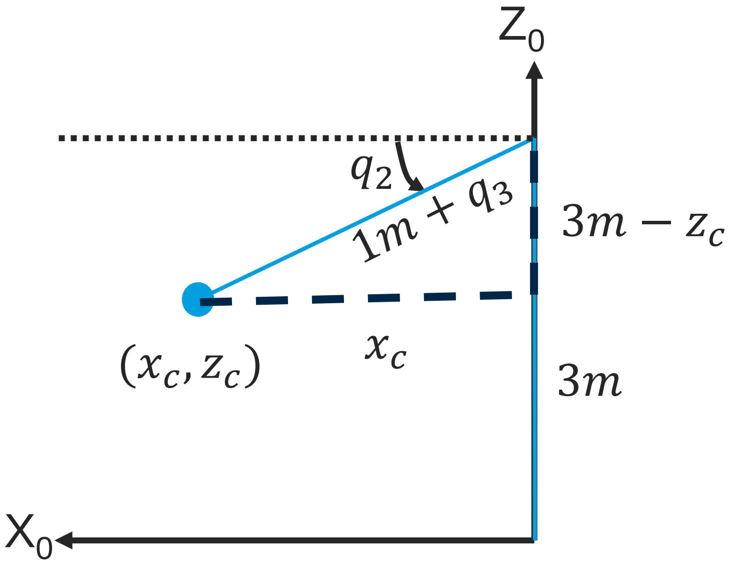
Differentiation of Trigonometric Functions

Function	Derivative
$\sin(q)$	$\cos(q)$
$\cos(q)$	$-\sin(q)$
$\sin(q_1+q_2)$ differentiated by q_1 or q_2	$\cos(q_1+q_2)$
$\cos(q_1+q_2)$ differentiated by q_1 or q_2	$-\sin(q_1+q_2)$
$q_3\sin(q_1+q_2)$ differentiated by q_3	$\sin(q_1+q_2)$
$q_3\cos(q_1+q_2)$ differentiated by q_3	$\cos(q_1+q_2)$
$(1+q_3)\sin(q_1+q_2)$ differentiated by q_3	$\sin(q_1+q_2)$
$(1+q_3)\cos(q_1+q_2)$ differentiated by q_3	$\cos(q_1+q_2)$

The Jacobian - Example

- Translational velocity Jacobian:
  - Lock joint 1 at the initial position (q_1=0) and only consider the planar motion of joint 2 and 3 (only investigate 2D problem for ease of demo)
  - I.e., given ($\dot{x}c$, $\dot{z}c$), find $\dot{q}2$ and $\dot{q}3$

$$\begin{cases} x_c = (1m + q_3)\cos(q_2) \\ z_c = 3m - (1m + q_3)\sin(q_2) \end{cases}$$



$$\Downarrow$$

$$J_v = \begin{bmatrix} \frac{\partial x_c}{\partial q_2} & \frac{\partial x_c}{\partial q_3} \\ \frac{\partial z_c}{\partial q_2} & \frac{\partial z_c}{\partial q_3} \end{bmatrix} = \begin{bmatrix} -(1m + q_3)\sin(q_2) & \cos(q_2) \\ -(1m + q_3)\cos(q_2) & -\sin(q_2) \end{bmatrix}$$

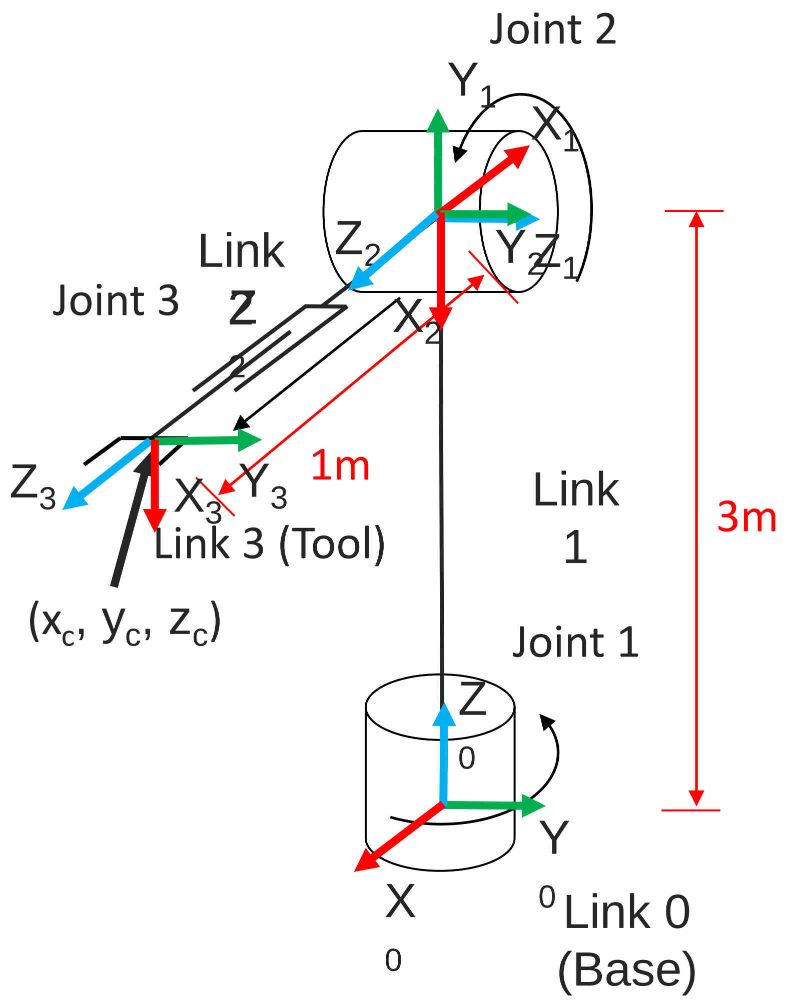
Forward Velocity Kinematics

- Given $\dot{q}$, find v
- Find the Jacobian matrix J_v first, then use the following equation to calculate v

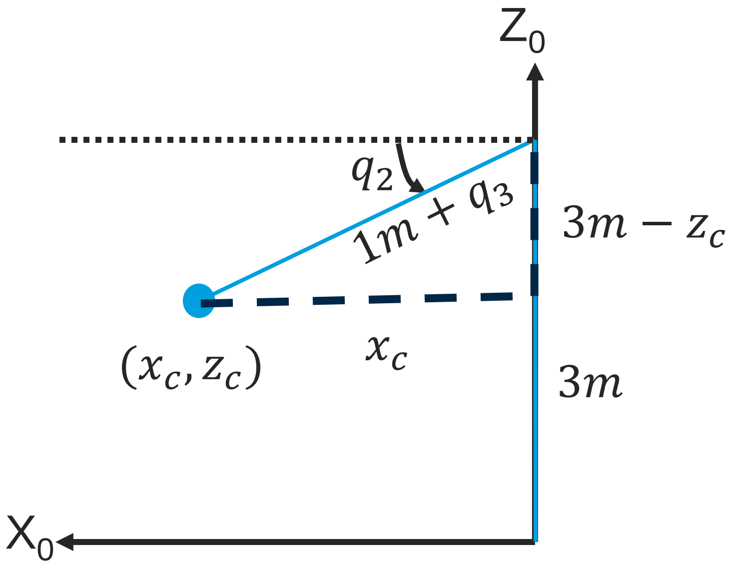
$$v = J_v(q)\dot{q}$$

Forward Velocity Kinematics - Example

- The arm is at its initial configuration (q_1=0, q_2=0, q_3=0).
- Assume joint 1 is locked in the initial position
- Find the tool translational velocity if: $\dot{q}_2 = 1\ rad/s$; $\dot{q}_3 = 2\ m/s$.



Forward Velocity Kinematics - Example



- The arm is at its initial configuration (q_1=0, q_2=0, q_3=0).
- Assume joint 1 is locked in the initial position
- Find the tool translational velocity if: $\dot{q}_2 = 1\ rad/s$; $\dot{q}_3 = 2\ m/s$.

$$\begin{bmatrix}\dot{x}_c \\ \dot{z}_c\end{bmatrix} = \begin{bmatrix}-(1m+q_3)\sin(q_2) & \cos(q_2) \\ -(1m+q_3)\cos(q_2) & -\sin(q_2)\end{bmatrix}\begin{bmatrix}\dot{q}_2 \\ \dot{q}_3\end{bmatrix}$$

$$\begin{bmatrix}\dot{x}_c \\ \dot{z}_c\end{bmatrix} = ?$$

$$\begin{bmatrix}\dot{x}_c \\ \dot{z}_c\end{bmatrix} = \begin{bmatrix}0 & 1 \\ -1 & 0\end{bmatrix}\begin{bmatrix}1 \\ 2\end{bmatrix} = \begin{bmatrix}2 \\ -1\end{bmatrix}$$

Inverse Velocity Kinematics

- Usually we have a desired toolpoint velocity, and want to find out the required joint velocities to achieve it.

- To compute the joint velocities for a given toolpoint velocity, we need to invert the Jacobian.

- Full Jacobian:

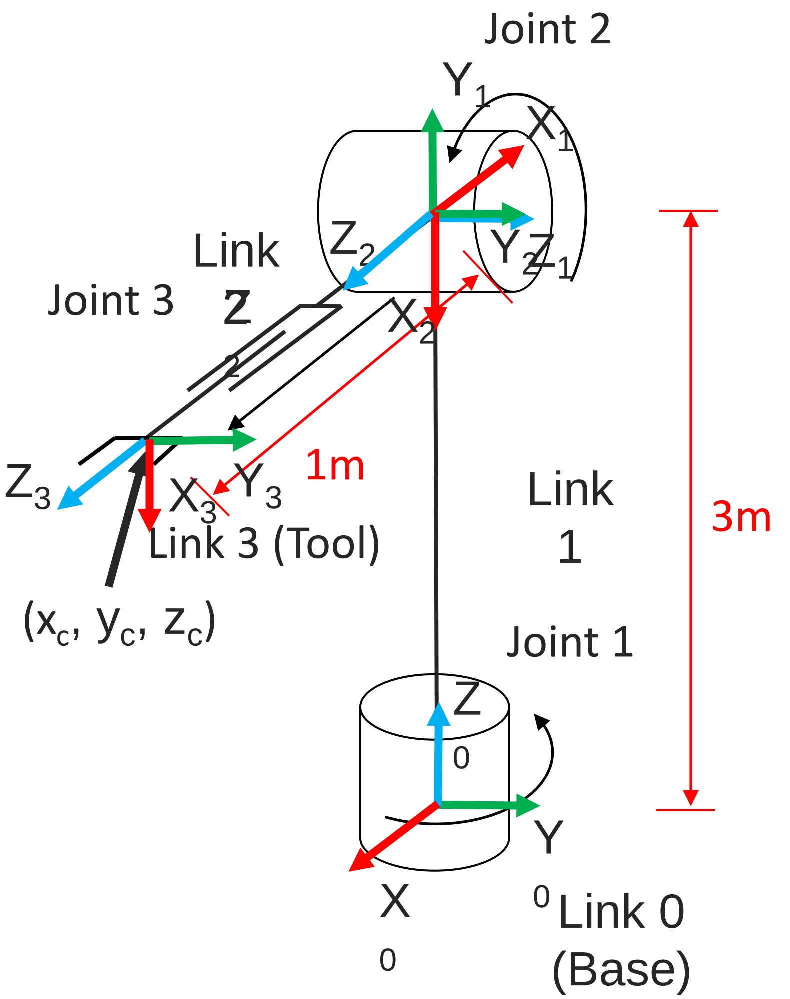
$$\dot{q} = J^{-1}(q)\dot{p} = J^{-1}(q)\begin{bmatrix}\omega \\ v\end{bmatrix}$$

- Translational velocity Jacobian:

$$\dot{q} = J_v^{-1}(q)v$$

Inverse Velocity Kinematics - Example

- The arm is in its initial configuration (q_1=0, q_2=0, q_3=0)
- Assume joint 1 is locked in the initial position
- Find the joint velocities $\dot{q}_2$ and $\dot{q}_3$ to move the toolpoint such that: $\dot{x}_c = 2\ m/s$; $\dot{z}_c = -1\ m/s$



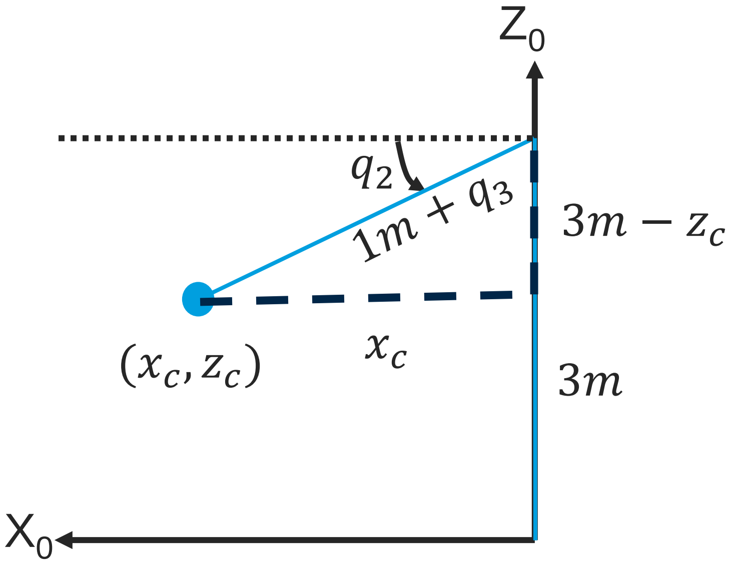
Inverse Velocity Kinematics - Example

- Find the joint velocities $(\dot{q}_2, \dot{q}_3)$ in terms of the tool velocity $(\dot{x}_c, \dot{z}_c)$.

$$\begin{bmatrix} \dot{x}_c \\ \dot{z}_c \end{bmatrix} = \begin{bmatrix} -(1+q_3)\sin(q_2) & \cos(q_2) \\ -(1+q_3)\cos(q_2) & -\sin(q_2) \end{bmatrix} \begin{bmatrix} \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} -(1+q_3)\sin(q_2) & \cos(q_2) \\ -(1+q_3)\cos(q_2) & -\sin(q_2) \end{bmatrix}^{-1} \begin{bmatrix} \dot{x}_c \\ \dot{z}_c \end{bmatrix}$$

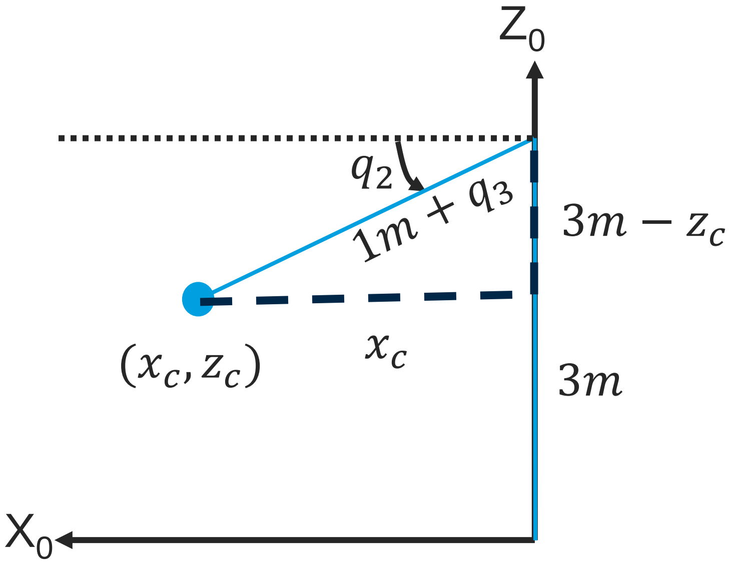
$$\begin{bmatrix} \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{1+q_3} \begin{bmatrix} -\sin(q_2) & -\cos(q_2) \\ (1+q_3)\cos(q_2) & -(1+q_3)\sin(q_2) \end{bmatrix} \begin{bmatrix} \dot{x}_c \\ \dot{z}_c \end{bmatrix}$$



$$\boxed{\mathbf{A}^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}$$

Inverse Velocity Kinematics - Example

- The arm is in its initial configuration (q_1=0, q_2=0, q_3=0)
- Assume joint 1 is locked in the initial position
- Find the joint velocities $\dot{q}_2$ and $\dot{q}_3$ to move the toolpoint such that: $\dot{x}_c = 2\ m/s$; $\dot{z}_c = -1\ m/s$



$$\begin{bmatrix}\dot{q}_2\\ \dot{q}_3\end{bmatrix} = \frac{1}{1+q_3}\begin{bmatrix}-\sin(q_2) & -\cos(q_2)\\ (1+q_3)\cos(q_2) & -(1+q_3)\sin(q_2)\end{bmatrix}\begin{bmatrix}\dot{x}_c\\ \dot{z}_c\end{bmatrix}$$

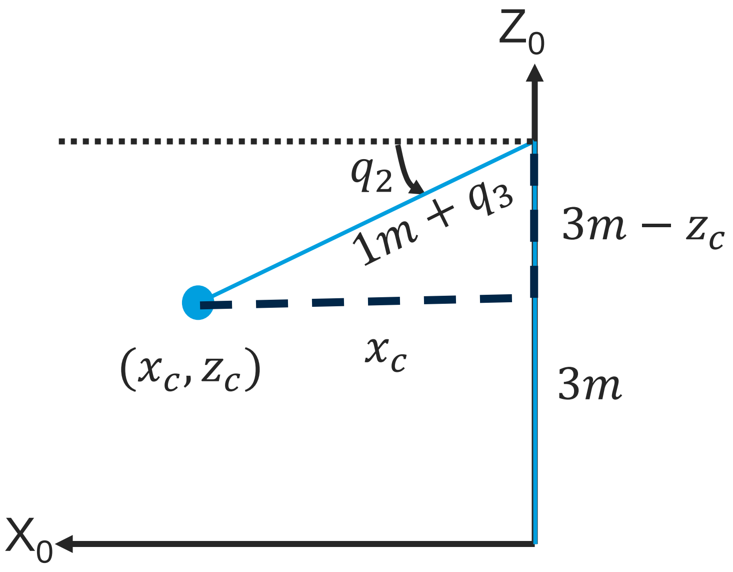
$$\begin{bmatrix}\dot{q}_2\\ \dot{q}_3\end{bmatrix} = \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}2\\ -1\end{bmatrix} = \begin{bmatrix}1\\ 2\end{bmatrix} \Rightarrow \dot{q}_2 = 1\ rad/s;\ \dot{q}_3 = 2\ m/s.$$

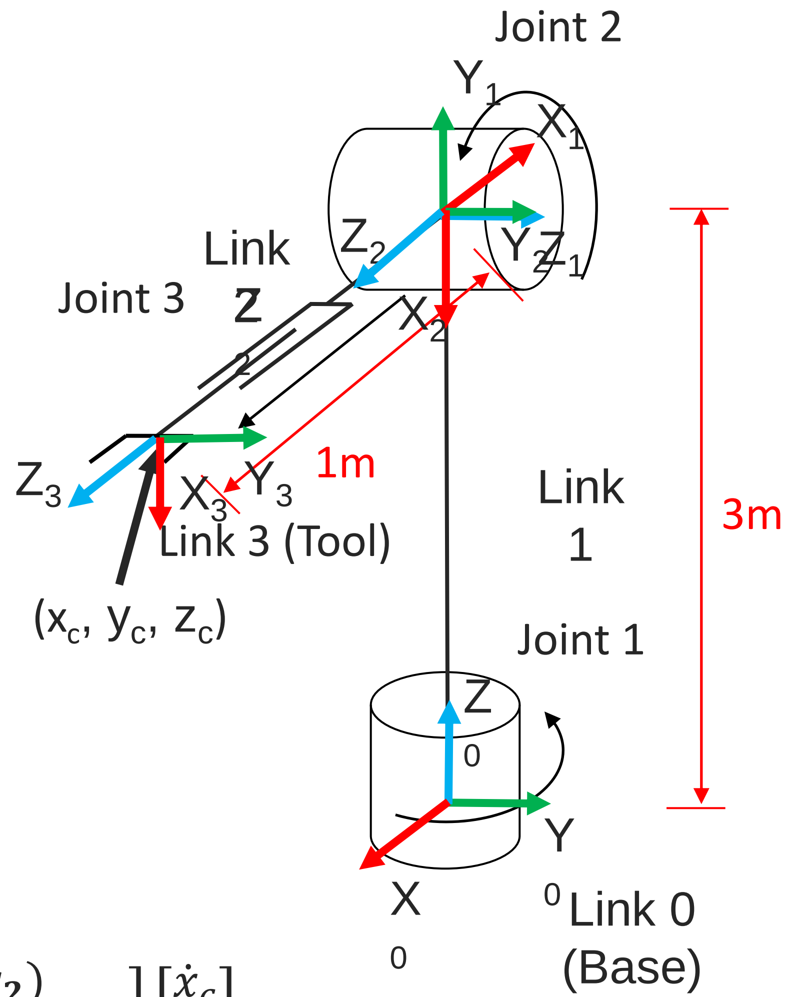
Other Uses of Velocity Kinematics

- **Singularity Analysis**
  - Singularity means a configuration of the robot in which the Jacobian matrix loses rank.
  - In singularity, the robot loses the ability to move in one or more directions.
  - Near singularity, small velocity of the tool may require large velocity of the joints.
- **General Inverse Kinematics**
- **Static Force/Torque Analysis**

Singularity - Example

- When q_3=-1m, there are no valid inverse velocity kinematics solutions.
- The robot can only move along the axis of joint 3, but loses the ability to move in another direction.





$$\begin{bmatrix}\dot{q}_2\\ \dot{q}_3\end{bmatrix} = \frac{1}{1+q_3}\begin{bmatrix}-\sin(q_2) & -\cos(q_2)\\ (1+q_3)\cos(q_2) & -(1+q_3)\sin(q_2)\end{bmatrix}\begin{bmatrix}\dot{x}_c\\ \dot{z}_c\end{bmatrix}$$

IK Challenges - Singularities

Near singularities, small velocity of the tool requires large velocity of the joints.

https://www.youtube.com/watch?v=zlGCurgsqg8

General Inverse Kinematics

Gauss-Newton Method

- Given current joint configuration q_c and the target pose p_t

- Find a temporal target pose (next pose) p_n that is near the current tool p_c pose and towards the target pose p_t

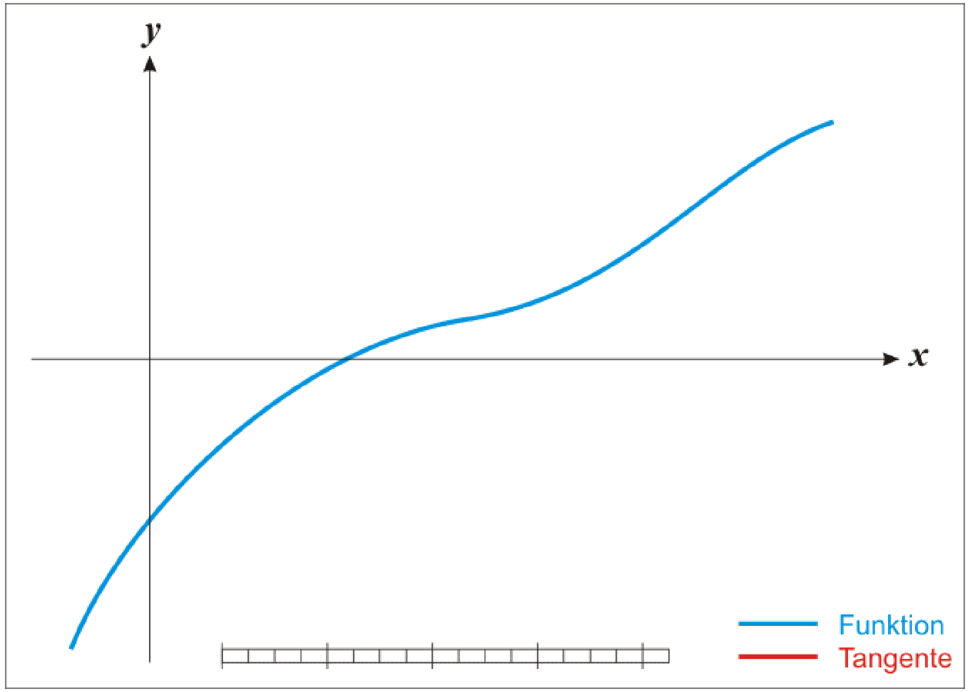
$$\Delta q \approx J^{-1}(q_c)\Delta p = J^{-1}(q_c)(p_n - p_c)$$

$$q_n = q_c + \Delta q$$

- Repeat the process until p_n is close enough to p_t

- Then q_n converges to the solution in which the robot reaches the target pose p_t

Newton's Method



Static Force/Torque Analysis

- When in equilibrium, work applied to the tool point is balanced by work done by joints (Virtual Work Principle):

"F" is force exerted on **toolpoint**

$$F \cdot \delta P = \tau \cdot \delta q$$

$$F^T \delta P = \tau^T \delta q$$

- Jacobian relates change in pose to change in joint variable: $\partial P = J \partial q$
- So:

$$F^T J \delta q = \tau^T \delta q$$

$$F^T J = \tau^T$$

$$\boxed{\tau = J^T F}$$

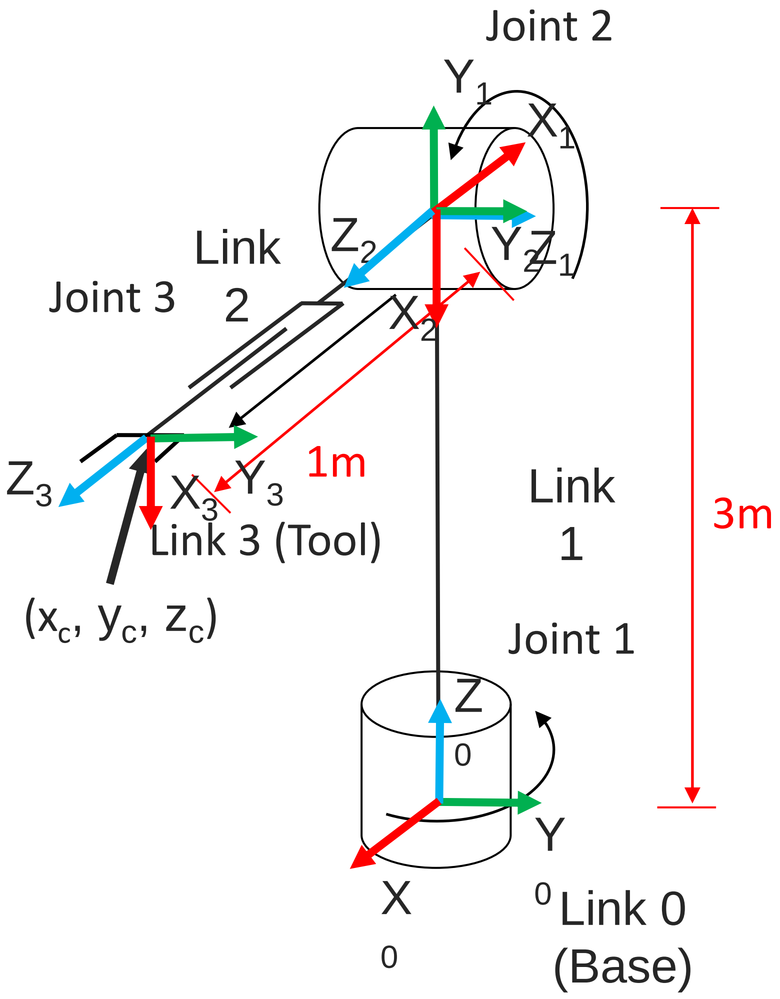
This equation finds the torques and forces ***experienced*** at the joints when the toolpoint is loaded.

Static Force/Torque - Example

- Find the force/torque required at the joints to support a 3 kg (30 N) load at the tool. (q_1=0, q_2=0, q_3=0) i.e., (ϑ_1=0, ϑ_2=-90, d_3=1)

$$J_v = \begin{bmatrix} d_3 s_1 s_2 & -d_3 c_1 c_2 & -c_1 s_2 \\ -d_3 c_1 s_2 & -d_3 s_1 c_2 & -s_1 s_2 \\ 0 & d_3 s_2 & -c_2 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$\tau = ?$

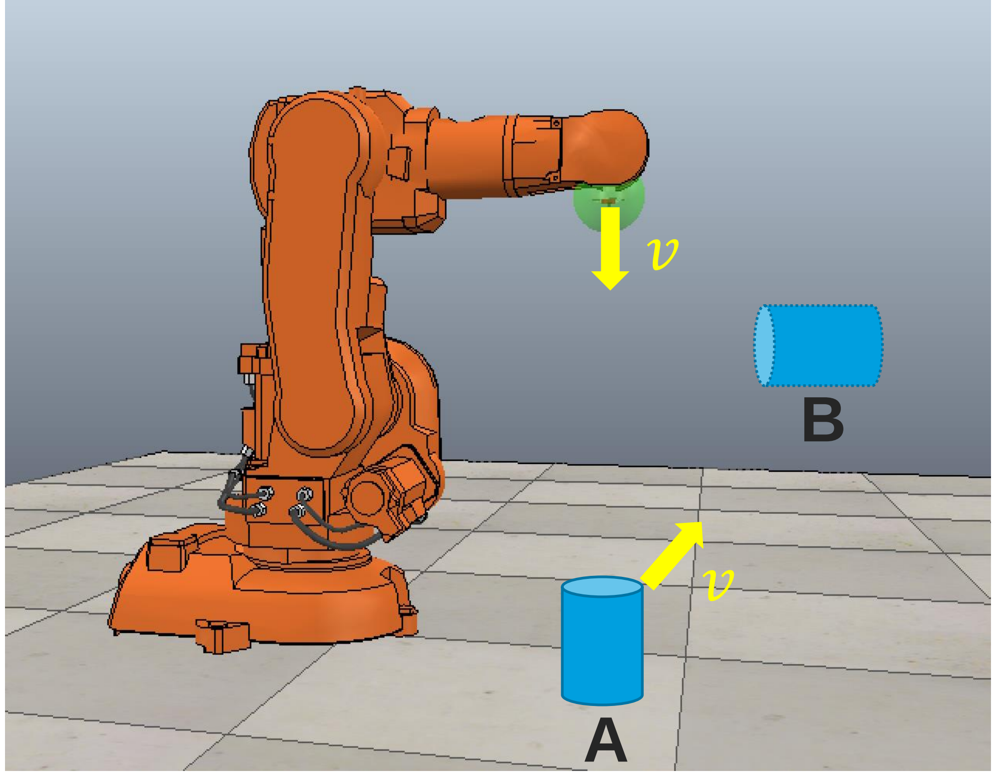


Summary

- The Jacobian is a matrix that is a function of joint position, that linearly relates joint velocity to toolpoint velocity.
- The Forward Velocity Kinematics maps the velocity of the joints to the velocity of the tool.
- The Inverse Velocity Kinematics maps the velocity of the tool to the velocity of the joints.
- From the viewpoint of velocity kinematics, singularity means a configuration of the robot in which the Jacobian matrix becomes rank-deficient.
- The static force/torque analysis can be done with the Jacobian matrix.

Motivating Problem - Revisit

- Imagine one of your arms is replaced by a robotic arm. You are supposed to move an object from A to B.
- Now you know where the object is in front of you (homogeneous transformation).
- You know where your "hand" is with respect to your "body" (forward kinematics).
- You also know how to move your "hand" to reach the object (inverse kinematics).
- Would you be able to move the object at a certain speed?
  - Inverse Velocity Kinematics



$$\dot{q} = J^{-1}(q)v$$

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Final Remarks

- Acknowledgements
  - Some material of the slides was developed by the previous lecturers of EGB339 - Introduction to Robotics (Michael Milford, Peter Corke, and Leo Wu)