Pros and cons of Bayesian Methods in Parameter Identification

Eduard Campillo-Funollet
with C.Venkataraman, A. Madzvamuse, S. Portet

23/11/2016

University of Sussex

The Leverhulme Trust

Bayesian approach to parameter identification[1]

Parameter identification

$$y = G(p) + \eta$$

- y experimental data
- G observation operator
- p parameters
- η noise

Bayes' theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$\pi(p|y) = \frac{\pi(y|p)\pi(p)}{\pi(y)}$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}\mu_0} \propto \exp(-\Phi(p;y))$$

Gaussian noise and Gaussian prior:

$$\pi(p|y) \propto \exp\left(-\frac{1}{2}\left(\|y - G(p)\|_\Sigma^2 + \|p\|_\Gamma^2\right)\right)$$

[1] Stuart 2010

Priors[1]

- X Banach space.
- $\{\phi_i\}_{i=1}^{\infty}$, $\phi_i \in X$
- $\{u_i\}_{i=1}^{\infty}$, $u_i \in \mathbb{R}$
- $m_0 \in X$

$$u = m_0 + \sum_{j=1}^{\infty} u_j \phi_j$$

Uniform priors

- Let $\{\gamma_i\}_{i=1}^{\infty} \in \ell^1$
- Let $\xi = \{\xi_i\}_{i=1}^{\infty}$, $\xi_i \sim \mathcal{U}(-1, 1)$, i.i.d.
- Take $u_i = \gamma_i \xi_i$

[1] Dashti-Stuart 2014

Pros and cons

Pros

- Rigorous modelling
- Choices connected to real world
- Additional information

Cons

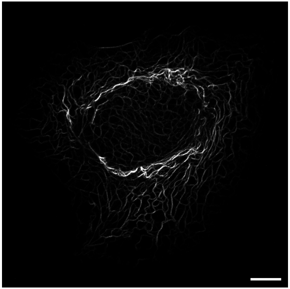
- Technically involved
- Choices due to technical reasons
- Computationally expensive

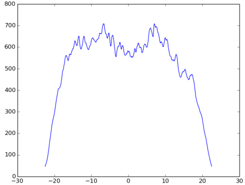
Parameter identification and data analysis

- Modelling
- Statistics
- *Big data*
- Machine learning

- Classification problems

Example: Dynamics of keratin network[1]





Model

- I concentration of assembled keratin.
- S concentration of soluble keratin.

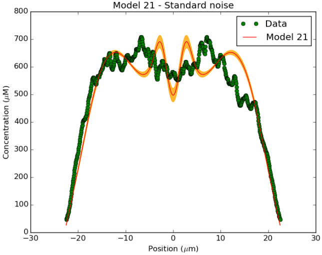
$$\begin{cases} \partial_t I + \mathbf{v} \cdot \nabla I - D_I \Delta I = R(I, S), \\ \partial_t S - D_S \Delta S = -R(I, S). \end{cases}$$

$$R(I, S) = \frac{k_{dis} I}{k_I + I} - \frac{k_{ass} S}{k_S + S}$$

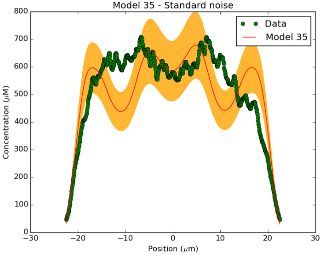
[1] Portet et al. 2015

Example: Dynamics of keratin network

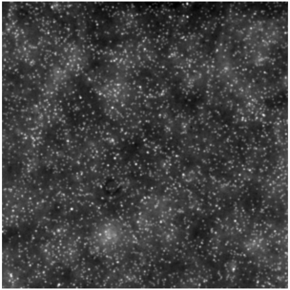
Akaike choice

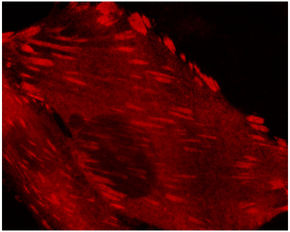


Experts choice



Example: Traction Force Microscopy

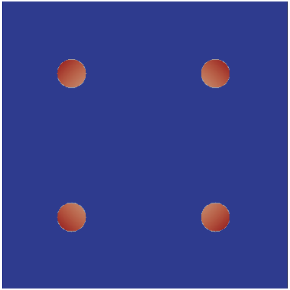




Model

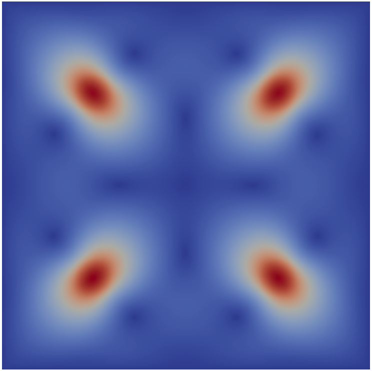
- u displacement
- f force

$$-\mathrm{div}(\mathbb{C}\nabla u) = f$$

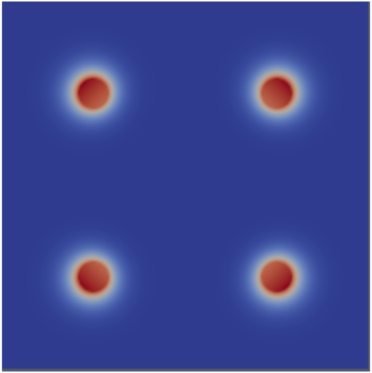


Example: Traction Force Microscopy

Displacement



Mean posterior



Thank you!

Parallel MCMC [1]

Idea

- Build a Markov chain with our target distribution as stationary distribution

MCMC

Given q proposal density, a acceptance probabiliy

1. Set $k = 0$. Pick $p^{(0)}$.
2. Propose $\tilde{p}^{(k)} \sim q(p^{(k)}, \cdot)$
3. Set $p^{(k+1)} = \tilde{p}^{(k)}$ with probability $a(p^{(k)}, \tilde{p}^{(k)})$. $p^{(k+1)} = p^{(k)}$ otherwise.
4. $k \leftarrow k + 1$ and go to 2.

- $a(p, \tilde{p}) = \min\{1, \exp(\phi(p) - \phi(\tilde{p}))\}$

Parallel MCMC

- Define a new probability $P = P(p_1, \ldots, p_{N+1}; i)$
- $P_i(p) = \pi(p_i) q(p_i, p_{-i})$

Given q proposal density, a acceptance probabiliy

1. Set $n = 0$. Pick $p^{(0)}$. Set $I = 1$.
2. Propose N points from $q(p^{(k)}, \cdot)$
3. Compute distribution of I: $P(I = i|p) \propto \pi(p_i)$
4. Sample N times from I.
5. $n \leftarrow n + 1$ and go to 2.

[1] Tjelmeland 2004, Calderhead 2014