Bayview Secondary School
Mathematics Department: Course Code: MDM4U1-01
Unit 4 - Assessment OF Probability Theory PART 1-Version 2

Name: _____ Alpha # _____

K-15	T-5

Instructions:

1) Scientific calculators are allowed but cannot be shared.
2) Duration of the assessment: 40 minutes
3) Show all work to obtain full marks for questions.
4) The use of cellphones, audio- or video-recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.
5) **Final answers must be in either improper faction to the lowest term or round to 3 decimal places.**

Knowledge and Understanding

1) A class is made up of 28 people. There are 17 boys and 11 girls. What is the probability of a study group of 6 with exactly 2 girls? [2 marks]

$$P(2\text{ girls}) = \frac{\binom{11}{2}\binom{17}{4}}{\binom{28}{6}} = \frac{935}{2691} \doteq 0.347$$

2) What is the probability that at least 2 out of a group of 12 students will have their birthday on the same day? Consider no students were born on February 29 and there are 365 days every year. [2 marks]

$$P(\text{birthday}) = 1 - \frac{365P_{12}}{365^{12}} \doteq 0.167$$

3) The odds in favour of Argentina winning the World Cup this year are 7: 5. What would be the winnings if you place a bet of $14 and Argentina win? [2 marks] (note: winnings means the profit)

$$\text{odds against} = \frac{5}{7} \qquad \frac{5}{7} = \frac{x}{14}$$

$x = 10$

∴ winnings of $10.

4) When drawing 5 cards from a standard deck of 52 cards, what is the probability of having exactly 2 even cards? [2 marks]

$$P(2\text{ even}) = \frac{\binom{20}{2}\binom{32}{3}}{\binom{52}{5}} \doteq 0.363$$

5) The word COUNTED has been spelled using Scrabble tiles. The two tiles are randomly chosen one at a time and placed in the order in which they were chosen. What is the probability that the tiles are both consonants? [2 marks]

$$P(\text{consonants}) = \frac{4P_2}{7P_2} = \frac{2}{7} \doteq 0.286$$

6) A 10-sided polygon has all of its edges (outer sides that form the shape) and diagonals drawn. What is the probability that a randomly chosen line is an edge? [2 marks]

$$P(\text{edge}) = \frac{10}{\binom{10}{2}} = \frac{2}{9} \doteq 0.222$$

7) The odds in favour of Brazil winning the World Cup this year is 1:6, while the odds against France winning the World Cup this year are 5:7. What are the odds in favour of Brazil OR France winning the World Cup this year? [3 marks]

$$P(\text{Brazil}) = \frac{1}{7} \qquad P(\text{France}) = \frac{7}{12}$$

$$P(\text{Brazil or France}) = \frac{1}{7} + \frac{7}{12} = \frac{61}{84}$$

∴ odds = 61 : 23

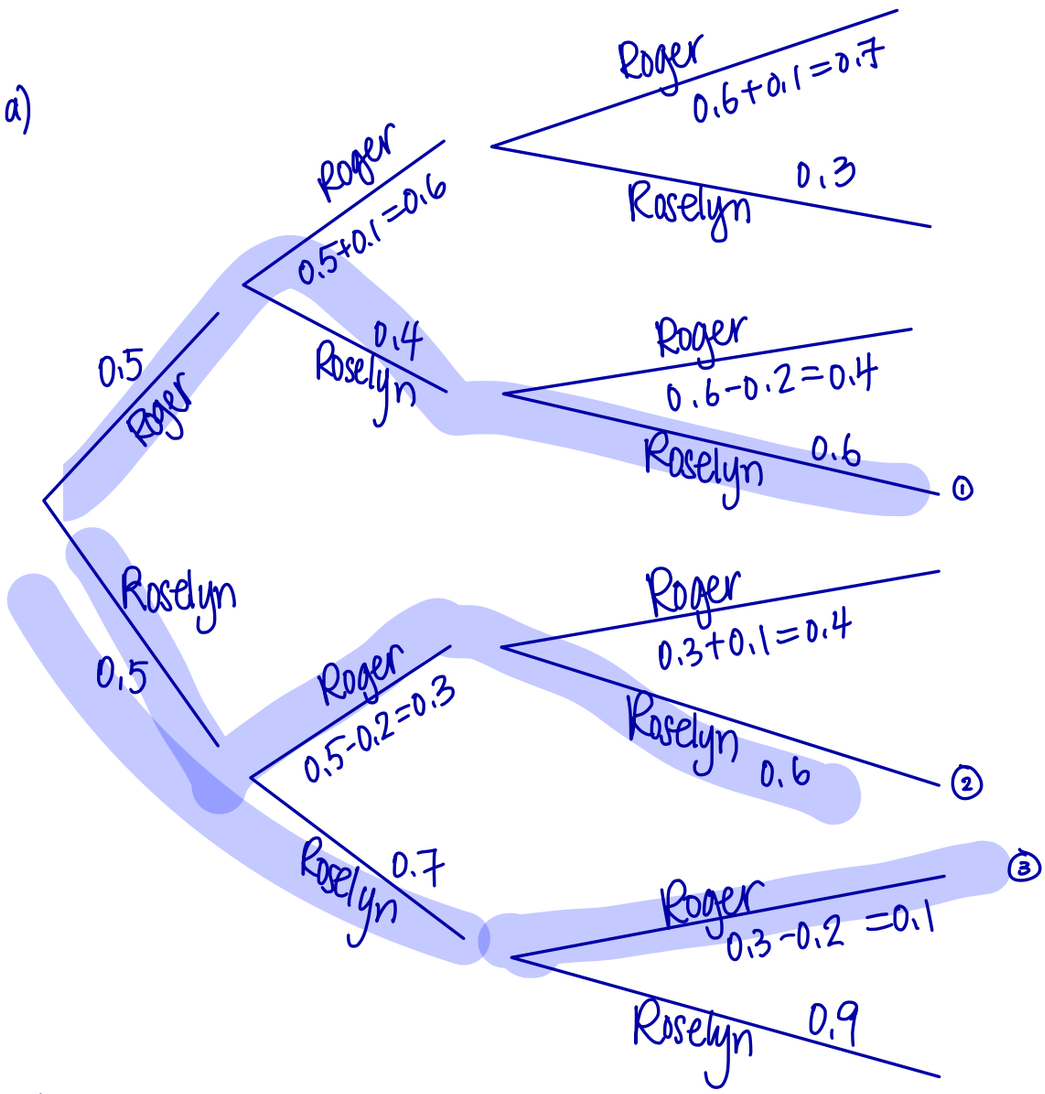
Thinking

1) Roger and Roselyn are evenly matched tennis players. However, each time Roger loses a game, his probability of winning the next game is decreased by 0.2 from the predicted winning probability of the previous game. But when he wins, his probability of winning the next game increases by 0.1 from the predicted winning probability of the previous game.

   a) Make a tree diagram for a three-game sequence and mark the probabilities on the branches of the diagram. [2 marks]

   b) Find the probability that Roselyn wins exactly two games. [3 marks]

a)



b) Case 1: P(Roger, Roselyn, Roselyn) = 0.5 × 0.4 × 0.6
= 0.12

Case 2: P(Roselyn, Roger, Roselyn) = 0.5 × 0.3 × 0.6
= 0.09

Case 3: P(Roselyn, Roselyn, Roger) = 0.5 × 0.7 × 0.1
= 0.035

∴ P(case 1 ∪ case 2 ∪ case 3)
= 0.12 + 0.09 + 0.035
= $\frac{49}{200}$ or = 0.245

Communication 2 marks will be awarded for proper mathematical forms throughout the assessments on both days.

--- End of Part 1 Assessment ---

Bayview Secondary School
Mathematics Department: Course Code: MDM4U1-01
Unit 4 – Assessment OF Probability Theory PART 2 – Version 2

Name: _____ Alpha # ___

A-18	T-3

Instructions:

1) Scientific calculators are allowed but cannot be shared.
2) Duration of the assessment: 40 minutes
3) Show all work to obtain full marks for questions.
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Application

1) There are 5 females and 2 males attending a conference. What is the probability that two of the females will arrive first? [3 marks]

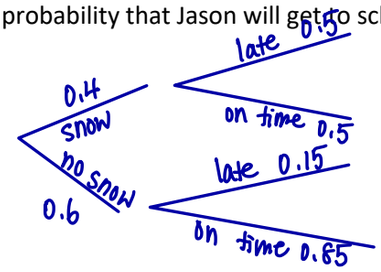
$$P(\text{2 females 1st}) = \frac{\binom{5}{2}2!5!}{7!}$$
$$= \frac{10}{21} \doteq 0.476$$

2) Given the set of integers $\{1, 2, 3, 4, 5, \ldots, 38, 39, 40\}$, what is the probability that a randomly drawn integer is a prime number? [2 marks]

$\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37\}$

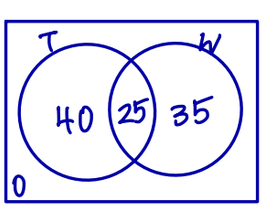
$$P(\text{prime}) = \frac{12}{40}$$
$$= \frac{3}{10}$$
$$= 0.3$$

3) The probability of Jason being late on a snowy day is 50%, and the probability of being late on a day that is not snowing is 15%. The weather forecast gives a 40% probability of snowing tomorrow. What is the probability that Jason will get to school on time? [3 marks]



$P(\text{snow} \cap \text{on time}) + P(\text{not snow} \cap \text{on time})$
$= 0.4(0.5) + 0.6(0.85)$
$= \frac{71}{100} = 0.71.$

4) Of 100 grade 12 students who applied to University of Toronto or University of Waterloo, 65 applied to University of Toronto. A quarter of the students applied to both universities. What is the probability that a student is selected randomly applied to only University of Waterloo? [2 marks]



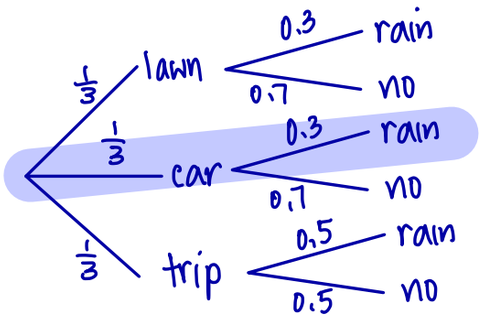
$$P(\text{Waterloo}) = \frac{100 - 65}{100}$$
$$= \frac{35}{100} = 0.35$$

5) Five playlists in Spotify are set on shuffle and randomly select songs to play from. What is the probability that during the first 5 songs played there will be three songs from paylist 1 or three songs from playlist 2? [3 marks]

Playlist	1	2	3	4	5
Number of songs	3	5	1	2	6

$$P(\text{playlist 1}) + P(\text{playlist 2})$$
$$= \frac{\binom{3}{3}\binom{14}{2} + \binom{5}{3}\binom{12}{2}}{\binom{17}{5}} = \frac{751}{6188} \doteq 0.121$$

6) Jessie estimates that there is a 30% chance of rain the next day if she wants to mow the lawn, a 30% chance if she washes the car, and 50% chance if she plans a trip to pick strawberries in the farm. Assuming that Jessie's estimates are accurate and these are the only three things she will equally likely do tomorrow, what are the odds against of rain tomorrow if she washes the car? [2 marks]



$1 - P(\text{car and rain})$

$= 1 - P(\text{car}) \times P(\text{rain}|\text{car})$

$= 1 - \frac{1}{3}(0.3)$

$= 1 - 0.1$

$= 0.9$

$\therefore$ odds: 9:1

7) A standard deck of 52 cards is shuffled and three cards are drawn. What is the probability that all three cards are even cards or all three cards are diamonds? [3 marks]

$P(\text{even}) + P(\text{diamond}) - P(\text{even} \cap \text{diamond})$

$= \frac{{}^{20}C_3 + {}^{13}C_3 - {}^{5}C_3}{\binom{52}{3}}$

$= \frac{354}{5525}$ or $\doteq 0.064$

Thinking

1) A distinct 4-letter word is formed using the letters available from the word TIGERMA with no repetition allowed. What is the probability that the T is used OR all vowels are used in a 4-letter word? [3 marks]

T I
G E
R A
M

$P(T) + P(\text{vowels}) + P(T \cap \text{vowels})$

$= \frac{1 \times 6 \times 5 \times 4 \times 4}{{}^{7}P_4} + \frac{3 \times 2 \times 1 \times 4 \times 4}{{}^{7}P_4} - \frac{3 \times 2 \times 1 \times 1 \times 4}{{}^{7}P_4}$

$= \frac{23}{35}$

$\doteq 0.657$