UNIT 1 ASSESSMENT OF LEARNING: LIMITS AND RATES OF CHANGE – DAY 2

Name: ________**Solutions**____________

Instructions:
- You MUST use concepts covered in this unit/course. Derivative or Instantaneous Rates of Change calculations MUST be done using **First Principles**. Show all steps for full marks.
- Non-graphing calculators may be used but not shared. Notebooks may not be used.
- The use of cellphones, audio- or video-recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

App	Comm.
/20	/5

Application - [20 marks]

1. Given the function $f(x)=\begin{cases} x^2-bx-2a & , x\in(-\infty,-1) \\ -5 & , x=-1 \\ \frac{a}{x}+b+3 & , x\in(-1,\infty) \end{cases}$, determine the values of a and b such that $f(x)$ is continuous. ❹

$$\lim_{x\to -1^-} f(x)=\lim_{x\to -1^+} f(x)=f(-1)$$
$$1+b-2a=-a+b+3=-5$$
$$-2a+b=-6 \quad (1)$$
$$a-b=8 \quad (2)$$
$$\oplus \; -a=2$$
$$\boxed{a=-2} \xrightarrow{\text{sub. into } (2)} -2-b=8$$
$$\boxed{b=-10}$$

2. Determine the equation (in standard form) of the tangent of the function, $f(x)=\sqrt{6-x}$ at x=2. ❹

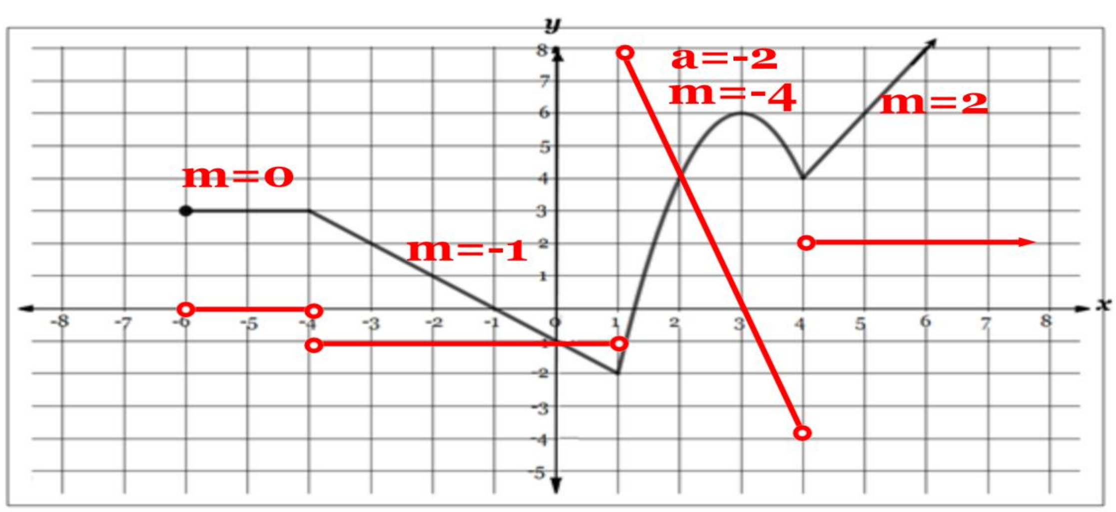
$$m_T=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$
$$m_T=\lim_{h\to 0}\frac{\sqrt{6-(2+h)}-2}{h}\times\frac{\sqrt{4-h}+2}{\sqrt{4-h}+2}$$
$$m_T=\lim_{h\to 0}\frac{4-h-4}{h\left(\sqrt{4-h}+2\right)}$$
$$m_T=\lim_{h\to 0}\frac{-\cancel{h}}{\cancel{h}\left(\sqrt{4-h}+2\right)}$$
$$m_T=\frac{-1}{4}$$

$\therefore$ **Equation of tangency line at $(2,2)$ is:**

$$y-2=\frac{-1}{4}(x-2)$$

or $\boxed{x+4y-10=0}$

3. Graph the derivative function for the function $f(x)$ given below on the same grid. ❹



4. A dead branch breaks off a tree located at the top of an 80 m cliff. After time *t*, in seconds, it has a fallen distance, *d*, in metres, where $d(t)=80-5t^2$,$0\le t\le 4$.

a) Determine the average rate of change of the distance the branch falls in the first 3 seconds.❷

$$\begin{aligned}\mathbf{AROC} &= \frac{d(3)-d(0)}{3-0}\\ &= \frac{35-80}{3}\\ &= -15\ \mathrm{m/sec}\end{aligned}$$

b) At **what time** is the instantaneous rate of change in distance -25m/sec?❸

$$\mathbf{IROC} = -25\,\mathrm{m/sec}$$

$$-25=\lim_{h\to 0}\frac{\left(80-5(t+h)\right)^2-80+5t^2}{h}$$

$$-25=\lim_{h\to 0}\frac{-5\left(t^2+2th+h^2\right)+5t^2}{h}$$

$$-25=\lim_{h\to 0}\frac{\cancel{-5t^2}-10th-5h^2+\cancel{5t^2}}{h}$$

$$-25=\lim_{h\to 0}\frac{\cancel{h}(-10t-5h)}{\cancel{h}}$$

$$-25=-10t$$

$$\boxed{t=2.5}\ \mathrm{sec}$$

5. If $\lim_{x\to 2} f(x)=4$ and $\lim_{x\to 2} g(x)=8$ use all applicable **properties of limits** to evaluate the following limit: $\lim_{x\to 2}\frac{[g(x)]^2-[f(x)]^2}{\sqrt[3]{2f(x)}}$. ❸

$$=\frac{\lim_{x\to 2}\left\{[g(x)]^2-[f(x)]^2\right\}}{\lim_{x\to 2}\sqrt[3]{2f(x)}}$$

$$=\frac{\left[\lim_{x\to 2}g(x)\right]^2-\left[\lim_{x\to 2}f(x)\right]^2}{\sqrt[3]{2\lim_{x\to 2}f(x)}}$$

$$=\frac{8^2-4^2}{\sqrt[3]{2(4)}}$$

$$=\frac{48}{2}$$

$$=24$$

Communication - [5 marks]

1. a) Provide a sketch of a function that is **not differentiable**, for having a **cusp** at x =-2 and a **vertical tangent** x = 3.

b) State whether or not your function is continuous at these points. Justify your answer ❸

Function is not differentiable at x=-2, since at this point function has a cusp

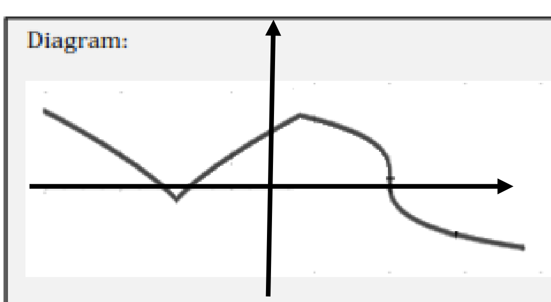
$\left(f'(-2^-)\to -\infty \text{ and } f'(-2^+)\to +\infty\right)$.

At x=3 there is a vertical tangent

$f'(3^-)\to -\infty$ and $\left(f'(3^+)\to -\infty\right)$.

Function is continues at these points

Diagram:



*** **2 marks** will be awarded in the Communication Category for proper mathematical form. ***