Bayview Secondary School Name Solutions Alpha #________
Mathematics Department – Course Code: MDM4U1
Unit 5 Probability Distribution Assessment OF Learning (Version 2)

KU/15	A /8

Instruction:

- Round all answers to **4 decimal places** unless otherwise indicated.
- **Show all necessary steps and work in a logical sequence to demonstrate the chain of thought to obtain full marks.**

KNOWLEDGE/UNDERSTANDING

Multiple Choice. Select the most appropriate answer and place it in the box provided on the right side of each question. [2 marks – 1 mark each]

1) Which of the following statement is true for uniform distribution?

E) Outcomes near the middle of the distribution are more likely than outcomes far from the middle
F) The left side of the distribution is not symmetric with the right side
G) All outcomes are equally likely
H) The probability of an outcome is read by determining the value of the function at a single point on the graph

G

2) What percent of values of a normally distributed variable lie between the mean and one standard deviations above the mean?

E) 95%
F) 68%
G) 47.5%
H) 34%

$68 \div 2 = 34$

H

3) A lottery has a $1,000,000 first prize, two $40,000 second prize, three $1,000 third prizes, and five $500 fourth prizes. A total of 3,500,000 tickets are sold. How much should the lottery operator set for ticket price to make this a fair game? [2 marks]

$$E(X) = \frac{1000000(1) + 40\,000(2) + 1000(3) + 500(5)}{3\,500\,000}$$

$$\doteq 0.3101$$

∴ the ticket price should be $0.31.

4) There are 7 males and 5 females participated in a lucky draw. Four winners are chosen at random. What is the probability that exactly 3 males will win? [3 marks]

$$P(3\text{ males}) = \frac{\binom{7}{3}\binom{5}{1}}{\binom{12}{4}}$$

$$= \frac{35}{99}$$

$$\doteq 0.3535$$

5) Supposed that 5% of the first batch of engines off a new production line have flaws. An inspector randomly selects 4 engines for testing. Determine the probability distribution for the number of flawed engines in the sample. [3 marks]

X = flawed engines	P(X)
0	$\binom{4}{0}(0.05)^0(0.95)^4 \doteq 0.8145$
1	$\binom{4}{1}(0.05)^1(0.95)^3 \doteq 0.1715$
2	$\binom{4}{2}(0.05)^2(0.95)^2 \doteq 0.0135$
3	$\binom{4}{3}(0.05)^3(0.95)^1 \doteq 0.0005$
4	$\binom{4}{4}(0.05)^4(0.95)^0 = \frac{1}{160\,000} = 0.00000625 \doteq 0$

6) A food bank collects food donations and distribute them by weights. The distribution of the donation to receivers is normally distributed with a mean of 3 kg and a standard deviation of 0.96 kg. What is the probability that a receiver will receive greater than 4 kg of food donation? [3 marks]

$$P(X>4)$$
$$=P\left(Z>\frac{4-3}{0.96}\right)$$
$$=P(Z>1.04)$$
$$=1-0.8508$$
$$=0.1492$$

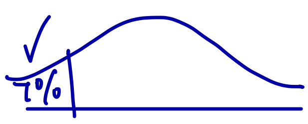
7) Earlier this year, 500 raccoons were caught and tagged. On a recent survey, 30 out of 125 raccoons had been tagged. Estimate the size of the raccoon population. [2 marks]

$$E(X)=\frac{ra}{n}$$
$$30=\frac{500(125)}{n}$$
$$n=2083.3333$$

$\therefore$ the size of the raccoon population is approximately 2083.

Application

1) The coach of a swim team can send only the top 7% of his swimmers to a regional swim meet. For the members of his team, times for a 50-m front stroke are normally distributed with a mean of 45 seconds and a standard deviation of 3 seconds. What is the cut-off time to determine which members of the team qualify for the regional meet? [4 marks]



$$P(Z<Z_1)=0.0700$$
$$Z_1=-\left(\frac{1.47+1.48}{2}\right)$$
$$Z_1\doteq-1.475$$

$$-1.475=\frac{x-45}{3}$$
$$x=40.575$$

$\therefore$ the cut off time should be 40.575 seconds.

2) A factory produces labels for prescription bottles in the pharmacy industry. 48% customers order the medium size label. Suppose 3000 orders have been received, what is the probability the factory will receive between 1400 and 1500 orders for medium size labels? [4 marks]

$$n=3000$$
$$\mu=np=3000(0.48)$$
$$=1440>5$$
$$nq=3000(0.52)$$
$$=1560>5$$
$$\sigma=\sqrt{npq}$$
$$=\sqrt{3000(0.48)(0.52)}$$
$$\doteq 27.3642102\ldots$$

$\therefore$ Normal Approximation can be used.

$P(1400<X<1500)$ apply continuity correction

$$=P(1400.5<X<1499.5)$$
$$=P\left(\frac{1400.5-1440}{\sqrt{3000(0.48)(0.52)}}<Z<\frac{1499.5-1440}{\sqrt{3000(0.48)(0.52)}}\right)$$
$$=P(-1.44<Z<2.17)$$
$$=0.9850-0.0749$$
$$\doteq 0.9101$$

----- End of Assessment ------