MHF 4U1 – am Name Yusuf Ali Jaffer Date _____

Logarithmic & Exponential Functions

Unit 6 & 7 TEST

Total: 33.5/35

1. Express as a sum or difference of logarithms, and then simplify. No exponents. State restrictions.

$\log \sqrt[5]{\frac{x^3y^2}{z^5}}$

(3)

$\log\left(\frac{x^3y^{2/5}}{z^5}\right)^{1/5}$

$\log\left(\frac{x^{3/5}y^{2/5}}{z}\right)$

$\log x^{3/5} + \log y^{2/5} - \log z$

$\frac{3}{5}\log x + \frac{2}{5}\log y - \log z$

$z > 0$
$y > 0$
$x > 0$

GREAT JOB!

2. Solve for x. Check for any extraneous roots. Where necessary, round to 2 decimal places.

a) $\log_7(x+2) + \log_7(x-4) = 1$

(2, 3)

$\log_7((x+2)(x-4)) = 1$

$(x+2)(x-4) = 7^1$

$x^2 - 4x + 2x - 8 = 7$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = +5, -3$

Extraneous.

b) $3^x + 1 - 56(3^{-x}) = 0$

$3^x + 1 - 56\left(\frac{1}{3^x}\right) = 0$

$3^x + 1 - \frac{56}{3^x} = 0$

Let $3^x = L$

$L + 1 - \frac{56}{L} = 0$

$L^2 + L - 56 = 0$

$(L-7)(L+8) = 0$

$3^x = 7$
$x = \log_3 7$
$x \approx 1.77$

$3^x = -8$
$x = \log_3(-8)$ ≠ ≥0
No Sol.

c) $\log_x 81 = \frac{4}{3}$

(2, 2, 2)

$\log_{81} x = 3/4$

$x = 81^{3/4}$

$x = (3^4)^{3/4}$

$x = 27$

Rule of log'. (Proven on back-page)

d) $\log\sqrt{x} = \log 1 - 2\log 3$

$0 = \log 1 - \log 9 - \log x^{1/2}$

$0 = \log\left(\frac{1}{9x^{1/2}}\right)$

$10^0 = \frac{1}{9x^{1/2}}$

$1 = \frac{1}{9x^{1/2}}$

$9x^{1/2} = 1$

$x^{1/2} = 1/9$

$x = \frac{1}{81}$ OR $x = 0.01$ (D form)

(F. form)

e) $\log_3(\log_2 x) = 1$

$\log_3\left(\frac{\log x}{\log 2}\right) = 1$

$\frac{\log x}{\log 2} = 3$

$\log x = 3\log 2$

$\log x = \log 8$

$10^{\log x} = 10^{\log 8}$

$x = 8$

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3. Solve $5^{3x+1} = 3^{2x-3}$. Provide an **exact** solution as a single logarithm.

(4)

$\log(5^{3x+1}) = \log(3^{2x-3})$

$3x\log 5 + \log 5 = 2x\log 3 - 3\log 3$

$\log 5 + 3\log 5 = 2x\log 3 - 3x\log 5$

$\log 5 + \log 27 = x(2\log 3 - 3\log 5)$

$\frac{\log 135}{\log(3/125)} = x$

$\log_{3/125} 135 = x$

4. Given that $\log_{x+1} y = 2$ and $9^{y-11} = 3^{10x}$ find x and y.

(3)

$\log_{x+1} y = 2$

$y = (x+1)^2$

$9^{y-11} = 3^{10x}$

$3^{2(y-11)} = 3^{10x}$

$2(y-11) = 10x$

$2y - 22 = 50$

$2y = 72$

$y = 36$

$2(y+1) = 10x$

$2((x+1)^2 - 11) = 10x$

$x^2 + 2x + 1 - 11 = 5x$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5, -2$

Extraneous, leads to - base

Thus no y comes from it.

5. The number of watts, w, provided by a space satellite's power supply after d days is given by the formula $w = 50(1.75)^{-0.004d}$.

a) How much power will be available after 2 years?

(1)

$2y = 356 \cdot 2$

$= 712$ in days

365 days.

$w = 50(1.75)^{-0.004 \cdot 712}$

$w = 10.16$

10.16 watts will be available

b) The satellite needs to be replaced if the number of watts falls below 10. How long is the life span of this satellite?

(2)

$10 = 50(1.75)^{-0.004d}$

$\frac{1}{5} = (1.75)^{-0.004d}$

$-0.004d = \log_{1.75}(\frac{1}{5})$

$d = 718.99$

The life span of the satellite is around 719 days

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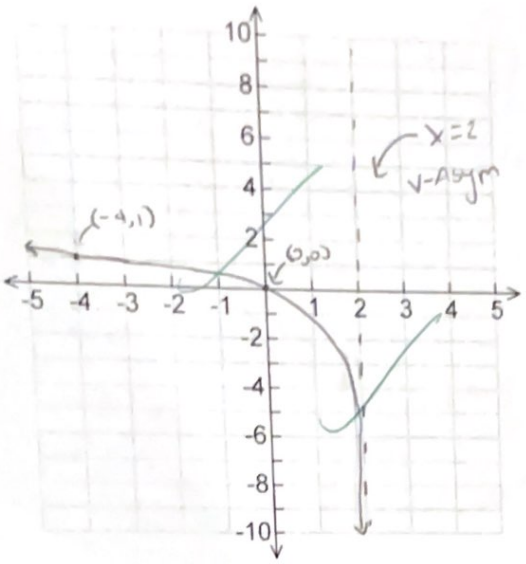
6. Graph $f(x) = log_3(-\frac{1}{2}x + 1)$ using the mapping rule. Include at least 3 points and label asymptotes. Complete the table below.

$h = -\frac{1}{2}$

-2

$\log_3(-\frac{1}{2}(x-2))$



map: $[-2x+2, y]$

Parent: $\log_3$ or $x = 3^y$

Reflection in y-axis
Horizontal Translation by 2 right
Horizontal Stretch by 2
Vertical asymptote at $x=2$

x	y	
1	0	→ [0, 0]
3	1	→ [-4, 1]
9	2	→ [-16, 2]

4.5 (5)

	Equation of Asymptote	Domain	Range
f(x)	$x=2$	$(x \in \mathbb{R} \mid x > 2)$	$(y \in \mathbb{R})$

0,8

7. Consider the expression $log_3 a$. (3)

a) For what values of a will this expression yield positive numbers?

$(a \in \mathbb{R} \mid a > 1)$

b) For what values of a will this expression yield negative numbers?

$(a \in \mathbb{R} \mid 1 > a > 0)$

$0 < a < 1$ ok, be careful about notation.

c) For what values of a will this expression be undefined?

$(a \in \mathbb{R} \mid a \leq 0)$

7.5

8. Show if $log_b a = c$ and $log_y b = c$ then, $log_a y = c^{-2}$

$\text{Log}_b a = c$

$\text{Log}_B a = c$

$\text{Log}_B a^k = 1$

$a^k = B$

$\text{Log}_y b = c$

$\text{Log}_B y = \frac{1}{c}$

$\text{Log}_B y^c = 1$

$y^c = B$

$a^k = y^c$

$\text{Log} a^k = \text{Log} y^c$

$\frac{1}{c}\text{Log} a = c\text{Log} y$

$\frac{\text{Log} a}{\text{Log} y} = c^2$

$\text{Log}_y a = c^2$

$\text{Log}_a y = \frac{1}{c^2}$ ✓

$\text{Log}_a y = c^{-2}$

$\text{Log}_y B = c$

$\frac{\text{Log} B}{\text{Log} y} = c$

$\frac{\text{Log} y}{\text{Log} B} = \frac{1}{c}$

$\text{Log}_B y = \frac{1}{c}$

(3)

Proof that this is a rule of log!

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