MHF4U1-V1
Leung/ Moshtagh/ Vincent

Name: Shakeel Nasarati Period: 2

Assessment of Learning: Unit 3 – Trigonometric Functions Part I –DAY 2

Application	Thinking	Communication
8 /17	2 /5	2 /2

Instructions:

- Non-graphing calculators may be used but not shared. Notebooks may not be used.
- Only methods taught in MHF4U1 will be accepted. Show all work in the space provided.
- The use of cellphones, audio- or video-recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

Application - [17 Marks]

1. Prove $(\sec(\theta)-\cos(\theta))(\csc(\theta)-\sin(\theta))=\frac{\tan(\theta)}{1+\tan^2(\theta)}$. **[5 Marks]**

$L.S = \left(\frac{1}{\cos(\theta)}-\cos(\theta)\right)\left(\frac{1}{\sin\theta}-\sin(\theta)\right)$

$R.S = \frac{\tan(\theta)}{1+\tan^2\theta}$

$= \frac{1}{\cos\theta\sin\theta} - \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta} + \cos\theta\sin\theta$

$= \frac{1-\sin^2\theta-\cos^2\theta+\cos^2\theta\sin^2\theta}{\cos\theta\sin\theta}$

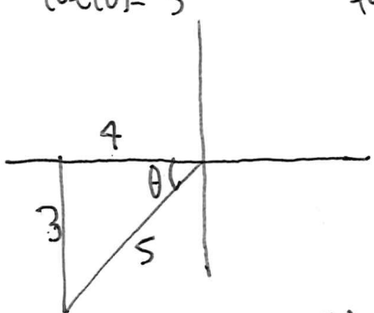
$= \frac{\sin^2\theta+\cos^2\theta-\sin^2\theta-\cos^2\theta+\cos^2\theta\sin^2\theta}{\cos\theta\sin\theta}$

$= \frac{\cos^2\theta\sin^2\theta}{\cos\theta\sin\theta} = \cos(\theta)\sin(\theta)$

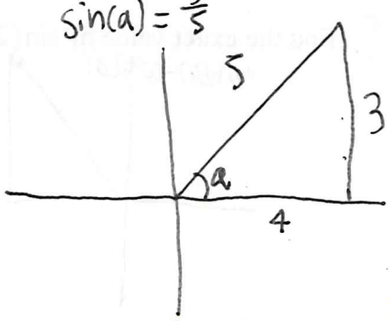
2. If $\cot(\theta)=\frac{4}{3}$, $\pi<\theta<\frac{3\pi}{2}$ and $\sin(\alpha)=\frac{3}{5}$, $0<\alpha<\frac{\pi}{2}$, determine the **exact** value of $\tan(2\theta-\alpha)$. **[5 Marks]**

A B

$\cot(\theta)=\frac{4}{3}$ $\tan\frac{3}{4}$



$\sin(a)=\frac{3}{5}$



$a^2+b^2=c^2$
$4^2+3^2=25$ $c=5$

$a^2+b^2=c^2$
$a^2=c^2-b^2$
$a^2=5^2-3^2$
$a^2=16$ $a=4$

$\tan(A-B)=\frac{\tan(a)-\tan(B)}{1+\tan(a)\tan(B)}$

$\tan(2\theta-a)=\frac{\tan(2\theta)-\tan(a)}{1+\tan(2\theta)\tan(a)}$

$= \frac{\frac{2\tan x}{1-\tan^2\pi}-}{}$

$\tan(2\theta)=\frac{2\tan(\theta)}{1-\tan^2(\theta)}$

$=\frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2}$

$=\frac{24}{7}$

3. If $\sin(\alpha)+\cos(\alpha)=\frac{6}{5}$, find the **exact** value of $\sin^3(\alpha)+\cos^3(\alpha)$. **[4 Marks]**

Isn't this like polynomial sum of cube or whatever...

$(A+b)(a^2+2ab+b^2)$

She is A

Cos is B?

$\sin(a)=\frac{6}{5}-\cos(a)$

$\cos(a)=\frac{6}{5}-\sin(a)$

$\sin^3(a)+\cos^3(a)$

$=(\sin(a)+\cos(a))(\sin^2(a)+2\sin(a)\cos(a)+\cos^2(a))$

$(\frac{6}{5})($

$(A+b)(a^2+2ab+b^2)$

4. Determine the **exact simplified** value of $\cos\left(\frac{3\pi}{8}\right)$. **[3 Marks]**

$\cos2(x)=2\cos^2(x)-1$

$\cos2(\frac{3\pi}{8})=2\cos^2(\frac{3\pi}{8})-1$

$\cos(\frac{3\pi}{4})=2\cos^2(\frac{3\pi}{8})-1$

$\frac{-\sqrt{2}+2}{2}=2\cos^2(\frac{3\pi}{8})$

$\cos^2(\frac{3\pi}{8})=\frac{-\sqrt{2}-2}{4}$

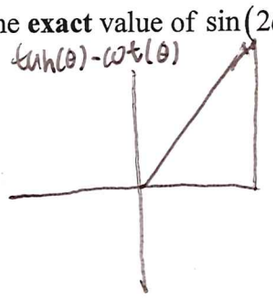
$\cos(\frac{3\pi}{8})=\pm\sqrt{\frac{-\sqrt{2}+2}{4}}$

$\cos(\frac{3\pi}{8})=\sqrt{0.1464}=0.3826$

∴ the exact simplified value of $\frac{3\pi}{8}$ is 0.38 which is also $\sqrt{\frac{-\sqrt{2}+2}{4}}$

Thinking – [5 Marks]

1. If $\tan(\theta)-\cot(\theta)=\frac{\sqrt{17}}{4}$, $0<\theta<\frac{\pi}{2}$, find the **exact** value of $\sin(2\theta)$.



*** 2 marks are awarded in the Communication Category for use of correct mathematical form. ***

MHF4U1-V1
Leung/ Moshtagh/ Vincent

Name: Shakeel Nasarali Period: 2

Assessment of Learning: Unit 3 – Trigonometric Functions Part I –DAY 1

Knowledge & Understanding	Thinking	Communication
14½ /17	1 /5	1½ /2

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Knowledge and Understanding – [17 Marks]

Multiple Choice: Write the **CAPITAL LETTER** corresponding to the correct answer on the line provided.
[1 Mark Each – 6 Marks Total]

1. In which quadrant is $\csc(\theta) > 0$ and $\sec(\theta) < 0$? sin cos B ✓

   A. 1 B. 2 C. 3 D. 4

2. Determine the approximate degree measure for an angle of 1.5 radians. C ✓

   1.5 × 180/π

   A. 136.4° B. 75.6° C. 85.9° D. 2.4°

3. If the central angle is $\frac{4\pi}{3}$ radians, what should the radius of a circle be to make the arc length 1m? B ✓

   A. 0.424 m B. 0.238 m C. 2.356 m D. 4.188 m

4. Which expression is equivalent to $\cos^2(2x) + \cot^2(2x) + \sin^2(2x)$? D ✓

   A. $2\csc^2(2x)$ B. $\cot^2(2x)$ C. $\tan^2(2x)$ D. $\csc^2(2x)$

5. A simplified expression for $\frac{2\cot\left(\frac{\pi}{2} - k\right)}{1 - \tan^2(k)}$ is: $\frac{2\tan k}{1-\tan^2 k}$ B ✓

   A. $\tan\left(\frac{\pi}{2} + k\right)$ B. $\tan(2k)$ C. $\cot(k)$ D. $-\tan(2k)$

6. Which expression is **NOT** equal to $1 + \sin(2t)$? A ✓

   A. $\sin^2(t) + \sin(2t)$
   B. $\sin^2(t) + 2\sin(t)\cos(t) + \cos^2(t)$ ✗
   C. $1 + 2\sin(t)\cos(t)$ ✗
   D. $(\sin(t) + \cos(t))^2$

$\cos^2 k + \cot^2 k + \sin^2 k$

b

7. Completely simplify the following expression. Show all steps. **[5 Marks]**

$$\frac{\sin(\pi+x)\cos\left(\frac{\pi}{2}+x\right)-\sin\left(\frac{3\pi}{2}-x\right)\cos(-x)}{\sec\left(\frac{\pi}{2}+x\right)\csc\left(\frac{3\pi}{2}-x\right)\cos(\pi-x)}.$$

$$\frac{-\sin x(-\sin x)-(-\cos x)\cos x}{\csc x(-\sec x)(-\cos x)}$$

$$= \frac{\sin^2 x+\cos^2 x}{\frac{1}{\sin x}\cdot\frac{1}{\cos x}\cdot\frac{\cos x}{1}}$$

$$= \frac{1}{\frac{1}{\sin x}}$$

$$= \sin x$$

8. Determine the **exact** simplified value of the following. Show all steps. **[6 Marks]**

a. $\sin\left(\frac{7\pi}{15}\right)\cos\left(\frac{6\pi}{5}\right)+\cos\left(\frac{7\pi}{15}\right)\sin\left(\frac{6\pi}{5}\right)$ **[3]**

$$=\sin\left(\frac{7\pi}{15}+\frac{6\pi}{5}\right)$$

$$=\sin\left(\frac{7\pi}{15}+\frac{18\pi}{15}\right)$$

$$=\sin\left(\frac{25\pi}{15}\right)$$

$$=\sin\left(\frac{4\pi}{3}+\frac{\pi}{3}\right)$$

$\frac{25\pi}{15}=300^\circ$ $\quad 240^\circ+60^\circ=300^\circ$

$\frac{4\pi}{3}+\frac{\pi}{3}=\frac{25\pi}{15}$

$\sin(A+B)=\sin A\cos B+\cos A\sin B$

$$=\sin\left(\frac{4\pi}{3}\right)\cos\left(\frac{\pi}{3}\right)+\cos\left(\frac{4\pi}{3}\right)\sin\left(\frac{\pi}{3}\right)$$

$$=\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)+\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$=-\frac{\sqrt{3}}{4}+\frac{-\sqrt{3}}{4}=\frac{-2\sqrt{3}}{4}=-\frac{\sqrt{3}}{2}$$

∴ the exact simplified value is $-\frac{\sqrt{3}}{2}$

b. $\cos\left(\frac{7\pi}{12}\right)$ **[3]**

Answer on scrap paper

$\cos(60+45)$

$\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

Thinking – [5 Marks]

1. Prove $\frac{1+\cos(4x)}{\sin(4x)}=\frac{\cot(x)-\tan(x)}{2}$. **[5 Marks]**

1

1½

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