Name: _Solutions_

Unit 6 – Combining Functions – Assessment of Learning

Knowledge & Understanding	Application	Comm.	Bonus
/15	/14	/2	

Instructions:

- Non-graphing calculators may be used but not shared. Notebooks may not be used.
- Only methods taught in MHF4U1 will be accepted. Show all work in the space provided.
- The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.
- **Please complete the assessment independently with <u>academic honesty as the guiding principle</u>.**

KNOWLEDGE AND UNDERSTANDING – [15 Marks]

Multiple Choice: Write the **CAPITAL LETTER** corresponding to the correct answer on the line provided.
[1 Mark Each – 5 Marks Total]

1. If $f(x) = x - 4$ and $g(x) = \dfrac{1}{2x+3}$, then $f(0) + g\left(\dfrac{3}{2}\right)$ equals _____D_____

   A. $\dfrac{3}{2}$ 　　　B. $-\dfrac{25}{6}$ 　　　C. -4 　　　D. $-\dfrac{23}{6}$

2. Given $f(x) = -2x + 1$, $0 \le x \le 7$ and $h(x) = x^3 - 2x + 5$, $-3 \le x < 6$, then the domain of _____B_____

   $(f + h)(x)$ is

   A. $0 \le x \le 7$ 　　B. $0 \le x < 6$ 　　C. $-3 \le x < 6$ 　　D. cannot be determined

3. Which of the following represents $(g \circ f)(x)$ if $f(x) = -2x$ and $g(x) = \sin(x)$? _____D_____

   A. $-\sin(2x)$ 　　B. $1 - 2\cos^2(x)$ 　　C. $-2\sin(x)\cos(x)$ 　　D. both A and C

4. If $f(x) = \dfrac{-3x^3 - 5x + 16}{\sqrt{x-4}}$, then $(f \circ f^{-1})(5)$ equals _____A_____

   A. 5 　　　B. -5 　　　C. 188 　　　D. cannot be determined

5. Given the functions $f(x) = \sin(x)$ and $g(x) = 3^x$, then the value of $g\left(f\left(\dfrac{3\pi}{2}\right)\right)$ is _____B_____

   A. 3 　　　B. $\dfrac{1}{3}$ 　　　C. 1 　　　D. 9

6. Given the following functions, determine the following in simplest **exact** form. [8 Marks]

$f(x) = \{(1, -2), (\sqrt{2}, 3), (3, 0), (\pi, 1), (14, \sqrt{7})\}$ $g(x) = \{(1, \pi), (\sqrt{2}, \sqrt{2}), (3, 1), (\pi, 3)\}$

$h(x) = x^2 + 5x - 6$ $k(x) = 3\sin\left[2\left(x + \frac{\pi}{2}\right)\right] - 1$

a. $(f \circ g)(x)$ [2]

b. $(h \circ k)(0)$ [2]

$= \{(1,1), (\sqrt{2}, 3), (3, -2), (\pi, 0)\}$

$= h(k(0))$
$= h(-1)$
$= -10$

c. $\left(\dfrac{g}{f}\right)(x)$ [2]

d. $(f^{-1} + g)(x)$ [2]

$= \{(1, -\frac{\pi}{2}), (\sqrt{2}, \frac{\sqrt{2}}{3}), (\pi, 3)\}$

$f^{-1} = \{(-2, 1), (3, \sqrt{2}), (0, 3), (1, \pi)(\sqrt{7}, 14)\}$

$f^{-1} + g = \{(3, \sqrt{2} + 1), (1, 2\pi)\}$

7. If $f = \{(-1, 4), (0, 2), (1, 0), (2, -2), (3, -8)\}$, determine the function g if
   $f - g = \{(-1, 1), (0, 4), (1, 6), (3, 3)\}$. [2 Marks]

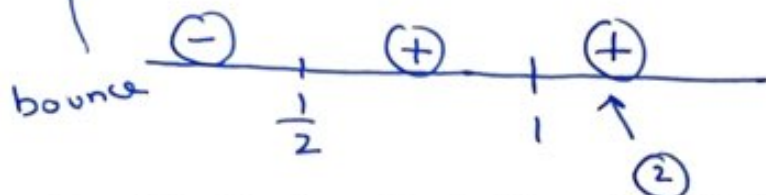$g = \{(-1, 3), (0, -2), (1, -6), (3, -11)\}$

Application – [14 Marks]

1. Solve: $2x^3 + 11x - 6 > 5x^2 + 7x - 5$. **[4 Marks]**

$$2x^3 - 5x^2 + 4x - 1 > 0$$

$$(x-1)^2 (2x-1) > 0$$

$$\therefore \text{Sol}^n : \left(\frac{1}{2}, 1\right) \cup (1, \infty)$$

bounce

(number line with $\ominus$, $\oplus$, $\oplus$ marked at $\frac{1}{2}$, 1) ②

2. Let $f(x) = x^2 - nx + 1$ and $g(x) = mx - 1$. The functions are combined to form the new function $h(x) = f(g(x))$. If $h(2) = 10$ and $h(0) = 2$, determine values of m and n. **[4 Marks]**

$h(2) = 10$

$f(g(2)) = 10$

$f(2m-1) = 10$

$(2m-1)^2 - n(2m-1) + 1 = 10$ ①

$h(0) = 2$

$f(g(0)) = 2$

$f(-1) = 2$

$1 + n + 1 = 2$

$\boxed{n = 0}$

sub $n = 0$ into ①

$(2m-1)^2 + 1 = 10$

$(2m-1)^2 = 9$

$2m - 1 = \pm 3$

$2m - 1 = 3 \quad | \quad 2m - 1 = 3$

$\boxed{m = 2} \quad \text{or} \quad \boxed{m = -1}$

3. Given the graph $y = k(x)$ below and the functions $g(x) = 3^{x-1}$, $h(x) = \dfrac{9 - x^2}{x - 4}$, $f(x) = (h \circ k \circ g)(x) - 4k(3)$ and $t(x) = (g \circ k \circ k)(x) + g(x+2)$, determine the **exact** value of the following: **[6 Marks]**
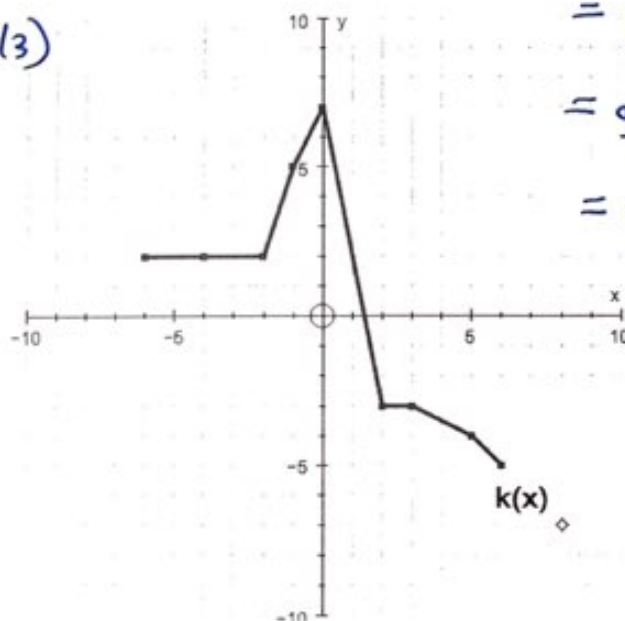
a. $f(2)$ **[3]**

$= h(k(g(2))) - 4k(3)$

$= h(k(3)) + 12$

$= h(-3) + 12$

$= 0 + 12$

$= \boxed{12}$

b. $t(-2)$ **[3]**

$= g(k(k(-2))) + g(0)$

$= g(k(2)) + \frac{1}{3}$

$= g(-3) + \frac{1}{3}$

$= \frac{1}{81} + \frac{1}{3}$

$= \boxed{\frac{28}{81}}$

k(x)

BONUS QUESTION - Correctly Answer One (1) of the following questions for three (3) Bonus Marks in a Category of your choice. Circle the number of the question you choose to answer.

1. Who invented the Ferris Wheel? Full name and proper spelling is required.

   George Washington Gale Ferris, Jr.

2. State a function that is both an even and an odd function.

   $f(x) = 0$

3. List the transformations that have been applied to the base rational function, $f(x) = \dfrac{1}{x}$ to obtain the function $f(x) = \dfrac{3x-2}{x+1}$. Correct order and wording of the transformations is required.

   $$\begin{array}{r} 3 \\ x+1 \overline{)\ 3x-2} \\ -(3x+3) \\ \hline -5\ R. \end{array}$$

   $\therefore f(x) = 3 - \dfrac{5}{x+1}$

   $\Rightarrow$ reflect in x-axis
   vertical stretch by a factor of 5
   horizontal shift 1 unit left
   vertical shift 3 units up

Circle the Category where you would like to apply the Bonus Marks.

Knowledge & Understanding   Application

Thinking   Communication

Two (2) marks will be awarded in the Communication Category for proper mathematical form and notation.