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Name: Solutions

Assessment of Learning: Unit 4 – Trigonometric Functions (PART 2) – DAY 1

Knowledge & Understanding	Thinking	Communication
/18	/5	/2

Instructions: Answer all questions in the space provided and **show all necessary steps.** Leave answers **exact** unless otherwise specified. The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

KNOWLEDGE & UNDERSTANDING – [18 MARKS]

Multiple Choice: Write the **CAPITAL LETTER** corresponding to the correct answer on the line provided. **[1 Mark Each – 5 Marks Total]**

1. The solution to $\tan(x)=1$ over the interval $0 \le x \le \pi$ is D

A. $x=\frac{\pi}{4},\frac{5\pi}{4}$ B. $x=-\frac{\pi}{4}$ C. $x=1$ D. $x=\frac{\pi}{4}$

2. The range of $f(x)=\sec(x)$ is C

A. $(-\infty,\infty)$ B. $(-\infty,-1)\cup(1,\infty)$ C. $(-\infty,-1]\cup[1,\infty)$ D. $[0,\infty)$

3. If $f(x)=4\cos\left(3x-\frac{\pi}{6}\right)$, then the phase shift of the function is C

A. $\frac{\pi}{6}$ units right B. $\frac{\pi}{18}$ units left C. $\frac{\pi}{18}$ units right D. $\frac{2\pi}{3}$ units left

4. One of the asymptotes of $f(\theta)=\cot(\theta)$ is A

A. $\theta=0$ B. $\theta=\frac{\pi}{2}$ C. $\theta=\frac{5\pi}{2}$ D. $\theta=-\frac{\pi}{4}$

5. The maximum value of the function $f(\theta)=-3\sin[4(\theta-\pi)]-1$ is D

A. -4 B. 3 C. 4 D. 2

6. Complete the table below for the **cosine** function. **[5 Marks]**

Equation	**Amplitude**	**Range**	**Period**	**Phase Shift**	**Equation of Axis**
$f(x)=-4\cos\left(-\frac{1}{6}x-\frac{\pi}{12}\right)+2$	4	$[-2,6]$	12π	$\frac{\pi}{2}$ units left	$y=2$

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7. Solve the following. **Exact** answers. **[6 Marks]**

a. $\sin(2x)+\sin(x)=0$, $x\in[0,\pi]$. **[3]**

$2\sin(x)\cos(x)+\sin(x)=0$

$\sin(x)(2\cos(x)+1)=0$

$\sin(x)=0$ $\quad$ $2\cos(x)+1=0$

$x=0,\pi$ $\quad$ $\cos(x)=-\frac{1}{2}$

$x=\frac{2\pi}{3}$

$\therefore x=0,\pi,\frac{2\pi}{3}$

b. $\sin^2(x)+2\cos^2(x)-1=0$, $x\in[0,2\pi]$. **[3]**

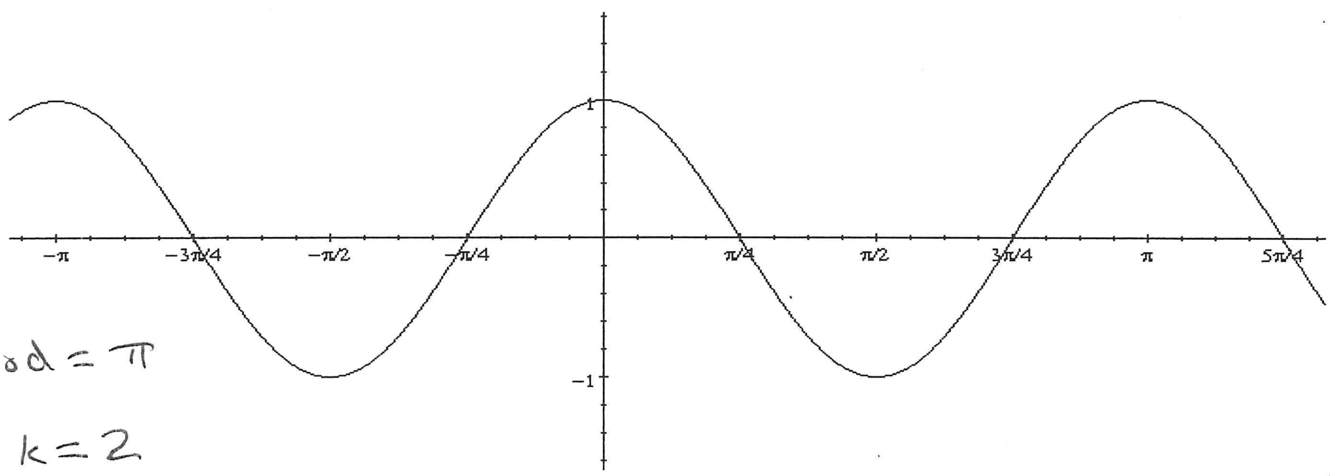
$1-\cos^2(x)+2\cos^2(x)-1=0$

$\cos^2(x)=0$

$\cos(x)=0$

$x=\frac{\pi}{2},\frac{3\pi}{2}$

8. Determine a **sine** function for the graph below. **[2 Marks]**



* Period $=\pi$

$\Rightarrow k=2$

* $a=1$ $\quad$ * $d=\frac{3\pi}{4}$

* $c=0$

Sine Function: $f(x)=\sin\left[2\left(x-\frac{3\pi}{4}\right)\right]$ $\quad$ $f(x)=\sin\left[2\left(x+\frac{\pi}{4}\right)\right]$

THINKING – [5 MARKS]

1. Sara is on a roller coaster where her height, in metres, above the ground over time, in seconds, is modelled by the equation $h(t)=a\cos\left[\frac{\pi}{12}(t-4)\right]+c$. Determine the values of a and c if her height is $2c-3a+17$ metres at 16 seconds and her height is $2c+5a-133$ metres at 12 seconds. **[5 Marks]**

$2c-3a+17=a\cos\left[\frac{\pi}{12}(16-4)\right]+c$

$2c-3a+17=-a+c$

$\boxed{c-2a=-17}$ (1)

$2c+5a-133=a\cos\left[\frac{\pi}{12}(12-4)\right]+c$

$2c+5a-133=-\frac{1}{2}a+c$

$c+\frac{11a}{2}=133$

$\boxed{2c+11a=266}$ (2)

(2) − 2×(1)

$2c+11a=266$

$-2(c-2a=-17)$

$15a=300$

$a=\frac{300}{15}$

$a=20$

sub $a=20$ in (1)

$c-40=-17$

$c=23$

*** 2 Marks are awarded in the Communication Category for the use of correct mathematical form. ***

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Assessment of Learning: Unit 4 – Trigonometric Functions (PART 2) – DAY 2

Application	Thinking	Communication
/18	/5	/2

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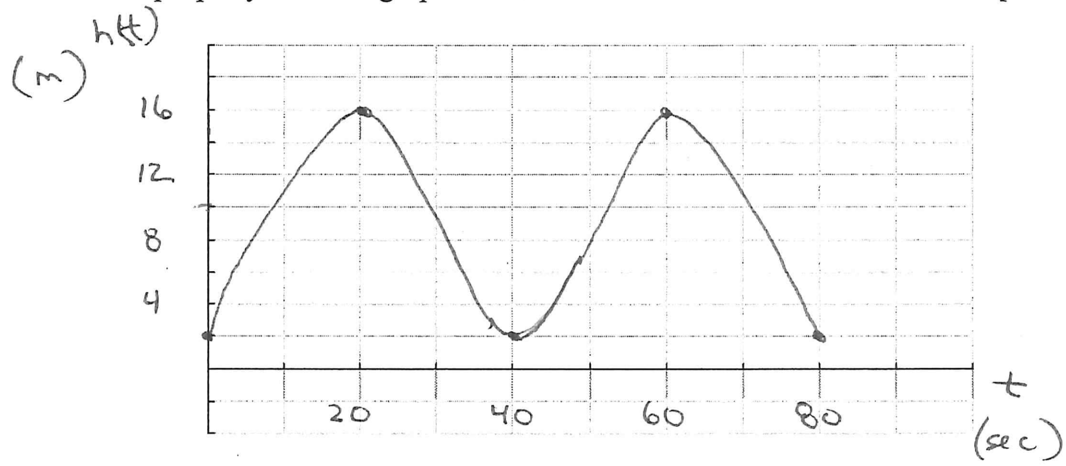
APPLICATION – [18 MARKS]

1. Jim is at the **bottom** of a Ferris wheel. The wheel has a radius of 7 metres and completes 1 cycle every 40 seconds. The bottom of the wheel is 2 metres above the ground.

a. Sketch a properly labelled graph for **two revolutions** of the Ferris Wheel. **[2 Marks]**



b. Determine a **sine function** that represents his height above the ground, in metres, as a function of time, in seconds. **[3 Marks]**

$a = 7$ $\quad d = \frac{1}{4}(40) = 10$

$c = 9$

$\frac{2\pi}{k} = 40$

$k = \frac{\pi}{20}$

Function: $h(t) = 7\sin\left[\frac{\pi}{20}(t-10)\right] + 9$

c. After the wheel starts moving, how many seconds will it take for Jim to be 13 metres above the ground for the first time? Round your answer to 2 decimal places. **[3 Marks]**

$13 = 7\sin\left[\frac{\pi}{20}(t-10)\right] + 9$

$\sin^{-1}\left(\frac{4}{7}\right) = \frac{\pi}{20}(t-10)$

$0.608 = \frac{\pi}{20}(t-10)$

$t = \frac{20}{\pi}(0.608) + 10$

$t \doteq 13.87\text{ s}$

d. What is Jim's vertical height above the ground after 47 seconds? Round your answer to 2 decimal places. **[2 Marks]**

$h(47) = 7\sin\left[\frac{\pi}{20}(47-10)\right] + 9$

$h(47) = 7\sin\left(\frac{37\pi}{20}\right) + 9$

$h(47) \doteq 5.82\text{ m}$

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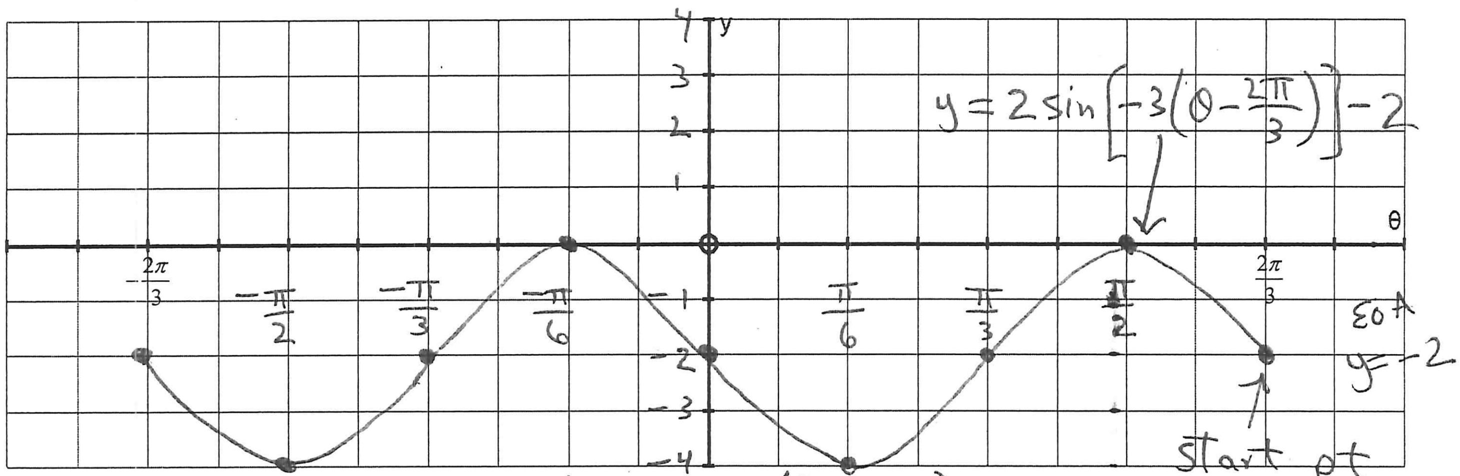
2. Sketch a properly labelled graph of $y=2\sin\left[-3\left(\theta-\frac{2\pi}{3}\right)\right]-2$ for $-\frac{2\pi}{3}\le\theta\le\frac{2\pi}{3}$. **[3 Marks]**



* $a=2$ * start pt. $\left(\frac{2\pi}{3},-2\right)$

* $c=-2 \Rightarrow$ EoA $y=-2$

* I.I $=\frac{2\pi}{3}\cdot\frac{1}{4}=\frac{\pi}{6}$

3. Determine the **first negative** and **first positive** x-intercepts for the function $f(x)=-2\sin\left[2(x-\frac{\pi}{2})\right]+1$.

Exact answers. **[5 Marks]**

$-2\sin\left[2\left(x-\frac{\pi}{2}\right)\right]+1=0$

$\sin\left[2\left(x-\frac{\pi}{2}\right)\right]=\frac{1}{2}$

$2\left(x-\frac{\pi}{2}\right)=\sin^{-1}\left(\frac{1}{2}\right)$

$2\left(x-\frac{\pi}{2}\right)=\frac{\pi}{6},\frac{5\pi}{6}$

$2\left(x-\frac{\pi}{2}\right)=\frac{\pi}{6}$ | $2\left(x-\frac{\pi}{2}\right)=\frac{5\pi}{6}$

$x=\frac{7\pi}{12}$ | $x=\frac{11\pi}{12}$

$\therefore$ 1st negative

$\frac{11\pi}{12}-\pi$

$=-\frac{\pi}{12}$

$\therefore$ 1st positive

$=\frac{7\pi}{12}$

THINKING – [5 MARKS]

1. Solve $\frac{2\cos(x)}{\tan^2(x)+1}-\frac{3}{2}\sin(2x)-3\cos(x)=0$, $x\in[0,2\pi]$. **Exact** answers. **[5 Marks]**

$\frac{2\cos(x)}{\sec^2(x)}-\frac{3}{2}\left(2\sin(x)\cos(x)\right)-3\cos(x)=0$

$2\cos^3(x)-3\sin(x)\cos(x)-3\cos(x)=0$

$\cos(x)\left[2\cos^2(x)-3\sin(x)-3\right]=0$

$\cos(x)\left[2\left(1-\sin^2(x)\right)-3\sin(x)-3\right]=0$

$\cos(x)\left[2\sin^2(x)+3\sin(x)+1\right]=0$

$\cos(x)=0$ | $2\sin^2(x)+3\sin(x)+1=0$

$x=\frac{\pi}{2},\frac{3\pi}{2}$ reject | $(2\sin(x)+1)(\sin(x)+1)=0$

$\hookrightarrow \tan^2(x)=\frac{\sin^2(x)}{\cos^2(x)}$, $\cos(x)\neq 0$

$2\sin(x)+1=0$

$\sin(x)=\frac{-1}{2}$

$x=\frac{7\pi}{6},\frac{11\pi}{6}$

$\sin(x)+1=0$

$\sin(x)=-1$

$x=\frac{3\pi}{2}$ reject

$\therefore x=\frac{7\pi}{6},\frac{11\pi}{6}$