Name: ________________________**Period:**______

Assessment of Learning: Unit 3 – Trigonometric Functions Part I – DAY 1

Knowledge & Understanding	Thinking	Communication
/17	/5	/2

Instructions:

- Non-graphing calculators may be used but not shared. Notebooks may not be used.
- Only methods taught in MHF4U1 will be accepted. Show all work in the space provided.
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Knowledge & Understanding – [17 Marks]

Multiple Choice: Write the **CAPITAL LETTER** corresponding to the correct answer on the line provided.
[1 Mark Each – 6 Marks Total]

1. Determine the approximate degree measure for an angle of 1.32 radians. ***B***

   A. 136.4° B. 75.6° C. 4.2° D. 2.4°

2. Simplify $\sin(\pi + x) - \sin(\pi - x)$: ***B***

   A. 0 B. $-2\sin(x)$ C. $-2\cos(x)$ D. $2\sin(x)$

3. The expression $1 - 2\sin^2\left(\frac{3}{2}\theta\right)$ expressed as a single trig functions is: ***B***

   A. $\cos\left(\frac{3}{2}\theta\right)$ B. $\cos(3\theta)$ C. $\sin(3\theta)$ D. $\cos\left(\frac{3}{4}\theta\right)$

4. When $\csc^2\left(\frac{\pi}{2} + \theta\right)$ is completely simplified the result is equal to ***C***

   A. $\csc^2(\theta)$ B. $-\csc^2(\theta)$ C. $\sec^2(\theta)$ D. $-\sec^2(\theta)$

5. Identify the equation below that is **not** an identity. ***D***

   A. $\sec\left(\frac{3\pi}{2} + \theta\right) = \csc\theta$ B. $\sec^2\theta - \tan^2\theta = 1$

   C. $1 + \cot^2\theta = \csc^2\theta$ D. $\tan^2\theta - \sec^2\theta = 1$

6. A circle has a radius of 15 cm . The **exact** length of arc that subtends by a central angle of 120° is: ***D***

   A. 1800 cm B. 30 cm C. 50 cm D. 10πcm

8. Completely simplify the following expression. **[5 Marks]**

$$\frac{\sin(-x)\cos\left(\frac{\pi}{2}+x\right)+\cot\left(\frac{3\pi}{2}-x\right)\cos(x-\pi)}{\sin(2\pi-x)\sin\left(\frac{3\pi}{2}+x\right)+\cos(x+\pi)}.$$

$$=\frac{\sin(x)\sin(x)-\tan(x)\cos(x)}{\sin(x)\cos(x)-\cos(x)}$$

$$=\frac{\sin^2(x)-\sin(x)}{\cos(x)[\sin(x)-1]}$$

$$=\frac{\sin(x)\,[\cancel{\sin(x)-1}]}{\cos(x)\,[\cancel{\sin(x)-1}]}$$

$$=\tan(x)$$

$$\sin(-x)=-\sin(x)$$

$$\cos\left(\frac{\pi}{2}+x\right)=-\sin(x)$$

$$\cot\left(\frac{3\pi}{2}-x\right)=\tan(x)$$

$$\cos(x-\pi)=\cos(\pi-x)=-\cos(x)$$

$$\sin(2\pi-x)=-\sin(x)$$

$$\sin\left(\frac{3\pi}{2}+x\right)=-\cos(x)$$

$$\cos(x+\pi)=-\cos(x)$$

9. Determine the **exact** simplified value of the following. **[6 Marks]**

a. $\cos\left(\frac{17\pi}{12}\right)$ **[3]**

$$\cos\left(\frac{17\pi}{12}\right)=\cos\left(\pi+\frac{5\pi}{12}\right)=-\cos\left(\frac{5\pi}{12}\right)$$

$$=-\cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$$

$$=-\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$

$$=-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$=\frac{\sqrt{2}-\sqrt{6}}{4}$$

b. $\sin\left(-\frac{9\pi}{8}\right)$ **[3]**

$$\sin\left(-\frac{9\pi}{8}\right)=-\sin\left(\pi+\frac{\pi}{8}\right)=\sin\left(\frac{\pi}{8}\right)$$

$$\cos\left(\frac{\pi}{4}\right)=1-2\sin^2\left(\frac{\pi}{8}\right)$$

$$2\sin^2\left(\frac{\pi}{8}\right)=1-\frac{\sqrt{2}}{2}$$

$$\sin^2\left(\frac{\pi}{8}\right)=\frac{-\sqrt{2}+2}{4}$$

$$\therefore \sin\left(\frac{\pi}{8}\right)=\frac{\sqrt{-\sqrt{2}+2}}{2}$$

Thinking – [5 Marks]

1. Prove $\frac{\cos(2x)}{1+\sin(2x)}=\frac{\cot(x)-1}{\cot(x)+1}$. **[5 Marks]**

$$\text{L.S}=\frac{\cos^2(x)-\sin^2(x)}{\cos^2(x)+\sin^2(x)+2\sin(x)\cos(x)}$$

$$=\frac{[\cos(x)-\sin(x)][\cos(x)+\sin(x)]}{[\cos(x)-\sin(x)]^2}$$

$$=\frac{\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\div\frac{\sin(x)}{\sin(x)}$$

$$=\frac{\cot(x)-1}{\cot(x)+1}$$

$$=\text{R.S}$$

*** 2 Marks are awarded in the Communication Category for use of correct mathematical form. ***

Name: ________________________**Period:**______

Assessment of Learning: Unit 3 – Trigonometric Functions Part I – DAY 2

Application	Thinking	Communication
/17	/5	/2

Instructions:
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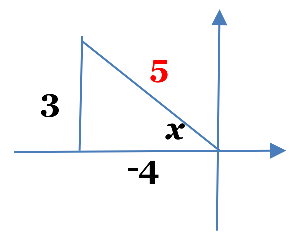
Application - [17 Marks]

1. If $\tan(x) = -\frac{3}{4}$, where $\frac{\pi}{2} < x < \pi$, determine the **exact** value of $\cos(4x)$. **[4 Marks]**

$$\cos(4x) = 2\cos^2(2x) - 1$$
$$= 2\left(2\cos^2(x) - 1\right)^2 - 1$$
$$= 8\cos^4(x) - 8\cos^2(x) + 1$$
$$= 8\left(\frac{-4}{5}\right)^4 - 8\left(\frac{-4}{5}\right)^2 + 1$$
$$= \frac{2048}{625} - \frac{128}{25} + 1$$
$$= -\frac{527}{625}$$



2. Determine the **exact** value of $\dfrac{\tan\left(\frac{\pi}{12}\right)}{\sec\left(\frac{7\pi}{6}\right)}$ (rationalize if necessary **[4 Marks]**

$$\tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$
$$= \frac{\tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{4}\right)}{1 + \tan\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{4}\right)} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}} = 2 - \sqrt{3}$$
$$\sec\left(\frac{7\pi}{6}\right) = \sec\left(\pi + \frac{\pi}{6}\right) = -\sec\left(\frac{\pi}{6}\right) = \frac{-1}{\cos\left(\frac{\pi}{6}\right)} = -\frac{2\sqrt{3}}{3}$$

$$\frac{\tan\left(\frac{\pi}{12}\right)}{\sec\left(\frac{7\pi}{6}\right)} = \frac{2 - \sqrt{3}}{-\frac{2\sqrt{3}}{3}}$$
$$= \frac{3\sqrt{3} - 6}{2\sqrt{3}}$$
$$= \frac{9 - 6\sqrt{3}}{6}$$
$$= \frac{3 - 2\sqrt{3}}{2}$$

3. Express each of the following as a completely simplified single trigonometric function. **[4 Marks]**

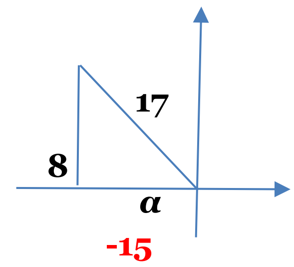
a. $\cos^2\left(\frac{45\pi}{14}\right) - \sin^2\left(\frac{45\pi}{14}\right)$ **[2]**

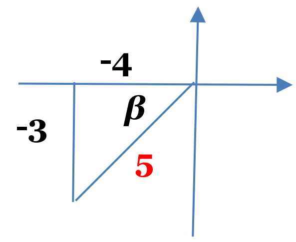
$$= \cos\left(2\left(\frac{45\pi}{14}\right)\right)$$
$$= \cos\left(\frac{45\pi}{7}\right)$$
$$= \cos\left(6\pi + \frac{3\pi}{7}\right)$$
$$= \cos\left(\frac{3\pi}{7}\right)$$
$$= \cos\left(\frac{3\pi}{7}\right)$$

b. $2\sin^2\left(\frac{3\pi}{4} - \frac{x}{2}\right) - 1$ **[2]**

$$= -\cos\left(2\left(\frac{3\pi}{4} - \frac{x}{2}\right)\right)$$
$$= -\cos\left(\frac{3\pi}{2} - x\right)$$
$$= \sin(x)$$

4. If $\sin(\alpha)=\dfrac{8}{17}$, $\dfrac{\pi}{2}<\alpha<\pi$, and $\tan(\beta)=\dfrac{3}{4}$, $\pi<\beta<\dfrac{3\pi}{2}$, determine the **exact** value of $\cos(\alpha-2\beta)$.
[5 Marks]





$$\cos(2\beta)=2\cos^2(\beta)-1$$
$$=2\left(\frac{-4}{5}\right)^2-1$$
$$=\frac{7}{25}$$
$$\sin(2\beta)=2\sin(\beta)\cos(\beta)$$
$$=2\left(\frac{-3}{5}\right)\left(\frac{-4}{5}\right)$$
$$=\frac{24}{25}$$

$$\cos(\alpha-2\beta)=\cos(\alpha)\cos(2\beta)+\sin(\alpha)\sin(2\beta)$$
$$=\left(\frac{-15}{17}\right)\left(\frac{7}{25}\right)+\left(\frac{8}{17}\right)\left(\frac{24}{25}\right)$$
$$=\frac{87}{425}$$

Thinking – [5 Marks]

1. If $\tan(x)-\tan(y)=m$, and $\cot(x)-\cot(y)=n$, prove that $\tan(x-y)=\dfrac{mn}{n-m}$. **[5 Marks]**

$$\cot(x)-\cot(y)=n$$
$$\frac{1}{\tan(x)}-\frac{1}{\tan(y)}=n$$
$$\frac{\tan(y)-\tan(x)}{\tan(x)\tan(y)}=n$$
$$\frac{-m}{\tan(x)\tan(y)}=n$$
$$\tan(x)\tan(y)=\frac{-m}{n}$$

$$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$$
$$=\frac{m}{1+\left(\frac{-m}{n}\right)}$$
$$=\frac{mn}{n-m}$$

*** 2 Marks are awarded in the Communication Category for use of correct mathematical form. ***