Name: Solutions

Unit 4: Trigonometric Functions – Part 2 Assessment of Learning – DAY 1

K & U	Application	Comm.
/15	/13	/2

Instructions:

- Non-graphing calculators may be used but not shared. Notebooks may not be used.
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KNOWLEDGE & UNDERSTANDING – [15 Marks]

Multiple Choice: Write the CAPITAL letter corresponding to the correct answer on the line provided. **[5 Marks]**

1. Which statement is **false** for the graph of $y = -3\sin\left[\frac{\pi}{10}(x-2)\right]+4$? B

I.	The amplitude is 3.
II.	The period is 10.
III.	The phase shift is 2 units to the right.
IV.	The vertical displacement is 4 units up.

A. I B. II C. III D. IV

2. The range of $y = \sec(x)$ is C

A. $(-\infty, \infty)$ B. $(-\infty, -1) \cup (1, \infty)$ C. $(-\infty, -1] \cup [1, \infty)$ D. $[0, \infty)$

3. The period of the function $f(\theta) = -3\tan(-7\theta) - 2$ is D

A. $\frac{\pi}{3}$ B. $-\frac{\pi}{7}$ C. $\frac{2\pi}{7}$ D. $\frac{\pi}{7}$

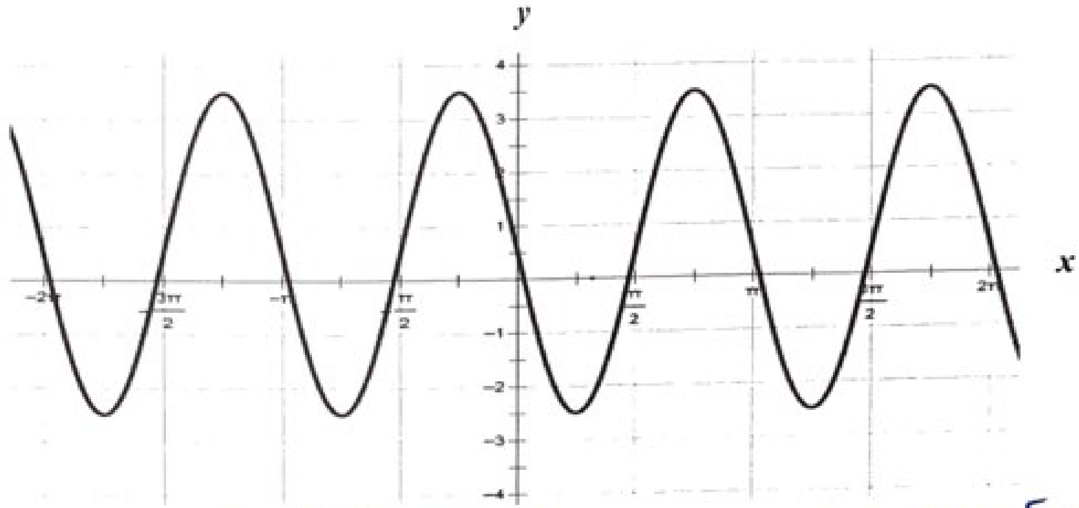
4. The minimum value of the function $y = 3\sin[5\pi(\theta-4)] - 1$ is A

A. -4 B. 3 C. 4 D. 5

5. The number of solutions to the equation $\sin(6\theta) = \frac{\sqrt{3}}{2}$, where $0 \le \theta \le 2\pi$, is D

A. 2 B. 4 C. 6 D. 12

6. Determine a **sine** function and a **cosine** function for the graph below. **[4 Marks]**



Sine Function: $y = 3\sin\left[2\left(x-\frac{\pi}{2}\right)\right]+\frac{1}{2}$ **Cosine Function:** $y = 3\cos\left[2\left(x-\frac{3\pi}{4}\right)\right]+\frac{1}{2}$

7. Complete the table below for the **cosine** function. **[4 Marks]**

Equation	**Amplitude**	**Range**	**Period**	**Phase Shift**
$f(x) = -7\cos\left(-\frac{1}{4}x-\frac{3\pi}{16}\right)+3$	7	$[-4, 10]$	8π	$\frac{3\pi}{4}$ to the left.

8. Solve $2\sin\left(x-\frac{\pi}{4}\right)-1=0$, $0 \le x \le 2\pi$. Exact answer(s). **[2 Marks]**

$\sin\left(x-\frac{\pi}{4}\right)=\frac{1}{2}$

$x-\frac{\pi}{4}=\sin^{-1}\left(\frac{1}{2}\right)$

$x-\frac{\pi}{4}=\frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{5\pi}{12}, \frac{13\pi}{12}$

APPLICATION – [13 Marks]

1. Alma is at the **bottom** of a Ferris wheel. The wheel has a radius of 6 metres and completes 1 cycle in 20 seconds. The bottom of the wheel is 4 metres above the ground.

   a. Determine a **cosine function** that represents his height above the ground, in metres, as a function of the time, in seconds. **[2 Marks]**

   Function: $h(t) = -6\cos\left[\frac{\pi}{10}t\right] + 10$

   b. After the wheel starts moving, how many seconds will it take for Alma to be 11 metres above the ground for the **first time**? Round your answer to 2 decimal places. **[3 Marks]**

   $11 = -6\cos\left[\frac{\pi}{10}t\right] + 10$

   $\frac{-1}{6} = \cos\left[\frac{\pi}{10}t\right]$

   $\frac{\pi}{10}t = \cos^{-1}\left(\frac{-1}{6}\right)$

   $t \doteq \frac{10\cos^{-1}\left[\frac{-1}{6}\right]}{\pi}$

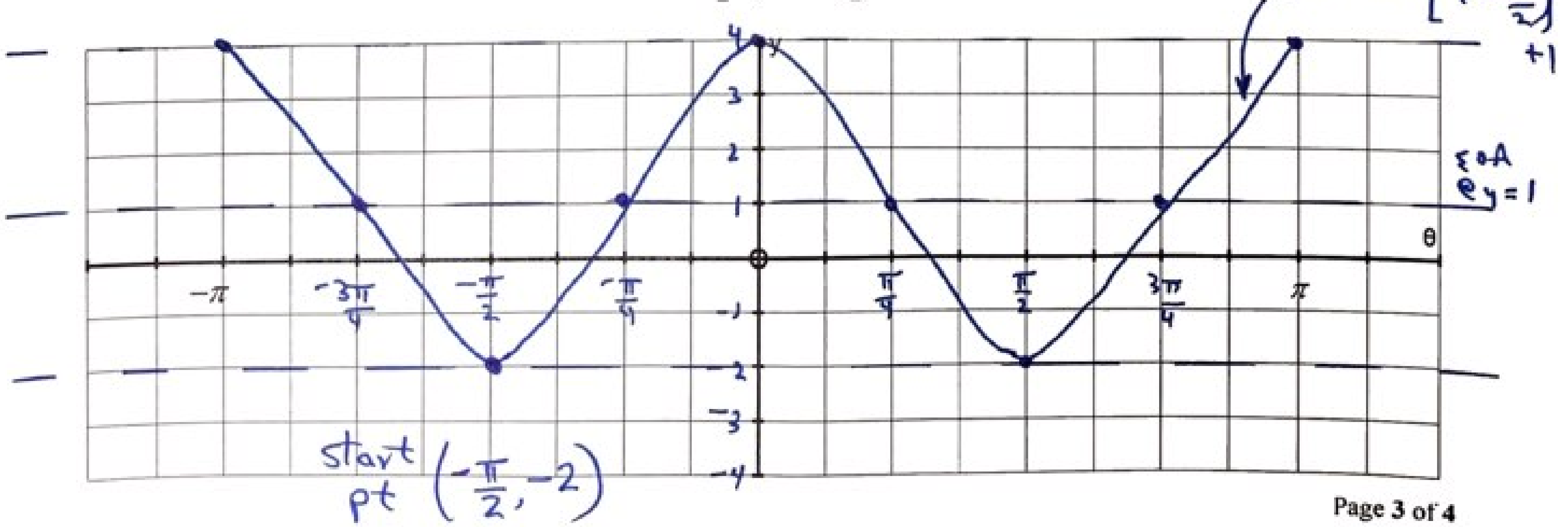
   $t \doteq 5.53$ seconds

   c. What is Alma's vertical height above the ground after 37 seconds? Round your answer to 2 decimal places. **[2 Marks]**

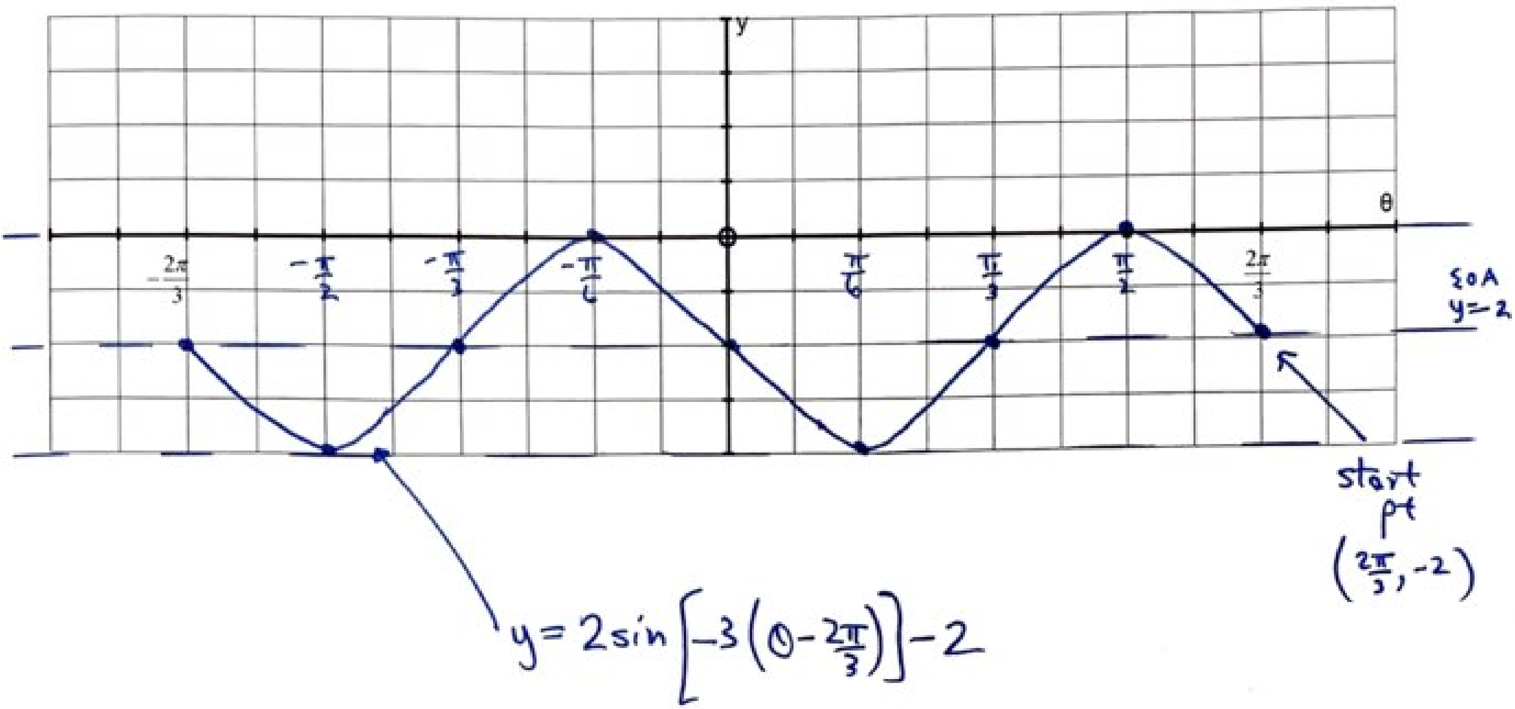
   $h(37) = -6\cos\left[\frac{37\pi}{10}\right] + 10$

   $\doteq 6.47$ m.

2. Sketch a properly labelled graph of $y = -3\cos\left[2\left(\theta + \frac{\pi}{2}\right)\right] + 1$ for $-\pi \le \theta \le \pi$. **[3 Marks]**



3. Sketch a properly labelled graph of $y = 2\sin\left[-3\left(\theta - \frac{2\pi}{3}\right)\right] - 2$ for $-\frac{2\pi}{3} \le \theta \le \frac{2\pi}{3}$. **[3 Marks]**



** Two (2) Marks will be awarded in the Communication Category for the use of proper mathematical form. **

Name: Solutions

Unit 4: Trigonometric Functions – Part 2 Assessment of Learning – DAY 2

Thinking	Comm.
/10	/2

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THINKING – [10 Marks]

1. Determine the **exact** intersection point(s) of the functions $f(x) = 4\sin^2(2x) + 7\sin(2x) + 6$ and $h(x) = 2\cos^2(2x) - 4\sin(2x) + 11$, where $x \in [0, 2\pi]$. **[5 Marks]**

$f(x) = h(x)$

$4\sin^2(2x) + 7\sin(2x) + 6 = 2\cos^2(2x) - 4\sin(2x) + 11$

$4\sin^2(2x) + 7\sin(2x) + 6 = 2\left(1 - \sin^2(2x)\right) - 4\sin(2x) + 11$

$6\sin^2(2x) + 11\sin(2x) - 7 = 0$

$6\sin^2(2x) - 3\sin(2x) + 14\sin(2x) - 7 = 0$

$3\sin(2x)\left(2\sin(2x) - 1\right) + 7\left(2\sin(2x) - 1\right) = 0$

$\left(3\sin(2x) + 7\right)\left(2\sin(2x) - 1\right) = 0$

$3\sin(2x) + 7 = 0$

$\sin(2x) = -\frac{7}{3}$

No solution

$2\sin(2x) = 1$

$\sin(2x) = \frac{1}{2}$ $[0, 2\pi]$

$\sin(t) = \frac{1}{2}$ $[0, 4\pi]$

$t = \sin^{-1}\left(\frac{1}{2}\right)$

$t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

2. Solve $4\cos(2x) - \sin(x)\csc^3(x) + 2 = 0$, where $0 \le x \le \pi$. Round answer(s) to 2 decimal places, if necessary. Otherwise, leave answer(s) exact. **[5 Marks]**

$$4\cos(2x) - \sin(x)\csc^3(x) + 2 = 0$$

$$4\left(1 - 2\sin^2(x)\right) - \frac{\sin(x)}{\sin^3(x)} + 2 = 0$$

$$4 - 8\sin^2(x) - \frac{1}{\sin^2(x)} + 2 = 0$$

$$\frac{4\sin^2(x) - 8\sin^4(x) - 1 + 2\sin^2(x)}{\sin^2(x)} = 0$$

$$8\sin^4(x) - 6\sin^2(x) + 1 = 0$$

$$8\sin^4(x) - 4\sin^2(x) - 2\sin^2(x) + 1 = 0$$

$$4\sin^2(x)\left(2\sin^2(x) - 1\right) - \left(2\sin^2(x) - 1\right) = 0$$

$$\left(4\sin^2(x) - 1\right)\left(2\sin^2(x) - 1\right) = 0$$

$4\sin^2(x) - 1 = 0$

$\sin^2(x) = \frac{1}{4}$

$\sin(x) = \pm\frac{1}{2}$

$x = \sin^{-1}\left(\pm\frac{1}{2}\right)$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$2\sin^2(x) - 1 = 0$

$\sin^2(x) = \frac{1}{2}$

$\sin(x) = \pm\frac{1}{\sqrt{2}}$

$x = \sin^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$

$x \doteq \frac{\pi}{4}, \frac{3\pi}{4}$

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