MHF4U1 – P4
Vincent

Name: Solutions

Unit 2: Rational Functions Assessment of Learning

K & U	Application	Thinking	Communication
/13	/15	/6	/2

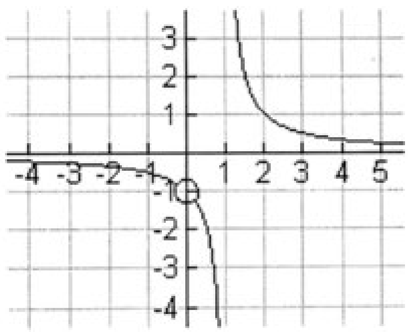
Instructions:

- Non-graphing calculators may be used but not shared. Notebooks may not be used.
- Only methods taught in MHF4U1 will be accepted. Show all work in the space provided.
- The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.
- **Please complete the assessment independently with academic honesty as the guiding principle.**

KNOWLEDGE & UNDERSTANDING – [13 Marks]

Multiple Choice: Write the CAPITAL letter corresponding to the correct answer on the line provided. **[4 Marks]**

1. Which of the following functions represent the given graph? A



A. $f(x)=\frac{x}{x^2-x}$ B. $f(x)=\frac{x^2-1}{x}$

C. $f(x)=\frac{x-1}{x^2-x}$ D. $f(x)=\frac{x^2-1}{x-1}$

2. Which of the following functions has vertical asymptotes at $x=-1$ and $x=3$ and a horizontal asymptote at $y=0$? D

A. $y=\frac{x^2-6x+9}{x^2-2x-3}$ B. $y=\frac{x^2}{x^2-2x-3}$ C. $y=\frac{x+1}{x-3}$ D. $y=\frac{x-9}{x^2-2x-3}$

3. Given $f(x)=\frac{-2(x+3)(x-12)}{(x+1)}$, which of the following statements is **false**? B

A. The $y-$intercept is at $(0, 72)$.
B. The function has a horizontal asymptote at $y=-2$.
C. There is a vertical asymptote at $x=-1$.
D. The $x-$intercepts are at $x=-3$ and $x=12$.

4. Which of the following is **true**? D

A. If a rational function originally looks like it has a hole and a vertical asymptote at the same value of x, then the hole takes precedent and there is no vertical asymptote.
B. A rational function cannot have both a horizontal asymptote and a hole.
C. A vertical asymptote can be crossed by the function.
D. A rational function can have more than one vertical asymptote.

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5. Determine the **exact** point(s) of intersection between $f(x) = x^2 + 5x + 3$ and its reciprocal function. **[5 Marks]**

$f(x) = \pm 1$

CASE 1

$x^2 + 5x + 3 = -1$

$x^2 + 5x + 4 = 0$

$(x+4)(x+1) = 0$

$\therefore x = -4, -1$

CASE 2

$x^2 + 5x + 3 = 1$

$x^2 + 5x + 2 = 0$

$x = \dfrac{-5 \pm \sqrt{25 - 4(1)(2)}}{2(1)}$

$x = \dfrac{-5 \pm \sqrt{17}}{2}$

Answer: $(-4,-1), (-1,-1), \left(\dfrac{-5 \pm \sqrt{17}}{2}, 1\right)$

6. Determine the **exact** point(s) where the function $f(x) = \dfrac{x^3 + 6}{x^3 + 2x^2 - 5x + 9}$ crosses its asymptote. **[4 Marks]**

$\cancel{x^3} + 6 = \cancel{x^3} + 2x^2 - 5x + 9$

$2x^2 - 5x + 3 = 0$

$(2x-3)(x-1) = 0$

$2x - 3 = 0 \quad | \quad x - 1 = 0$

$x = \dfrac{3}{2} \quad | \quad x = 1$

$\therefore$ crosses @ $x = \dfrac{3}{2}$ & 1

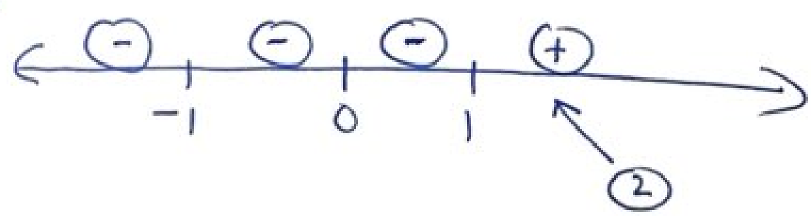
Answer: $\left(\dfrac{3}{2}, 1\right)$ & $(1, 1)$

APPLICATION – [15 Marks]

1. Solve $\dfrac{x^2(x-1)^2}{(x^2-1)(x+1)} < 0$. **[4 Marks]**

$\dfrac{x^2(x-1)\cancel{(x-1)}}{\cancel{(x-1)}(x+1)(x+1)} < 0$

$\dfrac{x^2(x-1)}{(x+1)(x+1)}$



Answer: $(-\infty, -1) \cup (-1, 0) \cup (0, 1)$

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2. Sketch and properly label the graph $f(x)=\dfrac{9(x+1)^2(x^2-4)}{(x^2-9)(x^2+3x+2)}$. **[11 Marks]**

① holes @ $(-1,0)$ & $\left(-2,-\frac{36}{5}\right)$

$$f(x)=\frac{9(x+1)\cancel{(x+1)}(x-2)\cancel{(x+2)}}{(x-3)(x+3)\cancel{(x+2)}\cancel{(x+1)}}$$

$$=\frac{9(x+1)(x-2)}{(x-3)(x+3)},\ x\neq -1,-2$$

② x-int @ $(2,0)$

Note: $(-1,0)$ is a hole

No x-int

③ y-int @ $(0,2)$

④ VA @ $x=\pm 3$

⑤ As $x\to -3^-$, $f(x)\to\infty$

As $x\to -3^+$, $f(x)\to -\infty$

As $x\to 3^-$, $f(x)\to -\infty$

As $x\to 3^+$, $f(x)\to\infty$

⑥ HA @ $y=9$

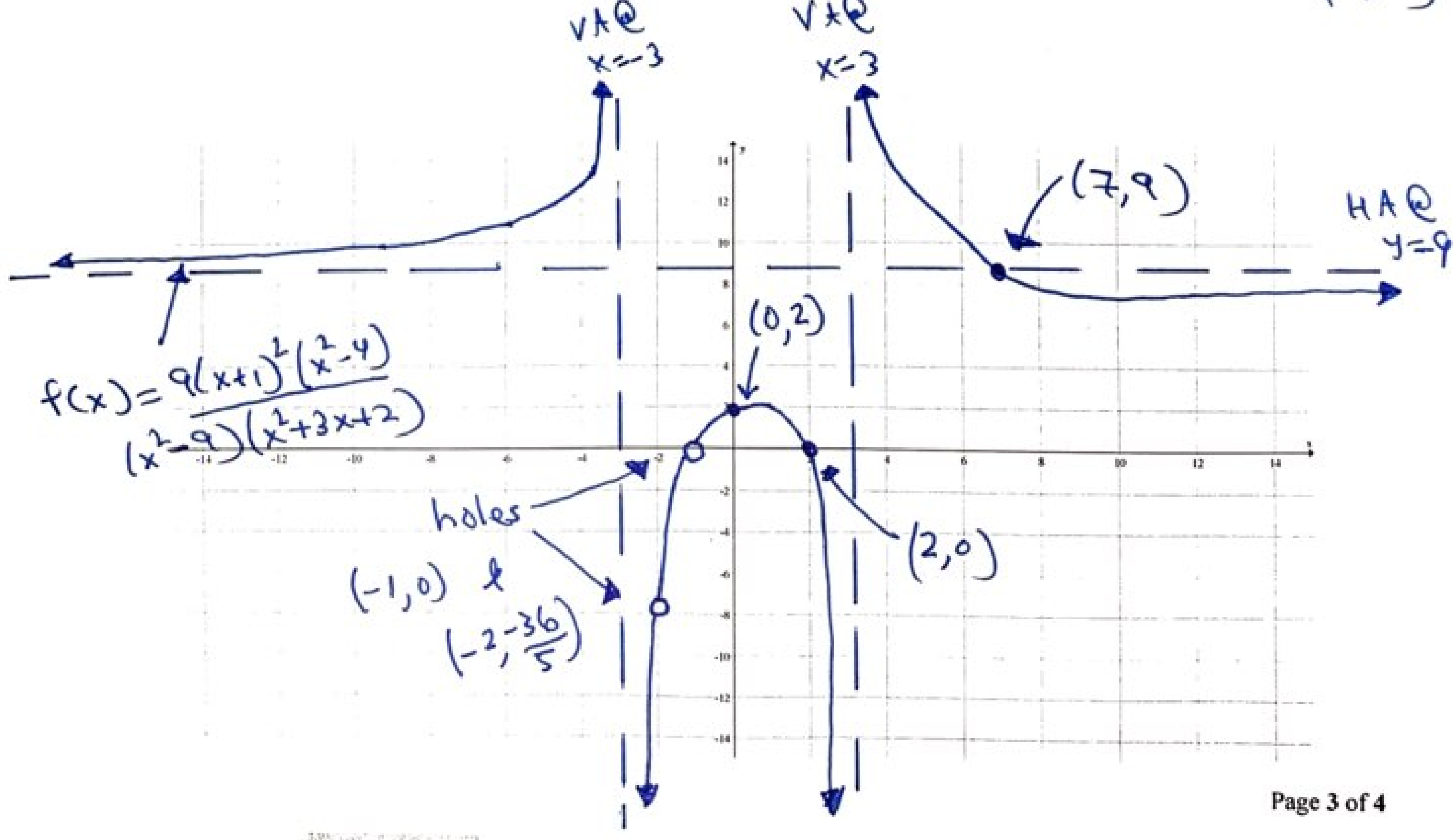
⑦ cross test

$$\frac{9(x+1)(x-2)}{(x-3)(x+3)}=9$$

$$\cancel{x^2}-x-2=\cancel{x^2}-9$$

$$-x=-7$$

$x=7$ crosses at $(7,9)$



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THINKING – [6 Marks]

1. $f(x)=\dfrac{x^3+a}{bx^2+7x+c}$ has a hole at $(-1,k)$ and a $y-$intercept at $\dfrac{1}{6}$. Determine the value of k. [6 Marks]

* If $f(x)$ has hole at $x=-1 \Rightarrow$ numerator $=0$ @ $x=-1$
denominator $=0$ @ $x=-1$

* x^3+a
$(-1)^3+a=0$
$\boxed{a=1}$

* bx^2+7x+c
$b(-1)^2+7(-1)+c=0$
$\boxed{b+c=7}$

* y-int @ $\frac{1}{6} \Rightarrow \dfrac{1}{6}=\dfrac{(0)^3+1}{b(0)^2+7(0)+c} \Rightarrow \dfrac{1}{6}=\dfrac{1}{c} \therefore \boxed{c=6}$
and $\boxed{b=1}$

$$\therefore f(x)=\frac{x^3+1}{x^2+7x+6}$$

$$=\frac{\cancel{(x+1)}(x^2-x+1)}{\cancel{(x+1)}(x+6)}$$

$$=\frac{x^2-x+1}{x+6},\ x\neq -1$$

location of hole $\rightarrow \dfrac{(-1)^2-(-1)+1}{(-1)+6}=\dfrac{3}{5} \quad \therefore k=\dfrac{3}{5}$

Answer: $k=\dfrac{3}{5}$

** Two (2) Marks will be awarded in the Communication Category for the use of proper mathematical form. **