Name: ______________________

Unit 6: Assessment of Learning Vectors in 2 Space and 3 Space – Day

Instructions:
Answer all questions in the space provided and **show all necessary steps**. Leave answers **exact** unless otherwise specified. Students must use the methods taught in MCV4U1 for all questions. The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

K & U	Thinking	Comm
/ 18	/5	/2

KNOWLEDGE & UNDERSTANDING [18 MARKS]

Multiple Choice: Write the CAPITAL LETTER corresponding to the correct answer on the line provided.
[6 marks – 1 mark each]

1. The vector $\vec{a}$ and $\vec{b}$ have the same magnitude. The angle between the vectors is 125°, and the magnitude of their cross product is 20. What is $|\vec{a}|$? _____ ***B*** ____
   (A) 4.08 (B) 4.94 (C) 16.4 (D) 4.4

2. If point A = (1, 3, 4) and point B = (-2, 2, 0), determine $\overrightarrow{AB}$. _____ ***B*** ____
   (A) [3, 1, 4] (B) [-3, -1, -4] (C) [-1, 5, 4] (D) [1, 5, 4]

3. A wagon is pulled a distance of 180 m. The amount of work done is 860 J and the magnitude of the force was 16 N applied at an angle θ to the ground. What is θ, to the nearest degree? ____ ***D*** _____
   (A) 17° (B) 33° (C) 57° (D) 73°

4. Three forces of 8 N, 11 N and ***x*** N are in a state of equilibrium. Which is a possible magnitude for the ***x***?
   (A) 2 N (B) $\sqrt{7}$ N (C) 10 N (D) 20 N ____ ***C*** ____

5. Which of the following expressions is meaningless? ____ ***C*** _____
   (A) $\vec{u}\bullet(\vec{v}\times\vec{w})$ (B) $(2\vec{u}-\vec{v})\bullet 5\vec{w}$ (C) $4\vec{u}\times(\vec{v}\bullet\vec{w})$ (D) $\dfrac{\vec{u}\times\vec{v}}{(\vec{u}-\vec{v})\bullet(\vec{u}+\vec{v})}$

6. If $\vec{a}=[5,0,2]$, then a unit vector collinear with $\vec{a}$ is ____ ***C*** ___
   (A) $[15,0,6]$ (B) $\dfrac{1}{\sqrt{29}}[2,0,5]$ (C) $\dfrac{-1}{\sqrt{29}}[5,0,2]$ (D) $\sqrt{29}[5,0,2]$

Part B: Write a complete solution to receive full marks.

7. Given the vectors $\vec{a}=[5,1,-2]$, $\vec{b}=\hat{\imath}+\hat{\jmath}-3\hat{k}$ and $\vec{c}=[-2,-4,1]$, determine the following. [8 marks]

(a) $2\vec{a}-\vec{c}$ ✓

$=[10,2,-4]-[-2,-4,1]$
$=[12,6,-5]$

(b) $\vec{b}\cdot\vec{a}$ ✓

$=[1,1,-3]\bullet[5,1,-2]$
$=12$

(c) Unit vector in the opposite direction of $\vec{b}$ ✓✓

$$-b=\frac{-\vec{b}}{|\vec{b}|}$$
$$=-\frac{\sqrt{11}}{11}[1,1,-3]$$

(d) $\vec{b}\times\vec{c}$, in component form ✓✓

$\vec{b}\times\vec{c}=[-11,5,-2]$

(e) The angle between $\vec{a}$ and $\vec{c}$, to the nearest degree. ✓✓

$$\cos(\theta)=\frac{\vec{a}\bullet\vec{c}}{|\vec{a}||\vec{c}|}$$
$$=\frac{-16}{(\sqrt{30})(\sqrt{21})}$$
$$\boxed{\theta\doteq 130^\circ}$$

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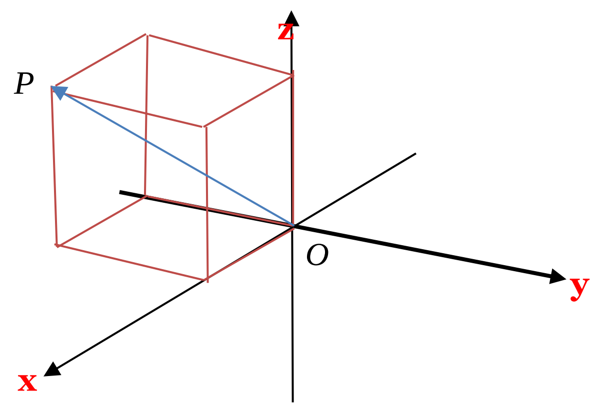
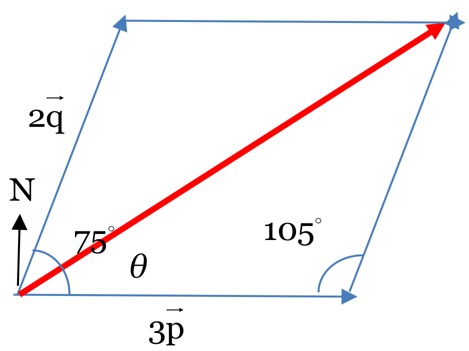
8. Draw the following position vector $\overrightarrow{OP} = [3, -4, 4]$ using a rectangular prism. **[1 mark]**



9. Given $|\vec{p}| = 5$ and $|\vec{q}| = 9$ and the angle between them is 75°, determine $3\vec{p} + 2\vec{q}$. [3 marks]



$$\left|3\vec{p}+2\vec{q}\right|^2 = \left|3\vec{p}\right|^2 + \left|2\vec{q}\right|^2 - 2\left|3\vec{p}\right|\left|2\vec{q}\right|\cos(105^\circ)$$

$$= 15^2 + 18^2 - 2(15)(18)\cos(105^\circ)$$

$$\left|3\vec{p}+2\vec{q}\right| = 26.24 \text{ units}$$

$$\frac{\sin(\theta)}{18} = \frac{\sin(105^\circ)}{26.24}$$

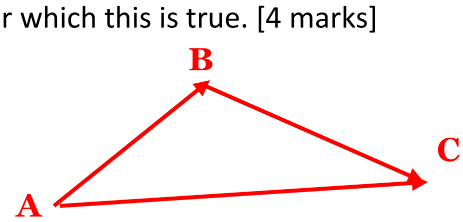
$$\theta = \sin^{-1}\left(\frac{18\sin(105^\circ)}{26.24}\right)$$

$$\theta = 41.5^\circ$$

$$\therefore 3\vec{p} + 2\vec{q} = 26.24 \text{ [E } 41.5^\circ \text{ N]}$$

THINKING [4 marks]

1. The points A(1, 0, 2), B(2, 0,1) and C(1, 2, **k**) form a triangle of area $\sqrt{6}$ units^2. Determine the value(s) of **k** for which this is true. [4 marks]



$$A_{\Delta ABC} = \frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{BC}\right| = \sqrt{6}$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = [1, 0, -1]$$

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = [-1, 2, k-1]$$

$$\overrightarrow{AB}\times\overrightarrow{BC} = [2, 2-k, 2]$$

$$\left|\overrightarrow{AB}\times\overrightarrow{BC}\right| = \sqrt{2^2 + (2-k)^2 + 2^2}$$

$$2\sqrt{6} = \sqrt{8 + (2-k)^2}$$

$$24 = 8 + (2-k)^2$$

$$16 = (2-k)^2$$

$$\pm 4 = 2 - k$$

$$\boxed{k = 6} \text{ or } \boxed{k = -2}$$

Communication - Two marks are awarded throughout the assessment for use of correct mathematical form.

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Assessment of Learning – Representing vectors in 2-space and 3-space (Day 2)

Instructions:
Answer all questions in the space provided and **show all necessary steps**. Leave answers **exact** unless otherwise specified. Students must use the methods taught in MCV4U1 for all questions. The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

App	Thinking	Comm
/17	/4	/5

APPLICATION [17 marks]

1. The scalar projection of vector $\vec{u} = [1, m, 0]$ onto vector $\vec{v} = [2,2,1]$ is 4. Determine the value of m. **[3 marks]**

$$\left|\mathrm{Proj}_{\vec{v}}\vec{u}\right| = 4$$

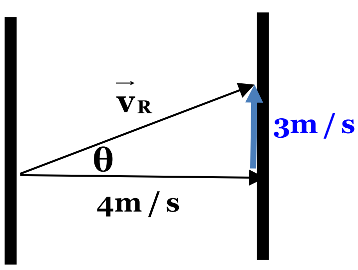
$$\frac{[1,m,0]\bullet[2,2,1]}{\sqrt{2^2+2^2+1^2}} = 4$$

$$\frac{2+2m}{3} = 4$$

$$2+2m = 12$$

$$\boxed{m = 5}$$

2. A motorboat traveling 4 m/s, East encounters a current traveling 3.0 m/s, North.
   (a) What is the resultant velocity of the motorboat?
   (b) If the width of the river is 80 meters wide, then how much time does it take the boat to travel shore to shore?
   (c) What distance downstream does the boat reach the opposite shore?
   Include a diagram. Round your answers to 1dp. **[3 marks]**



a)

$$\vec{v}_R = \sqrt{4^2+3^2}$$
$$= \sqrt{25}$$
$$= 5\text{m}/\text{s}$$

$$\tan(\theta) = \frac{3}{4}$$
$$\theta = 36.9^\circ$$
$\therefore$ The heading is [E 36.9° N]

b)

$$t = \frac{80\text{ m}}{4\text{ m}/\text{s}}$$
$$= 20\text{ s}$$

c)

$$\text{distance} = (3\text{ m}/\text{s})(20\text{ s})$$
$$= 60\text{ m}$$

3. If $\vec{a} = [1,3,-1]$, $\vec{b} = [2,1,5]$, $\vec{v} = [-3, \boldsymbol{y}, \boldsymbol{z}]$, and $\vec{a}\times\vec{v} = \vec{b}$, find $\boldsymbol{y}$ and $\boldsymbol{z}$. **[3 marks]**

$$\vec{a}\times\vec{v} = \vec{b}$$
$$[1,3,-1]\times[-3,y,z] = [2,1,5]$$
$$[3z+y, 3-z, y+9] = [2,1,5]$$
$$3z+y = 2$$
$$3-z = 1 \rightarrow \boxed{z = 2}$$
$$y+9 = 5 \rightarrow \boxed{y = -4}$$

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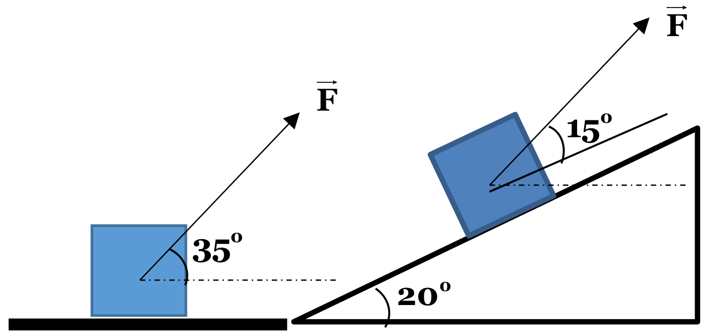
4. A box is dragged 16 m by a 75 N force applied at an angle of 35° to the level ground at all time**.** It is then dragged to the top of a 6 m ramp by the same force. If the ramp is inclined at 20° to the ground, find the total work done. Include a diagram. **[4 marks]**

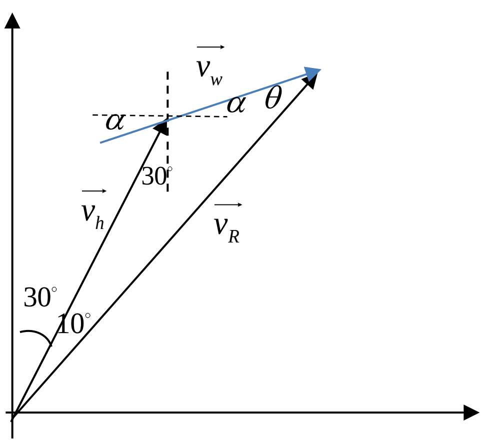


$$\mathbf{w_1} = |\vec{F}||\vec{d}|\cos\theta$$
$$= 75 \times 16 \times \cos(35^{o})$$
$$= 982.98 \text{ J}$$

$$\mathbf{w_2} = |\vec{F}||\vec{d}|\cos\alpha$$
$$= 75 \times 6 \times \cos(15^{o})$$
$$= 434.67 \text{ J}$$

$$\mathbf{w_T} = \mathbf{w_1} + \mathbf{w_2}$$
$$= 1417.65 \text{ J}$$

5. A plane has an airspeed of 330 km/h and is heading in a direction of N30°E. The pilot is travelling to a destination that is N40°E of his starting location at 400 km/h. Find the magnitude and direction of the wind. Include a clear diagram. **[4 marks]**



$$|\vec{v}_w|^2 = 330^2 + 400^2 - 2(330)(400)\cos(10^{\circ})$$
$$|\vec{v}_w| = 94.4\text{km/h}$$
$$\frac{\sin(10^{o})}{94.4} = \frac{\sin(\theta)}{330}$$
$$\theta = 37.4^{o}$$
$$\alpha = 180^{o} - 10^{o} - 90^{o} - 37.4^{o} - 30^{o}$$
$$\alpha = 12.6^{o}$$
$\therefore$ **The wind is 94.4km/h and blowing from [W 12.6° S]**

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THINKING [4 MARKS]

1. If the angle between the vectors $\left(\vec{a}-\vec{b}\right)$ and $\left(\vec{a}+\vec{b}\right)$ is 60°. Prove that

$$\left|\vec{a}+\vec{b}\right|\left|\vec{a}-\vec{b}\right| = \left|\vec{a}+\vec{b}\right|^2+\left|\vec{a}-\vec{b}\right|^2-4\left|\vec{b}\right|^2.$$ **[4 marks]**

$$\begin{aligned}\mathbf{R.S} &= \left|\vec{a}+\vec{b}\right|^2+\left|\vec{a}-\vec{b}\right|^2-4\left|\vec{b}\right|^2\\ &= \left(\vec{a}+\vec{b}\right)\bullet\left(\vec{a}+\vec{b}\right)+\left(\vec{a}-\vec{b}\right)\bullet\left(\vec{a}-\vec{b}\right)-4\left|\vec{b}\right|^2\\ &= \left|\vec{a}\right|^2+2\vec{a}\bullet\vec{b}+\left|\vec{b}\right|^2+\left|\vec{a}\right|^2-2\vec{a}\bullet\vec{b}+\left|\vec{b}\right|^2-4\left|\vec{b}\right|^2\\ &= 2\left|\vec{a}\right|^2-2\left|\vec{b}\right|^2\end{aligned}$$

If the angle between the vectors $\left(\vec{a}-\vec{b}\right)$ and $\left(\vec{a}+\vec{b}\right)$ is 60° , then

$$\cos\left(60^\circ\right)=\frac{\left(\vec{a}+\vec{b}\right)\bullet\left(\vec{a}-\vec{b}\right)}{\left|\vec{a}+\vec{b}\right|\left|\vec{a}-\vec{b}\right|}$$ **or**

$$\begin{aligned}\mathbf{L.S} &= \left|\vec{a}+\vec{b}\right|\left|\vec{a}-\vec{b}\right| = \frac{\left(\vec{a}+\vec{b}\right)\bullet\left(\vec{a}-\vec{b}\right)}{\cos\left(60^\circ\right)}\\ &= \frac{\left|\vec{a}\right|^2-\left|\vec{b}\right|^2}{\frac{1}{2}}\\ &= 2\left(\left|\vec{a}\right|^2-\left|\vec{b}\right|\right)\end{aligned}$$

$\therefore$ **L.S = R.S**

COMMUNICATION [5 MARKS]

1. Explain clearly how you would determine a unit vector that is perpendicular to both the vectors $\vec{u}=[1,-1,-1]$ and $\vec{v}=[2,-2,3]$. **[3 marks]**

The vector perpendicular to both vectors is the cross product of $\vec{u}$ and $\vec{v}$. Let $\vec{a}=\vec{u}\times\vec{v}$,
A unit vector on the same direction or opposite direction of $\vec{a}$ is $\mathbf{a}=\pm\frac{\vec{a}}{\left|\vec{a}\right|}$.

***2 marks will be awarded in the Communication category for the use of proper mathematical form ***