MHF4U1-V1
Leung, Moshtagh, Vincent

Name: Shakeel

Unit 1: Polynomial Functions - Assessment of Learning – DAY 1

K & U	Thinking	Comm.
16 /18	4 1/2 /5	2 /2

Instructions: Answer all questions in the space provided and **show all necessary steps**. Leave [illegible] unless otherwise specified. The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

KNOWLEDGE & UNDERSTANDING – [18 MARKS]

Multiple Choice: Write the CAPITAL letter corresponding to the correct answer on the line provided. **[5 Marks Total – 1 Mark Each]**

1. Which statement is **false** regarding $f(x)=-4(x-1)^3(x^2-9)$? B

   A. The leading coefficient is negative.
   B. $f(x)$ goes through the x-axis once and bounces on the x-axis twice.
   C. The constant value of the function's finite differences is equal to -480.
   D. As $x\to-\infty$, $f(x)\to\infty$.

2. Which of the following statements is/are **true** for $f(x)=2(1-x)(x+2)^2(x-3)$? C

   I. as $x\to\infty$, $f(x)\to-\infty$
   II. as $x\to\infty$, $f(x)\to\infty$
   III. as $x\to-\infty$, $f(x)\to-\infty$
   IV. as $x\to-\infty$, $f(x)\to\infty$

   A. I only.
   B. I and II only.
   C. I and III only.
   D. II and IV only.

3. If $x+1$ is a factor of $x^3+qx^2-10x+3$, then the value of q is B

   A. 12 B. -12 C. 6 D. -6

4. When completely factored, $(a+b)^3-(a-b)^3$ equals C

   A. $(2a)(a^2-3b^2)$
   B. $(2b)(3a^2+b^2)$
   C. $(a-b)(3a^2+4ab+3b^2)$
   D. $2(a+b))(3a^2+b^2)$

5. Which of the following functions is/are odd? C

   I. $f(x)=-2(x-1)^2(x+1)$
   II. $f(x)=4x^4+9x^2-16$
   III. $f(x)=-3x^5+9x^3+2x$
   IV. $f(x)=3x^3+8x+1$

   A. III and IV. X B. I, II and III. X C. III only D. IV only. X

6. Divide $(3x^3-10x+8)$ by $(x-5)$ using polynomial division and then write the division statement. **[3 Marks]**

$$x-5\overline{)\,3x^3+0x^2-10x+8}\quad 3x^2+15x+65\ R333$$

$3x^3-15x^2$

$15x^2-10x$

$15x^2-75x$

$65x+8$

$65x-325$

$R333$

$3x^2+15x+65+\frac{333}{x-5}$

quotient statement

Division Statement: $3x^3-10x+8=(3x^2+15x+65)(x-5)+333$

MHF4U1-V1
Leung, Moshtagh, Vincent

Name: Shulleel

7. Graph and properly label $f(x) = -2x^3(x^2+x-6)(x-2)$. **[4 Marks]**

$-2x^3(x-2)^2(x+3)$

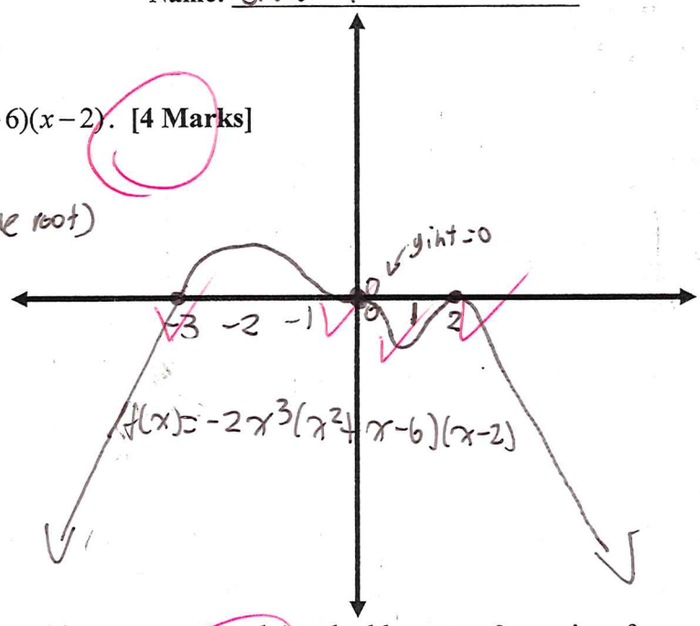
x ints: 0 (order 3), 2 (order 2), -3 (order 1/single root)

SLC: –

degree = 6

y int = 0

q3 – q4



8. State the family of polynomial functions of degree 7 with roots at -7 and 4, a double root at 3, a point of inflection at $x = 1$ and as $x \to -\infty$, $f(x) \to \infty$. **[3 Marks]**

$2\frac{1}{2}$

$f(x) = k(x-1)^3(x+7)(x-4)(x-3)^2$, $k < 0$

9. The height of a miniature rocket above the ground is modeled by $h(t) = -5t^2 + 5t + 120$, where $h(t)$ is the height in metres and t is time in seconds. Estimate the IROC at $t = 2$ seconds. **[3 Marks]**

$1\frac{1}{2}$

$IROC = \lim_{h \to 0} \frac{h(2+h) - h(2)}{h}$

$IROC = \frac{110 - 109.984995}{0.001} = 15.002 = \frac{7501}{500}$

$\therefore$ IROC at 2 seconds is 15.002 m/s

C

THINKING – [5 MARKS]

1. Determine the values of a, b and c given $P(x) = ax^3 + bx^2 + cx + 15$ and the following information:
   - $P(x)$ is divisible by $x+1$.
   - $P(x)$ has a remainder of -3 when divided by $x-2$.
   - The value of the third finite differences is 24.

[5 Marks]

$4\frac{1}{2}$

divisible by $x+1$ | $4x^3+bx^2+cx+15$ | $a3! = 24$

$a(3 \cdot 2 \cdot 1) = 24$

$6a = 24$

$a = 4$

$4(-1)^3 + b(-1)^2 + c(-1) + 15$

$-4 + b - c + 15$

$b - c + 11$

$\therefore$ the values of P(x) are $a = 4$, $b = -14$, $c = 3$

$4(2)^3 + b(2)^2 + c(2) + 15 = -3$

$4(8) + 4b + 2c + 15 = -3$

$32 + 4b + 2c + 15 = -3$

$4b + 2c = -50$

$2c = -50 - 4b$

$c = -25 - 2b$

(2, -3) Point to Plug in

$b = -25 - 2b + 11 = 0$

$-b - 25 + 11 = 0$

$b = -25 + 11$

$b = -14$ -12

$4(-14) + 2c = -50$

$-56 + 2c = -50$

$2c = 6$

$c = 3$ -1

*** 2 Marks will be awarded in the Communication Category for the use of proper mathematical form. ***

MCF4U1-V1
Leung, Moshtagh, Vincent

Name: Shukeel

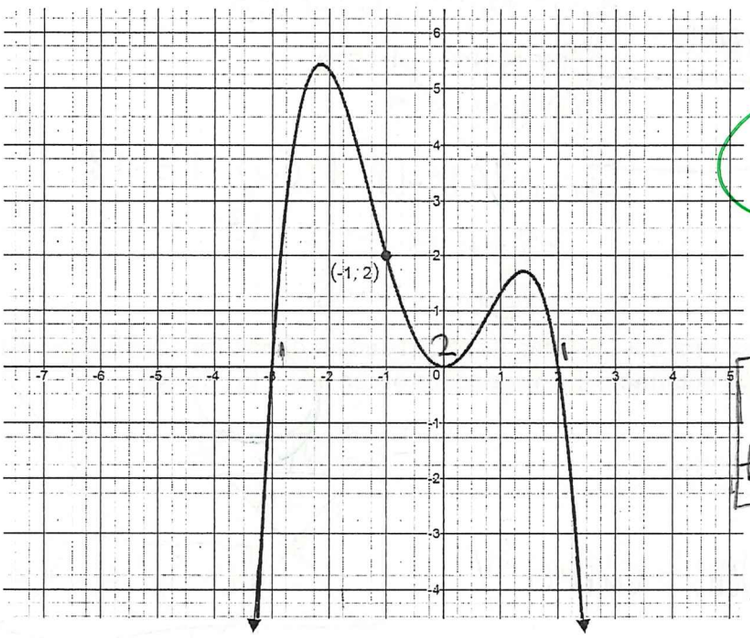
Unit 1: Polynomial Functions - Assessment of Learning – DAY 2

Application	Thinking	Comm.
19 /20	5 /5	4.5 /5

Instructions: Answer all questions in the space provided and **show all necessary steps**. Leave answers **exact** unless otherwise specified. The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

APPLICATION – [20 MARKS]

1. Determine the **specific equation** (assume lowest possible degree) of the function below. **[4 Marks]**



$f(x) = kx^2(x+3)(x-2)$

$2 = k(-1)^2(-1+3)(-1-2)$

$2 = k(1)(2)(-3)$

$2 = -6k$

$k = -\frac{1}{3}$

∴ equation is $f(x) = -\frac{1}{3}x^2(x+3)(x-2)$

C

$-30 \quad 1\ 36 \quad 2\ 15 \quad -3\ 10$

2. Solve the following. **Note:** For Part b) use interval notation. **[7 Marks]**

a) $4x^3 - x^2 - 11x - 6 = 0$ **[4]**

$\frac{-6}{4} \to \pm\left\{\frac{1,2,3,6}{1,2,4}\right\}$

$f(2) = 0$ ∴ $x-2$ is a factor

$2 \mid 4 \ -1 \ -11 \ -6$

$\quad\quad 8 \ \ 14 \ \ 6$

$4 \ \ 7 \ \ 3 \ \ 0$ ✓

$(4x^2+7x+3)(x-2) = 0$

$(4x+3)(x+1)(x-2) = 0$

$\therefore x = -\frac{3}{4}, -1, 2$ ✓

C

12

7

$3, 4$

$\frac{3}{4}, \frac{4}{4}$

$\frac{3}{4x}, \frac{1}{1x}$

b) $-(x^2+7x-30)(9-x^2) > 0$ **[3]**

$1\ 30 \quad 2\ 15 \quad 3\ 10 \quad -3\ 10$

$\frac{-3}{1x} \quad \frac{10}{1x}$

$(-x^2+9)$

$-(x^2-9)$

$(x+3)(x-3)$

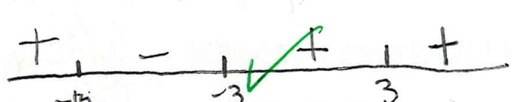
$-(x-3)(x+10)-(x+3)(x-3)$

$= (x-3)^2(x+10)(x+3) > 0$

roots: $x = 3$ (order 2), -10 (single root), -3 (single root)

$(5-3)^2(5+10)(5+3) = (4)(15)(8) = 480$

Fully factor to make roots do # line



$\therefore f(x) > 0 : (-\infty, -10) \cup (-3, \infty)$

3. Determine the values of m and n so that the polynomials $P(x) = 2x^3 + mx^2 + nx - 6$ and $Q(x) = x^3 - 3mx^2 + 2nx + 4$ are both divisible by $x+2$. **[4 Marks]**

3

$Q(x)$

$(-2)^3 - 3m(-2)^2 + 2n(-2) + 4 = 0$

$-8 - 3m(4) + 2n(-2) + 4 = 0$

$-8 - 12m + -4n + 4 = 0$

$-12m - 4n - 4 = 0$

$-12m - 4n = 4$

$-12(-11-h) - 4h = 4$

$132 + 12h - 4h = 4$

$8h = 128$ $h = -16$

$P(x)$

$2(-2)^3 + m(-2)^2 + n(-2) - 6 = 0$

$2(-8) + -2m + -2n - 6 = 0$

$-16 - 2m - 2n - 6$

$-2m - 2n = 22$

$2m - h = 11$

$m = -11 - h$

$m = -11 - 16$

$m = -27$

∴ the values of m and h are $h = -16$ and $m = -27$

14

MHF4U1-V1
Leung, Moshtagh, Vincent

Name: Shakeel

4. Determine the polynomial function that passes through the following points. **[5 Marks]**

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
−3	−151			
−2	−55	96		
−1	−13	42	−54	
0	−1	12	−30	24
1	5	6	−6	24
2	29	24	18	24
3	95	66	42	24

third difference

ax^3+bx^2+cx+d

$-1=a(0)^3+b(0)^2+c(0)+d$

$-1=d$

ax^3+bx^2+cx-1

$4x^3+bx^2+cx-1$

$4!=\Delta$

$a(3\cdot2\cdot1)=24$

$6a=24$

$a=4$

$4(1)^3+b(1)^2+c(1)-1=5$

$4+b+c-1=5$

① $b+c=2$

$4(-1)^3+b(-1)^2+c(-1)-1=-13$

$-4+b-c-1=-13$

② $b-c=-8$

② $b=-8+c$

① $b+c=2$

Sub 2 into 1

$-8+c+c=2$

$2c=10$

$c=5$

$b+c=2$

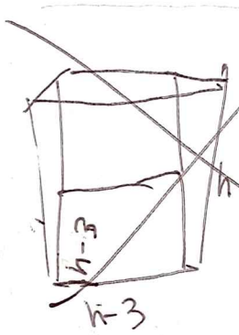
$b+5=2$

$b=-3$

∴ the equation is $f(x)=4x^3-3x^2+5x-1$

THINKING – [5 MARKS]

1. The width of a square-based storage tank is 3 metres less than its height and all dimensions are integer values. If the tank has a capacity of 20 cubic metres determine the dimensions of the tank. **[5 Marks]**



capacity = 20

$(3-h)(3-h)(h)=\text{capacity}$

$(3-h)(3-h)(h)=20$

$(9-3h-3h+h^2)(h)=20$

$(h^2-6h+9)(h)$

$h^3-6h^2+9h=20$

$h^3-6h^2+9h-20=0$

$\pm\{1,2,4,5,10,20\}$ $f(5)=0$ (h-5 is factor)

5	1	−6	9	−20
		5	−5	20
	1	−1	4	0

(h^2-h+4)

$h=5$

width = 3

Length = 3−h

height = h

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(4)}}{2(1)}$

$=\frac{1\pm\sqrt{1-16}}{2}$

$=$

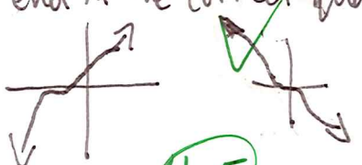
Refer to Scrap Paper attached for work and final answer

COMMUNICATION – [5 MARKS]

1. **"An odd degree polynomial function must have at least one x - intercept."**

Is the above statement **always true**, **sometimes true** or **never true**? Provide a thorough explanation and diagrams to justify your answer. **[3 Marks]**

The above statement is always true. As an odd degree function no matter what degree it is (as long as its odd), the function must cross the x int at least once. Which means it must always have at least one x intercept. If the SLC is +, then the function goes from q_3-q_1. If its − it goes from q_2-q_4. In both cases in order for the function to end in the correct quadrant, it must cross the x int, meaning it has at least one x int



*** 2 Marks will be awarded in the Communication Category for the use of proper mathematical form. ***

4/5

4. Divide x^3-6x^2+10 by x^2-x-2 and write the division statement. [3 marks]

$$\begin{array}{r} x-5 \\ x^2-x-2 \;\overline{)\; x^3-6x^2+0x+10} \\ x^3-x^2-2x \\ \hline -5x^2+2x+10 \\ -5x^2+5x+10 \\ \hline -3x \end{array}$$

Let $f(x)=x^3-6x^2$

$f(x)=(x^2-x-2)(x-5)-3x$

Division statement: $f(x)=(x^2-x-2)(x-5)-3x$

5. Write the equation of the family of polynomial functions of degree 7 with roots of order 1 at 15 and -1 and a point of inflection at -2. [3 marks]

$y=a(x-15)^2(x+1)^2(x+2)^3,\ a\neq 0,\ a\in\mathbb{R}$

$y=a(x-15)(x+1)(x+2)^5$

APPLICATION

6. Determine the interval(s) that satisfy the inequality: $8x^3+1\le 4x^2+2x$. Show all steps. [5 Marks]

$8x^3+1-4x^2-2x\le 0$

$8x^3-4x^2-2x+1\le 0$

$4x^2(2x-1)-(2x-1)\le 0$

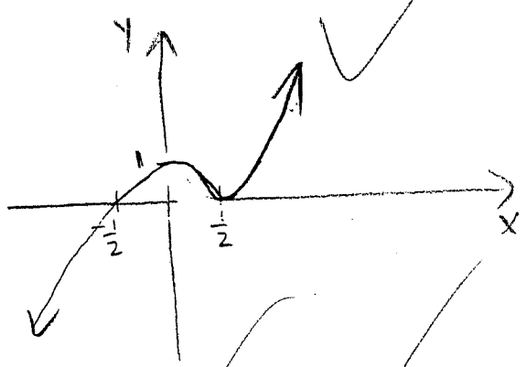
$(2x-1)(4x^2-1)\le 0$

$(2x-1)(2x+1)(2x+1)\le 0$

$(2x-1)^2(2x+1)\le 0$

Let $f(x)=(2x-1)^2(2x+1)$.

y int: 1



$\therefore\ 8x^3+1\le 4x^2+2x,\ x\in(-\infty,-\frac{1}{2}]\cup[\frac{1}{2}],\ x\in\mathbb{R}$

7. $f(x)=ax^3-x^2+bx-24$ has three factors. Two of these factors are $x-2$ and $x+4$. Determine the values of a and b. [4 marks]

$f(2)=a(2)^3-(2)^2+b(2)-24$

$=8a-4+2b-24$

$28=8a+2b$ ① $\times 2\Rightarrow$ ③.

$56=16a+4b$ ③

$f(-4)=a(-4)^3-(-4)^2+b(-4)-24$

$=-64a-16-4b-24$

$40=-64a-4b$ ②

③+②

$56=16a+4b$

$40=-64a-4b$

$96=-48a$

$-2=a$

∴ the value of a is -2

sub $a=-2$ into ①

$28=8a+2b$

$28=8(-2)+2b$

$28+16=2b$

$22=b$

8. Factor fully: $m^6 - 2m^3 + 1$ [3 marks]

$= (m^2 - 1)^3$

$= [(m+1)(m-1)]^3$

$[(m-1)(m^2+m+1)]^2$

2

9. Determine the polynomial function that passes through the points below. Show all steps. [6 Marks]

x	y	1st diff.	2nd diff.	3rd diff.	4th diff.
−3	289				
−2	81	−208			
−1	17	−64	144		
0	7	−10	54	−90	
1	9	2	12	−42	48
2	29	20	18	6	48
3	121	192	72	54	48

this is always your leading term

mislabelled

$a = \frac{FD}{n!}$

$a = \frac{48}{4!}$

$a = 2$

y int. 7 (given)

$\therefore$ the eq'n is $y = 2x^4 - 7x^3 + 24x^2 + 7$

$y = 2x^4 + ax^3 + bx^2 + cx + 7$

$17 = 2(-1)^4 + a(-1)^3 + b(-1)^2 + c(-1) + 7$

$17 = 2 - a - b - c + 7$

① $8 = -a - b - c$

$29 = 2(2)^4 + a(2)^3 + b(2)^2 + c(2) + 7$

$29 = 32 + 8a + 4b + 2c + 7$

② $-10 = 8a + 4b + 2c$

$121 = 2(3)^4 + a(3)^3 + b(3)^2 + c(3) + 7$

$121 = 162 + 27a + 9b + 3c + 7$

③ $-48 = 27a + 9b + 3c$

① × 3 $24 = -3a - 3b - 3c$ ④

③ + ④ $-24 = 24a + 6b$ ⑤

① × 2 $\rightarrow 16 = -2a - 2b - 2c$ ⑥

② + ⑥ $-10 = 8a + 4b + 2c$

$6a + 2b$ ⑦

⑤ × 3 $\Rightarrow 18 = 18a + 6b$ ⑧

⑧ − ⑦ $42 = -6a$

$-7 = a$

sub $a = -7$ into ⑥: $6 = 6(-7) + 2b$

$6 = -42 + 2b$

sub $a = -7$ into ①: $b = 24$

$-48 = 27(-7) + 9(24) + 3c$

$-48 = -189 + 216 + 3c$

$-25 = c$

you have already solved for a, b, c

THINKING

10. Given the equation $x^3 - 6x^2 + px + q = 0$, algebraically determine the values of p and q for which the equation has three equal roots? Show all steps. [4 Marks]

3 equal roots = all the same.

$x^3 - 6x^2 + px + q = 0$

$x^2(x-6) - 36(x-6) = 0$

$(x-6)(x^2-36) = 0$

$(x-6)(x-6)(x+6) = 0$

$(x-6)^2(x+6) = 0$

2 equal roots

$a = n!$

$(x \pm a)^3 = x^3 - 6x^2 + px + q$

must be, when added to 6.

guess & check

factors: $\pm 1, \pm 2, \pm 3$

$(x-6)(x-6)(x-6)$

$(x^2 - 6x - 6x + 36)(x-6)$

$(x^2 - 12x + 36)(x-6)$

$x^3 - 12x^2 + 36x - 6x^2 + 72x - 216$

$x^3 - 18x^2 + 108x - 216$

$1+1+1 = 3$

$2+2+2 = 6$ ✗

$3+3+3 = 9$

$(x-2)^3$

$(x-2)(x-2)(x-2)$

$(x^2 - 2x - 2x + 4)(x-2)$

$(x^2 - 4x + 4)(x-2)$

$x^3 - 4x^2 + 4x - 2x^2 + 8x - 8$

$x^3 - 6x^2 + 12x - 8$

$x^3 - 2^3 - 6x^2 + 12x$

$= (x-2)(x^2+2x+4) - 6x(x-2)$

$= (x-2)(x^2 + 2x + 4 - 6x)$

$= (x-2)(x^2 - 4x + 4)$ ← PST

$= (x-2)(x-2)^2$

$\therefore$ the value of p is 12 and the value of q is −8.

11. The height, width and length of a **small** box are consecutive integers with the height being the smallest of the three dimensions. If the width and length are increased by 1 *cm* each and the height is doubled, then the volume of the box is increased by 120 cm^3. Determine the dimensions of the original **small** box. Show all steps. [6 Marks]

Let x be the height, $x+1$ be length and $x+2$ be width.

$V = (x+1)(x+2)(x)$ ①

$V+120 = [(x+1)+1][(x+2)+1](2x)$

$V+120 = (x+2)(x+3)(2x)$ ②

$(x+1)(x+2)(x)$.
$(x^2+2x+x+2)(x)$.
x^3+3x^2+2x

$(x+2)(x+3)(2x)$.
$(x^2+3x+2x+6)(2x)$.
$(x^2+5x+6)(2x)$
$2x^3+10x^2+12x$.

$(x+2)(x+3)(2x) - (x+1)(x+2)(x) = 120$.
$2x^3+10x^2+12x - (x^3+3x^2+2x) = 120$.
$x^3+7x^2+10x = 120$.
$x^3+7x^2+10x-120 = 0$

Let $f(x) = x^3+7x^2+10x-120$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 8, \pm 10, \pm 12,$

$f(3) = 0$

$\therefore x-3$ is a factor, by FT.

$$\begin{array}{r} x^2+10x+40 \\ x-3 \overline{)\,x^3+7x^2+10x-120} \\ x^3-3x^2 \\ \hline 10x^2+10x \\ 10x^2-30x \\ \hline 40x-120 \\ 40x-120 \\ \hline 0 \end{array}$$

$f(x) = (x-3)(x^2+10x+40)$.

$$x = \frac{-(10) \pm \sqrt{(10)^2-4(1)(40)}}{2(1)}$$

$$x = \frac{-10 \pm \sqrt{-60}}{2}$$ ← inadmissible since there are imaginary.

$\therefore$ the dimensions are height 3cm, length 4cm and width 5cm.

$x = 3$ → height
length $= x+1$
$= 3+1$
$= 4$
width $= x+2$
$= 3+2$
$= 5$.

COMMUNICATION

12. Compare the average and instantaneous rates of change. [3 marks]

Both the average rate of change and instantaneous rate of change are slopes on graphs.

The average rate of change is a secant; a line between 2 points on a graph while the instantaneous rate of change is a tangent; a line that passes through 1 point.

Four marks are awarded for use of appropriate mathematical form throughout the assessment. [4 marks]

+4