# Unit 7 Assessment of Learning – Lines & Planes

| Knowledge & Understanding | Application | Thinking | Communication |
|---|---|---|---|
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**Instructions:** Answer all questions in the space provided and show all necessary steps. Leave answers exact unless otherwise specified. Students must use the methods taught in MCV4U1 for all questions. The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

1. Which of the following lines is perpendicular to $\vec{r}=[1,3,-4]+t[2,4,-3], t\in\mathbb{R}$? **A**

A. $\frac{x-3}{5}=7-y=\frac{z+7}{2}$
B. $\vec{r}=[2,6,-8]+t[2,4,-3], t\in\mathbb{R}$
C. $\vec{r}=[1,3,-4]+t[4,8,-6], t\in\mathbb{R}$
D. $x=1+t, y=3-t, z=-4-t, t\in\mathbb{R}$

2. The Cartesian equation of the plane with normal $\vec{n}=[-1,2,6]$ and containing the point $(1,2,2)$ is **D**

A. $x-2y-6z-15=0$
B. $x-2y-6z=0$
C. $x-2y-6z=0$
D. $x-2y-6z+15=0$

3. Which of the following is/are **true**? **B**

I. The plane $\pi: 3x-11y+2=0$ is parallel to the $z-$axis.
II. The vector equation of a line with slope of $-3$ and a $x-$intercept of 7 is $\vec{r}=[0,7]+t[1,-3], t\in\mathbb{R}$.
III. Two planes are perpendicular if their normals are parallel.
IV. The vector equation of a line with slope of $-\frac{3}{4}$ that passes trough point (2, 3) is $\vec{r}=[2,3]+t[4,-3], t\in\mathbb{R}$

A. I only
B. I and IV only
C. II, III and IV only
D. All are true

4. The point of intersection, if any, between the line $\frac{x-1}{2}=-4-y=\frac{z}{3}$ and the plane $\pi: 5x-y-4z-2=0$ is **B**

A. $(-15,11,-21)$
B. $(15,-11,21)$
C. $(-13,3,-21)$
D. They do not intersect

5. The acute angle that is formed by the intersection of the two planes $\pi_1: 2x+3y-z+9=0$ and $\pi_2: x+2y+4=0$ is approximately **C**

A. $33.2^\circ$
B. $83.4^\circ$
C. $17.0^\circ$
D. None of these

6. The value(s) of $q$ for which the lines $\vec{r}=[-1,-4,2]+t\left[\frac{12}{q},6,21\right], t\in\mathbb{R}$ and $\frac{x-4}{2}=\frac{y+3}{q}=\frac{z-5}{q+10}$ are parallel is/are **B**

A. 6
B. 4
C. 4 and 6
D. $-4$ and $-6$

7. The scalar equation of the plane containing the points $A(1,2,-3)$, $B(5,1,-4)$ and $C(-1,8,4)$ is **C**

A. $x-26y+22z+119=0$
B. $x+26y-22z+119=0$
C. $x+26y-22z-119=0$
D. $x-26y-22z-119=0$

8. The point of intersection, if any, for the lines $\vec{r} = [2, 5, 4] + s[5, -15, 10]$, $s \in \mathbb{R}$ and $x = 4 - 2t$, $y = 2 + 12t$, $z = 6 - 8t$, $t \in \mathbb{R}$ is **C**

A. $(-5, 4, -10)$ B. $(5, 4, -10)$ C. $(5, -4, 10)$ D. They do not intersect

9. A vector equation of the plane $\pi : 2x + 12y - z + 11 = 0$ is **D**

A. $\vec{r} = [0, 1, -1] + s[0, 1, 12] + t[1, 0, 2],\ s, t \in \mathbb{R}$ B. $\vec{r} = [0, -1, -1] + s[0, 2, 12] + t[1, 0, 1],\ s, t \in \mathbb{R}$

C. $\vec{r} = [0, -1, 1] + s[0, 1, 12] + t[1, 0, 1],\ s, t \in \mathbb{R}$ D. $\vec{r} = [0, -1, -1] + s[0, 1, 12] + t[1, 0, 2],\ s, t \in \mathbb{R}$

**APPLICATION - [5 Marks]**

1. Determine the intersection, if any, of the following and interpret the solution geometrically.

$$\pi_1 :\ 2x + y - z - 5 = 0$$
$$\pi_2 :\ x - y + 3z - 7 = 0$$
$$\pi_3 :\ 4x - y + 5z - 19 = 0$$

$$\vec{n}_1 \bullet (\vec{n}_2 \times \vec{n}_3) = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 3 \\ 4 & -1 & 5 \end{vmatrix} = 0$$

There may or may not be points of intersection

$2x + y - z = 5 \quad (1)$

$x - y + 3z = 7 \quad (2)$

$4x - y + 5z = 19 \quad (3)$

$(1) + (2): 3x + 2z = 12 \quad (4)$

$(1) + (3): 6x + 4z = 29 \quad (5)$

$2 \times (4) - (4): 0 = 0$

Let $x = t$

$3t + 2z = 12$

$z = 6 - \frac{3}{2}t$

sub. $x = t$ and $z = 6 - \frac{3}{2}t$ into $(1)$:

$2t + y - 6 + \frac{3}{2}t = 5$

$y = 11 + \frac{7}{2}t$

Equation of line of intersection is:

$$\begin{cases} x = t \\ y = 11 + \frac{7}{2}t \\ z = 6 - \frac{3}{2}t \end{cases}$$

or $\vec{r} = [0, 11, 6] + t[1, \frac{7}{2}, -\frac{3}{2}],\ t \in R$ or $\vec{r} = [0, 11, 6] + s[2, 7, -3],\ s \in R$

2. Determine the point of intersection, if it exists between the following the lines. [4 marks]

$l_1$: $\vec{r} = [4, 9, 2] + t[1, -1, -1]$, $t \in R$ and $l_2 : \frac{x-2}{5} = \frac{11-y}{3} = \frac{z-4}{-4}$

$$L_1 : \begin{cases} x = 4 + t \\ y = 9 - t \\ z = 2 - t \end{cases} \qquad L_2 : \begin{cases} x = 2 + 5s \\ y = 11 - 3s \\ z = 4 - 4s \end{cases}$$

$4 + t = 2 + 5s$ $\quad$ $9 - t = 11 - 3s$ $\quad$ $2 - t = 4 - 4s$

$5s - t = 2 \quad (1)$ $\quad$ $3s - t = 2 \quad (2)$ $\quad$ $4s - t = 2 \quad (3)$

$(2) - (1): 2s = 0$

$s = 0 \ \& \ t = -2$

$\text{Check}(3): 0 - (-2) = 2$

$x = 2 + 0 = 4$

$y = 11 - 3(0) = 11$

$z = 2 + 5(0) = 2$

$\therefore \text{P.O.I} = (2, 11, 4)$

**THINKING [5 Marks]**

1. Determine the value(s) of $k$ such that the line $\frac{x-1}{2} = \frac{y+5}{-1} = \frac{z+3}{-3}$ and the points $A(-1, -1, 4)$, $B(-2, 0, 4)$ and $C(4k, 1-2k, k^2)$ all lie on the same plane. **[5 marks]**

$\overrightarrow{AB} = [-1, 1, 0]$

$\vec{m} = [2, -1, -3]$

$$\vec{n} = \overrightarrow{AB} \times \vec{m} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ 2 & -1 & -3 \end{vmatrix} = [-3, -3, -1] = -[3, 3, 1]$$

Equation of plane with normal $\vec{n}$ is:

$3x + 3y + z + D = 0$

Sub. point $A(-1, -1, 4)$, we get:

$3(-1) + 3(-1) + 4 + D = 0$

$-3 - 3 + 4 + D = 0$

$D = 2$

$\because 3x + 3y + z + 2 = 0$

Sub. point $C(4k, 1-2k, k^2)$, we get:

$3(4k) + 3(1-2k) + k^2 + 2 = 0$

$12k + 3 - 6k + k^2 + 2 = 0$

$k^2 + 6k + 5 = 0$

$(k + 5)(k + 1) = 0$

$\therefore k = -5, \ k = -1$

+ 3 marks awarded for proper mathematical form used throughout the test.