Bayview Secondary School
Mathematics Department – Course Code: MDM4U1
Assessment of Organized Counting Version 2 Day 1

Name: _____Alpha #:_____

K-25	T-5

Instructions:

1) Show all work to obtain full marks for questions that are worth greater than 1 mark.
2) 3 marks will be awarded for communication in total for both days of the assessment.
3) The use of cellphones, audio- or video-recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.
4) Scientific calculators are allowed but cannot be shared.
5) Duration of the assessment on each day: 40 minutes

KNOWLEDGE AND UNDERSTANDING (K)

1. Using Pascal's method to express the following term as a sum. [1 mark]

$t_{n+r-2,3}$

$= t_{n+r-1,3} + t_{n+r-1,2}$

$= \binom{n+r-1}{3} + \binom{n+r-1}{2}$

$= {}_{n+r-1}C_3 + {}_{n+r-1}C_2$

2. Express the following as a single factorial expression. [1 mark]

$504(6!)$

$= 72 \times 7 \times 6!$

$= 72 \times 7!$

$= 9 \times 8 \times 7!$

$= 9!$

3. Let $U = \{a, b, c, d, e, f, g, h, i, o, u\}$;
$A = \{all\ vowels\}$,
$B = \{a, c, d, e, f, i, o\}$,
$C = \{c, d, g, h, u\}$.
List the elements of the following set.
$(A \cap B') \cap C$ [1 mark]

$A = \{aeiou\}$

$B' = \{bghu\}$

$(A \cap B') = \{u\}$

$= \{u\}$

4. A standard deck of cards has 52 cards, a game of which each player will have 6 cards in hand, how many ways can a player have a set of three of a kind, a pair, and a random card that is not the same number/face card as the three of a kind and the pair in hand? [2 marks]

3 of a kind: $\binom{13}{1}\binom{4}{3}$ × a pair: $\binom{12}{1}\binom{4}{2}$ × random: $\binom{11}{1}\binom{4}{1}$

$= 164736$ ways

5. Determine the total number of divisors of 8820. [2 marks]

$8820 = 2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 7$

Divisors:

$3 \times 3 \times 2 \times 3 - 1$

$= 53$ divisors

6. How many ways can three married couples be arranged around a circular table? [2 marks]

$(6-1)!$

$= 5!$

$= 120$ ways

7. How many four-digit odd numbers less than 5000 can be formed from the digits 123590 and no repetition is allowed? [3 marks]

Case 1: $\underset{1,2,3}{3} \times 4 \times \underset{5,9}{3} \times 2 = 72$

Case 2: $\underset{\cancel{1,2,3}}{2} \times 4 \times \underset{1,3}{3} \times 2 = 48$

Case 1 + Case 2

$= 72 + 48$

$= 120$ numbers

8. How many 5-permutation of the letters TURNHAM are there if both vowels must be used and A and U are adjacent to each other? [3 marks]

TRNHM (5 consonants) UA (2 vowels).

$\left(\underset{A\ U}{2 \times 1} \times 5 \times 4 \times 3\right) \times 4$ cases

A↔U

A↔U

A↔U

$= 480$ permutations.

9. How many even four-digit numbers can be formed if repetition is not allowed? [3 marks]

case 1: $9 \times 8 \times 7 \times 1$ (0)

case 2: $8 \times 8 \times 7 \times 4$ (2, 4, 6, 8; 0 not allowed)

case 1 + case 2

= 2296 numbers

10. There are 8 instrument players and 9 vocal singers came for audition for our school music showcase. How many ways can we choose 8 of them for solo performance in our showcase if at least one instrument player and one vocal singer must be included? [3 marks]

Indirect method:

All − no instrumental − no vocal

$\binom{8+9}{8} - \binom{8}{0}\binom{9}{8} - \binom{8}{8}\binom{9}{0}$

$= 24310 - 9 - 1$

$= 24300$ ways

11. There are 2 basketballs, 5 volleyballs, and 4 soccer balls. In how many ways can a teacher grab 3 balls to the class for activities with at least one volleyball? [2 marks]

2 Bball	5 Vball	4 Soccer
2	1	0
0	1	2
1	1	1
1	2	0
0	2	1
0	3	0

∴ 6 ways

12. Solve for n, $n \in \mathrm{N}$. [2 marks]

$$-5n\,P(n,3) = 24\,C(n,4) - 15$$

$$-5n\left(\frac{n!}{(n-3)!}\right) = 24\left(\frac{n!}{4!(n-4)!}\right) - 15$$

$$\frac{-5n(n)(n-1)(n-2)(n-3)!}{(n-3)!} = \frac{24(n)(n-1)(n-2)(n-3)(n-4)!}{24\,(n-4)!} - 15$$

$$-5n = n - 3 - 15$$

$$-5n = n - 18$$

$$18 = n + 5n$$

$$6n = 18$$

$$n = 3$$

Since $C(n,4)$, $n \geq 4$,

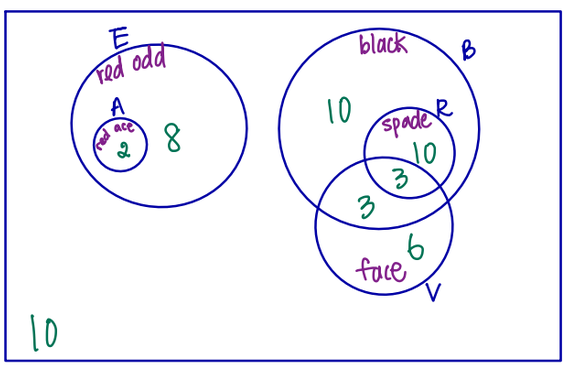
∴ there's no solutions.

THINKING (T)

1. A card game consists a full set of standard deck of cards.

Draw a Venn diagram to illustrate the relationship among the sets E, B, A, V, and R in S. [5 marks]

if S is the full set of standard deck of cards;
E is the set of all red odd number cards,
B is the set of all black cards,
A is the set of all red aces,
V is the set of all face cards, and
R is the set of all spade cards.



--- End of day 1 assessment ---

Bayview Secondary School
Mathematics Department – Course Code: MDM4U1
Assessment of Organized Counting Version 2 Day 2

Name: _____Alpha #:_____

A-26	T-5
	C-3

Instructions:

1) Show all work to obtain full marks for questions that are worth greater than 1 mark.
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APPLICATION (A)

1. How many distinct 4-letter words can be formed from the word CAREFUL if F is to be used OR all vowels are to be used? [4 marks]

CRFL (4 consonants) AEU (3 vowels.)

Case 1 "F": $\left(\frac{6}{F \text{ or } F \text{ or } F \text{ or } F} \times \frac{5}{} \times \frac{4}{} \times \frac{1}{F}\right) \times \binom{4}{1}$

⊕ Case 2 "all vowels": $\left(\frac{3}{V} \times \frac{2}{V} \times \frac{1}{V} \times \frac{4}{C}\right) \times \binom{4}{1}$

⊖ Case 3 "all vowels" and "F": $\left(\frac{3}{V} \times \frac{2}{V} \times \frac{1}{V} \times \frac{1}{F}\right) \times \binom{4}{1}$

case 1 + case 2 − case 3
= 480 + 96 − 24
= 552 words

2. How many 4-digit even numbers and the number 0 must be used if repetition is not allowed? [4 marks]

case 1: $\frac{9}{} \times \frac{8}{} \times \frac{7}{} \times \frac{1}{0}$

case 2: $\left(\frac{8}{} \times \frac{7}{0 \text{ or } 0} \times \frac{1}{} \times \frac{4}{2,4,6,8}\right) \times 2$

case 1 + case 2
= 504 + 448
= 952 numbers

3. How many diagonals there are in a 13-sided polygon? [2 marks]

$\binom{13}{2} - 13$
= 65 diagonals

4. A 3-number security code can be formed by using the numbers from 0-59 and no consecutive numbers can be the same. If Sachin forgot his code and he is trying to guess, it will take him 3 seconds to try one code. How many minutes will he need to try all of the codes? [3 marks]

$$\frac{60 \times 59 \times 59 \times 3 \text{ seconds}}{60 \text{ seconds}}$$

= 10 443 minutes

5. How many combinations are there of 12 things if four of which are identical and you need to choose five from these 12 things? [3 marks]

0 identical from + 1 identical from + 2 identical from + 3 identical from + 4 identical from

$(1) \times \binom{8}{5} + 1 \times \binom{8}{4} + 1 \times \binom{8}{3} + 1 \times \binom{8}{2} + 1 \times \binom{8}{1}$

= 56 + 70 + 56 + 28 + 8

= 218 combinations

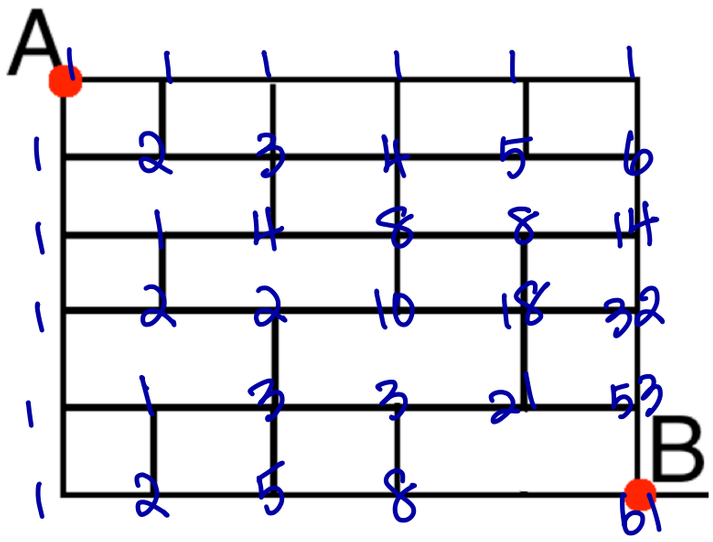
6. Elizabeth works at Indigo and is preparing for a display table for a collection of novels. There are 5 mysteries, 6 romance, 7 sci-fi and 4 thrillers. How many ways can she arrange the novels on the table if can only display two novels from each type? [3 marks]

mysteries romance sci-fi thrillers

$\binom{5}{2} \times \binom{6}{2} \times \binom{7}{2} \times \binom{4}{2} \times 8!$

= 762 048 000 ways

7. Evan is planning to travel from Point A to Point B. How many pathways can he have if he can only travel east or south along the streets? [3 marks]

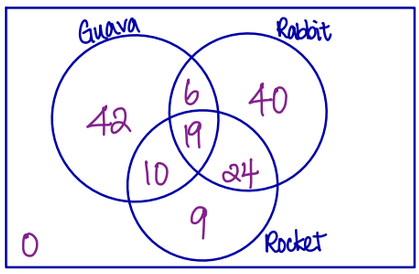


∴ 61 pathways

8. A poll about students' favourite candies collected the following data from Ms. Hung's 150 students in this semester. Assuming that all students like at least one type of candies:

- 77 students like guava candies
- 89 students like rabbit candies
- 62 students like rockets
- 25 students like guava and rabbit
- 29 students like guava and rockets
- 43 students like rabbit and rockets

a. Draw a Venn diagram to illustrate the scenario. [3 marks]



9. How many students like all three types of candies? [1 mark]

$150 = 77 + 89 + 62 - 25 - 29 - 43 + x$

$19 = x$

Thinking (T) [4 marks]

1. At a concert, four people are seated together. After the break, they return to the same four seats, but no one occupies the same seat as before. In how many ways can they seat themselves after the break? [5 marks]

Indirect method:

All − 1 return to the same seat − 2 return to the same seats − 3 return to the same seat (same as all return to the same)

$4! - \left(\frac{2}{\text{2 choices not to return to the same}} \times 1 \times 1 \times \frac{1}{\text{same}}\right)\binom{4}{1} - \left(\frac{1}{\text{1 choice not to be the same}} \times 1 \times \frac{1}{\text{same}} \times 1\right)\binom{4}{2} - \underbrace{1 \times 1 \times 1 \times 1}_{\text{all same}}$

$= 24 - 8 - 6 - 1$

$= 9$ ways

Communication (C) [3 marks]

3 marks for overall mathematical form throughout the test.

--- End of day 2 assessment ---