MHF4U1 – V2
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Full Name: Solutions

Assessment of Learning: Unit 2 – Rational Functions – DAY 1

Knowledge & Understanding	Thinking	Communication
/18	/5	/5

Instructions: Answer all questions in the space provided and **show all necessary steps**. Leave answers **exact** unless otherwise specified. The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

KNOWLEDGE & UNDERSTANDING – [18 MARKS]

Multiple Choice: Write the **CAPITAL LETTER** corresponding to the correct answer on the line provided. **[1 Mark Each – 4 Marks Total]**

1. Given the function $f(x)=\frac{-3x^2}{x^2-9}$, determine which of the following is/are **true**. D

   A. The horizontal asymptote is at $y=3$ ×
   B. A vertical asymptote is at $x=-3$ ✓
   C. The y-intercept is 0 ✓
   D. Both B & C ✓

2. Given the function $f(x)=\frac{x^a+k}{x^b+m}$, where k and m are integers, a horizontal asymptote of $y=0$ will occur when: B

   A. $a>b$ B. $b>a$ C. $a-b=1$ D. $a-b=2$ E. None of the above

3. The function $f(x)=\frac{(x-1)(x+1)(x+3)}{(x-4)(x-1)(2x+5)}$ has B

   A. vertical asymptotes at $x=4, x=1$ and $x=-\frac{5}{2}$ and a horizontal asymptote at $y=2$.
   B. vertical asymptotes at $x=4$ and $x=-\frac{5}{2}$ and a horizontal asymptote at $y=\frac{1}{2}$.
   C. vertical asymptotes at $x=4, x=1$ and $x=-\frac{5}{2}$ and a horizontal asymptote at $y=\frac{1}{2}$.
   D. vertical asymptotes at $x=4$ and $x=-\frac{5}{2}$ and a horizontal asymptote at $y=2$.

4. Given $f(x)=\frac{-2(x+3)(x-12)}{(x+1)}$, which of the following statements is **true**? A

   A. The $y-$intercept is at $(0, 72)$.
   B. The function has a horizontal asymptote at $y=2$.
   C. There is a vertical asymptote at $x=-2$.
   D. The $x-$intercepts are at $x=3$ and $x=-12$.

5. Consider the function $y=\frac{-4}{(-3x-8)^2}$ and determine the following. **[7 Marks]**

   $y=\frac{-4}{(3x+8)^2}$

   i) Asymptote(s)

   Vertical: $x=-8/3$

   Horizontal: $y=0$

   Oblique: none

   ii) x-int(s): none

   y-int: $-\frac{1}{16}$

   iii) Domain: $\{x\in\mathbb{R} \mid x\neq -8/3\}$

   Range: $\{y\in\mathbb{R} \mid y<0\}$

6. Determine the equation of a rational function, in factored form, that has the following properties: **[3 Marks]**
   - zeros at $x=\pm 2$
   - vertical asymptotes at $x=-3$ and $x=-1$.
   - horizontal asymptote at $y=3$

   Equation: $f(x)=\frac{3(x-2)(x+2)}{(x+3)(x+1)}$

7. Determine the exact point(s) of intersection between $f(x) = x^2 - 5x + 5$ and its reciprocal function.
[4 Marks]

$-1 = x^2 - 5x + 5$
$0 = x^2 - 5x + 6$
$0 = (x-3)(x-2)$
$\therefore x = 3$ or 2

$1 = x^2 - 5x + 5$
$0 = x^2 - 5x + 4$
$0 = (x-1)(x-4)$
$\therefore x = 1$ or 4

$\therefore$ point of intersections are $(3,-1)$, $(2,-1)$, $(1,1)$, and $(4,1)$

THINKING – [5 MARKS]

1. Given $f(x) = -x^2 + 4x + 3$, determine the equation (in standard form) of the line that joins the local extremum of $f(x)$ with the local extremum of the **reciprocal** function of $g(x) = x^2 + 4x + 6$. **[5 Marks]**

$f(x) = -(x^2 - 4x + 4 - 4) + 3$
$= -(x-2)^2 + 7$
$\therefore$ max at $(2,7)$

$g(x) = x^2 + 4x + 6$
$= (x^2 + 4x + 4 - 4) + 6$
$= (x+2)^2 + 2$
$\therefore$ min at $(-2,2)$
$\therefore$ reciprocal function has max at $(-2, \frac{1}{2})$.

slope of line:

$m = \dfrac{7 - \frac{1}{2}}{2-(-2)}$
$= \dfrac{\frac{13}{2}}{4}$
$= \dfrac{13}{8}$

Sub $x = 2, y = 7, m = \frac{13}{8}$ into $y = mx + b$

$7 = \frac{13}{8}(2) + b$
$7 - \frac{13}{4} = b$
$\frac{15}{4} = b$

$\therefore y = \frac{13}{8}x + \frac{15}{4}$

standard form: $8y = 13x + 30$
$\therefore 0 = 13x - 8y + 30$ is equation of line.

COMMUNICATION – [5 MARKS]

1. Explain how to find the intersection of a rational function and its horizontal asymptote. **[3 Marks]**

Set the rational function equal to its horizontal asymptote and solve for x. The x-value(s) is/are where they intersect.
If you cannot solve for the x-value(s) then the function doesn't intersect the asymptote.