/—

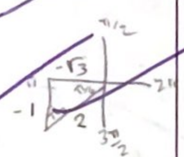
MHF 4U1- Unit 4 Test AM - Trigonometry

Name Yusuf Ali Juffer Date Nov 25

Total
38
/40

KNOWLEDGE

1. Determine the exact values of the following expressions. Be sure to include any necessary diagrams and show all work to support your final answer.

$cos(\frac{7\pi}{6})$ [2]	$sin(-\frac{\pi}{12})$ [2]
$\cos(\frac{7\pi}{6}) = \frac{a}{h}$  $= \frac{-\sqrt{3}}{2}$	$\sin(-\frac{4\pi}{12}+\frac{3\pi}{12})$  $\sin(\frac{\pi}{4}-\frac{\pi}{3})$ $= \sin\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\cos\frac{\pi}{4}$ $= (\frac{1}{\sqrt{2}})(\frac{1}{2}) - (\frac{\sqrt{3}}{2})(\frac{1}{\sqrt{2}})$ $= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$ $= \frac{1-\sqrt{3}}{2\sqrt{2}} = \frac{2\sqrt{2}-2\sqrt{6}}{8} = \frac{\sqrt{2}-\sqrt{6}}{4}$

2. Simplify the following expression:

$$2sin^2(x) + cos(2x) - tan(x)cot(x)$$

[2]

$= 2\sin^2 x + (1-2\sin^2 x) - \tan x \cdot \frac{1}{\tan x}$

$= 1 - \frac{\tan x}{\tan x}$

$= 1 - 1$

$= 0$

6

< 2.917
= 1.176 = x

CAST

3. Given $\csc x = \frac{13}{12}$ and $\cos y = -\frac{3}{5}$ with $\frac{\pi}{2} \le x \le \pi$, $\pi \le y \le \frac{3\pi}{2}$ evaluate the following:

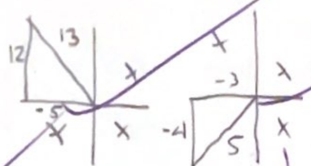
Use diagrams for full marks. Rationalize your answer.

$\csc x \rightarrow \frac{1}{\sin}$

$\csc x = \frac{13}{12}\,\frac{h}{o}$

CAST

a) $\tan(x - y)$ ~~[3]~~ 2.5



$\frac{\sin(x-y)}{\cos(x-y)}$

$\frac{\sin x \cos y - \sin y \cos x}{\cos x \cos y + \sin x \sin y}$

$= \frac{(\frac{12}{13})(-\frac{3}{5}) - (-\frac{4}{5})(-\frac{5}{13})}{(-\frac{5}{13})(-\frac{3}{5}) + (\frac{12}{13})(-\frac{4}{5})} \rightarrow = \frac{-36}{65} - \frac{20}{65} \div \frac{15}{65} + -\frac{48}{65}$

$= -\frac{56}{65} \div -\frac{33}{65} \rightarrow = \frac{56}{33}$

Math error!

b) $\cos 2x$ [3]

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = \cos x \cos x - \sin x \sin x$

$\cos 2x = (-\frac{5}{13})(-\frac{5}{13}) - (\frac{12}{13})(\frac{12}{13})$

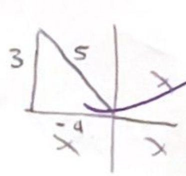
$\cos 2x = \frac{25}{169} - \frac{144}{169}$

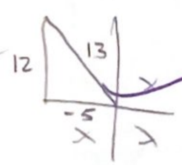
$= -\frac{119}{169}$

4. Angles "x" and "y" are located in the second quadrant such that $\sin(x) = \frac{3}{5}$ and $\cos(y) = \frac{-5}{13}$.

(a) Determine the exact value for $\cos(x)$ and $\sin(y)$.

$\sin x = \frac{3}{5}$ $\cos y = -\frac{5}{13}$





$13^2 - (-5)^2 = 12$

$5^2 - 3^2 = 4$

$\boxed{\cos x = -\frac{4}{5} \quad \sin y = \frac{12}{13}}$

[2]

(b) Using the information above, determine an exact value for 2 of the following (You pick which 2!).

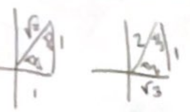
[4]

$\sin(x + y)$ $\sin x \cos y + \sin y \cos x$ $= (\frac{3}{5})(-\frac{5}{13}) + (\frac{12}{13})(-\frac{4}{5})$ $= \frac{-15}{65} + -\frac{48}{65}$ $\rightarrow -\frac{63}{65}$ Keep an eye out for signs!	$\cos(x + y)$
$\sin(x - y)$ $\sin x \cos y - \sin y \cos x$ $= (\frac{3}{5})(-\frac{5}{13}) - (\frac{12}{13})(-\frac{4}{5})$ $= -\frac{15}{65} + \frac{48}{65}$ $= \frac{33}{65}$	$\cos(x - y)$

11.5

APPLICATION

1. Determine the exact values of the following expressions. Be sure to show all your work and include any necessary diagrams.

$2\cos^2(\frac{5\pi}{24}) - 1$ [4]	$\sin(\frac{-10\pi}{9})\cos(\frac{13\pi}{9}) + \cos(\frac{-10\pi}{9})\sin(\frac{13\pi}{9})$ [3]
$2\cos^2 x - 1$ $= \cos 2x$ $= \cos 2(\frac{5\pi}{24})$ $= \cos(\frac{12\pi}{24})$ $= \cos(\frac{5\pi}{12})$ $= \cos(\frac{3\pi}{12} + \frac{2\pi}{12})$ $= \cos(\frac{\pi}{4} + \frac{\pi}{6})$ $= \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$ $= (\frac{1}{\sqrt{2}})(\frac{\sqrt{3}}{2}) - (\frac{1}{\sqrt{2}})(\frac{1}{2})$ $= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$ $= \frac{\sqrt{3}-1}{2\sqrt{2}}$ $= \frac{2\sqrt{6}-2\sqrt{2}}{8}$ $= \frac{\sqrt{6}-\sqrt{2}}{4}$ 	$\sin x\cos y + \sin y\cos x$ $\sin(x+y)$ $\sin(-\frac{10\pi}{9} + \frac{13\pi}{9})$ $\sin(\frac{3\pi}{9})$ $\sin(\frac{\pi}{3})$ $= \frac{\sqrt{3}}{2}$ 

2. Prove the following identities, if possible. Full marks will only be awarded for solutions that are presented clearly and follow a logical flow.

a) $\frac{1-\tan^2 x}{1+\tan^2 x} = \cos 2x$ ✓ [2]

LS: $1 - \frac{\sin^2 x}{\cos^2 x} \div 1 + \frac{\sin^2 x}{\cos^2 x}$

$\frac{\cos^2 x - \sin^2 x}{\cos^2 x} \div \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$

$\frac{\cos 2x}{1}$

$\cos 2x$ ✓

LS = RS

b) $\cos(4x) = 8\sin^4 x - 8\sin^2 x + 1$ ✓ [3]

LS: $\cos 2(2x)$

$1 - 2\sin^2(2x)$

$1 - 2(2\sin\cos)^2$

$1 - 2(4\sin^2\cos^2)$

$1 - 8\sin^2 x\cos^2 x$

$1 - 8\sin^2 x(1 - \sin^2 x)$

$1 - 8\sin^2 x + 8\sin^4 x$

$8\sin^4 x - 8\sin^2 x + 1$

LS = RS ✓

/12

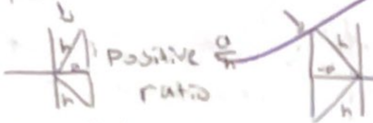
COMMUNICATION

1. Explain why there will be at most 2 solutions for $cos(x) = -\frac{a}{b}$, if $0 \le x \le 2\pi$ and $a > 0,\ b > 0$.

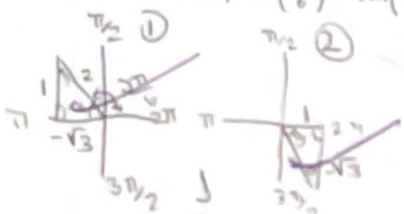
$\cos = -\frac{a}{b}\ \frac{a}{h}$

$a = a$
$b = h$

$\frac{a}{b}$ is negative given each value is over 0. The 4 graphs below are possible. As seen, two triangles give $-\frac{a}{h}$ (Q_2, Q_3). This is verified by the CAST rule where if cos is negative (ratio-wise), it must be in Q_2, Q_3. [2]



2. Explain why $cos\left(\frac{5\pi}{6}\right) = sin\left(-\frac{\pi}{3}\right)$. Use a diagram to support your answer. [3]

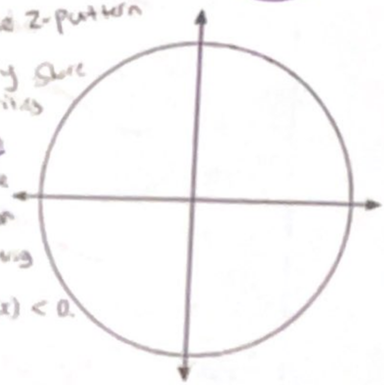


They are flipped! Same triangle but inverted!

$\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

Due to the Z-pattern rule, they share many similarities as a 30-60-90 triangle. Since they are the flipped version of each other, their opposing (trig ratios) are equivalent ($\frac{a}{h} = \frac{a}{h}$).



THINKING

1. Determine the value of $sec(x)$ if $cot(x) = -a$, where $a > 0$, and $sin(x) < 0$.

$\cot(x) = -a$ ② $\sec x = \frac{1}{\cos x}$

$\frac{\cos x}{\sin x} = \frac{\cos x}{-\sin x} = -a$ $\boxed{\sec x = \frac{1}{a \sin x}}$ ok, but we want this in terms of only "a".

① $\cos x = a \sin x$ ok

~~[2]~~ 0.5

2. Two gears work together. The smaller gear has a radius of 5cm and the larger gear has a radius of 12cm.

(a) Determine the arc length that the lower gear moves through when it rotates 300°.

g_s $r = 5cm$

g_l $r = 12cm$

$300° = \frac{300\pi}{180}$
$= \frac{12\pi}{6}$
$= \frac{5\pi}{3}$

$a = r\theta$
$a = 5\left(\frac{5\pi}{3}\right)$
$a = \frac{25\pi}{3}$ cm (26.18cm)

∴ The smaller gear rotates $\frac{25\pi}{3}$ cm in arc length

[1]

(b) Determine the number of radians that the larger gear rotates when the smaller gear rotates 300°.

$a = \frac{25\pi}{3}$ cm
$\theta = ?$
$r = 12cm$

$a = \theta r$
$\frac{a}{r} = \theta$
$\frac{25\pi}{3} \div 12 = \theta$
$\frac{25\pi}{36} = \theta$

$\frac{25\pi}{36}$ rad

∴ The larger gear rotates $\frac{25\pi}{36}$ rad

[2]

/8.5