 Name: Solutions

Unit 5: Exponential & Logarithmic Functions – Assessment of Learning – DAY 1

K & U	Thinking	Comm.
/18	/5	/2

Instructions:

- Non-graphing calculators may be used but not shared. Notebooks may not be used.
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KNOWLEDGE & UNDERSTANDING – [18 Marks]

Multiple Choice: Write the CAPITAL letter corresponding to the correct answer on the line provided. **[6 Marks]**

1. The inverse of $y = 3^{x-2}$ is C

   A. $y = \log_3(x) - 3$ B. $y = 1 - \log_3(x)$ C. $y = \log_3(x) + 2$ D. $y = 2\log_3(x) + 1$

2. The range of $y = -\log_5(x) + 3$ is D

   A. $y \in (3, \infty)$ B. $y \in (-\infty, 3)$ C. $y \in (-\infty, 0)$ D. $y \in (-\infty, \infty)$

3. The hydrogen-ion concentration of a solution with a pH level of 7.28 is A

   A. 0.000000052 B. 1,9054,607 C. 0.00000002 D. 0.0055

4. An earthquake that is 100 times as intense as an earthquake measuring 5.2 on the Richter scale has a magnitude of

   A. 7.2 B. 15.2 C. 6.2 D. 520 A

5. Which expression is equivalent to $\log\left(\frac{\sqrt{B}A}{C^4}\right)$? A

   A. $\frac{1}{2}\log(B) + \log(A) - 4\log(C)$ C. $\frac{1}{2}\log(B) + 4\log\left(\frac{A}{C}\right)$

   B. $\frac{\frac{1}{2}\log(A+B)}{4\log(C)}$ D. $\frac{\frac{1}{2}\log(B) + \log(A)}{4\log(C)}$

6. Which of the following is **false** regarding the graphs of $f(x) = \log(x)$ and $g(x) = 10^x$? C

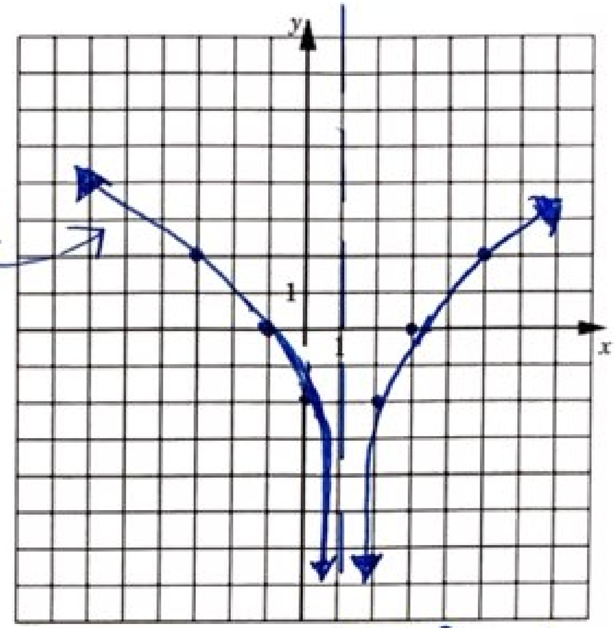
   A. Both graphs have an asymptote.
   B. Both functions are always increasing.
   C. The ranges of the $f(x)$ and $g(x)$ are different but the domains are the same.
   D. $f(x)$ is the inverse of $g(x)$.

7. Sketch a properly labelled graph of $y = \log_2(x-1)^2 - 2$. Clearly plot and label a **minimum of 6 points** on the graph. **[4 Marks]**

x	y
2	-2
3	0
5	2
0	-2
-1	0
-3	2

VA @ x=1

$y = \log_2(x-1)^2 - 2$



| VA @ x=1

8. Solve each of the following. Round your answers to 2 decimal places, if necessary. **[5 Marks]**

a. $7^{-3x+4} = 18$ **[2]**

$(-3x+4)\log 7 = \log 18$

$-3x+4 = \frac{\log 18}{\log 7}$

$-3x = \frac{\log 18}{\log 7} - 4$

$x = \frac{-1}{3}\left[\frac{\log 18}{\log 7} - 4\right]$

$x \doteq 0.84$

b. $\log_{12}(x-2) - \log_{12}(x-3) = 1$ **[3]**

$\frac{x-2}{x-3} = 12$

$x-2 = 12x - 36$

$11x = 34$

$x = \frac{34}{11}$

9. Write $3\log_{\frac{1}{2}}(xy) + \log_{\frac{1}{2}}(x) - \frac{1}{3}\log_{\frac{1}{2}}(y^6)$ as a single fully simplified logarithm. **[3 Marks]**

$= 3\left[\log_{\frac{1}{2}} x + \log_{\frac{1}{2}} y\right] + \log_{\frac{1}{2}} x - 2\log_{\frac{1}{2}} y$

$= 4\log_{\frac{1}{2}} x + \log_{\frac{1}{2}} y$

$= \log_{\frac{1}{2}}\left(\frac{x^4}{y}\right)$

THINKING – [5 Marks]

1. To try and simplify $\frac{\log a}{\log b}$, Mr. Vincent **incorrectly** '*crossed out*' the ' log ' getting $\frac{a}{b}$ which he then reduced to $\frac{4}{3}$. Amazingly, and coincidentally, the correct answer when $\frac{\log a}{\log b}$ is simplified is $\frac{4}{3}$. Determine the **exact** values of a and b. **[5 Marks]**

$$\frac{a}{b} = \frac{4}{3}$$

$$3a = 4b$$

$$a = \frac{4b}{3}$$

& $$\frac{\log a}{\log b} = \frac{4}{3}$$

$$3\log a = 4\log b$$

$$\log a^3 = \log b^4$$

$$\log\left(\frac{4b}{3}\right)^3 = \log b^4$$

$$\therefore \frac{64}{27}b^3 = b^4$$

$$\frac{64}{27} = b$$

$$a = \frac{4b}{3}$$

$$a = \frac{4\left(\frac{64}{27}\right)}{3}$$

$$a = \frac{256}{81}$$

** Two (2) Marks will be awarded in the Communication Category for the use of proper mathematical form. **

Name: Solutions

Unit 5: Exponential & Logarithmic Functions – Assessment of Learning – DAY 2

Application	Thinking	Comm.
/18	/5	/2

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APPLICATION – [18 Marks]

1. Determine the x – intercept(s) of the function $y = \log_{2022}(-x^2 + 4x + 4) - 3\log_{2022}(x)$. **[4 Marks]**

set $y = 0$

$\log_{2022}(-x^2+4x+4) - 3\log_{2022}(x) = 0$

$\frac{-x^2+4x+4}{x^3} = 1$

$x^3 + x^2 - 4x - 4 = 0$

$x^2(x+1) - 4(x+1) = 0$

$(x-2)(x+2)(x+1) = 0$

$\therefore x = \pm 2, -1$

reject -2 and -1 violates the domain.

$\therefore x = 2$

2. If $\log_a(b) = p^3$ and $\log_b(\sqrt{a}) = \frac{2}{p}$, determine the **exact** value of p. **[4 Marks]**

$\log_a(b) = p^3$

$\log_b(a) = \frac{1}{p^3}$

$\log_b \sqrt{a} = \frac{2}{p}$

$\log_b a^{\frac{1}{2}} = \frac{2}{p}$

$\frac{1}{2}\log_b a = \frac{2}{p}$

$\log_b a = \frac{4}{p}$

$\therefore \frac{1}{p^3} = \frac{4}{p} \Rightarrow 4p^3 = p$

$4p^3 - p = 0$

$p(4p^2 - 1) = 0$

$p(2p+1)(2p-1) = 0$

$p = 0, -\frac{1}{2}, \frac{1}{2}$ $\therefore p = \pm\frac{1}{2}$

reject $\div$ by 0.

3. A baby cries at a loudness of 70 dB. When the baby starts screaming the resulting sound is 25 times louder than when she cries. Determine the loudness of the scream in decibels. Round your answer to 2 decimal places. **[3 Marks]**

$$\beta_2 - \beta_1 = 10 \log\left(\frac{I_2}{I_1}\right)$$

$$\beta_2 - 70 = 10 \log(25)$$

$$\beta_2 = 10 \log(25) + 70$$

$$\beta_2 \doteq 83.98 \text{ dB}$$

4. Amy invested \$10,000 in a bank account that earns 4.5% per annum, compounded weekly. How many years will it take for her investment to **triple**? Round your answer to 2 decimal places. **[3 Marks]**

$$A = P(1+i)^n$$

$$30000 = 10000\left(1 + \frac{0.045}{52}\right)^{52\cdot n}$$

$$3 = \left(1 + \frac{0.045}{52}\right)^{52n}$$

$$\log 3 = 52n \cdot \log\left(1 + \frac{0.045}{52}\right)$$

$$n = \frac{\log 3}{52 \log\left(1 + \frac{0.045}{52}\right)}$$

$$n \doteq 24.42 \text{ years}$$

5. Solve: $\left(3^{2x}\right) + 2\left(3^{x+1}\right) - 27 = 0$. **Exact** answer(s). **[4 Marks]**

$$3^{2x} + 6\,3^x - 27 = 0$$

Let $t = 3^x$

$$t^2 + 6t - 27 = 0$$

$$(t+9)(t-3) = 0$$

$t = -9$	$t = 3$
$3^x = -9$	$3^x = 3$
No solution.	$\therefore x = 1$

THIINKING – [5 Marks]

1. Solve: $\log_{\sqrt{x}}\left(5^{\log_x(5)}\right)+\log_x(125)-2=0$. **Exact** answers(s). **[5 Marks]**

$\log_{\sqrt{x}}\left(5^{\log_x 5}\right)+\log_x 125-2=0$

$\log_x 5 \cdot \log_{\sqrt{x}} 5+\log_x 5^3-2=0$

$\log_x 5 \cdot 2\log_x 5+3\log_x 5-2=0$

$2(\log_x 5)^2+3\log_x 5-2=0$

let $t=\log_x 5$

$2t^2+3t-2=0$

$(2t-1)(t+2)=0$

$t=\frac{1}{2}$, $t=-2$

$\log_x 5=\frac{1}{2}$	$\log_x 5=-2$
$x^{\frac{1}{2}}=5$	$x^{-2}=5$
$x=25$	$x^2=\frac{1}{5}$

$x=\pm\frac{1}{\sqrt{5}}$

$\therefore x=25, \frac{1}{\sqrt{5}}$ reject $-\frac{1}{\sqrt{5}}$ violates base restriction

** Two (2) Marks will be awarded in the Communication Category for the use of proper mathematical form. **