MHF 4U1- Unit 2 Test am

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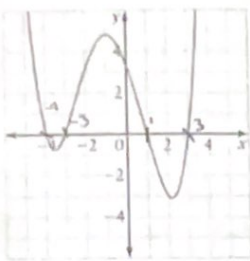
Total
28
/30

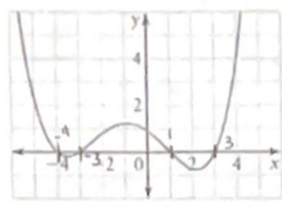
KNOWLEDGE: For Questions 1 to 5 Circle one answer: (5) 4

1. Which of the graphs does not belong to the same family?

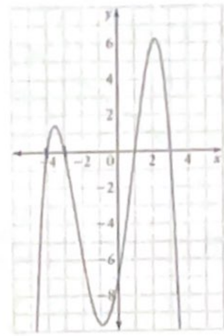
A



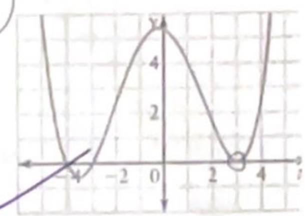
B



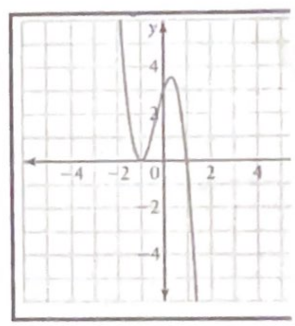
C



D



2. What is the equation for the cubic function represented by the graph to the right?



a) $y = 3(x-1)^2(x+1)$ b) $y = 3(x+1)^2(x-1)$

c) $y = -3(x-1)^2(x+1)$ d) $y = -3(x+1)^2(x-1)$

$y = k(x+1)^2(x-1)$

$3 = k(1)^2(-1)$

$3 = -k$

$-3 = k$

3. Which of the following is a factor of the polynomial, $P(x) = -2x^3 - x^2 + 3x - 6$?

a) $(x-1)$ b) $(x+2)$

c) $(x+1)$ d) $(x-2)$

X

4. What is the remainder when the polynomial $P(x) = 8x^3 - 4x^2 + 2x + 1$ is divided by $(2x+1)$?

a) 2 b) -4 c) -2 d) 4

$-\frac{1}{2}$

$-\frac{1}{2}$ | 3 -4 12 +1

↓ -4 +4 -3

3 -3 6 -2

5. Which of the following **does not** belong to the same family?

a) $y = 3.5(x+2)(x-1)(x-3)$ b) $y = (4x-12)(x+2)(x-1)$ $(x-3)$

c) $y = -0.2(x-3)(2x+4)(2x-3)$ d) $y = -7(x-1)(x-3)(x+2)$

6. The zeros of a cubic function are -8, 3i, and -3i.

Determine an equation in **standard form** for the member of the family that has a y-intercept of -24. (2) 1.5

$y = \frac{1}{3}(x^3 + 3x^2 - 9x - 72)$

$y = -\frac{1}{3}x^3 + \frac{3}{3}x^2 - 3x - 24$

$y = k(x-(-3))(\cancel{x-(3i)})(x-(-3i))$

$-24 = k(3)(-3i)(3i)$

$-3 = k(-9(-1))$

$-3 = 9k$

$-\frac{3}{9} = k$

$-\frac{1}{3} = k \longrightarrow y = \frac{1}{3}(x+3)(x-3i)(\cancel{x+3i})$

$x = 0$

$y = -24$

Let $-3i = L$

Let $3i = m$

$3 \cdot -3i$

$Lm = 9(-1)$

$Lm = -9$

$y = \frac{1}{3}(x+3)(x+L)(x+m)$

$y = \frac{1}{3}(x^2 + xL + 3x + 3L)(x+m)$

$y = \frac{1}{3}(x^3 + x^2m + x^2L + mxL + 3x^2 + 3xm + 3Lx + 3Lm)$

$y = \frac{1}{3}(x^3 + \cancel{x^2(3i)} + \cancel{x^2(-3i)} + (-9x) + 3x^2$

$+ \cancel{3x(3i)} + \cancel{3x(-3i)} + 3(-9))$

6.5

$10 = 8$ ✓

7. When $2x^3k - k^2x^2 + kx + 2$ is divided by $x + 2$, the remainder is 10. Find the value(s) of k. (3)

$10 = 2(-2)^3k - k^2(-2)^2 + k(-2) + 2$

$8 = -16k - 4k^2 - 2k$

$0 = -4k^2 - 18k - 8$ $\quad -16 \times -2 = 32$

$0 = -4k^2 - 16k - 2k - 8$ $\quad -16 + -2 = -18$

$= -4k(k+4) - 2(k+4)$

$= (-4k-2)(k+4)$

$-4k - 2 = 0$

$-4k = 2 \rightarrow k = -2/4 \rightarrow -1/2$

3

K can be either -4 or $-\frac{1}{2}$ (-0.5)

Check

$10 = 2(-2)^3(4) - (4)^2(-2$

✓ $10 = 64 \ldots + 8 + 2$

COMMUNICATION:

1. What is the difference between solving a polynomial equation and a polynomial inequality?

In an inequality, the goal is to find domain of x to satisfy the condition, in a normal polynomial equation the goal is to find the zeros (x-int) of the function. In an inequality question, we isolate the zeros and plot them. Then we find the intervals between or equal to zeros that satisfy the condition. In solving polynomial equations, we solve for the zeros and stop there.

2

2. Suppose the degree of a polynomial function is 9. What is the maximum number of real roots this polynomial can have?

The max number of real roots is 9, as seen in the example on the left. In this case, there are 9 1 degree roots, giving the maximum.

(2) 2

APPLICATION:

1. Graph the function $f(x) = 2x^3 + 3x^2 - 17x + 12$. Clearly label the x- and y- intercepts. Show all your work! (4)

4

$f(x) = 2x^3 + 3x^2 - 17x + 12$ $\quad$ Roots: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 12, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

$1 \,|\, 2 \quad 3 \quad -17 \quad 12$

$\quad\;\; 2 \quad 5 \quad -12$

$\quad\;\; 2 \quad 5 \quad -12$

$\rightarrow 2x^2 + 5x - 12$ $\quad 8 \times -3 = -24$

$2x^2 + 8x - 3x - 12$ $\quad -3 + 8 = 5$

$2x(x+4) - 3(x+4)$

$(2x-3)(x+4)$

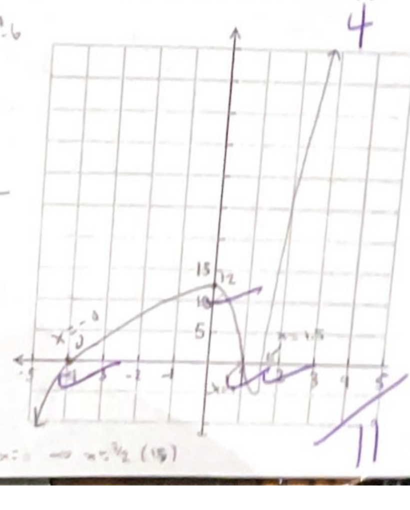
$y = k(x-1)(2x-3)(x+4)$

$\boxed{y = (x-1)(2x-3)(x+4)}$

$12 = k(-1)(-3)(4)$

$12 = 12k$

$\underline{1 = k}$



$2x-3 = 0 \quad 2x = 3 \rightarrow x = 3/2$ (1.5)

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2. Solve the following for all possible roots (please indicate real, irrational or complex) (4) 3,5

$8x^4 = x$

$(2x-1)(4x^2+2x+1)$ $2x-1=0$

$8x^3+4x^2+2x-4x^2-2x-1$ $2x=1$ $x=\frac{1}{2}$ $8x^4-x=0$

$x(8x^3-1)=0$

Real Roots: $x=0$, $x=0.5$

$x(2x-1)(4x^2+2x+1)=0$

Complex/imaginary: $x=\frac{-1-\sqrt{3}i}{4}$, $\frac{-1+\sqrt{3}i}{4}$ keep the roots!

$\frac{-2\pm\sqrt{(2)^2-4(1)(4)}}{2(4)} \rightarrow \frac{-2\pm\sqrt{-12}}{8} \rightarrow \frac{-2\pm 2\sqrt{3}}{8} \rightarrow \frac{-1\pm\sqrt{3}}{4}$ $\rightarrow \frac{-1+\sqrt{3}i}{4}$, $\frac{-1-\sqrt{3}i}{4}$

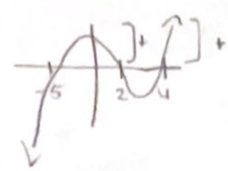
3. If $f(x) = x^4 + 4x^3 - 3x^2 - 16x + 20$ and $g(x) = x^4 + 3x^3 - 2x^2 + 6x - 20$, solve the inequality $f(x) \geq g(x)$ using an interval chart/table.

$x^4+4x^3-3x^2-16x+20 \geq x^4+3x^3-2x^2+6x-20$

$x^4-x^4+4x^3-3x^3-3x^2+2x^2-16x-6x+20+20 \geq 0$

Roots: $\pm1, \pm2, \pm4, \pm5$... $x^3-x^2-22x+40 \geq 0$ (5)

$(x+5)(x-4)(x-2) \geq 0$ 5



$$\begin{array}{r|rrrr} 2 & 1 & -1 & -22 & +40 \\ & \downarrow & 2 & 2 & -40 \\ \hline & 1 & 1 & -20 & \end{array}$$

x^2+x-20

$(x+5)(x-4)$

Root: 2, 4, -5

Intervals	$-x \leq -5$ (-6)	$-5 \leq x \leq 2$ (1)	$2 \leq x \leq 4$ (3)	$x \geq 4$ (5)
$x+5$	−	+	+	+
$x-4$	−	−	−	+
$x-2$	−	−	+	+
$(x+5)(x-2)(x-4) \geq 0$	−	+	−	+

Therefore, $f(x) \geq g(x)$ when : $(-5 \leq x \leq 2)$ or $(x \geq 4)$

Essentially between or equals to -5 and 2 or greater or equal than 4 is when the solution is greater or equal to zero.

/8.5

THINKING:

1. The solutions below correspond to an inequality involving a degree 8 function. Write **two** possible degree 8 polynomial inequalities in factored form that satisfy the solutions given. (3)

$x < -\frac{5}{2},\ \frac{3}{2} < x < \frac{5}{2},\ x > 7$

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$x > 7$ is positive

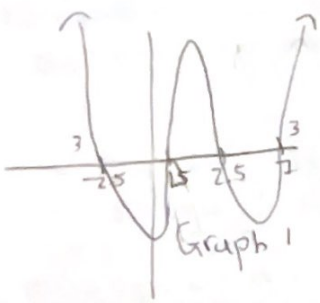
$x - 7 > 0$

$x + \frac{5}{2} < 0$

$x < -5/2$ is positive

Roots: 7, -2.5, 1.5

$2x + 5$



Graph 1

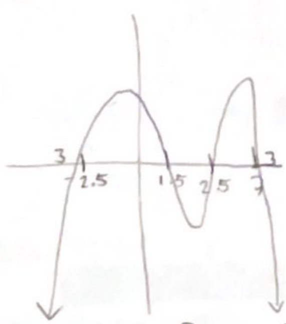
$(2x+5)^3(2x-3)^1(2x-5)^1(x-7)^3 \geq 0$

↓

$\{x \in \mathbb{R} \mid x \geq -2.5,\ 1.5 < x < 2.5,\ x > 3\}$

Inequality one

- All intervals are satisfied such that they are greater to zero



Graph 2

$-(2x+5)^3(2x-3)^1(2x-5)^1(x-7)^3 < 0$

↓

$\{x \in \mathbb{R} \mid x < -2.5,\ 1.5 < x < 2.5,\ x > 7\}$

Inequality Two

- All intervals are satisfied such that they are less less than zero

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