MHF4U1-V2
Leung, Moshtagh, Vincent

Name: Solutions

Unit 1: Polynomial Functions - Assessment of Learning – DAY 1

K & U	Thinking	Comm.
/18	/5	/2

Instructions: Answer all questions in the space provided and **show all necessary steps**. Leave answers **exact** unless otherwise specified. The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

KNOWLEDGE & UNDERSTANDING – [18 MARKS]

Multiple Choice: Write the CAPITAL letter corresponding to the correct answer on the line provided.
[5 Marks Total – 1 Mark Each]

1. The remainder, when x^3-4x^2+5 is divided by $x+3$ is **A**

   A. -58 B. -4 C. 14 D. 122

2. Which of the following statements is/are **true** for $f(x)=2(5-x)(x-1)(x-3)(x+2)^2$? **C**

   I. as $x\to\infty$, $f(x)\to-\infty$
   II. as $x\to\infty$, $f(x)\to\infty$
   III. as $x\to-\infty$, $f(x)\to-\infty$
   IV. as $x\to-\infty$, $f(x)\to\infty$

   A. I only.
   B. I and II only.
   C. I and IV only.
   D. II and III only.

3. Which statement is **true** regarding $f(x)=-2(3-x)(x^2-4)^2$? **B**

   A. The leading coefficient is negative.
   B. The constant value of the function's finite differences is equal to 240.
   C. $f(x)$ goes through the x-axis once and bounces on the x-axis once.
   D. As $x\to\infty$, $f(x)\to-\infty$

4. Which of the following functions is/are even? **C**

   I. $f(x)=-2(x-1)^2(x+1)$
   II. $f(x)=4x^4+9x^2-16$
   III. $f(x)=-3x^5+9x^3+2x$
   IV. $f(x)=3x^3+8x+1$

   A. III and IV. B. I and II. C. II only D. IV only.

5. When completely factored, $(125x)^3+(1331y)^3$ equals **B**

   A. $(5x-11y)(25x^2-55xy+121y^2)$
   B. $(5x+11y)(25x^2-55xy+121y^2)$
   C. $(5x-11y)(25x^2+55xy+121y^2)$
   D. $(5x+11y)(25x^2+55xy+121y^2)$

6. Divide $(2x^3-8x^2-3)$ by $(2x-4)$ using polynomial division and then write the division statement.
   [3 Marks]

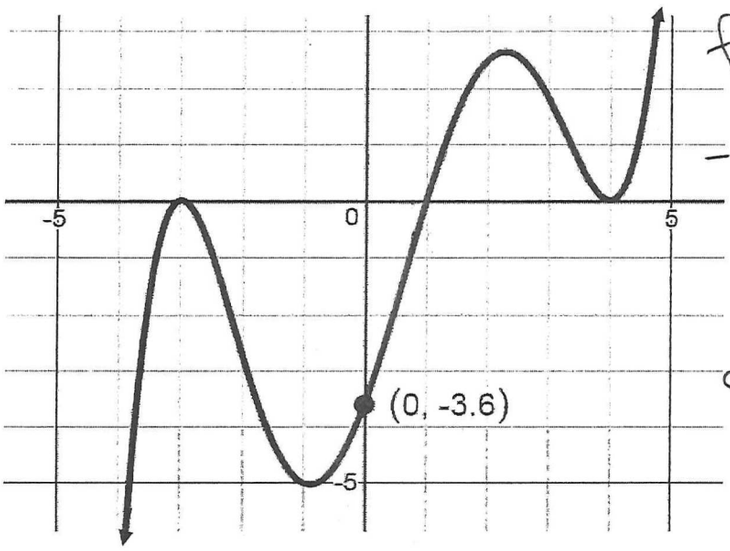
$$
\begin{array}{r|l}
2x-4 & x^2-2x-4 \\
\hline
 & 2x^3-8x^2+0x-3 \\
 & -(2x^3-4x^2) \\
 & \quad -4x^2+0x \\
 & \quad -(-4x^2+8x) \\
 & \quad\quad -8x-3 \\
 & \quad\quad -(-8x+16) \\
 & \quad\quad\quad -19 \text{ R}
\end{array}
$$

Division Statement: $2x^3-8x^2-3=(2x-4)(x^2-2x-4)-19$, $x\neq 2$

MHF4U1-V2
Leung, Moshtagh, Vincent

Name: _____

7. Determine the **specific equation** (assume lowest possible degree) of the function below. **[4 Marks]**



$f(x) = a(x+3)^2(x-1)(x-4)^2$

$-3.6 = a(9)(-1)(16)$

$a = \frac{-3.6}{-144}$

$a = \frac{1}{40}(x+3)^2(x-1)(x-4)^2$

8. State the family of polynomial functions of degree 8 with a double root at $x = 4$, a root at $x = -1$ and a point of inflection at $x = \frac{2}{3}$. **[3 Marks]**

$f(x) = a(x-4)^2(x+1)(3x-2)$ OR $f(x) = a(x-4)^4(x+1)(3x-2)^3$

$a \in \mathbb{R}, a \neq 0$ $\quad$ $a \in \mathbb{R}, a \neq 0$

9. The height of a toy plane above the ground is modeled by $h(t) = -5t^2 + 40t + 81$, where $h(t)$ is the height in metres and t is time in seconds. Estimate the IROC at $t = 3$ seconds. **[3 Marks]**

interval	AROC
$2.99 \le t \le 3$	10.05 ↓
$2.999 \le t \le 3$	10.005 ↓
$3 \le t \le 3.001$	9.995 ↑
$3 \le t \le 3.01$	9.95 ↑

∴ IROC is approx. 10 m/s

THINKING – [5 MARKS]

1. Determine the values of a, b, c, d, e and f for $f(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ given the following information:
   - $x - 1$ is a factor of $f(x)$.
   - $f(x)$ is an odd function. → $b, d, f = 0$
   - The average rate of change from $x = 0$ to $x = 3$ is equal to 0.
   - The point $(2, -30)$ lies on the curve.

[5 Marks]

$f(1) = 0 \Rightarrow a + c + e = 0$ (1)

$f(2) = -30 \Rightarrow 32a + 8c + 2e = -30$

$16a + 4c + e = -15$ (2)

AROC = 0 from 0 to 3 $\Rightarrow \frac{f(3) - f(0)}{3} = 0$

$\therefore f(3) = f(0)$

$f(3) = 0$

$243a + 27c + 3e = 0$

$81a + 9c + e = 0$ (3)

(2) − (1): $15a + 3c = -15$ (4)

(3) − (2): $65a + 5c = 15$ (5)

$5 \times (4) - 3 \times (5)$

$-120a = -120$

$a = 1$

sub $a = 1$ into (4)

$15 + 3c = -15$

$c = -10$

sub $a = 1$, $c = -10$ into (1)

$1 - 10 + e = 0$

$e = 9$

*** 2 Marks will be awarded in the Communication Category for the use of proper mathematical form. ***

$\therefore a = 1, b = 0, c = -10, d = 0, e = 9, f = 0$

MHF4U1-V2
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Unit 1: Polynomial Functions - Assessment of Learning – DAY 2

Application	Thinking	Comm.
/20	/5	/5

Instructions: Answer all questions in the space provided and **show all necessary steps**. Leave answers **exact** unless otherwise specified. The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

APPLICATION – [20 MARKS]

1. Graph and properly label $f(x)=\frac{1}{27}(x-3)^3(x^2+2x-3)(1-x)$. **[4 Marks]**

$(x+3)(x-1)$

Degree: 6

L.C: −ve.

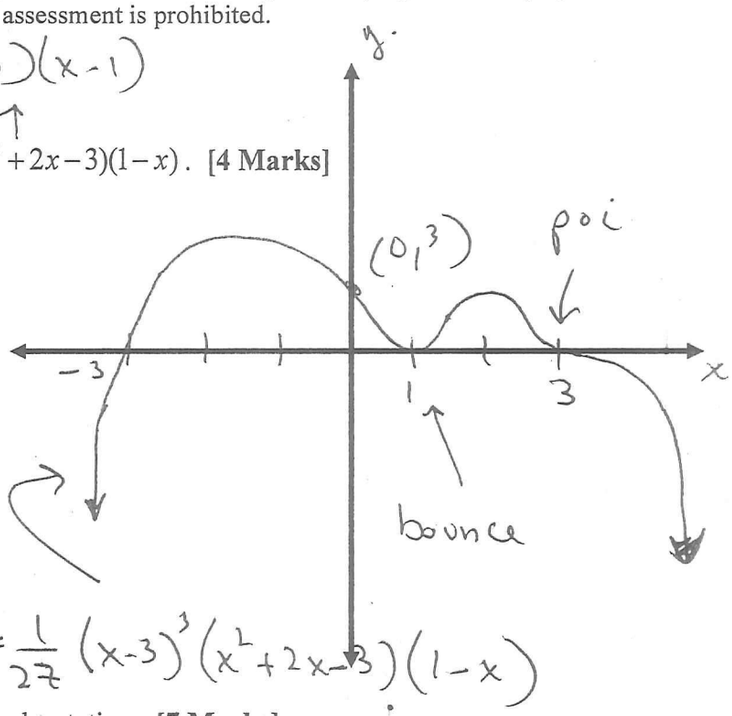
E.B: As $x\to\infty, y\to-\infty$

As $x\to-\infty, y\to-\infty$

y-int: (0, 3)

x-int(s): 3, 1, −3.

poi bounce.



$f(x)=\frac{1}{27}(x-3)^3(x^2+2x-3)(1-x)$

2. Solve the following. **Note:** For Part b) use interval notation. **[7 Marks]**

a) $6x^3+7x^2-x-2=0$ **[4]**

x+1 is a factor

$$\begin{array}{r} 6x^2+x-2 \\ x+1\,\overline{)\,6x^3+7x^2-x-2} \\ -(6x^3+6x^2) \\ \hline x^2-x-2 \\ -(x^2+x) \\ \hline -2x-2 \\ -(-2x-2) \\ \hline 0 \end{array}$$

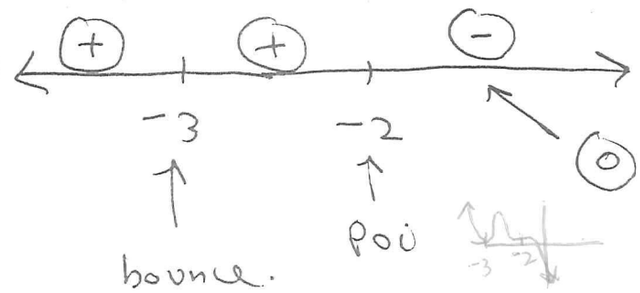
OR. $* \; 6x^2+x-2 = (3x+2)(2x-1)$

$\therefore x=-1, -\frac{2}{3}, \frac{1}{2}$

b) $-(x^2+5x+6)(x+2)^2(x+3)>0$ **[3]**

$-(x+3)(x+2)(x+2)^2(x+3)>0$

$-(x+2)^3(x+3)^2>0$



−3: bounce. −2: poi

Soln: $(-\infty,-3)\cup(-3,-2)$

3. The function $f(x)=ax^3+3x+b+5$ has a remainder of $2a$ when divided by x and a remainder of $2b$ when divided by $x-1$. Determine the values of a and b. **[4 Marks]**

$f(0)=2a$ $\quad f(1)=2b$

$b+5=2a$ $\quad a+3+b+5=2b$

$2a-b=5$ ① $\quad b-a=8$ ②

① + ②

$2a-b=5$

$+(-a+b=8)$

$a=13$

sub $a=13$ into ②

$b-13=8$

$b=21$

MHF4U1-V2
Leung, Moshtagh, Vincent

Name: _____

4. Determine the polynomial function that passes through the following points. **[5 Marks]**

x	y	
−1	−1	
0	5	6 > −2 > 12
1	9	4 > 10
2	23	14 > 22 > 12
3	59	36 > 22 > 12

y-int = 5

$\therefore n = 3$

$t_3 = 12$

$s = 1$

$t_3 = a \cdot 3!$

$12 = 6a$

$a = 2$ $\quad d = 5$

$f(x) = 2x^3 + bx^2 + cx + 5$

sub $(-1, -1)$

$-1 = -2 + b - c + 5$

$b - c = -4$ (1)

sub $(1, 9)$

$9 = 2 + b + c + 5$

$b + c = 2$ (2)

(1) + (2)

$b - c = -4$

$+(b + c = 2)$

$2b = -2$

$b = -1$

sub $b = -1$ into 2

$-1 + c = 2$

$c = 3$

$\therefore f(x) = 2x^3 - x^2 + 3x + 5$

THINKING – [5 MARKS]

1. The length, width and height of a small box are three consecutive integers, where the width is the least and the length is the greatest integer. If the width is doubled and the length and height are both increased by 2 cm, then the original volume is increased by 192 cm^3. Determine the dimensions of the original box. **[5 Marks]**

Let x rep the width

Let $x+1$ " " height

Let $x+2$ " " length

$(2x)(x+3)(x+4) = x(x+1)(x+2) + 192$

$2x^3 + 14x^2 + 24x = x^3 + 3x^2 + 2x + 192$

$x^3 + 11x^2 + 22x - 192 = 0$

$x - 3$ is a factor

$x - 3 \;\overline{)\; x^3 + 11x^2 + 22x - 192}$

$x^2 + 14x + 64$

$-(x^3 - 3x^2)$

$14x^2 + 22x - 192$

$-(14x^2 - 42x)$

$64x - 192$

$-(64x - 192)$

OR.

$* \; x^2 + 14x + 64$

$b^2 - 4ac$

$= 1 - 4(1)(64)$

$= -255 < 0$

$\Rightarrow$ 2 imaginary roots

$\therefore x = 3$ cm.

Dimensions of original box $= 3\text{cm} \times 4\text{cm} \times 5\text{cm}$

COMMUNICATION – [5 MARKS]

1. **"Even degree polynomial functions are even functions."**

   Is the above statement **always true**, **sometimes true** or **never true?** Provide a thorough explanation and diagrams to justify your answer. **[3 Marks]**

Sometimes True. If the exponents on each x-term is even then it is an even function (symmetry about y axis). If exponents are a mix of even and odd then there is no symmetry.

*** 2 Marks will be awarded in the Communication Category for the use of proper mathematical form. ***