MCV4U1 – Moshtagh Name: ___________________________

Unit 5 Assessment of Learning – Derivative of Trigonometric & Exponential Functions-Day 1

Instructions:

Please show all your work for full marks. You may only use methods taught in MCV4U for full marks. Express your final answers in **exact** form, unless otherwise indicated. The use of cellphones, audio-or video- recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

Knowledge & Understanding	Thinking	Comm
/ 18	/ 5	/2

KNOWLEDGE & UNDERSTANDING - [18 marks]

Multiple Choice: Write the CAPITAL LETTER corresponding to the correct answer on the line provided. ❺

1. The derivative of $y=\sec(e^x)$ is **B**

   A. $y'=\sec(e^x)\tan(e^x)$ B. $y'=e^x\sec(e^x)\tan(e^x)$

   C. $y'=e^x\tan(e^x)$ D. $y'=xe^x\sec(e^x)\tan(e^x)$

2. The slope of the tangent for the function $f(x)=\cos^2(x)$ at $x=\frac{\pi}{4}$ is equal to **A**

   A. -1 B. 1 C. $\frac{-1}{2}$ D. $\frac{1}{2}$

3. If $y=\ln\left(\sqrt[3]{x-1}\right)$ then $\left.\frac{d^2y}{dx^2}\right|_{x=2}$ is equal to **B**

   A. $\frac{1}{3}$ B. $-\frac{1}{3}$ C. $\frac{1}{9}$ D. 0

4. The $x-$coordinate of the point where the function $f(x)=\frac{2^x}{x}$ has a horizontal tangent is **C**

   A. $\ln 2$ B. 0 C. $\frac{1}{\ln 2}$ D. $-\ln 2$

5. Which of the following is/are **true**? **C**

   I. If $f(x)=e^{2x}-e^{-2x}$, then $f'(x)=2e^{2x}(1+e^{-4x})$.

   II. If $f(x)=\sin(x)\cos(x)$, then $f'(x)=\cos(2x)$.

   III. The function $f(x)=\frac{\ln x}{x}$ has a horizontal tangent at $x=1$.

   IV. $\lim_{h\to 0}\frac{e^h-1}{h}$ is equal to 1.

   A. I and II only. B. I and III only. C. I, II and IV only. D. All are true.

6. Determine the derivative for each of the following. Do **NOT** simplify your answers for Part a). **Completely simplify your answer for Part b)**. **[7 marks]**

   a) $f(x)=\dfrac{\tan(\sqrt{x})}{e^{5x+1}}-\ln(x^5-1)$ ❸

   $$f'(x)=\frac{\frac{1}{2\sqrt{x}}\sec^2(\sqrt{x})e^{5x+1}-5e^{5x+1}\tan(\sqrt{x})}{\left(e^{5x+1}\right)^2}-\frac{5x^4}{x^5-1}$$

   b) $f(x)=e^{x^3-6x+2}\sin^2\left(x^3-6x+2\right)+\ln\left(4^{x-1}\right)$ ❹

   $f(x)=e^{x^3-6x+2}\sin^2\left(x^3-6x+2\right)+(x-1)\ln(4)$

   $$f'(x)=(3x^2-6)e^{x^3-6x+2}\sin^2\left(x^3-6x+2\right)+2\sin\left(x^3-6x+2\right)\cos\left(x^3-6x+2\right)(3x^2-6)e^{x^3-6x+2}+\ln(4)$$

   $$=(3x^2-6)e^{x^3-6x+2}\left[\sin^2\left(x^3-6x+2\right)+\sin\left(2(x^3-6x+2)\right)\right]+\ln(4)$$

7. If $f(x) = (1+\ln x)^2$, determine the intervals of concavity. ❸

$f'(x) = 2(1+\ln x)\left(\frac{1}{x}\right)$

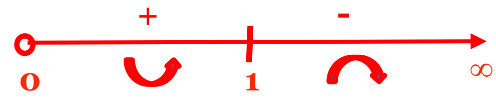
$f''(x) = 2\left(\frac{1}{x^2}\right) - \left(\frac{2}{x^2}\right)(1+\ln x)$

$f''(x) = \frac{-2\ln x}{x^2}$

$f''(x) = 0$

$\ln x = 0$

$x = 1$



concave up: $x \in (0,1)$

concave down: $x \in (1,\infty)$

8. If $f(x) = e^{\cos(2x)}$ on $x \in [-\pi, \pi]$, determine and classify all local extrema.❸

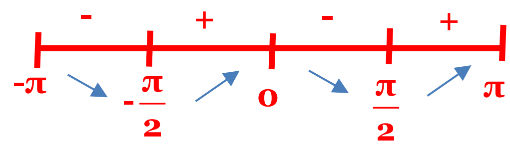
$f'(x) = -2\sin(2x)e^{\cos(2x)}$

$\sin(2x) = 0$

$2x = k\pi$

$x = \frac{k\pi}{2}, k = 0, \pm1, \pm2, \ldots$

$x = \frac{-\pi}{2}, -\pi, 0, \frac{\pi}{2}, \pi$



local min$(-\frac{\pi}{2}, \frac{1}{e})$

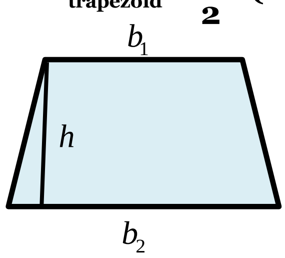
local max$(0, e)$

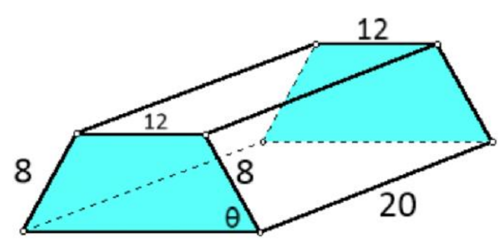
local min$(\frac{\pi}{2}, \frac{1}{e})$

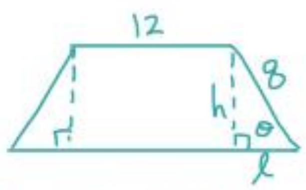
THINKING - [5 marks]

1. A group of campers are setting up a tent on flat ground in the shape of a trapezoid prism. The poles slanted to the ground have a length of 8 units, two poles along the roof of the tent have a length of 12 units, and the tent extends to have a base length of 20 units. At what angle to the ground should the campers place the 8 unit poles to maximize the volume of the tent? Round your answer to the nearest degree. **A second derivative test is not required.** ❺

Note: $\mathbf{A_{trapezoid} = \frac{1}{2}h(b_1 + b_2)}$







$A = 12h + 2\cdot\frac{1}{2}\ell h$

$A = 12h + \ell h$

$\sin(\theta) = \frac{h}{8}$ $\cos(\theta) = \frac{\ell}{8}$

$\therefore A = 96\sin\theta + 64\sin\theta\cos\theta$

$\therefore V = 20A$

$V = 1920\sin(\theta) + 1280\sin\theta\cos\theta$

or $V = 20[8\sin\theta]\left[\frac{2(12)+2(8\cos\theta)}{2}\right] = 640\sin\theta[3+2\cos\theta]$

$V' = 1920\cos(\theta) + 1280\cos^2\theta - 1280\sin^2\theta$ $\div 640$

set $V' = 0$ to find max/min

$0 = 3\cos(\theta) + 2\cos^2\theta - 2[1-\cos^2\theta]$

$0 = 4\cos^2\theta + 3\cos(\theta) - 2$

$\cos(\theta) = \frac{-3\pm\sqrt{41}}{10}$

since $0 \le \theta \le 90°$

$\theta = \cos^{-1}\left(\frac{-3+\sqrt{41}}{10}\right)$

$\boxed{\theta = 64.8°}$

*** **2 marks** will be awarded in the Communication Category for proper mathematical form. **

Unit 5 Assessment of Learning – Derivative of Trigonometric & Exponential Functions-Day 2

Instructions:
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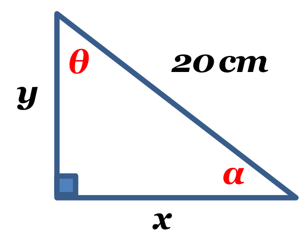
Application	Thinking	Comm
/ 17	/ 4	/5

APPLICATION - [17 marks]

1. The hypotenuse of a right triangle is 20 cm. Determine the exact measures (in radians) of the unknown angles that maximizes the perimeter of the triangle. ❹



$$P = 20 + x + y$$

$$P(\theta) = 20 + 20\sin(\theta) + 20\cos(\theta);\ 0 < \theta < \frac{\pi}{2}$$

$$P'(\theta) = 20\cos(\theta) - 20\sin(\theta)$$

$$P'(\theta) = 0$$

$$\cos(\theta) = \sin(\theta)$$

$$\tan(\theta) = 1$$

$$\boxed{\theta = \alpha = \frac{\pi}{4}}$$

2. For what value(s) of k does $y = ae^{-kx}$ satisfy $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = 0$, where a and k are constants and $a \neq 0$? ❹

$$\frac{dy}{dx} = -ake^{-kx}$$

$$\frac{d^2y}{dx^2} = ak^2e^{-kx}$$

$$\frac{d^3y}{dx^3} = -ak^3e^{-kx}$$

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = 0$$

$$-ak^3e^{-kx} - 4ak^2e^{-kx} - ake^{-kx} + 6ae^{-kx} = 0$$

$$-ae^{-kx}\left(k^3 + 4k^2 + k - 6\right) = 0 \quad (a \neq 0)$$

$$k^3 + 4k^2 + k - 6 = 0$$

$$(k-1)(k+2)(k+3) = 0$$

$$k = 1, -2, -3$$

3. Determine the equation (point-slope form) of the tangent to $y = x\ln\left(x^{e^2}\right)$ that is perpendicular to $\frac{1}{2e^3}x + \frac{1}{e}y - \sqrt{e} = 0$. ❹

$y = xe^2\ln(x)$

$y' = e^2(\ln(x)+1)$

$m_\perp = \dfrac{\frac{-1}{2e^3}}{\frac{1}{e}}$

$= \dfrac{-1}{2e^2}$

$m_t = 2e^2$

$e^2(\ln(x)+1) = e^2$

$\ln(x)+1 = 2$

$\ln(x) = 1$

$x = e\ ,y = e^3$

Equation of tangent line : $y - e^3 = 2e^2(x - e)$

4. When extended, the tangent to the curve $f(x) = \cos^2(2x) - \sin^2(2x)$ at $x = \frac{3\pi}{8}$ intersects the $x-$axis and the $y-$axis to form a triangle. Determine the **exact fully simplified** area of the triangle. ❺

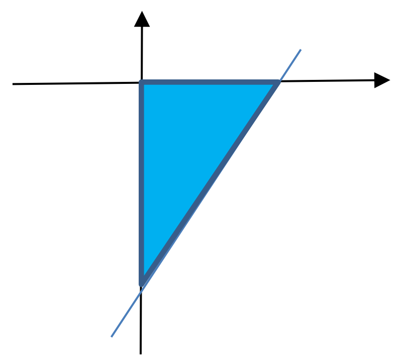
$f(x) = \cos(4x)$

$f'(x) = -4\sin(4x)$

$f'\left(\frac{3\pi}{8}\right) = -4\sin\left(4\left(\frac{3\pi}{8}\right)\right)$

$m_t = f'\left(\frac{3\pi}{8}\right) = -4\sin\left(\frac{3\pi}{2}\right)$

$m_t = 4$



Equation of tangent line : $y = 4(x - \frac{3\pi}{8})$

x-int, set $y = 0$

$A(\frac{3\pi}{8}, 0)$

y-int. ,set $x = 0$

$B(0, -\frac{3\pi}{2})$

$\text{Area} = \frac{1}{2}\left(\frac{3\pi}{8}\right)\left(\frac{3\pi}{2}\right)$

$A = \frac{9\pi^2}{32}\ \text{units}^2$

THINKING - [4 marks]

1. Given that $f(x) = a\,xe^{bx^2}$ has a maximum value of 1 when x= 2. Find the **exact** values of constants a and b.❹

$f'(x) = a\,e^{bx^2} + 2ab\,x^2\,e^{bx^2}$

$f(2) = 1$

$2a\,e^{4b} = 1 \quad (1)$

$f'(2) = 0$

$a\,e^{4b} + 8ab\,e^{4b} = 0$

$a\,e^{4b}(1 + 8b) = 0$

$a \neq 0\ ,\ \boxed{b = -\frac{1}{8}}$

Sub. $b = -\frac{1}{8}$ into (1) : $2a\,e^{4\left(\frac{-1}{8}\right)} = 1$

$2ae^{-\frac{1}{2}} = 1$

$\boxed{a = \frac{\sqrt{e}}{2}}$

COMMUNICATION - [5 marks]

1. Rita was asked to derive and fully factor his final answer. Her solution is incorrect, but she cannot find her mistake(s). Explain where she made her mistake(s) and correct them.❸

$y = (2^{\cos(x)})$

$y' = \cos(x)(2^{\cos(x)-1})(\ln 2)(\sin(x))$

The mistake she made is about using none-existing formula and mixing up the real formula with the power rule!

The correct formula uses the chain rue by differentiating the function in the exponent followed by multiply by the function and then by the ln of the base, i.e

$y = (2^{\cos(x)})$

$y' = -\sin(x)(2^{\cos(x)})(\ln 2)$

*** **2 marks** will be awarded in the Communication Category for proper mathematical form. ***