MHF 4U1 - Unit 5 Test AM

Name: Yusuf Ali Jaffer
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TOTAL
30.5
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KNOWLEDGE:

1. Sketch one cycle of $y = -\frac{3}{2}\cos(\frac{x}{2} + \frac{\pi}{12}) + 3$. You can use the mapping rule or 5 point method to sketch. Show all your work.

$2\pi \div x = 1$ $\pi = x$ $-\frac{3}{2}\cos(\frac{1}{2}(x+\frac{\pi}{6}))+3$

~~[5]~~ 4

$y = -\frac{3}{2}\cos(2(x+\frac{\pi}{6}))+3$

Map: $[\frac{x}{2} - \frac{\pi}{6}, -\frac{3y}{2}+3]$

$0, 1 \to -\frac{\pi}{6}, \frac{3}{2}$

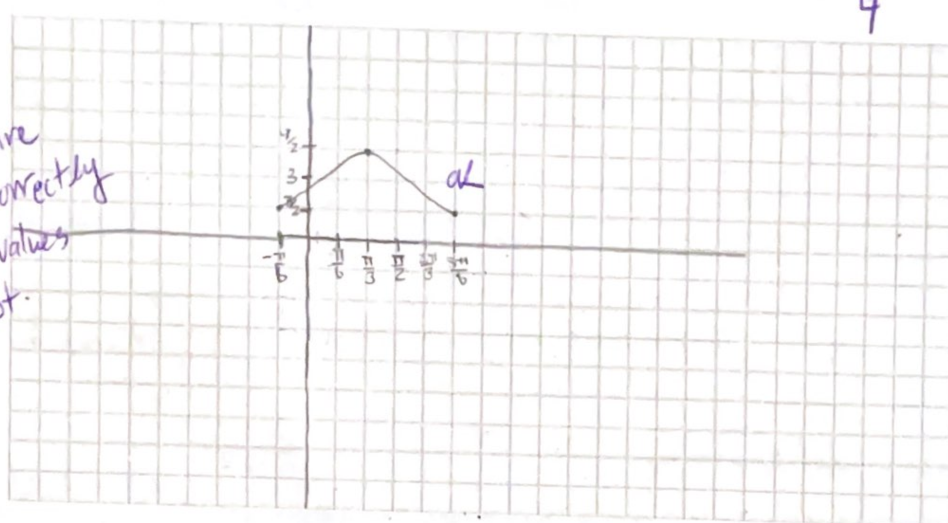
$\frac{\pi}{2}, 0 \to \frac{\pi}{12}, 3$

$\pi, -1 \to \frac{\pi}{3}, \frac{9}{2}$

$\frac{3\pi}{2}, 0 \to \frac{9\pi}{12}, 3$

$2\pi, 1 \to \frac{5\pi}{6}, \frac{3}{2}$

Y-values are mapped correctly but x-values are not.



2. Determine the exact solutions for $2tan^2x - \sqrt{12} = 0$ in the interval $x \in [0, 2\pi]$. [2]

$2\tan^2 x = \sqrt{12}$

$\tan^2 x = \frac{\sqrt{12}}{2}$

$\tan x = \sqrt{\frac{\sqrt{12}}{2}}$

CASE one $x = \tan^{-1}(\sqrt{\frac{\sqrt{12}}{2}})$

$x = ~~0.921~~$

CASE TWO

$x = \tan^{-1}(-\sqrt{\frac{\sqrt{12}}{2}})$

$x = -0.921$

CAST

$x_1 = 0.921$

$x_2 = \pi - 0.921 = 2.22$

$x_3 = \pi + 0.921 = 4.06$

$x_4 = 2\pi - 0.921 = 5.36$

COMMUNICATION:

1. Identify one distinguishing characteristic that y= tanx and y= cotx share in common and two that cause them to differ. Be specific! [2]

Both functions have asymptotes (vertical)

Tan has different asymptotes (vertical) than cot. Tan is also always increasing, cot is always decreasing (towards $+\infty$)

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2. a) Describe the transformations that when applied to the graph of y = sinx result in the graph shown. [3]

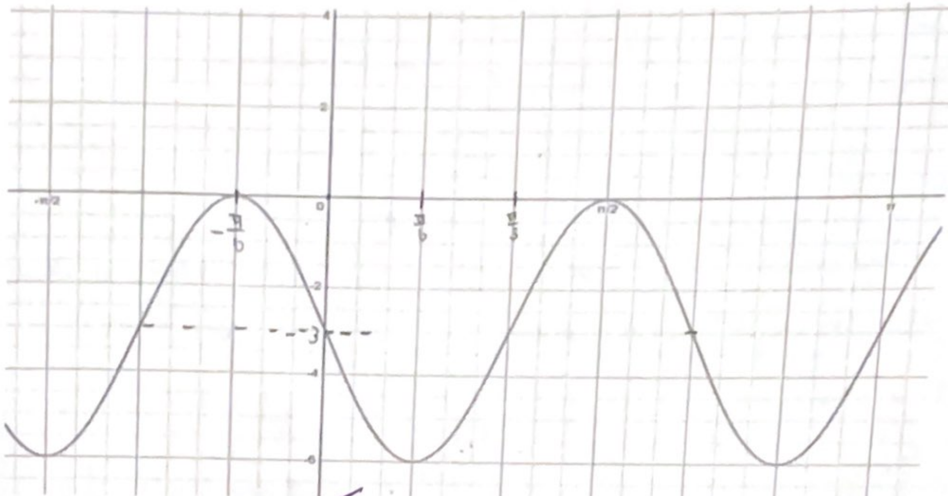
1.5

$\frac{\pi}{2}$

$\frac{\pi}{3} + \frac{\pi}{6}$

$\frac{2\pi}{6} + \frac{\pi}{6}$

$\frac{\pi}{2}$



reflection in x

v-t = -3

amp = 3

what does this mean about the vertical stretch?

$k = 2\pi \div \frac{2\pi}{3}$

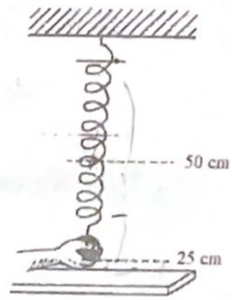
K = 3

What does this mean in terms of horizontal stretch?

period = $\frac{\pi}{2} + \frac{\pi}{6} = \frac{3\pi}{6} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$

b) Write the equation of the sinusoidal function above: $y = -3\sin(3(x)) - 3$ [2]

APPLICATION:



1. A weight is supported by a spring. The weight rests 50 cm above a tabletop. The weight is pulled down 25 cm and released at $t=0$. This creates a periodic up-and-down motion. It takes 1.6 s for the weight to return to the low position each time.

a) Sketch two cycles of the height of the weight above the tabletop against time. [4] $2\pi \div 1.6 = \frac{5\pi}{4}$

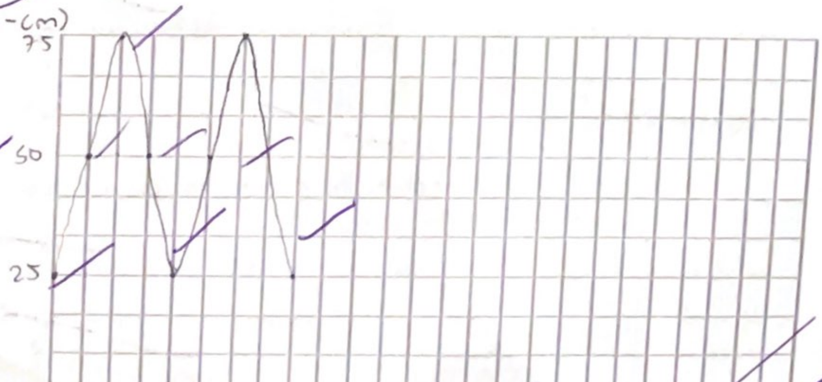
v-t = 50

amp = 25

Period = 1.6

f = -cos x

$y = -25\cos(\frac{5\pi}{4}x) + 50$



7.5

b) Determine an equation to describe the motion. [2]

$y = -25\cos\left(\frac{5\pi}{4}x\right) + 150$

c) At what time is the weight 60 cm above the table. List the first 3 times. [2]

$60 = -25\cos\theta + 50$

$10 = -25\cos\theta$

$-\frac{2}{5} = \cos\theta$

$\cos^{-1}\left(-\frac{2}{5}\right) = \theta$ (CAST)

$1.98 = \theta$

$\theta_1 = 1.98$ $\quad \theta_2 = 2\pi - 1.98$

$\frac{5\pi}{4}x = 1.98$ $\quad \frac{5\pi}{4}x = 2\pi - 1.98$

$x_1 = 0.5\,s$ $\quad x_2 = 1.1\,s$

$\theta_3 = 0.5 + 1.6\,s$

$\theta_3 = 2.1\,s$

First 3 times: $0.5\,s,\ 1.1\,s,\ 2.1\,s$

$4\pi = \sim 12$

2. Solve $10\cos^2 x - 3\cos x = 1$ on the interval $x \in [0, 4\pi]$. Round answers to the nearest hundredth of a radian where necessary. [4]

$\cos x = L$

$10L^2 - 3L - 1 = 0$

$10L^2 - 5L + 2L - 1 = 0$

$5L(2L-1) + 2L - 1 = 0$

$(5L+1)(2L-1)$

$2\cos x - 1$

$\cos x = \frac{1}{2}$ (CAST)

$x = \cos^{-1}\left(\frac{1}{2}\right)$

$x_1 = \frac{\pi}{3}$ $\quad x_2 = \frac{5\pi}{6}$ $\quad x_3 = \frac{7\pi}{3}$ $\quad x_4 = \frac{17\pi}{6}$

$4\pi \rightarrow x_3 = 2\pi + \frac{\pi}{3}$ $\quad x_4 = \frac{5\pi}{6} + 2\pi$

$= \frac{7\pi}{3}$ $\quad x_4 = \frac{17\pi}{6}$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{3}, \frac{17\pi}{6}, 1.77, 4.51, 8.05, 10.79}$

$5\cos x + 1$

$\cos x = -\frac{1}{5}$

$x = \cos^{-1}\left(-\frac{1}{5}\right)$ (CAST)

$x_1 = 1.772$ $\quad x_2 = 2\pi - 1.772$

$x_2 = 4.51$

$4\pi \rightarrow 1.772 + 2\pi = x_3$, $\quad 4.51 + 2\pi = x_4$

$8.05 = x_3$ $\quad 10.79 = x_4$

3. Solve $2\sin 4x - 1 = 0$ on the interval $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Express answers as exact value, if possible. [4]

$4x = \theta$

θ Domain: $[-2\pi, 2\pi]$

$2\sin 4x - 1 = 0$

$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$ (CAST)

$\theta = \frac{\pi}{6}$

$\theta_1 = \frac{\pi}{6}$ $\quad \theta_3 = \frac{\pi}{6} - 2\pi$ $\quad \theta_4 = \frac{5\pi}{6} - 2\pi$

$4x = \frac{\pi}{6}$ $\quad 4x = -\frac{11\pi}{6}$ $\quad 4x = -\frac{7\pi}{6}$

$x_1 = \frac{\pi}{24}$ $\quad x_3 = -\frac{11\pi}{24}$ $\quad x_4 = -\frac{7\pi}{24}$

$\theta_2 = \pi - \frac{\pi}{6}$

$\theta_2 = \frac{5\pi}{6}$

$4x = \frac{5\pi}{6}$

$x_2 = \frac{5\pi}{24}$

$\boxed{x = \frac{\pi}{24}, \frac{5\pi}{24}, -\frac{11\pi}{24}, -\frac{7\pi}{24}}$

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THINKING:

1. Determine the point(s) of intersection between the functions $y = 2cos(3x - 3\pi)$ and $y = 4sin[3(x - \pi)]$ on the interval $x \epsilon [0, 2\pi]$. Round your answer(s) to 2 decimal places and be sure to show all necessary work.

[4]

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$y = 2\cos(3(x-\pi))$ $\quad y = 4\sin(3(x-\pi))$

$\theta = 3(x-\pi)$

$2\cos(3(x-\pi)) = 4\sin(3(x-\pi))$

$2\cos\theta = 4\sin\theta$

Domain θ:

$1 = 2\tan\theta$

$\frac{1}{2} = \tan\theta$

$\tan^{-1}(\frac{1}{2}) = \theta$ $\quad$ C(A)S(T)

$\frac{5\pi}{4} + 3\pi$

$0.463 = \theta$

$\frac{17}{4}\pi$

$\theta_1 = 0.463$ $\qquad \theta_2 = \pi + 0.463$

$3x - 3\pi = 0.463$ $\qquad 3x - 3\pi = \pi + 0.463$

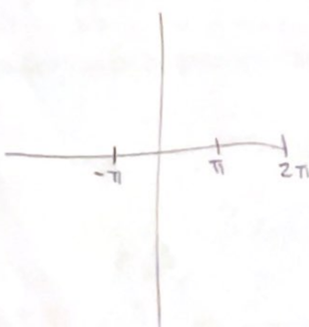
$x = 3.296$ $\qquad x = 4.34$

$x = 3.29$ $\qquad y = -1.79$

$y = 1.78$

Need to find all points. Good start.

$\therefore$ The 2 graphs intercept at $(3.29, 1.78)$ and at $(4.34, -1.79)$



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