

Assessment of Learning: Unit 2 – Rational Functions – DAY 2

Application	Thinking	Communication	
/18	/5	/2	

Instructions: Answer all questions in the space provided and **show all necessary steps**. Leave answers **exact** unless otherwise specified. The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

APPLICATION – [18 MARKS]

1. Sketch and properly label the graph $f(x) = \frac{5(x-1)(x+1)^2}{(x^2-4x-5)(x-8)}$. [10 Marks]

$f(x) = \frac{5(x-1)(x+1)^2}{(x+1)(x-5)(x-8)}$

$= \frac{5(x-1)(x+1)}{(x-5)(x-8)}$, $x \neq -1$
hole at $x = -1$

x -int: $x = 1$

y -int: $-\frac{1}{8}$

VA: $x = 5, x = 8$

HA: $y = 5$

hole at $x = -1$
 $f(-1) = 0$
 \therefore hole at $(-1, 0)$

HA cross test:

$\cancel{5} = \frac{\cancel{5}(x-1)(x+1)}{(x-5)(x-8)}$

$x^2 - 13x + 40 = x^2 - 1$

$-13x = -41$

$x = \frac{41}{13}$

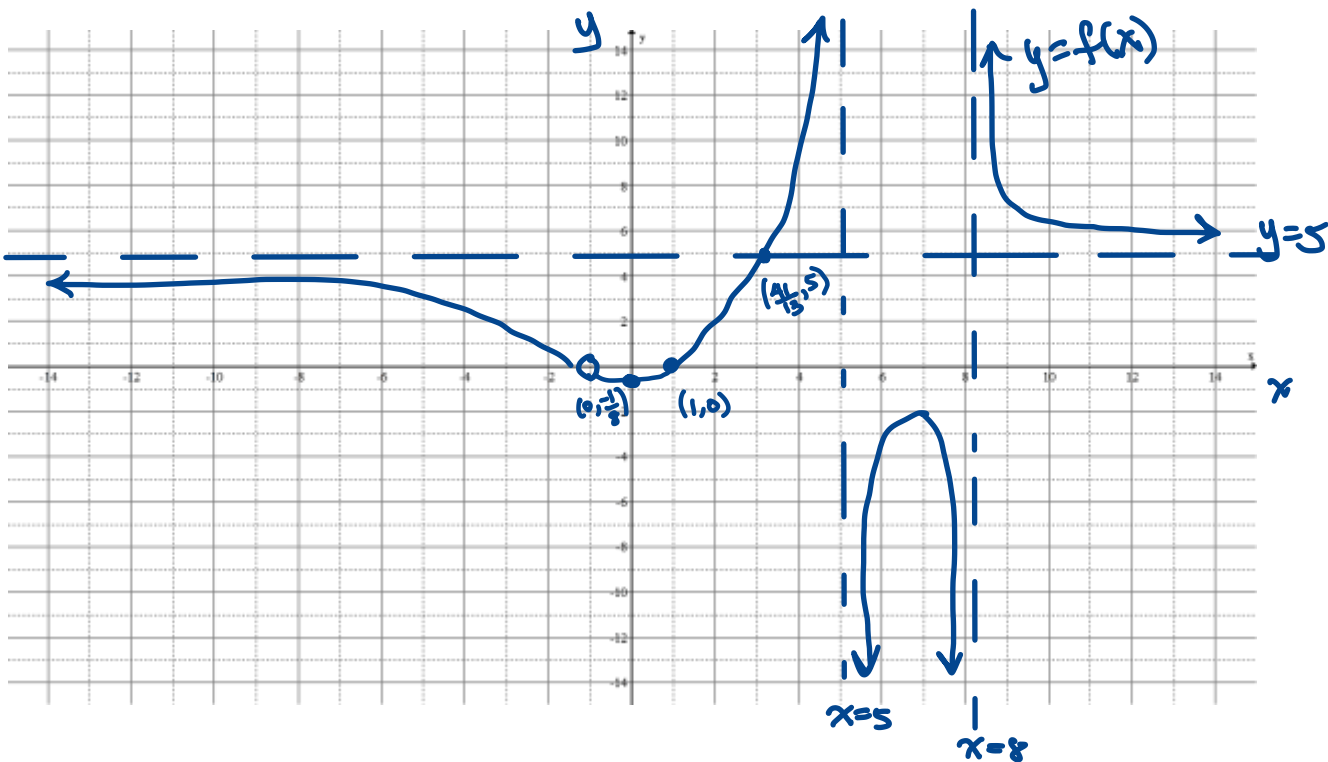
≈ 3.15

\therefore crosses HA at approx $x \approx 3.15$

Sign chart:

$\leftarrow \begin{array}{cccccc} + & - & + & - & + \\ | & | & | & | & | \\ -\infty & -1 & 1 & 5 & 8 & \infty \end{array} \rightarrow$

hole



2. Solve $\frac{2x-1}{x+7} \geq \frac{x+1}{x+3}$. [4 Marks]

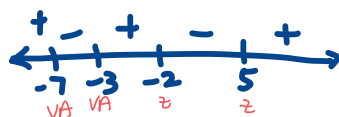
$$\frac{2x-1}{x+7} - \frac{x+1}{x+3} \geq 0$$

$$\frac{(2x-1)(x+3) - (x+1)(x+7)}{(x+7)(x+3)} \geq 0$$

$$\frac{2x^2+5x-3-x^2-8x-7}{(x+7)(x+3)} \geq 0$$

$$\frac{x^2-3x-10}{(x+7)(x+3)} \geq 0$$

$$\frac{(x-5)(x+2)}{(x+7)(x+3)} \geq 0$$



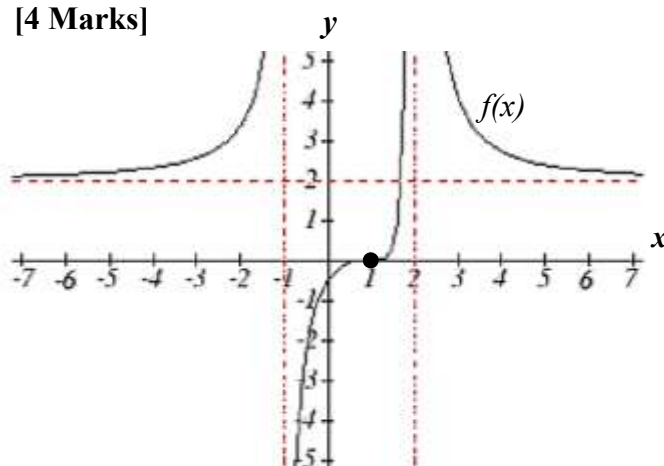
$$\therefore x \in (-\infty, -7) \cup (-3, -2] \cup [5, \infty), x \in \mathbb{R}$$

3. Determine the equation of the given function $f(x)$. [4 Marks]

$$f(x) = \frac{a(x-1)^3}{(x+1)(x-2)^2}$$

$$\text{HA: } y=2 \therefore a=2$$

$$\therefore f(x) = \frac{2(x-1)^3}{(x+1)(x-2)^2}$$



THINKING – [5 MARKS]

1. Sketch the graph of a rational function with the following properties:

[5 Marks]

- The leading term of the original unsimplified numerator is $2x^2$
- The denominator of the unsimplified function is $2x^2 - 2x - 24$
- $f(0) = 1$
- $f(x) > 0$ when $x < -3$, $-2 < x < 4$, and $x > 6$
- $f(x)$ increases from $x < -3$, $-3 < x < 4$, and $x > 4$
- The function crosses the horizontal asymptote once.

$$\text{HA: } y=1$$

$$\text{VA: } x=-3, x=4$$

$$\begin{aligned} f(x) &= \frac{2x^2+bx+c}{2x^2-2x-24} \\ &= \frac{2x^2+bx+c}{2(x^2-x-12)} \\ &= \frac{2x^2+bx+c}{2(x-4)(x+3)} \end{aligned}$$

$$f(0)=1$$

$$1 = \frac{2(0)^2+b(0)+c}{2(-4)(3)}$$

$$c = -24$$

$$f(x) = \frac{2x^2+bx-24}{2(x-4)(x+3)}$$

$$= \frac{2(x-6)(x+2)}{2(x-4)(x+3)}$$

