Assessment of Learning: Unit 4 – Trigonometric Functions (PART 2) – DAY 1

Knowledge & Understanding	Thinking	Communication
18 /18	1 /5	2 /2

Instructions: Answer all questions in the space provided and **show all necessary steps.** Leave answers **exact** unless otherwise specified. The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

KNOWLEDGE & UNDERSTANDING – [18 MARKS]

Multiple Choice: Write the **CAPITAL LETTER** corresponding to the correct answer on the line provided. **[1 Mark Each – 5 Marks Total]**

1. A function $f(x) = \tan(k\theta)$ has consecutive asymptotes of $\theta = \pi$ and $\theta = 3\pi$. Hence, k equals __B__

   A. 2   B. $\dfrac{1}{2}$ C. $\dfrac{2}{3}$ D. $\dfrac{3}{4}$

2. The range of $f(x) = \csc(x)$ is   __C__

   A. $(-\infty, \infty)$ B. $(-\infty, -1) \cup (1, \infty)$ C. $(-\infty, -1] \cup [1, \infty)$ D. $[0, \infty)$

   $-3-b = -(3+b)$
   $-3\sin\left(\frac{\pi}{2}\right) - b$

3. The y–intercept of the graph represented by $f(\theta) = -3\sin\left(k\theta + \dfrac{\pi}{2}\right) - b$ is   __B__

   A. $-b$ B. $-(b+3)$ C. $\dfrac{3-b}{k}$ D. $\dfrac{-3-b}{k}$

4. The period of the function $f(x) = 5\cot(-4x) + 2$ is   __A__

   A. $\dfrac{\pi}{4}$ B. $-\dfrac{\pi}{4}$ C. $\dfrac{\pi}{2}$ D. $-\dfrac{\pi}{2}$

   $-1-3$
   $= -4$

5. The minimum value of the function $g(\theta) = 3\cos[4(\theta - \pi)] - 1$ is   __A__

   A. -4 B. 3   C. 4   D. 2

6. Complete the table below for the **sine** function. **[5 Marks]**

Equation	Amplitude	Range	Period	Phase Shift	Equation of Axis
$f(x) = -2\sin\left(-\dfrac{3}{5}x + \dfrac{\pi}{10}\right) - 3$ $-2\sin\dfrac{3}{5}\left(x - \dfrac{1}{6}\pi\right) - 3$	2	$y \in [-5, -1]$	$\dfrac{10\pi}{3}$	$\dfrac{1}{6}\pi$ (to the right)	-3

$\dfrac{2\pi}{3}$ $\dfrac{2\pi \times 5}{3}$

$\dfrac{\frac{2\pi}{3}}{5}$

7. Solve the following. **Exact** answers. **[6 Marks]**

a. $4\sin^2(x) - 3 = 0$, $x \in [0, 2\pi]$. **[3]**

$4\sin^2(x) = 3$

$\sin^2(x) = \frac{3}{4}$

$\sin(x) = \pm\frac{\sqrt{3}}{2}$

RAA is $\frac{\pi}{3}$

$(+)$ in QI and II

$\pi - \theta_R = \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$ $x = \frac{\pi}{3}, \frac{2\pi}{3}$

$(-)$ in Q III and IV (four)

$\pi + \theta_R$	$2\pi - \theta_R$
$\pi + \frac{\pi}{3}$	$2\pi - \frac{\pi}{3}$
$= \frac{4\pi}{3}$	$= \frac{5\pi}{3}$

$\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

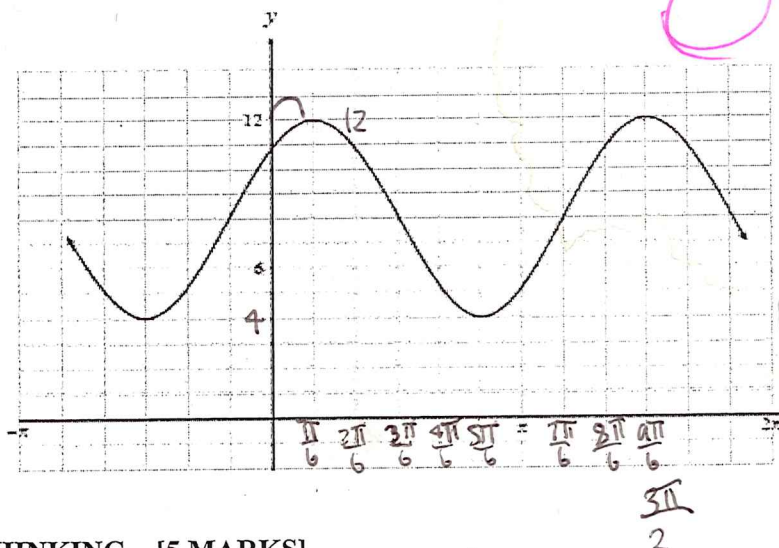b. $\cos(2x) = \frac{1}{2}$, $x \in [0, \pi]$. **[3]**

RAA is $\frac{\pi}{3}$

$2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$2x = \frac{\pi}{3} + 2\pi k, k \in \mathbb{Z}$ $2x = \frac{5\pi}{3} + 2\pi k, k \in \mathbb{Z}$

$x = \frac{\pi}{6}, \pi k, k \in \mathbb{Z}$ $x = \frac{5\pi}{6} + \pi k, k \in \mathbb{Z}$

$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}$

8. Determine a **cosine** function for the graph below. **[2 Marks]**

Max = 12 $A = \frac{max - min}{2}$

Min = 4 $A = 4$

$C = \frac{max + min}{2} = 8$

$P = 2(\text{min value} - \text{max x value})$

$P = 8\frac{\pi}{6} = \frac{4\pi}{3}$

Cosine Function: $4\cos\frac{4\pi}{3}(x - \frac{\pi}{6}) + 8$

THINKING – [5 MARKS]

1. The point $\left(\frac{\pi}{2}, 1\right)$ on the base graph of $f(x) = \sin(x)$ maps to the point $\left(\frac{2\pi}{3}, 6\right)$ on an unknown transformed sine function. Similarly, the point $\left(\frac{\pi}{6}, \frac{1}{2}\right)$ on the base graph of $f(x) = \sin(x)$ maps to the point $\left(\frac{5\pi}{9}, \frac{13}{2}\right)$ on the same unknown transformed sine function. If the point $(p, 8)$ is on the unknown transformed sine function, then determine the value(s) of p, if $p \in [0, \pi]$. **[5 Marks]**

Some work is on scrap paper

Elimination? Make them equal?

x	y
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

$\frac{\pi}{6} \cdot \frac{3\pi}{18}$ or	$\frac{1}{2}$
$\frac{\pi}{2}$ or $\frac{9\pi}{18}$	1

$\frac{2\pi}{3}$ or $\frac{12\pi}{18}$	6
$\frac{5\pi}{9}$ or $\frac{10\pi}{18}$	$\frac{13}{2}$
p	8

$\frac{\frac{\pi}{2}}{\frac{3}{4}} = \frac{\pi}{2} \times \frac{4}{3}$

$= \frac{4\pi}{6} = \frac{2\pi}{3}$

$\frac{\frac{\pi}{6}}{\frac{3}{4}} = \frac{\pi}{6} \times \frac{4}{3}$?

$= \frac{4\pi}{18} =$

*** 2 Marks are awarded in the Communication Category for the use of correct mathematical form. ***

Assessment of Learning: Unit 4 – Trigonometric Functions (PART 2) – DAY 2

Application	Thinking	Communication
$16\frac{1}{2}$ /18	2 /5	2 /2

Instructions: Answer all questions in the space provided and **show all necessary steps**. Leave answers **exact** unless otherwise specified. The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

APPLICATION – [18 MARKS]

1. Risa is at the **top of** a Ferris wheel. The wheel has a radius of 8 metres and completes 1 cycle every 30 seconds. The bottom of the wheel is 2 metres above the ground.

    a. Sketch a properly labelled graph for **two revolutions** of the Ferris Wheel. **[2 Marks]**
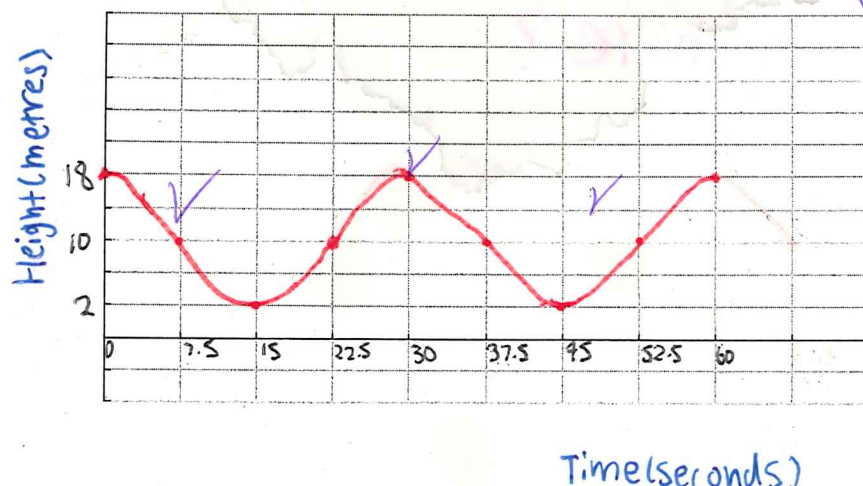
Amplitude = 8

$30 \div 4 =$

0	18
7.5	10
15	2
22.5	10
30	18

Time (seconds)

    b. Determine a **cosine function** that represents her height above the ground, in metres, as a function of time, in seconds. **[3 Marks]**

$K = \frac{2\pi}{b}$

$K = \frac{2\pi}{30}$

$K = \frac{\pi}{15}$

Function: $h(t) = 8\cos\frac{\pi}{15}(t) + 10$

    c. After the wheel starts moving, how many seconds will it take for Risa to be 13 metres above the ground for the first time? Round your answer to 2 decimal places. **[3 Marks]**

$13 = 8\cos\frac{\pi}{15}(t) + 10$

$3 = 8\cos\frac{\pi}{15}(t)$

$\frac{3}{8} = \cos\frac{\pi}{15}(t)$

$\cos^{-1}\left(\frac{3}{8}\right) = \frac{\pi}{15}(t)$

$\frac{\pi}{15}(t) = 1.186390552$

$\frac{\pi}{15} t = 5.667690597$

∴ It will take her 5.66 seconds to be 13 metres above the ground

    d. What is Risa's vertical height above the ground after 51 seconds? Round your answer to 2 decimal places. **[2 Marks]**

$h(t) = 8\cos\frac{\pi}{15}(t) + 10$

$h(t) = 8\cos\left(\frac{\pi}{15}t\right) + 10$

Plug in 51 to t
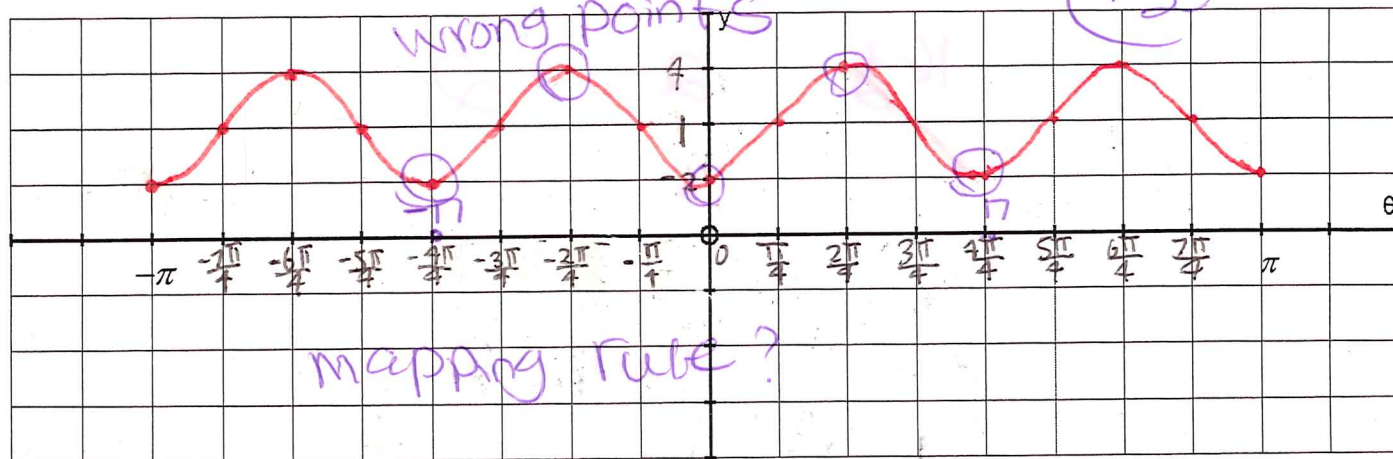
$h(t) = 7.527864045$

∴ Risa's vertical height above the ground after 51 seconds will be 7.53 metres.

10

2. Sketch a properly labelled graph of $y = -3\cos\left[2\left(\theta + \dfrac{\pi}{2}\right)\right] + 1$ for $-\pi \le \theta \le \pi$. **[3 Marks]**

wrong points (handwritten annotation)
$1\frac{1}{2}$ (handwritten annotation, circled)
mapping rule? (handwritten annotation)

Some work on Scrap Paper (handwritten)
where? (handwritten annotation)

3. Determine the **first four positive zeros** for the function $f(x) = 1 - 2\cos\left(2x - \dfrac{\pi}{3}\right)$. **Exact** answers. **[5 Marks]**

$\cos\left(2x - \dfrac{\pi}{3}\right) = \dfrac{1}{2}$

RAA is $\dfrac{\pi}{3}$ cos is (+) in Q I and IV

1st quadrant	4th quadrant
OR $\dfrac{\pi}{3}$	2π – OR
	$2\pi - \dfrac{\pi}{3} = \dfrac{5\pi}{3}$
$2x - \dfrac{\pi}{3} = \dfrac{\pi}{3}$	$2x - \dfrac{\pi}{3} = \dfrac{5\pi}{3}$
$2x = \dfrac{2\pi}{3} + 2\pi K$	$2x = 2\pi + 2\pi K$
$x = \dfrac{2\pi}{6} + \pi K$	$x = \pi + \pi K$

$\dfrac{2\pi}{6} + \pi = \dfrac{8\pi}{6} = \dfrac{4\pi}{3}$

$\pi + \pi = 2\pi$

$x = \pi, 2\pi$

$x = \dfrac{\pi}{3}, \dfrac{4\pi}{3}$

∴ the first four positive zeros are

$x = \dfrac{\pi}{3}, \pi, \dfrac{4\pi}{3}, 2\pi$

THINKING – [5 MARKS]

1. Determine the **exact** intersection point(s) of the functions $f(x) = \sin^2(2x) + 17\sin(2x) + 6$ and $h(x) = 5\cos^2(2x) + 6\sin(2x) + 8$, where $x \in [0, 2\pi]$. **[5 Marks]**

$f(x) = h(x)$

$\sin^2(2x) + 17\sin(2x) + 6 = 5\cos^2(2x) + 6\sin(2x) + 8$

$\sin^2(2x) + 17\sin(2x) + 6 = 5[1 - 2\sin^2(x)]^2 + 6\sin 2x) + 8$

$\sin^2(2x) + 17\sin(2x) + 6 = 5(1 + 4\sin^4(x)] + 6\sin(2x) + 8$

$\sin^2(2x) + 17\sin(2x) + 6 = 20\sin^4(x) + 5 + 6\sin(2x) + 8$

$0 = 20\sin^4 - \sin^2(2x) - 11\sin(2x) + 7$

$20\sin^4 - 1(2\sin x\cos x)^2 - 11(2\sin x\cos x) + 7$

$20\sin^4 - 4\sin^2 x\cos^2 x - 22\sin x\cos x + 7$

$20\sin^4 - 4\sin^2 x(1 - \sin^2 x)$

$+ 4\sin^4 - 4\sin^2 x$

(right side annotations:)
$-7\sin^2(2x) + 17\sin(2x) + 6 = 5\cos^2(2x) + 6\sin(2x) + 8$
$= \sin^2(2x) + 17\sin(2x) + 6 = 5[1 - 2\sin^2(x)][1 - 2\sin^2(x)] + 6\sin 2x + 8$
$= \sin^2(2x) + 17\sin(2x) + 6 =$

what are you doing? (handwritten annotation)

Work is on Scrap Paper (handwritten annotation)

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