Assessment of Learning: Unit 5 – Exponential & Logarithmic Functions – DAY 1

Knowledge & Understanding	Thinking	Communication
/19	/4	/5

Instructions: Answer all questions in the space provided and **show all necessary steps**. Leave answers **exact** unless otherwise specified. The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

KNOWLEDGE AND UNDERSTANDING

Multiple Choice: Write the **CAPITAL LETTER** corresponding to the correct answer on the line provided.
[1 Mark Each – 5 Marks Total]

1. The range of the function is $f(x) = 2^{x+5} - 4$ is _____D_____

    A. $y \in (0, \infty)$ B. $y \in [-4, \infty)$ C. $y \in (-\infty, -4)$ D. $y \in (-4, \infty)$

2. $6^{\log_6(3x)} + 5x$ simplifies to _____D_____

    A.   8   B. $\log(6)$ C. 6^{3x} D. $8x$

3. $\log_{\frac{3}{2}}\left(\frac{2}{3}\right)$ equals _____D_____

    A.   0   B.   1   C. $\frac{2}{3}$ D. -1

4. The x-intercept of the function $g(x) = 3\log_3(x+4) - 6$ is _____D_____

    A. -6 B. -5 C.   3   D.   5

5. Approximately how many times more intense is an earthquake of magnitude 6.2 than an earthquake of magnitude 5.5? _____C_____

    A.   5.5   B.   0.70   C.   5.01   D.   6.2

6. **Simplify**, and then evaluate. Do not use the Change of Base Formula.

    a) $\log_{36}(2) - \frac{1}{2}\log_{\frac{1}{6}}(3)$ [3 marks]

    $= -\frac{1}{2}\log_6(2) + \frac{1}{2}\log_6(3)$
    $= \log_6(2^{-\frac{1}{2}}) + \log_6(3^{\frac{1}{2}})$
    $= \log_6(\sqrt{2} \cdot \sqrt{3})$
    $= \log_6(\sqrt{6})$
    $= \frac{1}{2}$

    b) $\frac{5}{5^{-2\log(100)}}$ [2 marks]

    $= \frac{5}{5^{-2\log 10^2}}$
    $= \frac{5}{5^{-4}}$
    $= 5^5$
    $= 3125$

7. Write as a single logarithm in fully simplified form: $\log_8(\sqrt{x})^3 + \log_8(x^2) - \log_8(\sqrt{x})$ [3 marks]

    $= \log_8\left(\frac{x^{\frac{3}{2}} \cdot x^2}{x^{\frac{1}{2}}}\right)$
    $= \log_8(x^3)$
    $= 3\log_8(x), \; x \neq 0$

8. Solve for the value of the unknown using the most efficient method.

a) $16^{m+5} = 64^{m-3}$ [3 marks]

$(2^4)^{m+5} = (2^6)^{m-3}$

$4m + 20 = 6m - 18$

$2m = 38$

$m = 19$

b) $\log_7(x^2 + 6) = 1$ [2 marks]

$7 = x^2 + 6$

$x^2 - 1 = 0$

$(x-1)(x+1) = 0$

$x = 1$ or -1

check:

$(1)^2 + 6 > 0$ ✓

$(-1)^2 + 6 > 0$ ✓

c) $\log_4(x) = 5$ [1 mark]

$4^5 = x$

$x = 1024$

check:

$1024 > 0$ ✓

THINKING

1. Solve: $5^{\log_x(9)} = 25^{\log_3(x)^{\frac{1}{2}}} \cdot \log_{9x^2}(3x)^{10}$. [4 marks]

$5^{\frac{1}{\log_3(9)}} = 5^{2\log_3(x^{\frac{1}{2}})} \cdot 5$

$5^{\frac{1}{\frac{1}{2}\log_3(x)}} = 5^{\log_3(x)} \cdot 5$

$\frac{1}{\frac{1}{2}\log_3(x)} = \log_3(x) + 1$

$\log_{9a^2}(3a)^{10}$

$= \log_{(3a)^2}(3a)^{10}$

$= \frac{10}{2}$

$= 5$

$1 = \frac{1}{2}[\log_3(x)]^2 + \frac{1}{2}\log_3(x)$

$2 = [\log_3(x)]^2 + \log_3(x)$

Let $A = \log_3(x)$

$2 = A^2 + A$

$0 = A^2 + A - 2$

$0 = (A+2)(A-1)$

$\therefore A = -2$ or $A = 1$

$\therefore \log_3(x) = -2$ or $\log_3(x) = 1$

$3^{-2} = x$ $x = 3$

$\therefore x = \frac{1}{9}$

check:

$\frac{1}{9} > 0$ ✓

$3 > 0$ ✓

$\therefore x = \frac{1}{9}, x = 3$

COMMUNICATION

1. Explain how you would find the graph of $f(x) = 3\log_2(x) + 1$ given $y = 2^x$. [3 marks]

Find the inverse of $y = 2^x$ to get the base graph of $y = \log_2(x)$.

Next, for the base graph of $y = \log_2(x)$, vertically expand it by a factor of 3 and translate it 1 unit up.

Two marks are awarded for the appropriate use of mathematical form throughout the test. [2 marks]
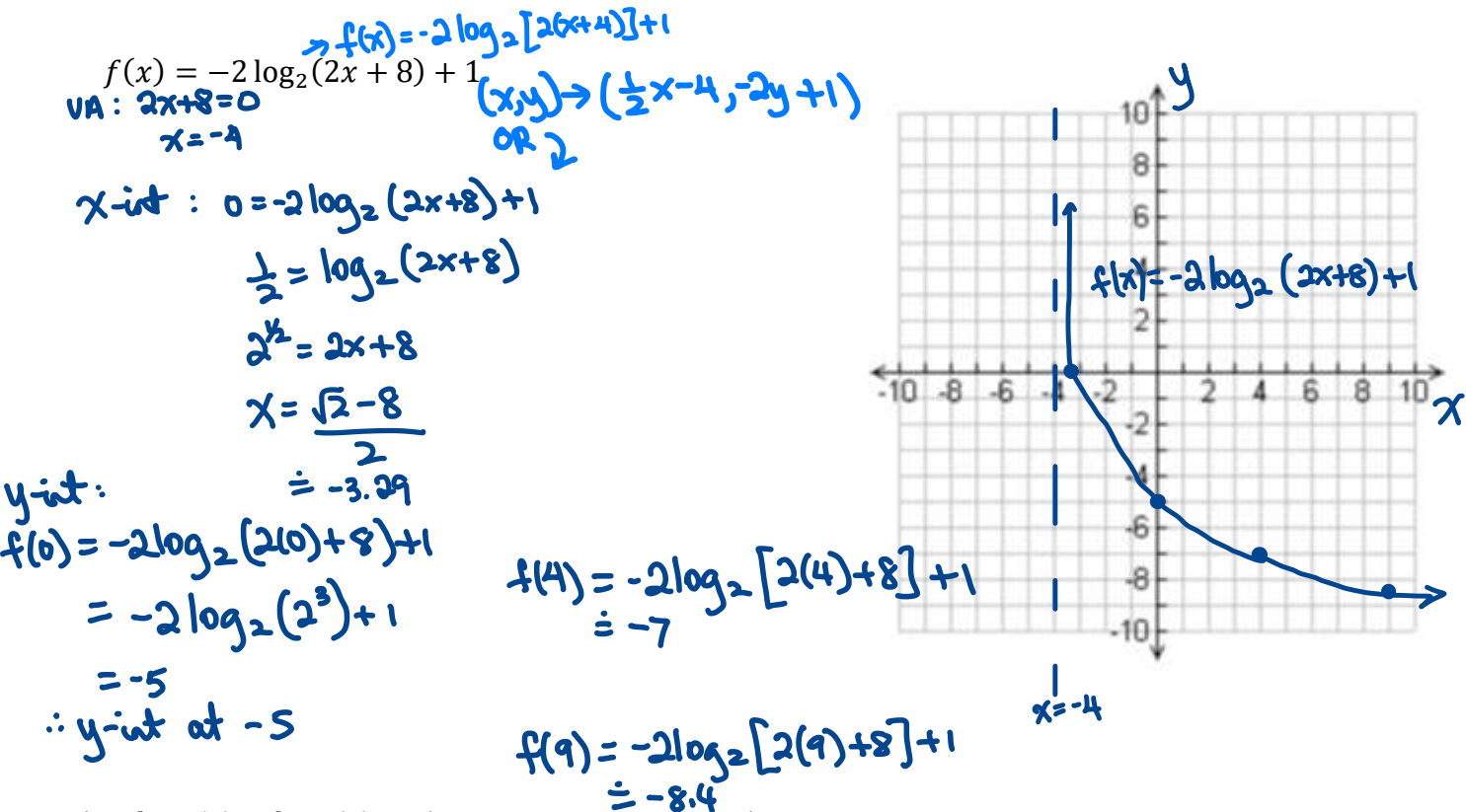
Assessment of Learning: Unit 5 – Exponential & Logarithmic Functions – DAY 2

Knowledge & Understanding	Thinking	Communication
/17	/4	/2

Instructions: Answer all questions in the space provided and **show all necessary steps**. Leave answers **exact** unless otherwise specified. The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

APPLICATION

1. Graph the following relation in the grid provided. <u>Include at least 4 points</u>. Show your work by either mapping or graphing the steps.   [4 marks]

$$f(x) = -2\log_2(2x + 8) + 1$$

$\rightarrow f(x) = -2\log_2[2(x+4)]+1$

VA: $2x+8=0$
   $x=-4$

$(x,y) \rightarrow (\frac{1}{2}x-4, -2y+1)$
   OR 2

x-int: $0 = -2\log_2(2x+8)+1$

$\frac{1}{2} = \log_2(2x+8)$

$2^{\frac{1}{2}} = 2x+8$

$x = \frac{\sqrt{2}-8}{2}$

$\doteq -3.29$

y-int:
$f(0) = -2\log_2(2(0)+8)+1$
$= -2\log_2(2^3)+1$
$= -5$
$\therefore$ y-int at -5

$f(4) = -2\log_2[2(4)+8]+1$
$\doteq -7$

$f(9) = -2\log_2[2(9)+8]+1$
$\doteq -8.4$

$f(x)=-2\log_2(2x+8)+1$

$x=-4$

2. Solve: $\log_2(x) + \log_4(x) = 5$. Exact answers. [4 marks]

$\log_2(x) + \frac{1}{2}\log_2(x) = 5$

$\frac{3}{2}\log_2(x) = 5$

$\log_2(x) = \frac{10}{3}$

$x = 2^{\frac{10}{3}}$

3. Determine the hydrogen ion concentration of an egg with a pH of 7.8? [2 marks]

$pH = -\log[H^+]$

$7.8 = -\log[H^+]$

$-7.8 = \log[H^+]$

$H^+ = 10^{-7.8}$
$= 1.58 \times 10^{-8}$

$\therefore$ The hydrogen ion concentration is approx 1.58×10^{-8} mol/L

4. The half-life of a certain substance is 3.6 days. How long will it take for 20 grams of the substance to decay to 7 grams? Round final answers to 2 decimal places. [3 marks]

$$A = A_0 R^{\frac{t}{a}}$$

$$7 = 20\left(\frac{1}{2}\right)^{\frac{t}{3.6}}$$

$$\frac{7}{20} = \left(\frac{1}{2}\right)^{\frac{t}{3.6}}$$

$$\log\left(\frac{7}{20}\right) = \frac{t}{3.6}\log\left(\frac{1}{2}\right)$$

$$t = \frac{3.6\log\left(\frac{7}{20}\right)}{\log\left(\frac{1}{2}\right)}$$

$$\doteq 5.45$$

∴ It will take approx 5.45 days.

5. Determine the domain of $y = \log_7(2x^3 - 14x^2 - 8x + 56)$. [4 marks]

Let $f(x) = 2x^3 - 14x^2 - 8x + 56$
$= 2(x^3 - 7x^2 - 4x + 28)$
$= 2[x^2(x-7) - 4(x-7)]$
$= 2(x^2 - 4)(x-7)$
$= 2(x-2)(x+2)(x-7)$

Domain: $2(x-2)(x-7)(x+2) > 0$

$$\frac{=}{-2} \quad \frac{+}{} \quad \frac{-}{2} \quad \frac{+}{7}$$

∴ $D = \{x \in \mathbb{R} \mid (-2, 2) \cup (7, \infty), x \in \mathbb{R}\}$

THINKING

6. If $\log_a(2) = x$ and $\log_a(3) = y$, find the value of $\log_{\sqrt{6}}(12)$ in terms of x and y. [4 marks]

$$= \frac{\log_a(12)}{\frac{1}{2}\log_a(6)}$$

$$= \frac{\log_a(3) + \log_a(2) + \log_a(2)}{\frac{1}{2}\log_a(6)}$$

$$= \frac{\log_a(3) + 2\log_a(2)}{\frac{1}{2}[\log_a(3) + \log_a(2)]}$$

$$= \frac{y + 2x}{\frac{1}{2}(y + x)}$$

$$= \frac{4x + 2y}{x + y}$$

COMMUNICATION
Two marks are awarded for the appropriate use of mathematical form throughout the test. [2 marks]