

UNIT 2 ASSESSMENT OF LEARNING: EXPLORING DERIVATIVES– DAY 1

Instructions:

Name: _____

- Non-graphing calculators may be used but not shared. Notebooks may not be used.
- The use of cellphones, audio- or video-recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

K & U	Thinking	Comm.
/18	/10	/2

Knowledge and Understanding - [18 marks]

Multiple Choice: Write the CAPITAL LETTER corresponding to the correct answer on the line provided.
[1 mark Each]

1.

If $f(x) = (\sqrt{3x-1}-4)(\sqrt{3x-1}+4)$, then $f'(3)$ is

D

- A. 0

B. 1

C. 2

D. 3
2.

If $y = (3x^2)f(x)$, then $\frac{dy}{dx}$ is

C

- A. $3x^2 f'(x)$

B. $3x[x f(x) + 2f'(x)]$

C. $3x[x f'(x) + 2f(x)]$

D. $18x f'(x)$
3.

The position function of an object is given by $s(t) = t^3 - \frac{3}{2}t^2$, where time, t , is in seconds and distance, s , is in metres. When is the acceleration of the object equal to zero?

A

- A. $\frac{1}{2}$ seconds

B. 2 seconds

C. 4 seconds

D. 6 seconds
4.

Determine $\left.\frac{d^2y}{dx^2}\right|_{x=-1}$ for the function $y = 3x^5 + 13x^3 - \frac{5}{x}$. [3 marks]

$$\frac{dy}{dx} = 15x^4 + 39x^2 + \frac{5}{x^2}$$
$$\frac{d^2y}{dx^2} = 60x^3 + 78x - \frac{10}{x^3}$$
$$\left.\frac{d^2y}{dx^2}\right|_{x=-1} = -128$$

5. Differentiate the following with respect to x . Leave all answers with positive exponents. **Completely simplify** your answer for **part c only**. [12 marks]

a.

$y = \frac{\pi^2 x + \sqrt[3]{x^2}}{\sqrt{x}}$

[3]

$$y = \pi^2 x^{\frac{1}{2}} + x^{\frac{1}{6}}$$
$$y' = \frac{1}{2}\pi^2 x^{-\frac{1}{2}} + \frac{1}{6}x^{-\frac{5}{6}}$$
$$y' = \frac{\pi^2}{2\sqrt{x}} + \frac{1}{6x^{\frac{5}{6}}}$$

b.

$f(x) = \sqrt[3]{2x^2 + (4x^3 - x)^3}$

[4]

$$f(x) = \left[2x^2 + (4x^3 - x)^3\right]^{\frac{1}{3}}$$
$$f(x) = \frac{1}{3}\left[2x^2 + (4x^3 - x)^3\right]^{\frac{2}{3}}\left[4x + 3(4x^3 - x)^2(12x^2 - 1)\right]$$

c. $f(x) = (2x^2 + 1)^3 (5x^3 + 4)^4$ [5]

$$\begin{aligned}
 f'(x) &= 3(2x^2 + 1)^2 (4x)(5x^3 + 4)^4 + 4(5x^3 + 4)^3 (15x^2)(2x^2 + 1)^3 \\
 &= 3(2x^2 + 1)^2 (4x)(5x^3 + 4)^4 + 4(5x^3 + 4)^3 (15x^2)(2x^2 + 1)^3 \\
 &= 12x(2x^2 + 1)^2 (5x^3 + 4)^3 [(5x^3 + 4) + 5x(2x^2 + 1)] \\
 &= 12x(2x^2 + 1)^2 (5x^3 + 4)^3 (15x^3 + 5x + 4)
 \end{aligned}$$

Thinking - [10 marks]

1. The graph of $y = \frac{1}{2}\sqrt{4-x^2}$ is as shown. This graph models a racetrack. Mrs. Moshtagh is in her car travelling too fast (clockwise) on the track and can't stay on the track. She leaves the track and travels along a tangent and hits a tree located at (4, 0). At which point did she leave the road? [5 marks]

$$y' = \frac{1}{2} \times \frac{-2x}{2\sqrt{4-x^2}}$$

$$y' = \frac{-x}{2\sqrt{4-x^2}}$$

$$y - \frac{1}{2}\sqrt{4-a^2} = \frac{-a}{2\sqrt{4-a^2}}(x-a)$$

sub. pt(4,0), we get

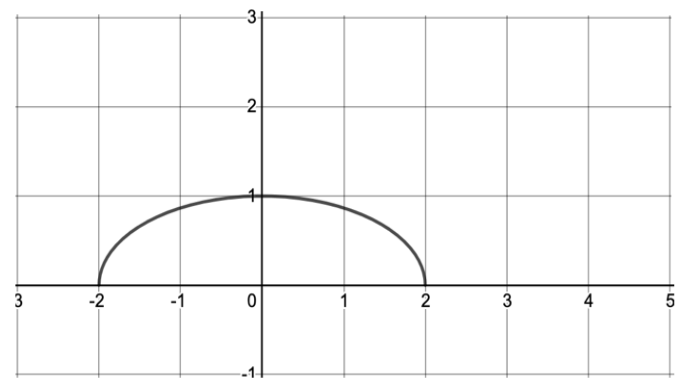
$$0 - \frac{1}{2}\sqrt{4-a^2} = \frac{-a}{2\sqrt{4-a^2}}(4-a)$$

$$4-a^2 = a(4-a)$$

$$4-a^2 = 4a-a^2$$

$$a = 1, y = \frac{\sqrt{3}}{2}$$

∴ She gets off the road at point $\left(1, \frac{\sqrt{3}}{2}\right)$



2. Given $f(x) = ax^3 + bx^2 + cx + d$, find values of a, b, c and d if $f''(0) = 2$, there are horizontal tangents at $x = -2$ and $x = 1$ and the function has a y -intercept equal to $g^{(4)}(4)$ where $g(x)$ is given by equation $g(x) = \frac{x^5}{120} - 6x^3 + 3x^2 + 1$. [5 marks]

$$f'(x) = 3ax^2 + 2bx + c$$

$$f''(x) = 6ax + 2b$$

$$f''(0) = 2$$

$$f'(-2) = 0$$

$$f'(1) = 0$$

$$2b = 2$$

$$12a - 4 + c = 0$$

$$3a + 2 + c = 0$$

$$b = 1$$

$$\begin{cases} 12a + c = 4 \\ 3a + c = -2 \end{cases}$$

$$9a = 6$$

$$a = \frac{2}{3}, c = -4$$

$$g'(x) = \frac{5x^4}{120} - 18x^2 + 6x$$

$$g''(x) = \frac{20x^3}{120} - 36x + 6$$

$$g'''(x) = \frac{60x^2}{120} - 36$$

$$g^{(4)}(x) = \frac{120x}{120} = x$$

$$y\text{-int} = g^{(4)}(4) = 4$$

$$\therefore f(x) = \frac{2}{3}x^3 + x^2 - 4x + 4$$