

**Unit 1: Polynomial Functions Assessment of Learning**

K & U	Application	Thinking	Communication
/15	/16	/5	/2

**Instructions:**

- Non-graphing calculators may be used but not shared. Notebooks may not be used.
- Only methods taught in MHF4U1 will be accepted. Show all work in the space provided.
- The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.
- Please complete the assessment independently with academic honesty as the guiding principle.

**KNOWLEDGE & UNDERSTANDING – [15 Marks]**

**Multiple Choice:** Write the CAPITAL letter corresponding to the correct answer on the line provided.  
[4 Marks]

1. The remainder, when  $3x^3 + 4x^2 + 4x$  is divided by  $x - 1$  is

A. -11

B. 11

C. -5

D. 0

B

2. Which of the following statements is (are) true for  $f(x) = 6(x-1)(x+5)^3(x-9)$ ?

I) as  $x \rightarrow \infty$ ,  $f(x) \rightarrow -\infty$

III) as  $x \rightarrow -\infty$ ,  $f(x) \rightarrow -\infty$

II) as  $x \rightarrow \infty$ ,  $f(x) \rightarrow \infty$

IV) as  $x \rightarrow -\infty$ ,  $f(x) \rightarrow \infty$

A. I)

B. II) and III)

C. I) and II)

D. I) and IV)

B

3. Which statement is false regarding  $f(x) = -4(x+2)^3(x^2+9)$ ?

A. The leading coefficient is negative.

B.  $f(x)$  goes through the  $x$ -axis once and bounces on the  $x$ -axis twice.

C. The constant value of the function's finite differences is equal to  $-480$ .

D. As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow \infty$ .

B

4. When factored,  $8x^3 + y^6$  equals:

A.  $(2x + y^2)(4x^2 - 2xy^2 + y^4)$

B.  $(2x + y^3)(4x^2 - 2xy + y^2)$

C.  $(2x - y^2)(4x^2 + 2xy^2 + y^4)$

D.  $(2x - y^3)(4x^2 + 2xy + y^2)$

A

5. State the family of functions of degree 8 with two equal roots at  $x = -5$ , a triple root at  $x = 3$  and a point of inflection at  $x = 1$ . [2 Marks]

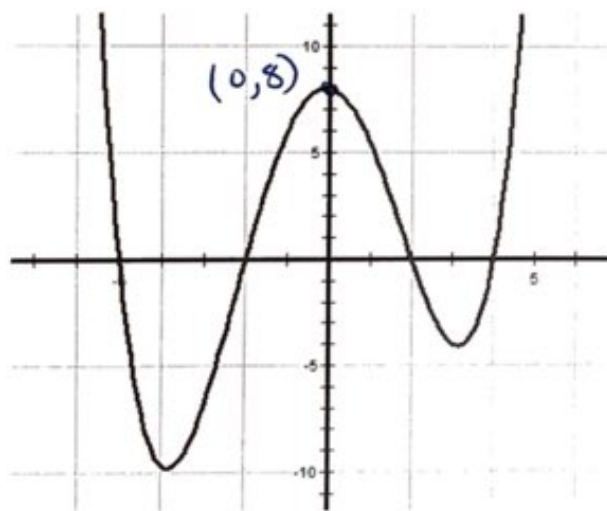
Answer:  $f(x) = a(x+5)^2(x-3)^3(x-1)^3$   $a \in \mathbb{R}$ ,  $a \neq 0$

6. Divide  $2x^3 - 4x^2 + 9x - 4$  by  $x+1$  using **long division** and write the division statement. [3 Marks]

$$\begin{array}{r}
 2x^2 - 6x + 15 \\
 x+1 \overline{) 2x^3 - 4x^2 + 9x - 4} \\
 \underline{-(2x^3 + 2x^2)} \phantom{-4} \\
 -6x^2 + 9x - 4 \\
 \underline{-(-6x^2 - 6x)} \phantom{-4} \\
 15x - 4 \\
 \underline{-(15x + 15)} \\
 -19
 \end{array}$$

Division Statement:  $2x^3 - 4x^2 + 9x - 4 = (x+1)(2x^2 - 6x + 15) - 19, x \neq -1$

7. Determine the **specific equation** (assume lowest possible degree) of the function below. [3 Marks]



$$f(x) = a(x+5)(x+2)(x-2)(x-4)$$

$a \in \mathbb{R}, a \neq 0$

$$8 = a(5)(2)(-2)(-4)$$

$$a = \frac{8}{80}$$

$$a = \frac{1}{10}$$

Answer:  $f(x) = \frac{1}{10}(x+5)(x+2)(x-2)(x-4)$

8. The height of a rock above the water is modeled by  $h(t) = -5t^2 - 5t + 80$ , where  $h(t)$  is the height in metres and  $t$  is time in seconds. Estimate the IROC at  $t = 1$  second. [3 Marks]

$$IROC \doteq \frac{h(1.01) - h(0.99)}{1.01 - 0.99}$$

$$IROC \doteq \frac{-5(1.01)^2 - 5(1.01) + 80 - [-5(0.99)^2 - 5(0.99) + 80]}{0.02}$$

$$IROC \doteq -15$$

Answer:  $-15 \text{ m/s}$

**APPLICATION - [16 Marks]**

1. Solve the following. **Note:** For Part b. use interval notation to express your final answer. [7 Marks]

a.  $4x^3 - x^2 - 11x - 6 = 0$  [4]

$f(-1) = 0 \Rightarrow x+1$  is a factor

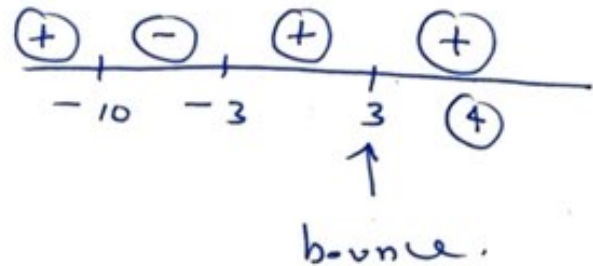
$$\begin{array}{r} 4x^2 - 5x - 6 \\ x+1 \overline{) 4x^3 - x^2 - 11x - 6} \\ \underline{-(4x^3 + 4x^2)} \phantom{-6} \\ -5x^2 - 11x - 6 \\ \underline{-(-5x^2 - 5x)} \phantom{-6} \\ -6x - 6 \\ \underline{-(-6x - 6)} \\ 0 \end{array}$$

$= (4x+3)(x-2)$  OR

Answer:  $\therefore x = -1, -\frac{3}{4}, 2$

b.  $(x^2 + 7x - 30)(x^2 - 9) > 0$  [3]

$(x+10)(x-3)(x-3)(x+3) > 0$



Answer:  $\therefore (-\infty, -10) \cup (-3, 3) \cup (3, \infty)$

2. Determine the polynomial function that passes through the following points. [5 Marks]

x	y	1st diff	2nd diff	3rd diff
-1	-1	6		
0	5		-2	
1	9	4		12
2	23	14	10	
3	59	36	22	12

$\Rightarrow$  cubic; degree 3  
 $n=3, t_3=12$

$t_n = s^n n! \cdot a$

$12 = 1^3 3! \cdot a$

$d=5$

$\therefore f(x) = 2x^3 + bx^2 + cx + 5$

Sub.  $(-1, -1)$

$-1 = -2 + b - c + 5$

$b - c = -4$  (1)

Sub.  $(1, 9)$

$9 = 2 + b + c + 5$

$b + c = 2$  (2)

(1) + (2)

$b - c = -4$

$+(b + c = 2)$

$b = -1$

$c = 3$

Answer:  $f(x) = 2x^3 - x^2 + 3x + 5$



3. Graph and properly label  $f(x) = -\frac{1}{8}(x-1)^3(x+4)^2(x+1)$ . [4 Marks]

\* Poi @  $x = 1$

\* bounce @  $x = -4$

\* leading coefficient: -ve

\* y-int

$$y = -\frac{1}{8}(-1)(16)(+1)$$

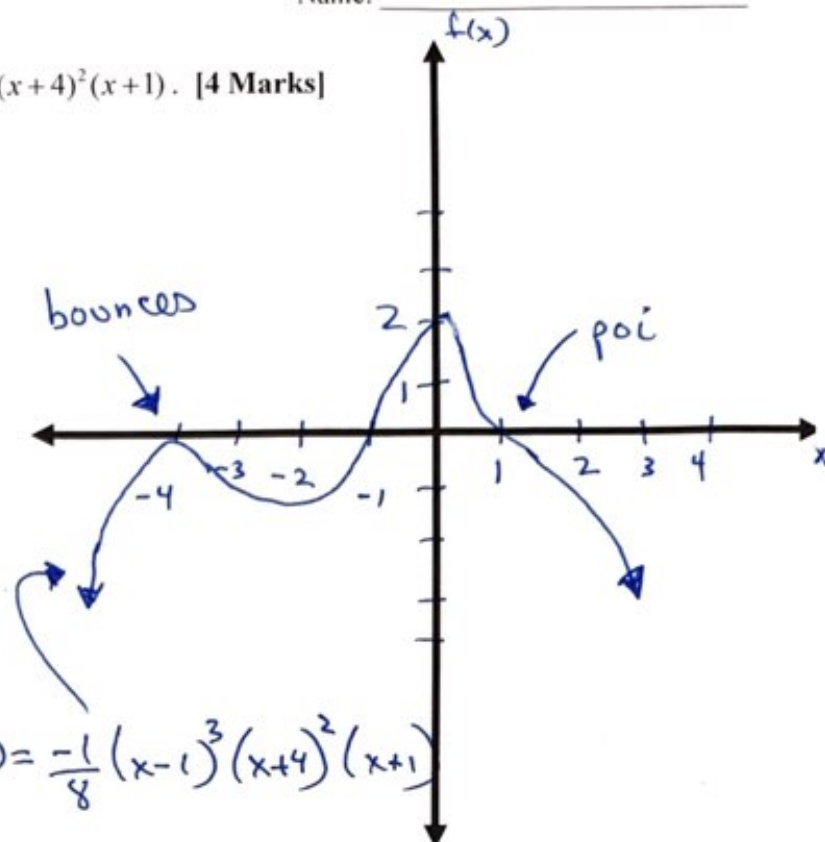
$$y = +2 \quad (0, +2)$$

\* end behaviour:

starts Q3

ends Q4

$$f(x) = -\frac{1}{8}(x-1)^3(x+4)^2(x+1)$$



### THINKING - [5 Marks]

1. A package sent by FedEx has the shape of a square base prism with a side length of  $x$  centimetres. The sum of the height of the prism and the perimeter of its base is 100 cm. Determine the dimensions of the package if the volume is  $368 \text{ cm}^3$ . [5 Marks]

let  $x$  rep the length & width.

$$\text{let } 4x + h = 100 \Rightarrow h = 100 - 4x$$

$$V = l \cdot w \cdot h$$

$$x^2(100 - 4x) = 368$$

$$-4x^3 + 100x^2 - 368 = 0$$

$$x^3 - 25x^2 + 92 = 0$$

$$f(2) = 0 \Rightarrow x - 2 \text{ is a factor}$$

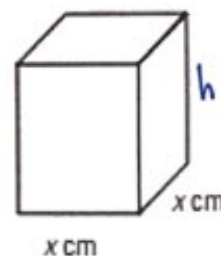
$$* x^2 - 23x - 46$$

$$x = \frac{23 \pm \sqrt{713}}{2}$$

$$x = 24.851 \text{ or } -1.851 \quad \left. \vphantom{x = 24.851} \right\} \text{violates domain of box.}$$

$$\therefore \underline{x = 2}$$

Answer: Dimensions are:  $2 \text{ cm} \times 2 \text{ cm} \times 92 \text{ cm}$ .



$$\begin{array}{r} x^2 - 23x - 46 \\ x - 2 \overline{) x^3 - 25x^2 + 0x + 92} \\ \underline{-(x^3 - 2x^2)} \phantom{+ 0x + 92} \\ -23x^2 + 0x + 92 \\ \underline{-(-23x^2 + 46x)} \phantom{+ 92} \\ -46x + 92 \\ \underline{-(-46x + 92)} \\ 0 \end{array}$$