MHF4U1-V2
Leung/ Moshtagh/ Vincent

Name: ___________________________ Period: ______

Date: Tuesday, January 17th, 2023

Assessment of Learning for Unit 6 – Combining Functions

Knowledge	Application	Thinking	Communication
/15	/10	/5	/2

Instructions:
- Non-graphing calculators may be used but not shared. Notebooks may not be used.
- Only methods taught in MHF4U1 will be accepted. Show all work in the space provided.
- The use of cellphones, audio- or video-recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

Knowledge and Understanding – [15 Marks]

Multiple Choice: Place the CAPITAL letter of the correct answer on the provided line.

1. Given the functions $f(x) = 2\log_3(x+3)$ and $g(x) = \sqrt{5-x}$, then the domain of $(f \div g)(x)$ is **D**

A. $-3 < x \leq 5$ B. $x \leq -3$ C. $x \geq 5$ D. $-3 < x < 5$

2. If $f(x)$ and $g(x)$ are both odd functions, where $g(x) \neq 0$, then $h(x) = \frac{f(x)}{g(x)}$ is **B**

A. an odd function B. an even function C. neither even nor odd D. cannot be determined

3. Given the following functions, determine the following in simplest **exact** form. **[10 Marks]**

$f(x) = \{(-5,2),\ (\sqrt{2},3),\ (3,0),\ (\pi,-1),\ (14,\sqrt{7})\}$ $g(x) = \{(-1,\pi),\ (2,\sqrt{2}),\ (3,-1),\ (4,3)\}$

$h(x) = 2x^2 - 18$ $k(x) = 2\cos(2x) - 3$ $m(x) = x^2 - x - 6$

a. $(f \circ g)(x)$ **[2]**

$f \circ g = \{(-1,-1),\ (2,3),\ (4,0)\}$

b. $f\left(k\left(\frac{\pi}{2}\right)\right)$ **[2]**

$= f(-5)$

$= 2$

c. $\left(\frac{m}{h}\right)(x)$ **[2]**

$\frac{m(x)}{h(x)} = \frac{x^2-x-6}{2x^2-18}$

$= \frac{(x-3)(x+2)}{2(x-3)(x+3)}$

$= \frac{x+2}{2(x+3)}$

d. $(f^{-1} \times g)(x)$ **[2]**

$= \{(-1,\pi^2),(2,-5\sqrt{2}),\ (3,-\sqrt{2})\}$

e. Domain of $\left(\frac{g}{h}\right)(x)$ **[2]**

$= \{-1,2,4\}$

4. Given $f(x) = 3x$ and $g(x) = 2\sqrt{x-1}$. Find the value(s) of x that make $(f \circ g)(x) = (g \circ f)(x)$. **[3 Marks]**

$(f\circ g)(x) = 6\sqrt{x-1}$

$(g\circ f)(x) = 2\sqrt{3x-1}$

$6\sqrt{x-1} = 2\sqrt{3x-1}$

$3\sqrt{x-1} = \sqrt{3x-1}$

$9(x-1) = 3x-1$

$6x = 8$

$\boxed{x = \frac{4}{3}}$

APPLICATION – [10 Marks]

1. Let $f(x)=x^4+bx^2-1$ and $g(x)=ax^2+2x-1$. The functions are combined to form the new function, $h(x)=f(x)\cdot g(x)$. Points $(1,16)$ and $(-1,-8)$ satisfy the new function. Determine the values of a and b. **[5 Marks]**

$$f(x)g(x)=(x^4+bx^2-1)(ax^6+2x^3-1)$$

$h(1)=10 \qquad (b)(a+1)=10 \qquad (1)$

$h(-1)=-10 \qquad (b)(a-3)=-10 \qquad (2)$

$$\frac{(1)}{(2)}:\frac{a+1}{a-3}=-1$$

$$a+1=-a+3$$

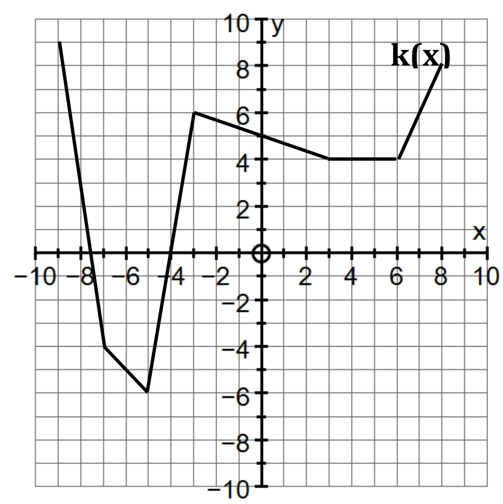
$$2a=2$$

$$\boxed{a=1}$$

Sub. $a=1$ into (1): $2b=10$

$$\boxed{b=5}$$

2. Given the graph $y=k(x)$ below and the functions $m(x)=\sqrt{3-5x}$, $h(x)=\dfrac{7x+1}{x^2-2x-5}$, $f(x)=m(h(k(x)))$ and $n(x)=4h(-10x)-k(k(m(x)+1))$, determine the following



a. $f(-4)$ **[3]**

$$f(-4)=m(h(k(-4)))$$
$$=m(h(0))$$
$$=m\left(\frac{-1}{5}\right)$$
$$=2$$

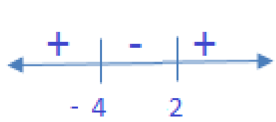
b. $n\left(-\frac{1}{5}\right)$ **[3]**

$$n\left(-\frac{1}{5}\right)=4h(-10x)-k(k(m(x)+1))$$
$$=4h(2)-k\left(k\left(m\left(-\frac{1}{5}\right)+1\right)\right)$$
$$=4(-3)-k(k(3))$$
$$=-12-4$$
$$=-16$$

Thinking – [5 Marks]

1. Solve $f(g(x))\le(h^{-1}\circ h)(x)-p(x)$, given $f(x)=x+2, g(x)=x^2+5x-18, h(x)=\dfrac{12}{x-7}$ and $p(x)=\log\left(10^{x^2}\right)$. **[5 Marks]**

$$f(g(x))=x^2+5x-16$$
$$(h^{-1}oh)(x)=x$$
$$p(x)=\log\left(10^{x^2}\right)$$
$$=x^2$$
$$f(g(x))\le(h^{-1}oh)(x)-p(x)$$
$$x^2+5x-16\le x-x^2$$
$$2x^2+4x-16\le 0$$
$$x^2+2x-8\le 0$$
$$(x+4)(x-2)\le 0$$



$$\therefore \boxed{x\in[-4,2]}$$

Two (2) marks will be awarded in the Communication Category for proper mathematical form and notation.