UNIT 3 ASSESSMENT OF LEARNING: CURVE SKETCHING – DAY 1

Name: _____

Instructions: You MUST use concepts covered in this unit/course. Show all steps for full marks.

K & U	Thinking	Comm.
/20	/5	/2

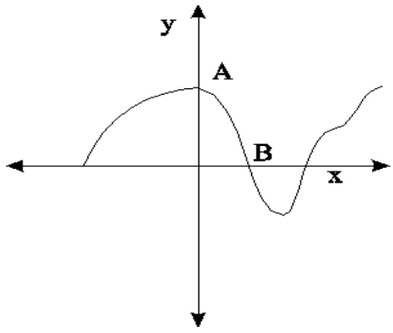
Knowledge and Understanding - [20 marks]

Multiple Choice: Write the CAPITAL LETTER corresponding to the correct answer on the line provided. **[4 marks]**

1. A critical number is: ***C***

   A. a number c in the domain of a function $f(x)$ such that $f'(c) \geq 0$.
   B. a number c in the domain of a function $f(x)$ such that $f'(c) \leq 0$.
   C. a number c in the domain of a function $f(x)$ such that $f'(c) = 0$ or $f'(c)$ does not exist.
   D. either a local maximum point or a local minimum point.

2. The graph of the function $f(x)$ is shown. For the section from point A to point B, indicate whether the derivative is positive or negative and whether the derivative is increasing or decreasing. ***D***



   A. positive and increasing.
   B. positive and decreasing.
   C. negative and increasing.
   D. negative and decreasing.

3. For the function $f(x) = x^3 - 3x + 1$, state the intervals on which the function is increasing. ***A***

   A. $(-\infty, -1) \cup (1, \infty)$ B. $(-\infty, -1)$ C. $(-1, 1)$ D. $(1, \infty)$

4. If $f'(c) = 0$, $f(x)$ **may** have ***D***

   A. a local minimum or a local maximum at $x = c$.
   B. a point of inflection at $x = c$.
   C. neither A nor B is possible.
   D. both A or B are possible.

5. Use the **2nd Derivative Test** to determine the coordinates of the local extrema of the function $f(x) = 3x^4 + 8x^3 - 6x^2 - 24x + 3$. **[4 marks]**

$f'(x) = 12x^3 + 24x^2 - 12x - 24x$

$= 12x^2(x+2) - 12(x+2)$

$= 12(x+2)(x^2 - 1)$

$= 12(x+2)(x+1)(x-1)$

$f'(x) = 0 : x = -2, -1, 1$

$f''(x) = 36x^2 + 48x - 12x$

$f''(1) = 72 > 0$ **local min (1,-16)**

$f''(-2) = 36 > 0$ **local min (-2,-11)**

$f''(-1) = -24 < 0$ **local max (-1,16)**

6. Answer the following as **True** or **False**. Place your answer on the space provided. **[3 marks]**

a. If $f''(x) < 0$ for $x < 7$ and $f''(x) > 0$ for $x > 7$, then $f(x)$ must have a point of inflection at $x = 7$. **False**

b. If 5 $f'(-6)$ does not exist and $f''(-6)$ does not exist, then there must be either a vertical tangent or a cusp at $x = -6$. **True**

c. If a function has a point of inflection at $x = \frac{1}{2}$, then there must be a vertical tangent at $x = \frac{1}{2}$. **False**

7. Determine the absolute maximum and minimum points of the function $f(x) = x^3 - 6x^2 + 4$ over the interval $[-3, 5]$. **[5 marks]**

$f'(x) = 3x^2 - 12x$

$0 = 3x(x - 4)$

$x = 0, \; x = 4$

$f(-3) = -111$ abs.min

$f(0) = 4$ abs.max

$f(4) = -28$

$f(5) = -21$

8. The point $(-1, 5)$ is a point of inflection on the graph of $f(x) = 2x^3 + mx^2 - 3x + n$. Determine the values of m and n. **[4 marks]**

$f(-1) = 5$ and $f''(1) = 0$

$-2 + m + 3 + n = 5$

$m + n = 4 \quad (1)$

$f'(x) = 6x^2 + 2mx - 3$

$f''(x) = 12x + 2m$

$12 + 3m = 0$

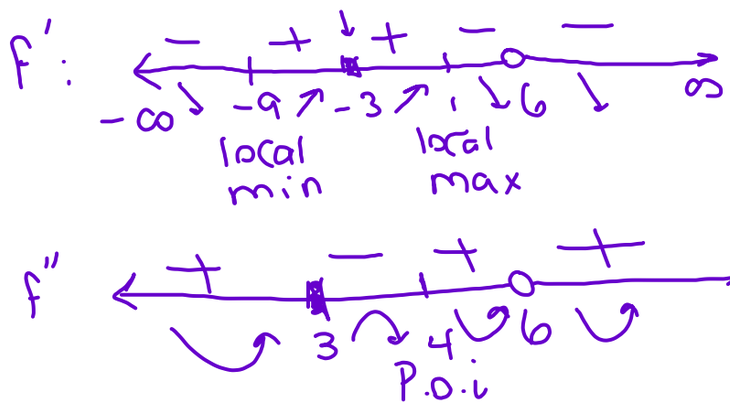
$3m = -12$

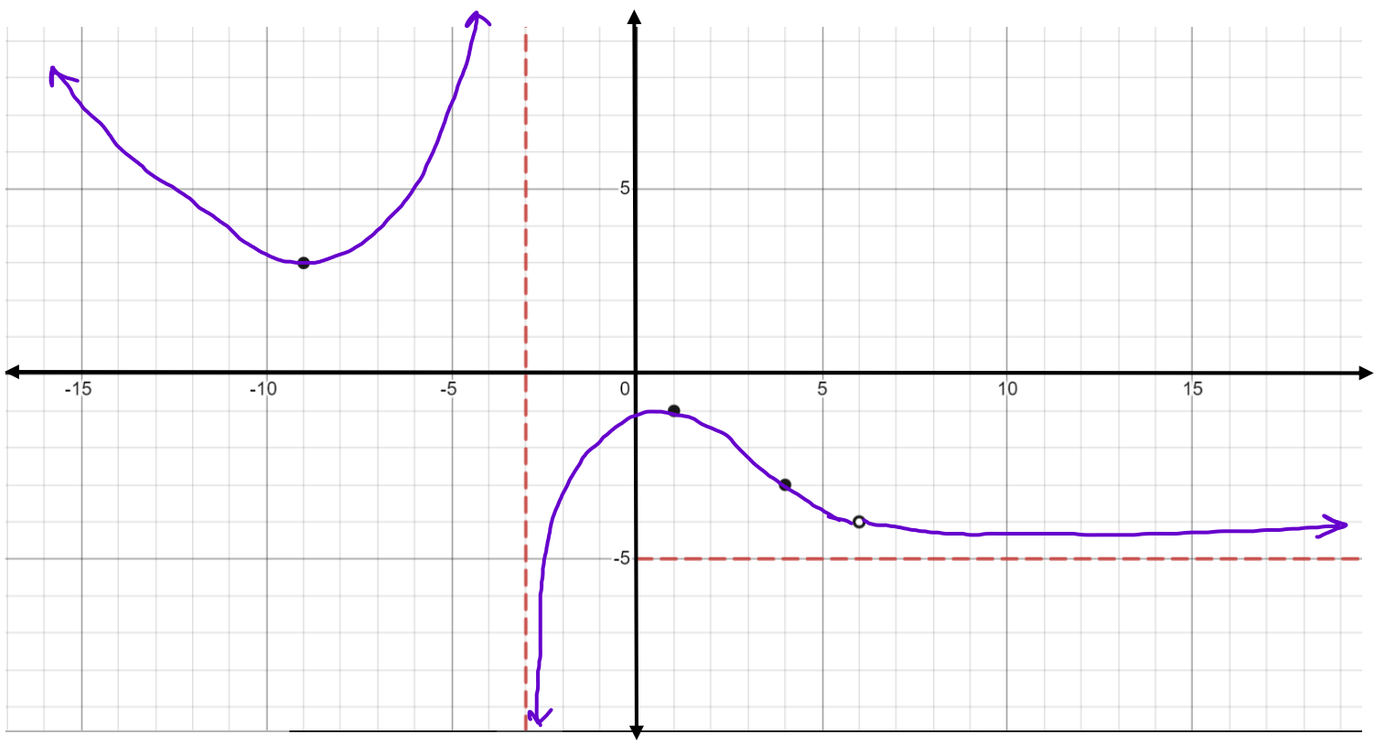
$\boxed{m = -4}$ sub. m=-4 into (1) $\boxed{n = 8}$

Thinking - [5 marks]

1. Sketch the graph of **any** function having **all** of the following properties. **[5 marks]**

- $f(1) = -1$, $f(4) = -3$, $f(6)$ is undefined
- $f'(x) < 0$ for $x \in (-\infty, -9)$, $f'(x) < 0$ for $x \in (1, 6) \cup (6, \infty)$
- $f'(x) > 0$ for $x \in (-9, -3)$, $f'(x) > 0$ for $x \in (-3, 1)$
- $f'(-9) = f'(1) = 0$
- $f''(x) > 0$ for $x \in (-\infty, -3)$, $f''(x) > 0$ for $x \in (4, 6) \cup (6, \infty)$
- $f''(x) < 0$ for $x \in (-3, 4)$
- $f''(4) = 0$
- $\lim_{x \to -3^+} f(x) = -\infty$, $\lim_{x \to -3^-} f(x) = +\infty$
- $\lim_{x \to +\infty} f(x) = -5$
- $\lim_{x \to -\infty} f(x) = +\infty$





*** **2 marks** will be awarded in the Communication Category for proper mathematical form. ***