Name: Solutions

Unit 3: Trigonometric Functions – Part 1 Assessment of Learning – DAY 1

K & U	Application	Comm.
/13	/15	/2

Instructions

- Non-graphing calculators may be used but not shared. Notebooks may not be used.
- Only methods taught in MHF4U1 will be accepted. Show all work in the space provided.
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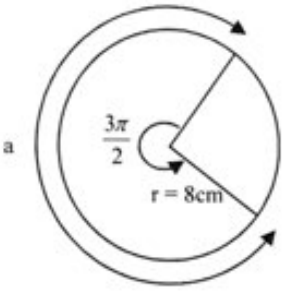
KNOWLEDGE & UNDERSTANDING – [13 Marks]

Multiple Choice: Write the CAPITAL letter corresponding to the correct answer on the line provided. **[5 Marks]**

1. Given $\sin(x) = m$, the expression for $\cos(2x)$, in terms of m, is A

   A. $1-2m^2$ B. $1-2m$ C. $2m^2-1$ D. $2m-1$

2. The arc length below is approximately equal to C



   A. 0.6 cm

   B. 1.7 cm

   C. 37.7 cm

   D. 75.4 cm

3. 110° converted to radians is approximately equal to A

   A. 1.92 B. 0.61 C. 1.64 D. 6,302.54

4. One negative and one positive co-terminal angle to $-\frac{15\pi}{7}$ is B

   A. $-\frac{13\pi}{7}$ and $\frac{\pi}{7}$ B. $-\frac{\pi}{7}$ and $\frac{13\pi}{7}$

   C. $-\frac{\pi}{7}$ and $\frac{\pi}{7}$ D. $-\frac{29\pi}{7}$ and $\frac{15\pi}{7}$

5. When $\cos\left(-\frac{4\pi}{3}\right)$ is written in terms of its co-function angle the result is C

   A. $\sin\left(\frac{3\pi}{2}\right)$ B. $\cos\left(\frac{3\pi}{2}\right)$ C. $-\sin\left(\frac{\pi}{6}\right)$ D. $\sin\left(\frac{\pi}{6}\right)$

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6. Express each of the following as a **completely simplified single** trigonometric expression. **[4 Marks]**

a. $\cos^2\left(\frac{37x}{14}\right)-\sin^2\left(\frac{37x}{14}\right)$ **[1]**

$= \cos\left(2\left(\frac{37x}{14}\right)\right)$

$= \cos\left(\frac{37x}{7}\right)$

b. $2\sin^2\left(\frac{3\pi}{4}-\frac{x}{2}\right)-1$ **[3]**

$= -\cos\left(2\left(\frac{3\pi}{4}-\frac{x}{2}\right)\right)$

$= -\cos\left(\frac{3\pi}{2}-x\right)$

$= -\left(-\sin(x)\right)$

$= \sin(x)$

7. **Completely** simplify the following expression: $\dfrac{\sec^2\left(\frac{\pi}{2}-x\right)\tan\left(\frac{3\pi}{2}+x\right)}{\sin^2(2\pi-x)+\cos^2(\pi+x)+\tan^2\left(\frac{3\pi}{2}+x\right)}$. **[4 Marks]**

$= \dfrac{-\csc^2(x)\cot(x)}{\sin^2(x)+\cos^2(x)+\cot^2(x)}$

$= \dfrac{-\csc^2(x)\cot(x)}{1+\cot^2(x)}$

$= \dfrac{-\csc^2(x)\cot(x)}{\csc^2(x)}$

$= -\cot(x)$

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APPLICATION – [15 Marks]

1. Determine the **exact** value of the following. **[6 Marks]**

a. $\cos\left(-\frac{13\pi}{12}\right)$ **[3 Marks]**

$= \cos\left(\frac{13\pi}{12}\right)$

$= \cos\left(\frac{3\pi}{4}+\frac{\pi}{3}\right)$

$= \cos\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$

$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= -\frac{(\sqrt{2}+\sqrt{6})}{4}$

b. $\sin\left(\frac{11\pi}{8}\right)$ **[3 Marks]**

$\sin^2\left(\frac{11\pi}{8}\right) = \frac{1-\cos\left(\frac{11\pi}{4}\right)}{2}$

$\sin^2\left(\frac{11\pi}{8}\right) = \frac{1+\frac{\sqrt{2}}{2}}{2}$

$\sin^2\left(\frac{11\pi}{8}\right) = \frac{2+\sqrt{2}}{4}$

$\sin\left(\frac{11\pi}{8}\right) = -\sqrt{\frac{2+\sqrt{2}}{4}}$

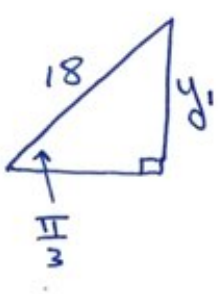
$\sin\left(\frac{11\pi}{8}\right) = -\frac{\sqrt{2+\sqrt{2}}}{2}$

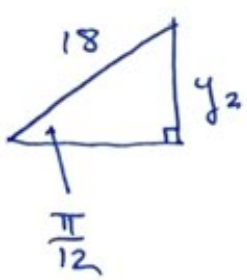
2. The arm of a crane has a length of 18 metres. The crane has a minimum inclination of $\frac{\pi}{12}$ and a maximum inclination of $\frac{\pi}{3}$. Determine the **exact** vertical displacement at the end of the crane's arm from the minimum inclination to the maximum inclination. **[4 Marks]**

Vertical displacement $= y_1 - y_2$

Vertical displacement $= \frac{18\sqrt{3}}{2} - 18\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$

Vertical displacement $= \frac{18\sqrt{3}-9\sqrt{6}+9\sqrt{2}}{2}$ m.





$\sin\left(\frac{\pi}{3}\right) = \frac{y_1}{18}$ $\sin\left(\frac{\pi}{12}\right) = \frac{y_2}{18}$

$y_1 = 18\sin\left(\frac{\pi}{3}\right)$ $y_2 = 18\sin\left(\frac{\pi}{12}\right)$

$y_1 = \frac{18\sqrt{3}}{2}$ $y_2 = 18\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$

Compound angle.

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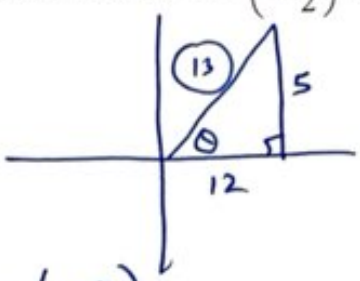
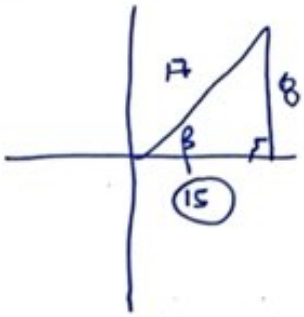
3. If $\tan(\theta)=\frac{5}{12}$ and $\sin(\beta)=\frac{8}{17}$ and θ and β are in the interval $\left(0,\frac{\pi}{2}\right)$, determine the **exact** value of $\sin(2\theta+\beta)$. **[5 Marks]**





$\sin(2\theta+\beta)$

$= \sin(2\theta)\cos(\beta) + \sin(\beta)\cos(2\theta)$

$= 2\sin(\theta)\cos(\theta)\cos(\beta) + \sin(\beta)\left[1-2\sin^2(\theta)\right]$

$= 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right)\left(\frac{15}{17}\right) + \left(\frac{8}{17}\right)\left(1-2\left(\frac{5}{13}\right)^2\right)$

$= \frac{1800}{2873} + \left(\frac{8}{17}\right)\left(1-\frac{50}{169}\right)$

$= \frac{1800}{2873} + \frac{952}{2873}$

$= \frac{2752}{2873}$

** Two (2) Marks will be awarded in the Communication Category for the use of proper mathematical form. **

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Unit 3: Trigonometric Functions – Part 1 Assessment of Learning – DAY 2

Thinking	Comm.
/9	/2

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THINKING – [9 Marks]

1. Prove: $\frac{\sin(2x)}{1+\cos(2x)} = \frac{\sec^2(x)-1}{\tan(x)}$. **[5 Marks]**

$$L.S. = \frac{\sin(2x)}{1+\cos(2x)}$$

$$= \frac{2\sin(x)\cos(x)}{1+2\cos^2(x)-1}$$

$$= \frac{2\sin(x)\cos(x)}{2\cos^2(x)}$$

$$= \frac{\sin(x)}{\cos(x)}$$

$$= \tan(x)$$

$$= \tan(x).\frac{\tan(x)}{\tan(x)}$$

$$= \frac{\tan^2(x)}{\tan(x)}$$

$$= \frac{\sec^2(x)-1}{\tan(x)}$$

$$= R.S.$$

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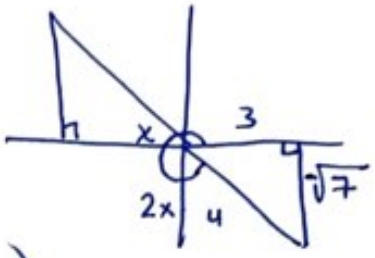
2. If $\cos(2x) = \frac{3}{4}$, where $x \in \left[\frac{\pi}{2}, \pi\right]$, determine the **exact** value of $\cos(3x)$. **[4 Marks]**

$\therefore 2x$ is in Q4



$\cos(3x) = \cos(2x + x)$

$\cos(3x) = \cos(2x)\cos(x) - \sin(2x)\sin(x)$

$\cos(3x) = \left(\frac{3}{4}\right)\left(-\frac{\sqrt{14}}{4}\right) - \left(-\frac{\sqrt{7}}{4}\right)\left(\frac{\sqrt{2}}{4}\right)$

$\cos(3x) = \frac{-3\sqrt{14} + \sqrt{14}}{16}$

$\cos(3x) = \frac{-\sqrt{14}}{8}$

* $\cos(2x) = 2\cos^2(x) - 1$

$\frac{3}{4} = 2\cos^2(x) - 1$

$\cos^2(x) = \frac{7}{8}$

$\cos(x) = -\frac{\sqrt{14}}{4}$

* $\cos^2(x) + \sin^2(x) = 1$

$\sin^2(x) = 1 - \cos^2(x)$

$\sin^2(x) = 1 - \frac{7}{8}$

$\sin^2(x) = \frac{1}{8}$

$\sin(x) = \frac{\sqrt{2}}{4}$

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