Name: ___________________

Day 1: Assessment of Learning
Unit 4: Application of Derivatives

Please show all your work for full marks. You may only use methods taught in MCV4U for full marks. Good Luck!
**Note: Your communication mark will be determined by how well you answer all of the questions; including, proper mathematical form, notation and narration (where appropriate).*

Knowledge	Thinking	Comm.
/ 18	/ 5	/3

KNOWLEDGE AND UNDERSTANDING [18 marks]

Multiple Choice: Write the CAPITAL LETTER corresponding to the correct answer on the line provided.
[1 Mark Each – 2 Marks Total]

1. The demand function for a product is given by $p(x)=\frac{1}{12}x^2-10x+300$, $0\le x\le 60$, where x is the number of units of the product sold and p is the price, in dollars. What is the value of x that maximize the revenue? ____ E ____

   A) 20 B) 30 C) 40 D) 50 E) 60

2. If $y=2x-8$, the minimum value of the product xy is ___ B _____

   A) -16 B) -8 C) - 4 D) 0 E) 2

Full Solutions

3. An object starts at rest and moves along a horizontal trail. Its position, S , in metres, after t seconds is given by $s(t)=2t^3-21t^2+60t+6, t\ge 0$.

   a) When is the particle at rest? **[2 marks]**

$$\begin{aligned} v(t)=s'(t)&=6t^2-42t+60\\ &=6(t^2-7t+10)\\ &=6(t-5)(t-2)\end{aligned}$$

$$v(t)=0: \boxed{t=5s} \text{ or } \boxed{t=2s}$$

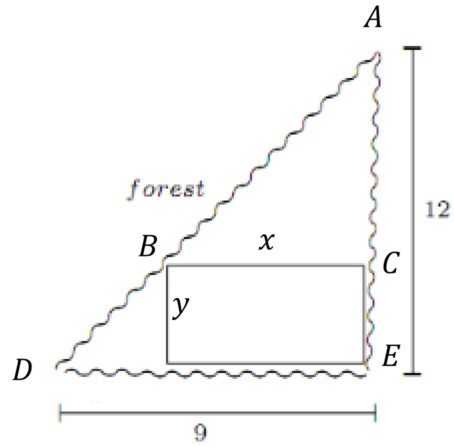
   b) Was the particle speeding up or slowing down at t = 3s? **[2 marks]**

$$v(t)=6(t^2-7t+10)$$
$$a(t)=6(2t-7)$$
$$v(3)=6((3)^2-7(3)+10)=-12<0$$
$$a(3)=6(2(3)-7)=-6<0$$
$$\because v(3)a(3)>0$$
$$\therefore \text{object is speeding up at } t=3s.$$

   c) Determine the total distance travelled by the particle during the first 6s. **[2 marks]**

$$\begin{aligned}\text{Total distance}&=|s(2)-s(0)|+|s(5)-s(2)|+|s(6)-s(5)|\\ &=|58-6|+|31-58|+|42-31|\\ &=52+27+11\\ &=90\,m\end{aligned}$$

4. A farmer wants to build a rectangular pig pen in a triangular lot that is bounded by a 12 m stretch of river, a 9 m stretch of wall, and a 15 m stretch of forest. Find the dimensions of the pen that he can build that maximizes the area enclosed by the fence. **[5 marks]**



$\Delta ADE \sim \Delta ABC$

$\angle A$ is common

$\angle E = \angle C = 90^{\circ}$

$$\frac{12-y}{12} = \frac{x}{9} \Rightarrow 3(12-y) = 4x$$

$$x = \frac{3}{4}(12-y)$$

Area = xy

$$A(y) = \frac{3}{4}y(12-y)\ ;\ 0 < y < 12$$

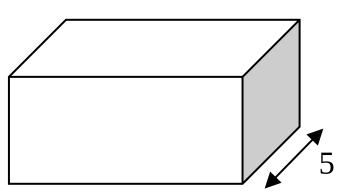
$$A'(y) = \frac{3}{4}(12-2y)$$

$$A'(y) = 0 \Rightarrow 12-2y = 0$$

$$\boxed{y = 6m} \rightarrow x = \frac{3}{4}(6)$$

$$\boxed{x = 4.5m}$$

5. A large aquarium tank in the shape of an open-topped rectangular box is to measure 5 m from front to back and have a volume of 300 m^3. The base, sides, and back of the tank are to be made of slate which cost \$42 per square metre. The front is to be made of glass which costs \$31.50 per square metre. What dimensions will minimize the cost of the tank? **(Second derivative test is not required.) [5 Marks]**



Let the width of the tank be w , the height be h , the depth be d, the volume be V and the cost be C

$V = 300 m^3$

$V = dwh$

$300 = 5wh$

$\frac{60}{w} = h$

$C = 42(\text{base} + \text{sides} + \text{back}) + 31.50(\text{front})$

$C = 42(5w + 2\cdot 5h + wh) + 31.50(hw)$

sub $h = \frac{60}{w}$ into C:

$\therefore C = 42\left[5w + 10\left(\frac{60}{w}\right)w + w\left(\frac{60}{w}\right)\right] + 31.50\left(\frac{60}{w}\right)w$

$C = 210w + \frac{25200}{w} + 2520 + 91.5$

$= 210w + \frac{25200}{w} + 2611.5$

$C' = 210 - \frac{25200}{w^2}$

$C' = 0 \Rightarrow \frac{25200}{w^2} = 210$

$w^2 = 120$

$w = \pm 2\sqrt{30}$

$\therefore w > 0 \therefore 2\sqrt{30}$

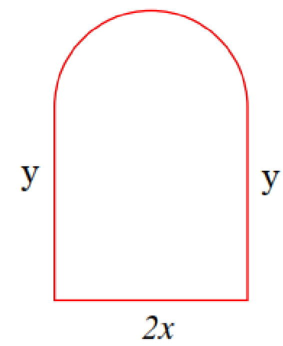
$h = \frac{60}{2\sqrt{30}}$

$= \sqrt{30}$

$\therefore$ The width should be $2\sqrt{30}$ m (approx. 11.0 m) and the height $\sqrt{30}$ m (approx. 5.48m).

THINKING [5 marks]

1. The **area** of a Norman window (a window consists of a rectangle surmounted by a semicircle) must be $2m^2$.What **exact** value of radius for the semi-circle will minimize the **total length** of the frame needed? **[5 Marks]**



$$A = 2xy + \frac{1}{2}\pi x^2 = 2$$

$$4xy + \pi x^2 = 4$$

$$y = \frac{4 - \pi x^2}{4x}$$

$$P = 2x + 2y + \pi x$$

$$P(x) = 2x + \pi x + 2\left(\frac{4 - \pi x^2}{4x}\right)$$

$$P(x) = \left(2 + \frac{\pi}{2}\right)x + \frac{2}{x} \;; 0 < x < \frac{2\sqrt{\pi}}{\pi}$$

$$P'(x) = \left(2 + \frac{\pi}{2}\right) - \frac{2}{x^2}$$

$$P'(x) = 0$$

$$\left(2 + \frac{\pi}{2}\right) = \frac{2}{x^2}$$

$$x^2 = \frac{4}{4 + \pi}$$

$$x = \frac{2}{\sqrt{4 + \pi}}$$

$$\boxed{x = \frac{2\sqrt{4 + \pi}}{4 + \pi}}$$

+ 3 marks will awarded for proper mathematical form used throughout the assessment

Day 2: Assessment of Learning
Unit 4: Application of Derivatives

Application	Thinking	Comm.
/15	/4	/2

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APPLICATION [15 marks]

1. Let A(w) be the area of the triangle formed in the first quadrant by the x-axis, y-axis, and a tangent line to the graph of $f(x)=\dfrac{x}{x-1}$ at point $(w, f(w)), w>1$ .For what value of w is $A(w)$ a minimum? **[5 marks]**

$$f'(x)=\frac{-1}{(x-1)^2}$$

$$y-\frac{w}{w-1}=\frac{-1}{(w-1)^2}(x-w)$$

$$x\text{-int}: \; y=0$$

$$-\frac{w}{w-1}=\frac{-1}{(w-1)^2}(x-w)$$

$$w(w-1)=x-w$$

$$x=w^2$$

$$A(w^2,0)$$

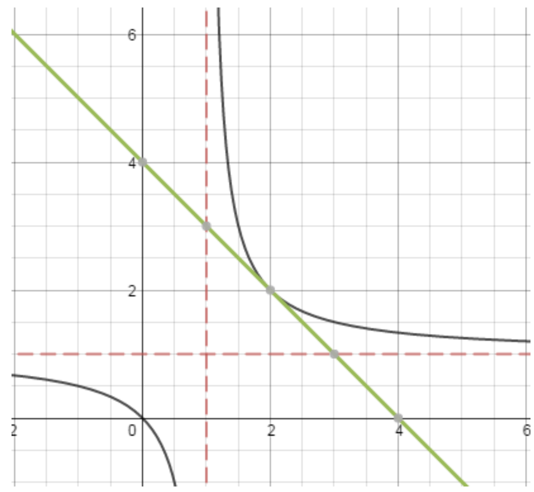
$$y\text{-int}: \; x=0$$

$$y-\frac{w}{w-1}=\frac{-1}{(w-1)^2}(-w)$$

$$y=\frac{w}{(w-1)^2}+\frac{w}{w-1}$$

$$y=\frac{w+w(w-1)}{(w-1)^2}$$

$$B(0,\frac{w^2}{(w-1)^2})$$



$$A_{\Delta OAB}=\frac{1}{2}(w^2)(\frac{w^2}{(w-1)^2})$$

$$A(w)=\frac{w^4}{2(w-1)^2}$$

2.

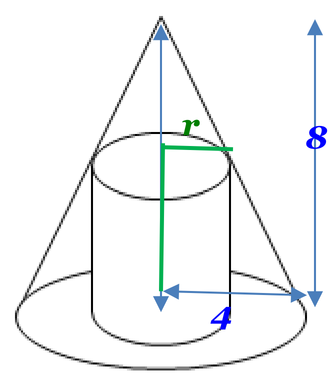
$$A'(w)=\frac{1}{2}\times\frac{4(w^3)(w-1)^2-2(w-1)w^4}{(w-1)^4}$$

$$A'(w)=0: 2w^3(w-1)(2w-2-w)$$

$$2w^3(w-1)(w-2)=0$$

$$\boxed{w=2}$$

3. Determine the surface area of the right circular cylinder of greatest volume that can be inscribed in a right circular cone of radius 4 cm and height 8 cm. Express final answer in **exact** form. **[5 marks]**



$$\frac{r}{4}=\frac{8-h}{8}$$

$$2r=8-h$$

$$h=8-2r$$

$$V=\pi r^2 h$$

$$V(r)=\pi r^2(8-2r) \quad ,0<r<4$$

$$V(r)=\pi(8r^2-2r^3)$$

$$V'(r)=0$$

$$16r-6r^2=0$$

$$2r(8-3r)=0$$

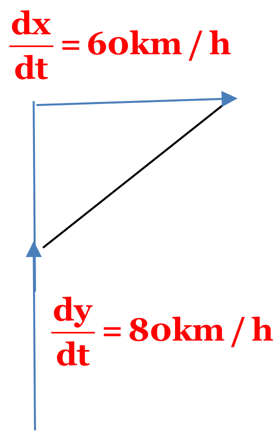
$$\underset{\text{inadmissidle}}{r=0} \quad \text{or} \quad \boxed{r=\frac{8}{3}\text{cm}} \quad \text{therefore} \quad \boxed{h=\frac{8}{3}\text{ cm}}$$

$$SA=2\pi r^2+2\pi rh$$

$$SA=2\pi\left[\left(\frac{8}{3}\right)^2+\left(\frac{8}{3}\right)\left(\frac{8}{3}\right)\right]$$

$$\boxed{SA=\frac{256\pi}{9}\text{cm}^2}$$

4. North-South highway intersects an East-West highway at point P. A vehicle crosses P at 1:00 pm, traveling east at 60 km/h. At the same instant, another vehicle is 5 km south of P, traveling north at 80 km/h. Determine the time when the two vehicles are closest to each other. What is the shortest distance between them? **[5 Marks]**



Let t represent the time the two vehicles are closest to each other.

$$d(t) = \sqrt{(60t)^2 + (5-80t)^2}$$

$$d'(t) = 0:$$

$$2(60t)(60) + 2(5-80t)(-80) = 0$$

$$(60t)(3) + (5-80t)(-4) = 0$$

$$180t - 20 + 320t = 0$$

$$500t = 20$$

$$t = 0.04 \text{ hour}$$

$$0.04 \times 60 = 2.4 \text{ min}$$

two carrs are closest at 1:02 pm

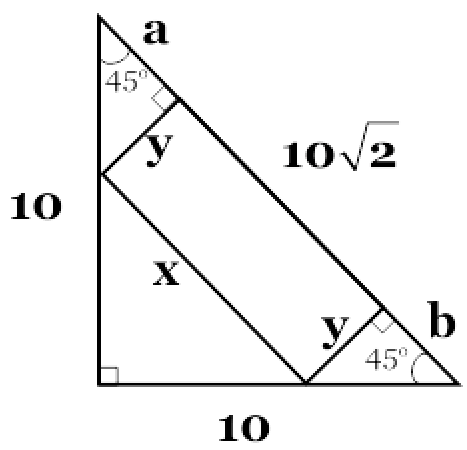
$$d(0.04) = \sqrt{(60 \times 0.04)^2 + (5-80(0.04))^2}$$

$$d(0.04) = \sqrt{9}$$

$$= 3 \text{ km}$$

THINKING [5 marks]

1. A rectangle is inscribed in an isosceles right angle triangle of two sides each equal to 10 cm. One side of the rectangle rests on the hypotenuse and the other two vertices on the shorter sides. Find the dimension of the largest rectangle. **[5 marks]**



$\tan(45^\circ) = \frac{y}{a}$

$1 = \frac{y}{a}$

$y = a$

$\tan(45^\circ) = \frac{y}{b}$

$1 = \frac{y}{b}$

$y = b$

$\therefore y = a = b$

$a + b + x = 10\sqrt{2}$

$2y + x = 10\sqrt{2}$

$x = 10\sqrt{2} - 2y$

$A(y) = (10\sqrt{2} - 2y)y \quad , 0 < y < 5\sqrt{2}$

$A'(y) = 10\sqrt{2} - 4y$

$A'(y) = 0 : 10\sqrt{2} - 4y = 0$

$\boxed{y = \frac{5\sqrt{2}}{2} \text{ cm}}$

$x = 10\sqrt{2} - 2\left(\frac{5\sqrt{2}}{2}\right)$

$\boxed{x = 5\sqrt{2} \text{ cm}}$

Marks allotted for proper form throughout the test +2 marks