MCV4U1-Moshtagh

Name: ______________________

UNIT 2 ASSESSMENT OF LEARNING: DERIVATIVES– DAY 2

Instructions: You MUST use concepts covered in this unit/course. Show all steps for full marks. The use of cellphones, audio or video recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

Application	Comm.
/20	/5

Application - [20 marks]

1. Determine the equation of the normal to the function $f(x)=\dfrac{4}{\sqrt{x^2-2x+1}}$ at $x=4$. **[4 marks]**

$$f(x)=\frac{4}{x-1}$$

$$f'(x)=\frac{-4}{(x-1)^2}$$

$$m_t=\frac{-4}{9}$$

$$m_{\wedge}=\frac{9}{4}$$

Equation of normal at $(4,\frac{4}{3})$ is:

$$y-\frac{4}{3}=\frac{9}{4}(x-4)$$

2. A particle moves along a horizontal line so that its position is given by the function $s(t)=t^3-6t^2+9t$, $0\le t\le 4$ where s is the position in meters and t is the time in seconds.

(a) When is the particle at rest? **[2 marks]**

$$v(t)=s'(t)=3t^2-12t+9\ ,0\le t\le 4$$

$$v(t)=0$$

$$3(t^2-4t+3)=0$$

$$3(t-3)(t-1)=0$$

$$\boxed{t=1\text{sec}}\ ,\ \boxed{t=3\text{sec}}$$

(b) What is the position of the particle when the acceleration is 12 m/s^2? **[3 marks]**

$$a(t)=s''(t)=6t-12\ ,0\le t\le 4$$

$$6t-12=12$$

$$6t=24$$

$$t=4\text{sec}$$

$$s(4)=4^3-6(4)^2+9(4)$$

$$=4\text{m}$$

3. Find the values of the real numbers a and b if $y=ax+b$ is a tangent to the curve $f(x)=2x+(3x-2)^3$ at the point $(1,3)$. **[4 marks]**

$$f(1)=3:\quad a+b=3\quad (1)$$

$$f'(1)=a$$

$$f'(x)=2+9(3x-2)^2$$

$$f'(1)=2+9(3(1)-2)^2=a$$

$$\boxed{a=11}$$

sub. $a=8$ into (1), we get: $\boxed{b=-8}$

4. Given $h(x) = f(x^2)[g(x)]^3$, where $h'(1) = 24$, $f(1) = 2$, $f'(1) = 3$ and $g(1) = -2$, determine the value of $g'(1)$. **[4 marks]**

$$h'(x) = 2xf'(x^2)[g(x)]^3 + 3[g(x)]^2 g'(x)f(x^2)$$

$$h'(1) = 2f'(1)[g(1)]^3 + 3[g(1)]^2 g'(1)f(1)$$

$$24 = 2(3)(-2)^3 + 3(-2)^2(2)g'(1)$$

$$24 = -48 + 24g'(1)$$

$$\boxed{g'(1) = 3}$$

5. Using **Leibniz's notation**, find the **exact** value of $\left.\dfrac{dy}{dx}\right|_{x=4}$ given: $y = u - \dfrac{50}{u}$, and $u = x - \sqrt[3]{2x}$.**[3 marks]**

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \left[1 + \frac{50}{u^2}\right] \times \left[1 - \frac{2}{3}(2x)^{\frac{-2}{3}}\right]$$

$$\left.\frac{dy}{dx}\right|_{x=4} = \left.\left[1 + \frac{50}{u^2}\right]\right|_{u=2} \times \left.\left[1 - \frac{2}{3}(2x)^{\frac{-2}{3}}\right]\right|_{x=4}$$

$$= \left(\frac{27}{4}\right)\left(\frac{5}{6}\right)$$

$$= \frac{45}{4}$$

<u>Communication – [5 marks]</u>

1. Given $f(x) = (x+1)(x^2-3)(2x^3+7)$, **clearly explain** using the rules of differentiation **how** to determine $f'(x)$. **Do not solve for** $f'(x)$. **[3 marks]**

We can see that the original function is a product of three functions, and its derivative is the sum of three products. We have :

$$f'(x) = \frac{d(x+1)}{dx}(x^2-3)(2x^3+7) + \frac{d(x^2-3)}{dx}(x+1)(2x^3+7) + \frac{d(2x^3+7)}{dx}(x^2-3)(x+1)$$

We take the derivative using product rule, we take the derivative of one function at a time, multiplying by the other two original functions. To be more specific, we take the derivative of (x+1), and multiply it by $(x^2-3)(2x^3+7)$, then we add to that the derivative of (x^2-3) multiplied by $(x+1)(2x^3+7)$. Then we take the derivative of $(2x^3+7)$,and multiplying by multiplied by $(x^2-3)(x+1)$.

*** 2 marks will be awarded in the Communication Category for proper mathematical form. ***