UNIT 1 ASSESSMENT OF LEARNING: LIMITS AND RATES OF CHANGE – DAY 1

Name: _____**Solutions**_____

Instructions:

- You MUST use concepts covered in this unit/course. Derivative or Instantaneous Rates of Change calculations MUST be done using **First Principles**. Show all steps for full marks.
- Non-graphing calculators may be used but not shared. Notebooks may not be used.
- The use of cellphones, audio- or video-recording devices, digital music players or email or text-messaging devices during the assessment is prohibited.

K & U	Thinking	Comm.
/20	/9	/2

Knowledge and Understanding:

Multiple Choice: Write the CAPITAL LETTER corresponding to the correct answer on the line provided. [1 mark each]

1. $\lim\limits_{x\to\infty}\dfrac{2x^2-5x^3}{(x+3)^3}$ is ___**A**___

   A) -5 B) $\frac{2}{3}$ C) 2 D) $-\infty$ E) ∞

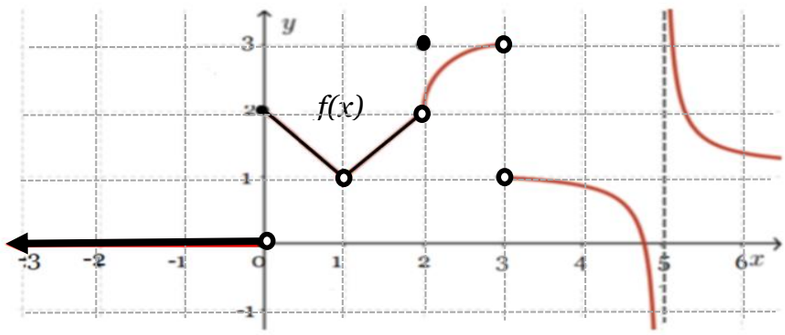
2. The average rate of change of $f(x)=\sqrt{3-x}$ over the interval $-6\le x\le -1$ is: ___ **B** ___

   A) $\frac{1}{5}$ B) $-\frac{1}{5}$ C) 1 D) -1 E) 2

3. If $f(x)=x^2-4$, the value of $\left.\dfrac{dy}{dx}\right|_{x=1}$ is ____ **C** __

   A) -4 B) ∞ C) 2 D) -2 E) 3

4. Given the function $y=f(x)$, determine the following: [0.5 mark each]



a) $\lim\limits_{x\to 2} f(x)=$ ___2____ b) $\lim\limits_{x\to 5+} f(x)=$ _____∞_____ c) $\lim\limits_{x\to 1} f(x)=$ _____1_____

d) $f(2)=$ _____3_____ e) $\lim\limits_{x\to -\infty} f(x)=$ ____0_____ f) type of discontinuity at $x=2$ __**removable**

5. Evaluate the following limits. [14 marks]

a) $\lim\limits_{x\to 3}\dfrac{\frac{1}{x+3}-\frac{1}{6}}{x^2-9}$ ❸

$=\lim\limits_{x\to 3}\dfrac{6-(x+3)}{6(x^2-9)(x+3)}$

$=\lim\limits_{x\to 3}\dfrac{-(x-3)}{6(x-3)(x+3)^2}$

$=-\dfrac{1}{216}$

b) $\lim\limits_{x\to 1}\dfrac{x^3-1}{x^3+5x^2-x-5}$ ❸

$=\lim\limits_{x\to 1}\dfrac{(x-1)(x^2+x+1)}{(x-1)(x^2+6x+5)}$

$=\dfrac{3}{12}$

$=\dfrac{1}{4}$

	1	5	-1	-5
1	↓	1	6	5
	1	6	5	0

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c) $\lim\limits_{x\to 0}\dfrac{(x+64)^{\frac{1}{3}}-4}{x}$ ❹

Let $(x+64)^{\frac{1}{3}}=u$

$x+64=u^3$

$x=u^3-64$

$\lim\limits_{x\to 0}\dfrac{(x+64)^{\frac{1}{3}}-4}{x}$

$=\lim\limits_{u\to 4}\dfrac{u-4}{u^3-64}$

$=\lim\limits_{u\to 4}\dfrac{(u-4)}{(u-4)(u^2+4u+16)}$

$=\dfrac{1}{48}$

d) $\lim\limits_{x\to 2}\left(\dfrac{1}{x-2}\right)\left(\dfrac{1}{x+3}-\dfrac{2}{3x+4}\right)$ ❹

$=\lim\limits_{x\to 2}\left[\dfrac{1}{x-2}\left(\dfrac{(3x+4)-2(x+3)}{(x+3)(3x+4)}\right)\right]$

$=\lim\limits_{x\to 2}\left[\dfrac{1}{x-2}\left(\dfrac{x-2}{(x+3)(3x+4)}\right)\right]$

$=\dfrac{1}{50}$

Thinking: [9 marks]

1. Determine the value of the constants a and b if $\lim\limits_{x\to -1}\dfrac{2x^2-ax-14}{(x-b)(x+1)}=4$. ❹

$2(-1)^2-a(-1)-14=0$

$\boxed{a=12}$

$\lim\limits_{x\to -1}\dfrac{2x^2-12x-14}{(x-b)(x+1)}=4$

$\lim\limits_{x\to -1}\dfrac{2(x-7)(x+1)}{(x-b)(x+1)}=4$

$\dfrac{2(-8)}{-1-b}=4$

$1+b=4$

$\boxed{b=3}$

2. Determine the values of a and b such that $f(x)=ax^2+\dfrac{b}{x}$ has a horizontal tangent at (1,3). ❺

$m_t=\lim\limits_{h\to 0}\dfrac{f(1+h)-f(1)}{h}$

$m_t=\lim\limits_{h\to 0}\dfrac{\left[a(1+h)^2+\dfrac{b}{(1+h)}\right]-[3]}{h}$

$0=\lim\limits_{h\to 0}\dfrac{a(1+h)^3+b-3(1+h)}{h(1+h)}$

$0=\lim\limits_{h\to 0}\dfrac{a(1+3h+3h^2+h^3)+b-3-3h}{h(1+h)}$

$0=\lim\limits_{h\to 0}\dfrac{a+3ah+3ah^2+ah^3+b-3-3h}{h}$

$0=\lim\limits_{h\to 0}\dfrac{h(3a+3ah+ah^2-3)}{h}$

$0=3a-3$

$\boxed{a=1}$ $\xrightarrow{\text{sub. into (1)}}$ $\boxed{b=2}$

$f(1)=3$

$a+b=3\ \ (1)$

***2 marks will be awarded in the Communication category for the proper mathematical form ***