UNIT 3 ASSESSMENT OF LEARNING: CURVE SKETCHING – DAY 2

App	Thinking	Comm.
/17	/5	/7

Name: ______________________

Instructions: You MUST use concepts covered in this unit/course. Show all steps for full marks.

Application - [17 marks]

1. Sketch and properly label a graph of $f(x)=\frac{4x^2-1}{x^3}$. **Note:** $f'(x)=\frac{3-4x^2}{x^4}$ and $f''(x)=\frac{8x^2-12}{x^5}$.

[12 marks]

$D=\{x\in R \mid x\neq 0\}$

cross over: $(\frac{1}{2},0),(-\frac{1}{2},0)$

x-int: $(\frac{1}{2},0),(-\frac{1}{2},0)$

y-int: none

Symmetry ; odd function

HA: $y=0$

$\lim_{x\to\infty} f(x)=0$ (above)

$\lim_{x\to-\infty} f(x)=0$ (below)

VA: $x=0$

$\lim_{x\to 0^-} f(x)=\infty$

$\lim_{x\to 0^+} f(x)=-\infty$

$f'(x)=0$

$4x^2=3$

$x=\pm\frac{\sqrt{3}}{2}\approx\pm0.8$

$f'(x)$ dne

$x=0$(not in domain)

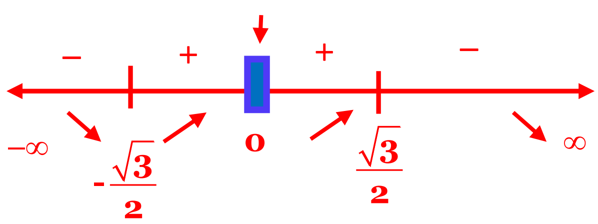
$f''(x)=0$

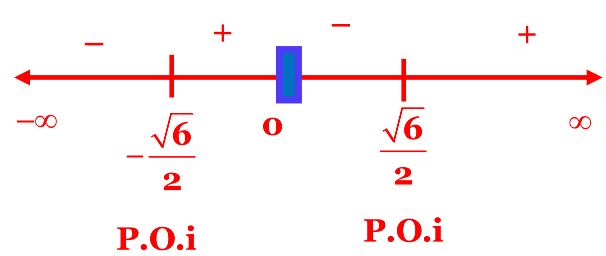
$8x^2=12$

$x=\pm\frac{\sqrt{6}}{2}$

$f''(x)$ dne

$x=0$(not in domain)



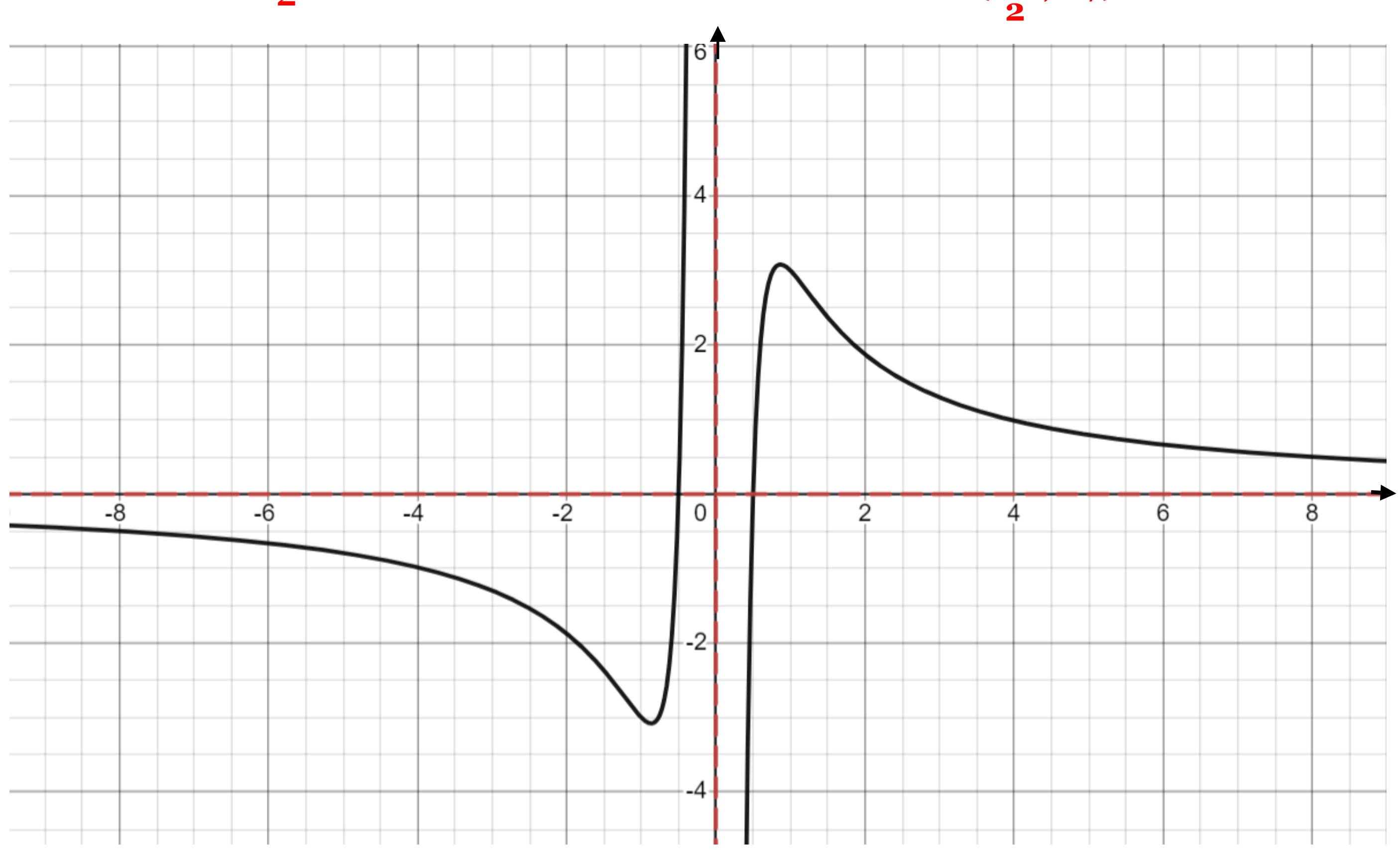


local min: $(-\frac{\sqrt{3}}{2},-3.1)$

local max: $(\frac{\sqrt{3}}{2},3.1)$

P.O.i: $(-\frac{\sqrt{6}}{2},-2.7)$

P.O.i: $(\frac{\sqrt{6}}{2},2.7)$



2. Determine the values of a, b and c such that the slope of the tangent to the function $f(x)=ax^3+bx^2+c$ is equal to 6 at the function's point of inflection of $(1,5)$. **[5 marks]**

$$\mathbf{f(1)=5}$$
$$\mathbf{f''(1)=0}$$
$$\mathbf{f'(1)=6}$$
$$\mathbf{f'(x)=3ax^2+2bx}$$
$$\mathbf{f''(x)=6ax+2b}$$

$$\mathbf{a+b+c=5 \quad (1)}$$
$$\mathbf{3a+2b=6 \quad (2)}$$
$$\mathbf{6a+2b=0 \quad (3)}$$
$$\mathbf{(3)\text{-}(2)\text{ we get}: 3a=\text{-}6}$$
$$\boxed{\mathbf{a=\text{-}2}} \xrightarrow{\text{sub. into (1)}} \mathbf{\text{-}6+2b=6}$$
$$\boxed{\mathbf{b=6}}$$

$$\mathbf{\text{-}2+6+c=5 \quad (1)}$$
$$\boxed{\mathbf{c=1}}$$

Thinking - [5 marks]

1. Given the function $f(x)=\dfrac{ax^2+bx+c}{x-2}$, determine the values of a, b, c and d and the **exact** $x-$intercepts of $f(x)$ if there is a local extremum at $(-1,1)$ and an oblique asymptote at $y=x+d$. **[5 marks]**

$$\mathbf{f(\text{-}1)=1}$$
$$\mathbf{f'(\text{-}1)=0}$$
$$\mathbf{f(x)=(ax+(2a+b))+\frac{4a+2b+c}{x\text{-}2}}$$
$$\mathbf{O.A: y=ax+(2a+b)}$$
$$\mathbf{y=x+d}$$
$$\boxed{\mathbf{a=1}}\mathbf{,\ 2+b=d}$$
$$\mathbf{b\text{-}d=\text{-}2}$$
$$\mathbf{f(\text{-}1)=1}$$
$$\mathbf{1=\frac{1\text{-}b+c}{\text{-}3}}$$
$$\mathbf{\text{-}b+c=\text{-}4 \quad (1)}$$

$$\mathbf{f'(x)=\frac{(2x+b)(x\text{-}2)\text{-}x^2\text{-}bx\text{-}c}{(x\text{-}2)^2}}$$
$$\mathbf{f'(\text{-}1)=0:(\text{-}2+b)(\text{-}3)\text{-}1+b\text{-}c=0}$$
$$\mathbf{\text{-}2b\text{-}c=\text{-}5 \quad (2)}$$
$$\mathbf{\begin{cases}\text{-}b+c=\text{-}4 & (1)\\ \text{-}2b\text{-}c=\text{-}5 & (2)\end{cases}}$$
$$\mathbf{\oplus\ \text{-}3b=\text{-}9}$$
$$\boxed{\mathbf{b=3}}\ ,\boxed{\mathbf{c=\text{-}1}}$$
$$\mathbf{b\text{-}d=\text{-}2}$$
$$\mathbf{3\text{-}d=\text{-}2}$$
$$\boxed{\mathbf{d=5}}$$

$$\mathbf{ax^2+bx+c=0}$$
$$\mathbf{x^2+3x\text{-}1=0}$$
$$\mathbf{x=\frac{\text{-}3\pm\sqrt{13}}{2}} \qquad \mathbf{\therefore x\text{-}int:\left(\frac{\text{-}3\pm\sqrt{13}}{2},0\right)}$$

Communication - [7 marks]

1. List **two** situations where $f(x)$, $f'(x)$ and $f''(x)$ are all undefined at $x=c$. **[2 marks]**

- **When the function has vertical asymptote at x=c**
- **When the function has a hole at x=c**

2. Write a concise step by step explanation on how to determine whether or not a function has a vertical tangent. **[3 marks]**

- **Find the derivative of the function.**
- **Find a value of x that makes dy/dx infinite and the sign of dy/dx does not change at that point.**
- **Find the d²y/dx², we are looking for an infinite slope, so the vertical tangent of the curve is a vertical line at this value of x providing that the concavity changes at this point.**

*** **2 marks** will be awarded in the Communication Category for proper mathematical form. ***