MHF 4U1-am Unit 3 Test - Rational Equations and Inequalities

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Total
23.5
/30

KNOWLEDGE

For questions 1-5, circle one answer (1 mark each):

1. What are the asymptotes for the function $f(x) = \frac{-5}{x^2+2x-3}$?

   a. $x = 3, x = -1, y = 0$

   b. $x = -3, x = -1, y = -5$

   c. $x = 3, x = 1, y = -5$

   d. $x = -3, x = 1, y = 0$

2. Which function has a y-intercept of $\frac{2}{3}$?

   a. $f(x) = -\frac{3x+2}{5x-1}$

   b. $f(x) = \frac{3x+2}{5x-1}$

   c. $f(x) = \frac{6x+5}{9x-7}$

   d. $f(x) = -\frac{5x+6}{7x-9}$

3. Where is the hole located in the function $f(x) = \frac{x^2-4x-21}{x^2-6x-7}$?

   a. $\left(-7, \frac{2}{3}\right)$

   b. $\left(7, \frac{5}{4}\right)$

   c. $(-7, 3)$

   d. $\left(7, \frac{-5}{4}\right)$

4. What is the behaviour of the function $f(x) = \frac{3}{x^2-6x+5}$, as $x \to -\infty$?

   a. $f(x) \to 0^+$

   b. $f(x) \to 0^-$

   c. $f(x) \to \infty$

   d. $f(x) \to -\infty$

5. Where is the function $f(x) = \frac{2x}{x-7}$ increasing?

   a. f(x) is never increasing

   b. $x < 7$

   c. $x > 7$

   d. $x \in \mathbb{R}, x \neq 7$

/3

6. Solve for x in the following rational equations. Find exact solutions.

a) $x+2=\frac{5x}{x-4}-\frac{20}{x-4}$

(2)

Restriction: $x \neq 4$

$x+2=\frac{5x-20}{x-4}$

$x+2-\frac{5x-20}{x-4}=0$

$\frac{((x+2)(x-4))-5x-20}{x-4}=0$

$\frac{x^2-2x-8-5x+20}{x-4}=0 \rightarrow \frac{x^2-7x+12}{x-4} \rightarrow \frac{(x-4)(x-3)}{(x-4)} \rightarrow x=3$

$\therefore x=3$ The value of x is 3, there is a hole at $x=4$.

b) $\frac{x+1}{x-1}-\frac{x-1}{x+1}=\frac{5}{6}$

(3)

$\frac{(x+1)^2-(x-1)^2}{(x-1)(x+1)}=\frac{5}{6}$

$\frac{x^2+2x+1-(x^2-2x+1)}{(x-1)(x+1)}=5/6$

$\frac{4x}{(x-1)(x+1)}=5/6$

$\frac{4x}{(x-1)(x+1)}-\frac{5}{6}=0$

$\frac{24x-(5(x-1)(x+1))}{6(x-1)(x+1)}=0$

$\frac{24x-(5x-5)(x+1)}{6(x-1)(x+1)}=0$

$\frac{24x-(5x^2+5x-5x-5)}{6(x-1)(x+1)}=0$

$\frac{24x-5x^2+5}{6(x-1)(x+1)} \rightarrow -5x^2+24x+5$

$-5x^2+25x-5$

$-5x(x-5)-(x-5)$

$0=\frac{(5x-1)(x-5)}{6(x-1)(x+1)}$

$\therefore x$ is equal to $5, -1/5$

APPLICATION

1. Solve the inequality $\frac{x^2+15x+36}{x^2-11x+28} \geq 0$ algebraically.

(4)

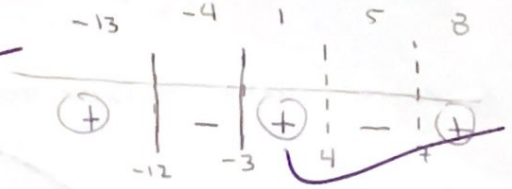
factoring $\rightarrow$ $x^2+15x+36$ $\quad$ $x^2-11x+28$

$(x+3)(x+12)$ $\quad$ $(x-7)(x-4)$

$\frac{(x+3)(x+12)}{(x-7)(x-4)} \geq 0$

x-int: $-3, -12$

VA: $7, 4$



Conclusion: $x \leq -12, -3 \leq x < 4, x > 7$

9

2. Graph the function $f(x) = \frac{x^3-3x-2}{x^2+x-20}$. Show all the key features, asymptotes, intercepts, behaviors, and discontinuous point(s) if any.

Factor → x^3-3x-2 Roots: $(\pm 1, \pm 2)$

$$\begin{array}{c|cccc} 2 & 1 & 0 & -3 & -2 \\ & \downarrow & 2 & 4 & 2 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

x^2+2x+1

$(x+1)^2$

x^2+x-20

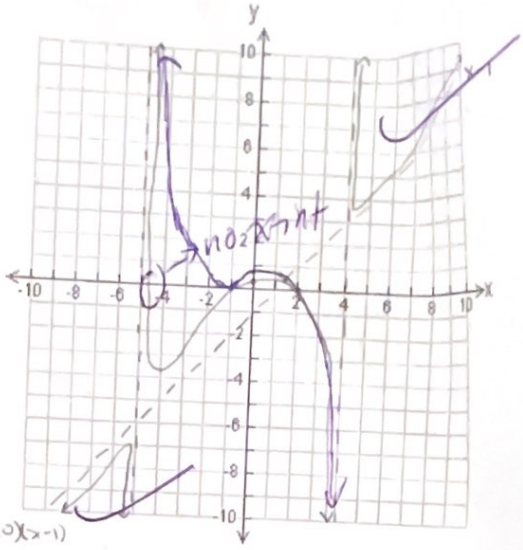
$(x+5)(x-4)$

(6)

4.5

$f(x) = \frac{(x+1)^2(x-2)}{(x+5)(x-4)}$

$\frac{(1)^2(-2)}{(5)(-4)} = \frac{-2}{-20}$



(VA) Restrictions: $x \neq -5, 4$ Cross: $\frac{23}{18} = x$

Oblique: $x-1$

x-int: $2, -1$

y-int: $1/10$

Test case at $x=5$ → $y = 10.8$

Test case at $x=-6$ → $y = -10.5$

$x^3-3x-2 = (x^2+x-20)(x-1)$

$x^3-3x-2 = x^3-x^2+x^2-x-20x+20$

$-3x-2 = -x-20x+20$

$-22 = -13x$

$(1.7) \frac{23}{18} = x$

$$\begin{array}{r} x-1 \\ x^2+x-20 \overline{)\ x^3+0x^2-3x-2} \\ -x^3-x^2+20x \downarrow \\ \hline -x^2+17x-2 \\ x^2+x-20 \\ \hline 13x-22 \end{array}$$

b) Determine when $f(x) < 0$. $-5 < x < -1$, $x < -5$, $2 < x < 4$

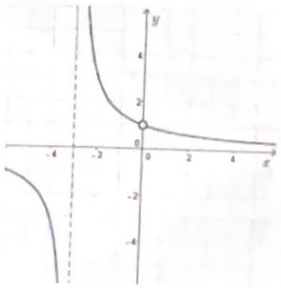
(4)

0.5

X

COMMUNICATION

1. Circle the equation below that represents the graph shown. Justify your choice.



a) $f(x) = \frac{3x}{x^2+3x}$ b) $f(x) = \frac{x^2+3x}{x^2+9}$ c) $f(x) = \frac{x-3}{x^2-3x}$ → $\frac{x-3}{x(x-3)}$ → $\frac{1}{x}$

$x(x+3)$

Hole: 3

(3)

HA: 0

VA: -3

x-int: 0

Hole: 0

$\frac{3x}{x(x+3)}$

$= \frac{3}{x+3}$

$\frac{x(x+3)}{(x-3)(x+3)}$

Hole: -3

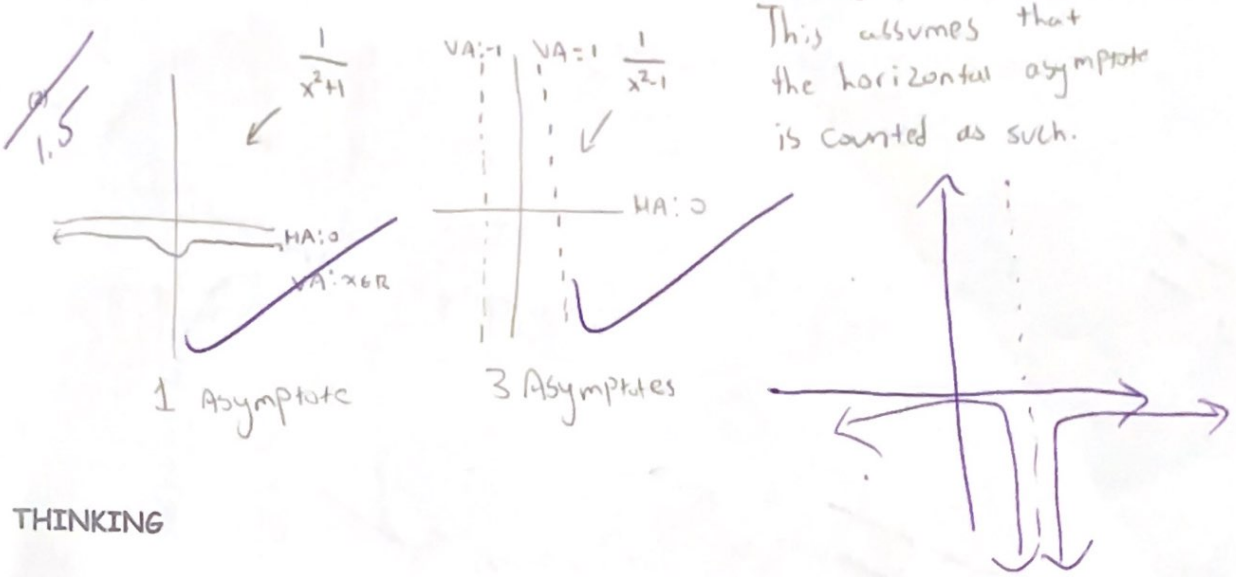
∴ option A is correct because the hole is at 0 (some others had a hole at 0), and the VA is also in the correct spot, -3. The HA is also 0, as shown on the graph.

8

2. How many asymptotes can the reciprocal of a parabola have? Draw examples of each type.

A reciprocal of a parabola can have 1 or 3 asymptotes

This assumes that the horizontal asymptote is counted as such.



1.5

THINKING

1. Determine, with support, an equation for a rational function of the form $y = \frac{g(x)}{h(x)}$ that satisfies the given conditions: g(2) = 0, h(2) = 0, a vertical asymptote at x = 2, and a hole at $(\frac{1}{5}, \frac{5}{6})$. Leave answer in expanded non-factored form.

x = 2

VA: +2

x-int: 2

Holes: 1/5, 5/6 → no hole here!

$y = \frac{(5x-1)(6x-5)(x-2)}{(5x-1)(6x-5)(x-2)(x-2)}$ ×

$(5x-1)(6x-5)(kx) = 0$

$6 \cdot 5 \cdot k \cdot x = 0$

$k = 0$

2

y of 1/5 (0.2) approx

input (0.21) = -1.17 → 0.5536

y of 5/6 (0.33̄)

input (0.35) = -1.15 → 0.8625

$y = a\frac{1}{x-2}$

$a = \frac{y_2 - y_1}{x_2 - x_1}$ → $a = \frac{0.8625 - (-0.5536)}{0.33 - 0.2}$

$a = 2.26$

I have implemented 5x-1, 6x-5, x-2 on both equations since holes should be on both top and bottom. The pair of x-2 is there to ensure the equation = 0 when x is 2. The final x-2 is to ensure the final equation has a VA of x=2. I also found the a-value from the y-values of the holes.

×

$y = 2.26\frac{1}{x-2}x$

3.5