AI1110 Assignment 1

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Q2 (C): In an Arithmetic Progression, the fourth and sixth terms are 8 and 16 respectively.Find :
i) common difference
ii) first term
iii) sum of the first 20 terms

Solution: Let a_i denote the i th term of the AP , d denote the common diff,
S_{20} denote the sum of first 20 terms

TABLE I
VARIABLES

Symbol	value
i	4
j	6
a_i	8
a_j	14
a_1	?
d	?
S_n	?
S_{20}	?

For any general a_i , a_j :

$$(i) \qquad a_i + (j-i)d = a_j$$
$$\Rightarrow \qquad (j-i)d = a_j - a_i$$
$$\Rightarrow \qquad d = \frac{a_j - a_i}{(j-i)} \tag{1}$$

Substituting $i = 4, j = 6, a_i = 8$ and $a_j = 14$ in eq.(1):

$$\Rightarrow \qquad d = \frac{14-8}{(6-4)}$$
$$\Rightarrow \qquad d = \frac{6}{2}$$
$$\therefore \qquad d = 3 \tag{2}$$

$$(ii) \qquad a_1 + (i-1)d = a_i$$
$$\Rightarrow \qquad a_1 = a_i - (i-1)d$$
$$\Rightarrow \qquad a_1 = a_i - \frac{(i-1)(a_j - a_i)}{(j-i)}$$
$$\Rightarrow \qquad a_1 = \frac{a_i(j-1) + a_j(1-i)}{(j-i)} \tag{3}$$

Substituting $d = 3$ and $a_i = 8$ in eq.(3):

$$\Rightarrow \qquad a_1 = \frac{8(6-1)+14(1-4)}{(6-4)}$$
$$\Rightarrow \qquad a_1 = \frac{8(5)+14(-3)}{(2)}$$
$$\Rightarrow \qquad a_1 = \frac{40-42}{(2)}$$
$$\therefore \qquad a_1 = \frac{-2}{2} = -1 \tag{4}$$

Now calculating S_n for general n:

$$iii) S_n = a_1 + a_2 + \dots\dots\dots + a_n$$
$$= \frac{n \times [2a_1 + (n-1)d]}{2}$$
$$= \frac{n}{2} \times \left[2 \times \frac{a_i(j-1)+a_j(1-i)}{(j-i)} + \frac{(n-1)(a_j-a_i)}{(j-i)}\right]$$
$$= \frac{n}{2} \times \left[\frac{2a_i(j-1)+2a_j(1-i)+(n-1)(a_j-a_i)}{(j-i)}\right]$$
$$= \frac{n}{2} \times \left[\frac{a_i(2j-n-1)+a_j(1+n-2i)}{(j-i)}\right] \tag{5}$$

Substituting the values of $n = 20, i, j, a_i$ and a_j in eq.(5)

$$S_{20} = \frac{20}{2} \times \left[\frac{8(2\times6-20-1)+14(1+20-2\times4)}{(6-4)}\right]$$
$$\Rightarrow \qquad S_{20} = \frac{20}{2} \times \left[\frac{8(2\times6-20-1)+14(1+20-2\times4)}{(2)}\right]$$
$$\Rightarrow \qquad S_{20} = 10 \times \left[\frac{8(12-21)+14(21-8)}{2}\right]$$
$$\Rightarrow \qquad S_{20} = \frac{10}{2} \times [8(12-21)+14(21-8)]$$
$$\Rightarrow \qquad S_{20} = 5 \times [8(-9)+14(13)]$$
$$\Rightarrow \qquad S_{20} = 5 \times [182-72]$$
$$\therefore \qquad S_{20} = 5 \times [110] = 550$$