AI1110 Assignment 1

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Q2 (C): In an Arithmetic Progression, the fourth and sixth terms are 8 and 16 respectively.Find :
i) common difference
ii) first term
iii) sum of the first 20 terms

Solution: Let a_i denote the i th term of the AP , d denote the common diff,
S_{20} denote the sum of first 20 terms

TABLE I
VARIABLES

Symbol	value
a_4	8
a_6	14
a_1	?
d	?
S_{20}	?

$$\begin{aligned}(i) \qquad a_4 + 2d &= a_6\\ \Rightarrow \qquad 2d &= a_6 - a_4\\ &= 14 - 8\\ &= 6\\ \Rightarrow \qquad d &= \tfrac{6}{2}\\ d &= 3\ (Ans)\end{aligned}$$

$$\begin{aligned}(ii) \qquad a_1 + 3d &= a_4\\ \Rightarrow \qquad a_1 &= a_4 - 3d\\ &= 8 - 3(3)\\ &= 8 - 9\\ a_1 &= -1\ (Ans)\end{aligned}$$

$$\begin{aligned}n &= 20\\ iii) S_{20} &= a_1 + a_2 + ... + a_{20}\\ &= \tfrac{n \times [2a_1 + (n-1)d]}{2}\\ &= \tfrac{20 \times [2(-1) + (20-1)3]}{2}\\ &= \tfrac{20 \times [-2 + (19)3]}{2}\\ &= \tfrac{20 \times [55]}{2}\\ &= 550(Ans)\end{aligned}$$

Hence the common difference , first term , sum of first 20 terms are 3 , -1 , 550 respectively.

TABLE II
ANSWERS

Symbol	value
a_1	-1
d	3
S_{20}	550

For any general a_i , a_j :

$$\begin{aligned}(i) \qquad a_i + (j-i)d &= a_j\\ \Rightarrow \qquad (j-i)d &= a_j - a_i\\ \Rightarrow \qquad d &= \tfrac{a_j - a_i}{(j-i)}\end{aligned}$$

$$\begin{aligned}(ii) \qquad a_1 + (i-1)d &= a_i\\ \Rightarrow \qquad a_1 &= a_i - (i-1)d\\ \Rightarrow \qquad a_1 &= a_i - \tfrac{(i-1)(a_j - a_i)}{(j-i)}\\ \Rightarrow \qquad a_1 &= \tfrac{a_i(j-1) + a_j(1-i)}{(j-i)}\end{aligned}$$

$$\begin{aligned}iii) S_{20} &= a_1 + a_2 + ... + a_{20}\\ &= \tfrac{n \times [2a_1 + (n-1)d]}{2}\\ &= \frac{20 \times \left[2 \times \frac{a_i(j-1) + a_j(1-i)}{(j-i)} + 19 \times \frac{(a_j - a_i)}{(j-i)}\right]}{2}\\ &= \tfrac{20[\, a_i(j-1) + a_j(1-i)\,] + 190(a_j - a_i)}{(j-i)}\end{aligned}$$