

AI1110 Assignment 1

Donal Loitam(AI21BTECH11009)

ICSE class 10 paper 2019

Q2 (C): In an Arithmetic Progression, the fourth and sixth terms are 8 and 16 respectively. Find :

i) common difference

ii) first term

iii) sum of the first 20 terms

$$\begin{aligned}
 (ii) \quad a_1 + (i-1)d &= a_i \\
 \Rightarrow a_1 &= a_i - (i-1)d \\
 \Rightarrow a_1 &= a_i - \frac{(i-1)6}{(j-i)} \\
 \Rightarrow a_1 &= \frac{a_i(j-i) - 6(i-1)}{(j-i)} \quad (3)
 \end{aligned}$$

Substituting $i = 4, j = 6$ and $a_i = 8$ in eq.(3):

$$\begin{aligned}
 \Rightarrow a_1 &= \frac{8(6-4) - 6(4-1)}{(6-4)} \\
 \Rightarrow a_1 &= \frac{8(2) - 6(3)}{(2)} \\
 \Rightarrow a_1 &= \frac{16 - 18}{2} \\
 \therefore a_1 &= \frac{-2}{2} = -1 \quad (4)
 \end{aligned}$$

Solution: Let a_i denote the i th term of the AP ,
 d denote the common diff,
 S_{20} denote the sum of first 20 terms

TABLE I
VARIABLES

Symbol	value
i	4
j	6
a_i	8
a_j	14
a_1	?
d	?
S_n	?
S_{20}	?

Now calculating S_n for general n :

$$\begin{aligned}
 iii) S_n &= a_1 + a_2 + \dots + a_{20} \\
 &= \frac{n \times [2a_1 + (n-1)d]}{2} \quad (5)
 \end{aligned}$$

Using values from eq.(2) and eq.(4) in eq.(5):

$$\begin{aligned}
 S_n &= \frac{n \times [2(-1) + (n-1)3]}{2} \\
 \therefore S_n &= \frac{n \times [3(n-1) - 2]}{2} \quad (6)
 \end{aligned}$$

For any general a_i, a_j :

$$\begin{aligned}
 (i) \quad a_i + (j-i)d &= a_j \\
 \Rightarrow (j-i)d &= a_j - a_i \\
 \Rightarrow (j-i)d &= 14 - 8 \\
 \Rightarrow (j-i)d &= 6 \\
 \Rightarrow d &= \frac{6}{(j-i)} \quad (1)
 \end{aligned}$$

Substituting $n = 20$ in eq.(6)

$$\begin{aligned}
 S_{20} &= \frac{20 \times [3(20-1) - 2]}{2} \\
 \Rightarrow S_{20} &= 10 \times [3(19) - 2] \\
 \Rightarrow S_{20} &= 10 \times [57 - 2] \\
 \therefore S_{20} &= 10 \times [55] = 550
 \end{aligned}$$

Substituting $i = 4$ and $j = 6$ in eq.(1):

$$\begin{aligned}
 \Rightarrow d &= \frac{6}{6-4} \\
 \Rightarrow d &= \frac{6}{2} \\
 \therefore d &= 3 \quad (2)
 \end{aligned}$$