Assignment 2

AI1110: Probability and Random Variables

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Question 1(vi) Prove that the function $f(x) = x^3 - 6x^2 + 12x + 5$ is increasing on $\mathbb{R}$

Solution. Given function is:

$$f(x) = x^3 - 6x^2 + 12x + 5 \tag{1}$$

Using the first principle of differentiation,the first derivative of $f(x)$:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \tag{2}$$

$$= \lim_{h\to 0} \left[\frac{(x+h)^3-6(x+h)^2+12(x+h)-x^3}{h} + \frac{6x^2-12x}{h} \right] \tag{3}$$

$$= \lim_{h\to 0} \frac{\left[(x+h)^3-x^3\right]-6\left[(x+h)^2-x^2\right]}{h} + \frac{12[(x+h)-x]}{h} \tag{4}$$

$$\because (x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3 \tag{5}$$

$$f'(x) = \lim_{h\to 0} \frac{\left[h^3+3x^2h+3xh^2\right]-6\left[2xh+h^2\right]+12h}{h} \tag{6}$$

$$= \lim_{h\to 0} \frac{h^3+3x^2h+3xh^2-12xh-6h^2+12h}{h} \tag{7}$$

$$= \lim_{h\to 0}(h^2 + 3x^2 + 3xh - 12x - 6h + 12) \tag{8}$$

$$(\because h \to 0, h^2 \to 0) \tag{9}$$

$$f'(x) = \lim_{h\to 0}(3x^2 - 12x + 12) \tag{10}$$

$$\implies f'(x) = 3x^2 - 12x + 12 \tag{11}$$

$$\implies f'(x) = 3(x^2 - 4x + 4) \tag{12}$$

$$\implies f'(x) = 3(x-2)^2 \tag{13}$$

$$\therefore \quad f'(x) \geq 0 \quad , \forall\ x \in \mathbb{R} \tag{14}$$

Since the slope of $f(x)$ i.e $f'(x) \geq 0, \ \forall\ x \in \mathbb{R}$
$\therefore\ f(x) = x^3 - 6x^2 + 12x + 5$ is always increasing on $\mathbb{R}$

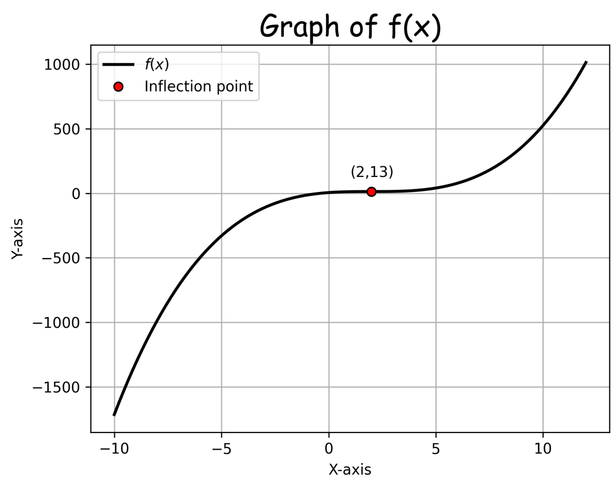

Fig. 1. Graph of $f(x) = x^3 - 6x^2 + 12x + 5$