Assignment 2

AI1110: Probability and Random Variables

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Question 1(vi) Prove that the function $f(x) = x^3 - 6x^2 + 12x + 5$ is increasing on $\mathbb{R}$

Solution. A function is said to be increasing if $\forall x_1, x_2$ that satisfies $x_2 > x_1$, then $f(x_2) \geq f(x_1)$ Given function can be simplified as:

$$f(x) = x^3 - 6x^2 + 12x + 5 \tag{1}$$
$$= (x^3 - 6x^2 + 12x - 8) + 13 \tag{2}$$
$$= (x-2)^3 + 13 \tag{3}$$

Let $x_2 > x_1$ and $y_1 = (x_1 - 2), y_2 = (x_2 - 2)$ then clearly $y_2 > y_1$. We have :-

$$f(x_2) - f(x_1) = (x_2 - 2)^3 - (x_1 - 2)^3 \tag{4}$$
$$= (y_2)^3 - (y_1)^3 \geq 0 \tag{5}$$
$$\because (y_2 > y_1 \implies y_2^3 \geq y_1^3) \tag{6}$$

Since we proved $\forall x_2 > x_1 \implies f(x_2) \geq f(x_1)$
$\therefore$ $f(x) = x^3 - 6x^2 + 12x + 5$ is always increasing on $\mathbb{R}$

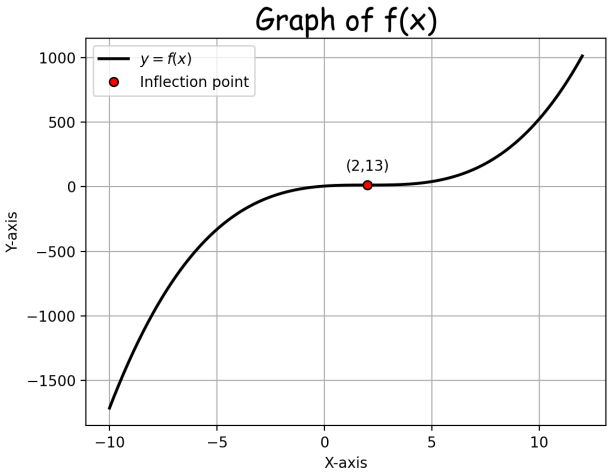

Fig. 1. Graph of $f(x) = x^3 - 6x^2 + 12x + 5$