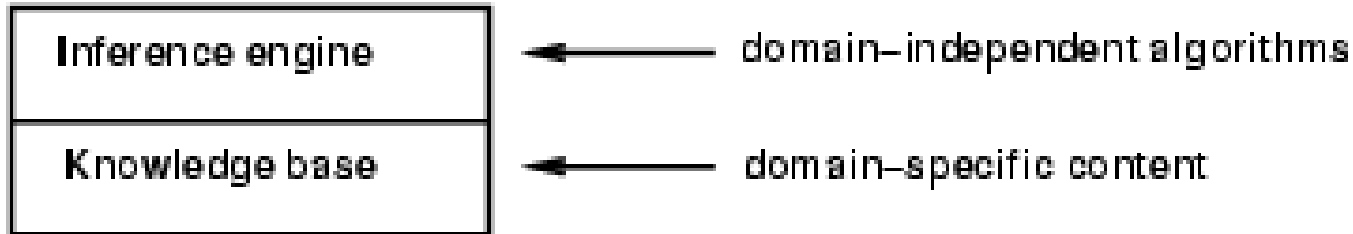


Logical Agents - Review

CS4881 Artificial Intelligence

Jay Urbain

Knowledge bases



- Knowledge base = set of **sentences** in a **formal** language
- **Declarative** approach to building an agent (or other system):
 - **Tell** it what it needs to know
- Then it can **Ask** itself what to do - answers should follow from the KB

A simple knowledge-based agent

```
function KB-AGENT(percept) returns an action
  static: KB, a knowledge base
         t, a counter, initially 0, indicating time

  TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
  action ← ASK(KB, MAKE-ACTION-QUERY(t))
  TELL(KB, MAKE-ACTION-SENTENCE(action, t))
  t ← t + 1
  return action
```

- The agent must be able to:
 - Represent states, actions, etc.
 - Incorporate new percepts
 - Update internal representations of the world
 - Deduce hidden properties of the world
 - Deduce appropriate actions

Logic in general

- **Logics** are formal languages for representing information such that conclusions can be drawn
- **Syntax** defines the sentences in the language
- **Semantics** define the "meaning" of sentences;
 - i.e., define **truth** of a sentence in a world

Entailment

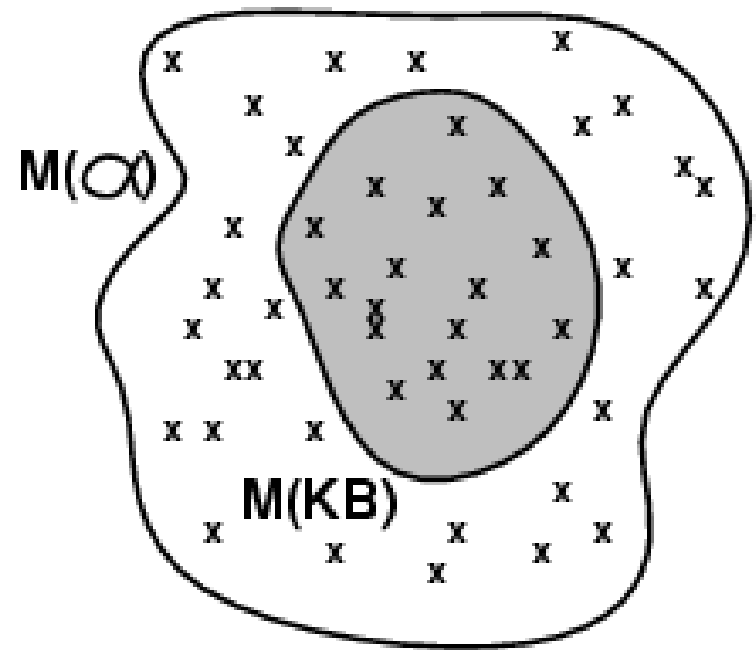
- Entailment means that one thing follows from another:

$$KB \models \alpha$$

- Knowledge base KB entails sentence α if and only if α is true in all worlds where KB is true
 - E.g., the KB containing “the Bulls won” and “the Bucks won” entails “Either the Bulls won or the Bucks won”

Models

- Logicians typically think in terms of **models**, which are formally structured worlds with respect to which truth can be evaluated
- We say m is a **model** of a sentence α if α is true in m
- $M(\alpha)$ is the set of all models of α
- Then $KB \models \alpha$ iff $M(KB) \subseteq M(\alpha)$
 - E.g. $\alpha = x + y = 4$
 - $KB1 : x=2, y=2 \rightarrow \alpha$ entailed
 - $KB2 : x=-2, y=6 \rightarrow \alpha$ entailed
 - $KB3 : x=1, y=2 \rightarrow \alpha$ not entailed



Inference

- $KB \vdash_i \alpha$ = sentence α can be derived from KB by inference procedure i
- **Soundness**: i is sound if whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$ (only derives entailed sentences)
- **Completeness**: i is complete if whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$ (can derive any entailed sentence)
- That is, the procedure will answer any question whose answer follows from what is known by the KB .

Propositional logic: Syntax

- Propositional (boolean) logic is the simplest logic – illustrates basic ideas
- The proposition symbols P_1, P_2 etc are sentences
 - If S is a sentence, $\neg S$ is a sentence (negation)
 - If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)
 - If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)
 - If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)
 - If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g. $P_{1,2}$ $P_{2,2}$ $P_{3,1}$
 false true false

With these symbols, 8 possible models, can be enumerated automatically.

Rules for evaluating truth with respect to a model m :

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff	S_1 is true or	S_2 is true
$S_1 \Rightarrow S_2$	is true iff	S_1 is false or	S_2 is true
i.e.,	is false iff	S_1 is true and	S_2 is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$ is true and	$S_2 \Rightarrow S_1$ is true

Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = \text{true} \wedge (\text{true} \vee \text{false}) = \text{true} \wedge \text{true} = \text{true}$$

Truth tables for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>
<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>
<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>
<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i, j]$.

Let $B_{i,j}$ be true if there is a breeze in $[i, j]$.

$$\neg P_{1,1}$$

$$\neg B_{1,1}$$

$$B_{2,1}$$

- "Pits cause breezes in adjacent squares"

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

$$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	KB	α_1
<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>
<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>true</i>
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<u><i>true</i></u>	<u><i>true</i></u>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<u><i>true</i></u>	<u><i>true</i></u>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<u><i>true</i></u>	<u><i>true</i></u>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>false</i>

Inference by enumeration

- Depth-first enumeration of all models is sound and complete \square

```
function TT-ENTAILS?(KB,  $\alpha$ ) returns true or false
    symbols  $\leftarrow$  a list of the proposition symbols in KB and  $\alpha$ 
    return TT-CHECK-ALL(KB,  $\alpha$ , symbols, [])

function TT-CHECK-ALL(KB,  $\alpha$ , symbols, model) returns true or false
    if EMPTY?(symbols) then
        if PL-TRUE?(KB, model) then return PL-TRUE?( $\alpha$ , model)
        else return true
    else do
        P  $\leftarrow$  FIRST(symbols); rest  $\leftarrow$  REST(symbols)
        return TT-CHECK-ALL(KB,  $\alpha$ , rest, EXTEND(P, true, model)) and
            TT-CHECK-ALL(KB,  $\alpha$ , rest, EXTEND(P, false, model))
```

- For n symbols, time complexity is $O(2^n)$, space complexity is $O(n)$ \square

Logical equivalence

- Two sentences are **logically equivalent** iff true in same models: $\alpha \equiv \beta$ iff $\alpha \models \beta$ and $\beta \models \alpha$

$$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge$$

$$(\alpha \vee \beta) \equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee$$

$$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge$$

$$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee$$

$$\neg(\neg\alpha) \equiv \alpha \quad \text{double-negation elimination}$$

$$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition}$$

$$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta) \quad \text{implication elimination}$$

$$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination}$$

$$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta) \quad \text{de Morgan}$$

$$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta) \quad \text{de Morgan}$$

$$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee$$

$$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge$$

Validity and satisfiability

A sentence is **valid** if it is true in **all** models,
e.g., *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the **Deduction Theorem**:
 $KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is **satisfiable** if it is true in **some** model
e.g., $A \vee B$, C

A sentence is **unsatisfiable** if it is true in **no** models
e.g., $A \wedge \neg A$

Satisfiability is connected to inference via the following:
 $KB \models \alpha$ if and only if $(KB \wedge \neg \alpha)$ is unsatisfiable

Proof methods

- Proof methods divide into (roughly) two kinds:
 - Application of inference rules
 - Legitimate (sound) generation of new sentences from old
 - **Proof** = a sequence of inference rule applications
 - Can use inference rules as operators in a standard search algorithm
 - Typically require transformation of sentences into a **normal form**
 - Model checking
 - truth table enumeration (always exponential in n)
 - improved backtracking, e.g., Davis--Putnam-Logemann-Loveland (DPLL)
 - heuristic search in model space (sound but incomplete)
 - e.g., min-conflicts-like algorithms

Resolution

Conjunctive Normal Form (CNF)

conjunction of disjunctions of literals
clauses

E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

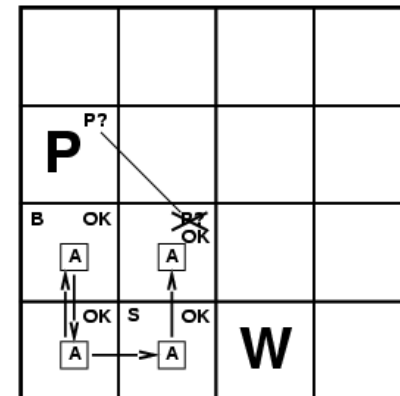
- Resolution inference rule (for CNF):

$$\frac{\begin{array}{c} \ell_i \vee \dots \vee \ell_k, \\ \ell_i \vee \dots \vee \ell_{i-1} \vee \ell_{i+1} \vee \dots \vee \ell_k \vee m_1 \vee \dots \vee m_{j-1} \vee m_{j+1} \vee \dots \vee m_n \end{array}}{\square}$$

where ℓ_i and m_j are complementary literals.

$$\text{E.g., } \frac{P_{1,3} \vee P_{2,2}, \quad \neg P_{2,2}}{P_{1,3}}$$

- Resolution is sound and complete for propositional logic



Conversion to CNF

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

1. Eliminate \Leftrightarrow , replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.

$$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

2. Eliminate \Rightarrow , replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$$

3. Move \neg inwards using de Morgan's rules and double-negation:

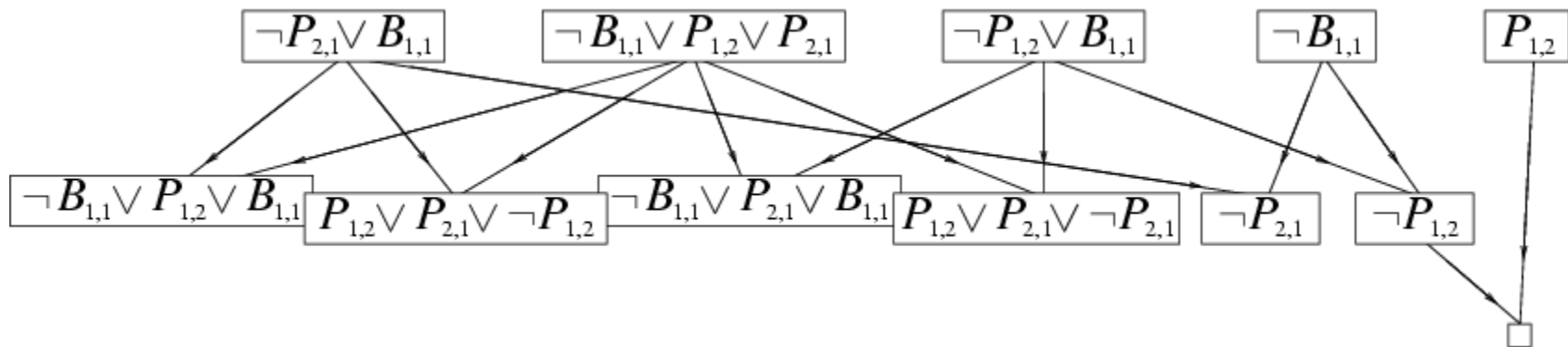
$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \vee \neg P_{2,1}) \vee B_{1,1})$$

4. Apply distributivity law (\wedge over \vee) and flatten:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$$

Resolution example

- $KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1} \quad \alpha = \neg P_{1,2}$



Forward and backward chaining

- **Horn Form** (restricted)

KB = **conjunction** of **Horn clauses**

- Horn clause =

proposition symbol; or (conjunction of symbols) \Rightarrow symbol

- E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

- **Modus Ponens** (for Horn Form): complete for Horn KBs

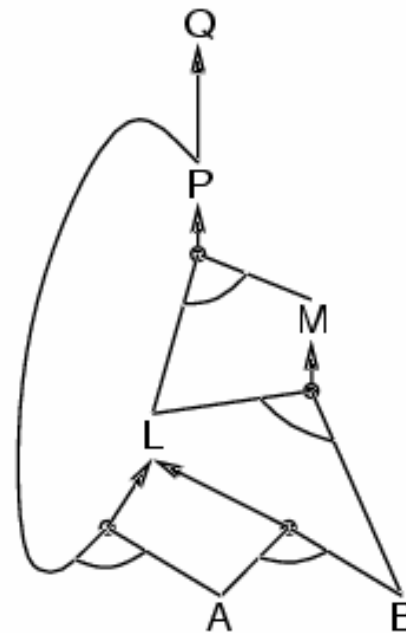
$$\frac{\alpha_1 \wedge \dots \wedge \alpha_n \Rightarrow \beta \quad \alpha_1, \dots, \alpha_n}{\beta}$$

- Can be used with **forward chaining** or **backward chaining**.
- These algorithms are very natural and run in **linear** time

Forward chaining

- Idea: fire any rule whose premises are satisfied in the *KB*,
 - add its conclusion to the *KB*, until query is found

$P \Rightarrow Q$
 $L \wedge M \Rightarrow P$
 $B \wedge L \Rightarrow M$
 $A \wedge P \Rightarrow L$
 $A \wedge B \Rightarrow L$
 A
 B



Forward chaining algorithm

```
function PL-FC-ENTAILS?(KB, q) returns true or false
  local variables: count, a table, indexed by clause, initially the number of premises
                  inferred, a table, indexed by symbol, each entry initially false
                  agenda, a list of symbols, initially the symbols known to be true

  while agenda is not empty do
    p ← POP(agenda)
    unless inferred[p] do
      inferred[p] ← true
      for each Horn clause c in whose premise p appears do
        decrement count[c]
        if count[c] = 0 then do
          if HEAD[c] = q then return true
          PUSH(HEAD[c], agenda)

  return false
```

- Forward chaining is sound and complete for Horn KB

Proof of completeness

- FC derives every atomic sentence that is entailed by KB
 1. FC reaches a **fixed point** where no new atomic sentences are derived
 2. Consider the final state as a model m , assigning true/false to symbols
 3. Every clause in the original KB is true in m
$$a_1 \wedge \dots \wedge a_k \Rightarrow b$$
 4. Hence m is a model of KB
 5. If $KB \models q$, q is true in **every** model of KB , including m

Backward chaining

Idea: work backwards from the query q :

to prove q by BC,

check if q is known already, or

prove by BC all premises of some rule concluding q \square

Avoid loops: check if new sub-goal is already on the goal stack

Avoid repeated work: check if new sub-goal

1. has already been proved true, or
2. has already failed

Forward vs. backward chaining

- FC is **data-driven**, automatic, unconscious processing,
 - e.g., object recognition, routine decisions
- May do lots of work that is irrelevant to the goal
- BC is **goal-driven**, appropriate for problem-solving,
 - e.g., Where are my keys? How do I get into a PhD program?
- Complexity of BC can be **much less** than linear in size of KB

Efficient propositional inference

Two families of efficient algorithms for propositional inference:

Complete backtracking search algorithms

- DPLL algorithm (Davis, Putnam, Logemann, Loveland)
- Incomplete local search algorithms
 - WalkSAT algorithm

The DPLL algorithm

Determine if an input propositional logic sentence (in **CNF**) is satisfiable.

Improvements over truth table enumeration:

1. Early termination

A clause is true if any literal is true.

A sentence is false if any clause is false.

2. Pure symbol heuristic

Pure symbol: always appears with the same "sign" in all clauses.

e.g., In the three clauses $(A \vee \neg B)$, $(\neg B \vee \neg C)$, $(C \vee A)$, A and B are pure, C is impure.

Make a pure symbol literal true.

3. Unit clause heuristic

Unit clause: only one literal in the clause

The only literal in a unit clause must be true.

The DPLL algorithm

function DPLL-SATISFIABLE?(*s*) **returns** *true* or *false*

inputs: *s*, a sentence in propositional logic

clauses \leftarrow the set of clauses in the CNF representation of *s*

symbols \leftarrow a list of the proposition symbols in *s*

return DPLL(*clauses*, *symbols*, [])

function DPLL(*clauses*, *symbols*, *model*) **returns** *true* or *false*

if every clause in *clauses* is true in *model* **then return** *true*

if some clause in *clauses* is false in *model* **then return** *false*

P, *value* \leftarrow FIND-PURE-SYMBOL(*symbols*, *clauses*, *model*)

if *P* is non-null **then return** DPLL(*clauses*, *symbols* - *P*, [*P* = *value* | *model*])

P, *value* \leftarrow FIND-UNIT-CLAUSE(*clauses*, *model*)

if *P* is non-null **then return** DPLL(*clauses*, *symbols* - *P*, [*P* = *value* | *model*])

P \leftarrow FIRST(*symbols*); *rest* \leftarrow REST(*symbols*)

return DPLL(*clauses*, *rest*, [*P* = *true* | *model*]) **or**
DPLL(*clauses*, *rest*, [*P* = *false* | *model*])

The WalkSAT algorithm

- Incomplete, local search algorithm
- Evaluation function: The min-conflict heuristic of minimizing the number of unsatisfied clauses
- Balance between greediness and randomness

The WalkSAT algorithm

```
function WALKSAT(clauses, p, max-flips) returns a satisfying model or failure
  inputs: clauses, a set of clauses in propositional logic
         p, the probability of choosing to do a “random walk” move
         max-flips, number of flips allowed before giving up

  model ← a random assignment of true/false to the symbols in clauses
  for i = 1 to max-flips do
    if model satisfies clauses then return model
    clause ← a randomly selected clause from clauses that is false in model
    with probability p flip the value in model of a randomly selected symbol
      from clause
    else flip whichever symbol in clause maximizes the number of satisfied clauses
  return failure
```

Inference-based agents in the wumpus world

A wumpus-world agent using propositional logic:

$$\begin{aligned}& \neg P_{1,1} \\& \neg W_{1,1} \\& B_{x,y} \Leftrightarrow (P_{x,y+1} \vee P_{x,y-1} \vee P_{x+1,y} \vee P_{x-1,y}) \\& S_{x,y} \Leftrightarrow (W_{x,y+1} \vee W_{x,y-1} \vee W_{x+1,y} \vee W_{x-1,y}) \\& W_{1,1} \vee W_{1,2} \vee \dots \vee W_{4,4} \\& \neg W_{1,1} \vee \neg W_{1,2} \\& \neg W_{1,1} \vee \neg W_{1,3} \\& \dots\end{aligned}$$

\Rightarrow 64 distinct proposition symbols, 155 sentences

```

function PL-WUMPUS-AGENT(percept) returns an action
  inputs: percept, a list, [stench, breeze, glitter]
  static: KB, initially containing the “physics” of the wumpus world
           x, y, orientation, the agent’s position (init. [1,1]) and orient. (init. right)
           visited, an array indicating which squares have been visited, initially false
           action, the agent’s most recent action, initially null
           plan, an action sequence, initially empty

  update x, y, orientation, visited based on action
  if stench then TELL(KB,  $S_{x,y}$ ) else TELL(KB,  $\neg S_{x,y}$ )
  if breeze then TELL(KB,  $B_{x,y}$ ) else TELL(KB,  $\neg B_{x,y}$ )
  if glitter then action  $\leftarrow$  grab
  else if plan is nonempty then action  $\leftarrow$  POP(plan)
  else if for some fringe square [i, j], ASK(KB, ( $\neg P_{i,j} \wedge \neg W_{i,j}$ )) is true or
           for some fringe square [i, j], ASK(KB, ( $P_{i,j} \vee W_{i,j}$ )) is false then do
           plan  $\leftarrow$  A*-GRAPH-SEARCH(ROUTE-PB([x, y], orientation, [i, j], visited))
           action  $\leftarrow$  POP(plan)
  else action  $\leftarrow$  a randomly chosen move
  return action

```


Expressiveness limitation of propositional logic

- KB contains "physics" sentences for every single square
- For every time t and every location $[x,y]$,
$$L_{x,y}^t \wedge \textit{FacingRight}^t \wedge \textit{Forward}^t \Rightarrow L_{x+1,y}^t$$
- Rapid proliferation of clauses

Summary

- Logical agents apply **inference** to a **knowledge base** to derive new information and make decisions
- Basic concepts of logic:
 - **syntax**: formal structure of **sentences**
 - **semantics**: **truth** of sentences with respect to **models**
 - **entailment**: necessary truth of one sentence given another
 - **inference**: deriving sentences from other sentences
 - **soundness**: derivations produce only entailed sentences
 - **completeness**: derivations can produce all entailed sentences
- Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.
- Resolution is complete for propositional logic
Forward, backward chaining are linear-time, complete for Horn clauses
- Propositional logic lacks expressive power