

### Cluster Analysis

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### Cluster Analysis

- What is Cluster Analysis?
- Types of Data in Cluster Analysis
- A Categorization of Major Clustering Methods
- Partitioning Methods
- Hierarchical Methods
- Density-Based Methods
- Grid-Based Methods
- Model-Based Clustering Methods
- Outlier Analysis
- Summary

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### What is Cluster Analysis?

- Cluster: a collection of data objects
  - Similar to one another within the same cluster
  - Dissimilar to the objects in other clusters
- Cluster analysis
  - Grouping a set of data objects into clusters
- Clustering is unsupervised classification: no predefined classes
- Typical applications
  - As a stand-alone tool to get insight into data distribution
  - As a preprocessing step for other algorithms

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### **General Applications of Clustering**

- Pattern Recognition
- Spatial Data Analysis
  - create thematic maps in GIS by clustering feature spaces
  - detect spatial clusters and explain them in spatial data mining
- Image Processing
- Economic Science (especially market research)
- WWW
  - Document/text classification
  - Cluster Weblog data to discover groups of similar access patterns

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### **Examples of Clustering Applications**

- Marketing: Help marketers discover distinct groups in their customer bases, and then use this knowledge to develop targeted marketing programs
- <u>Land use:</u> Identification of areas of similar land use in an earth observation database
- Insurance: Identifying groups of motor insurance policy holders with a high average claim cost
- <u>City-planning:</u> Identifying groups of houses according to their house type, value, and geographical location
- <u>Earth-quake studies</u>: Observed earth quake epicenters should be clustered along continent faults

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### What Is Good Clustering?

- A good clustering method will produce high quality clusters with
  - high intra-class similarity
  - low <u>inter-class</u> similarity
- The <u>quality</u> of a clustering result depends on both the similarity measure used by the method and its implementation.
- The <u>quality</u> of a clustering method is also measured by its ability to discover some or all of the <u>hidden</u> patterns.



### Requirements of Clustering

- Scalability
- Ability to deal with different types of attributes
- Discovery of clusters with arbitrary shape
- Minimal requirements for domain knowledge to determine input parameters
- Able to deal with noise and outliers
- Insensitive to order of input records
- High dimensionality
- Incorporation of user-specified constraints
- Interpretability and usability

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### **Data Structures**

- Data matrix
  - (two modes)

$$\begin{bmatrix} x_{11} & \cdots & x_{1f} & \cdots & x_{1p} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{i1} & \cdots & x_{if} & \cdots & x_{ip} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{n1} & \cdots & x_{nf} & \cdots & x_{np} \end{bmatrix}$$

- Dissimilarity matrix
- (one mode)

$$\begin{bmatrix} 0 \\ d(2,1) & 0 \\ d(3,1) & d(3,2) & 0 \\ \vdots & \vdots & \vdots \\ d(n,1) & d(n,2) & \dots & \dots & 0 \end{bmatrix}$$

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### Measure the Quality of Clustering

- Dissimilarity/Similarity metric: Similarity is expressed in terms of a distance function, which is typically metric: d(i, j)
- There is a separate "quality" function that measures the "goodness" of a cluster.
- The definitions of distance functions are usually very different for interval-scaled, boolean, categorical, ordinal and ratio variables.
- Weights should be associated with different variables based on applications and data semantics.
- It is hard to define "similar enough" or "good enough"
  - the answer is typically highly subjective.

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### Type of data in clustering analysis

- Interval-scaled variables:
- Binary variables:
- Nominal, ordinal, and ratio variables:
- Variables of mixed types:

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### Interval-valued variables

- Standardize data
  - Calculate the *mean absolute deviation*:

$$x_{nf} - m_f \mid$$

 $s_f = \frac{1}{n}(|x_{1f} - m_f| + |x_{2f} - m_f| + ... + |x_{nf} - m_f|)$ 

where  $m_f = \frac{1}{n} (x_{1f} + x_{2f} + \dots + x_{nf})$  • Calculate the standardized measurement (*z-score*)

$$z_{if} = \frac{x_{if} - m_f}{s} \qquad z = \frac{x - \mu}{\sigma}$$

Using mean absolute deviation is more robust than using standard deviation



# Similarity and Dissimilarity Between Objects

- <u>Distances</u> are normally used to measure the <u>similarity</u> or <u>dissimilarity</u> between two data objects
- Some popular ones include: Minkowski distance:

$$d(i,j) = \sqrt{\left(|x_{i_1} - x_{j_1}|^q + |x_{i_2} - x_{j_2}|^q + ... + |x_{i_p} - x_{j_p}|^q\right)}$$

where  $i = (x_{j1}, x_{j2}, ..., x_{jp})$  and  $j = (x_{j1}, x_{j2}, ..., x_{jp})$  are two  $\rho$ -dimensional data objects, and q is a positive integer

• If q = 1, d is Manhattan distance

$$d(i,j) = |x_{i_1} - x_{j_1}| + |x_{i_2} - x_{j_2}| + ... + |x_{i_p} - x_{j_p}|$$

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# Similarity and Dissimilarity Between Objects (Cont.)

• If q = 2, d is Euclidean distance:

$$d(i,j) = \sqrt{(|x_{i1} - x_{j1}|^2 + |x_{i2} - x_{j2}|^2 + ... + |x_{ip} - x_{jp}|^2)}$$

- Properties
  - d(i,j) ≥ 0
  - d(i,i)=0
  - $\bullet \ d(i,j) = d(j,i)$
  - $d(i,j) \leq d(i,k) + d(k,j)$
- Also one can use weighted distance, parametric Pearson product moment correlation, or other disimilarity measures.

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### **Binary Variables**

A contingency table for binary data

5 .	,	ı	Object j		
		1	0	sum	
	1	а	b	a+b	
Object i	0	c	d	c+d	
	sum	a+c	b+d	a+b c+d p	

- Simple matching coefficient (invariant, if the binary variable is  $\underline{symmetric}$ ):  $d\left(i,j\right) = \frac{b+c}{a+b+c+d}$
- Jaccard coefficient (noninvariant if the binary variable is  $\underbrace{asymmetric}$ ):  $d(i,j) = \underbrace{\frac{b+c}{a+b+c}}$

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# Dissimilarity between Binary Variables

Example

Name	Gender	Fever	Cough	Test-1	Test-2	Test-3	Test-4
Jack	M	Y	N	P	N	N	N
Mary	F	Y	N	P	N	P	N
Jim	M	Y	P	N	N	N	N

- gender is a symmetric attribute
- the remaining attributes are asymmetric binary
- let the values Y and P be set to 1, and the value N be set to 0

$$d (jack ,mary ) = \frac{0+1}{2+0+1} = 0.33$$

$$d (jack ,jim ) = \frac{1+1}{1+1+1} = 0.67$$

$$d (jim ,mary ) = \frac{1+2}{1+1+2} = 0.75$$

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### Nominal Variables

- A generalization of the binary variable in that it can take more than 2 states, e.g., red, yellow, blue, green
- Method 1: Simple matching
  - m: # of matches, p: total # of variables

$$d(i,j) = \frac{p-m}{p}$$

- Method 2: use a large number of binary variables
  - creating a new binary variable for each of the M nominal states

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### **Ordinal Variables**

- An ordinal variable can be discrete or continuous
- order is important, e.g., rank
- Can be treated like interval-scaled
  - replacing  $x_{if}$  by their rank  $r_{if} \in \{1,..., M_f\}$
  - map the range of each variable onto [0, 1] by replacing *i*-th object in the *f*-th variable by

$$z_{if} = \frac{r_{if} - 1}{M_{f} - 1}$$

 compute the dissimilarity using methods for intervalscaled variables



### **Ratio-Scaled Variables**

- <u>Ratio-scaled variable</u>: a positive measurement on a nonlinear scale, approximately at exponential scale, such as Ae<sup>Bt</sup> or Ae<sup>-Bt</sup>
- Methods:
  - treat them like interval-scaled variables not a good choice! (why?)
  - apply logarithmic transformation

$$y_{if} = log(x_{if})$$

 treat them as continuous ordinal data treat their rank as interval-scaled.

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### Variables of Mixed Types

- A database may contain all the six types of variables
- symmetric binary, asymmetric binary, nominal, ordinal, interval and ratio.
- One may use a weighted formula to combine their effects.
   \(\sum\_{P} = \delta \) (f) d (f)

 $d(i,j) = \frac{\sum_{f=1}^{p} \delta_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^{p} \delta_{ij}^{(f)}}$ 

• f is binary or nominal:

$$d_{ii}^{(f)} = 0$$
 if  $x_{if} = x_{if}$ , or  $d_{ii}^{(f)} = 1$  o.w.

- f is interval-based: use the normalized distance
- f is ordinal or ratio-scaled
  - compute ranks  $r_{if}$  and  $z_{if} = \frac{r_{if} 1}{M_{i-1}}$
  - and treat z<sub>if</sub> as interval-scaled

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### Major Clustering Approaches

- <u>Partitioning algorithms</u>: Construct various partitions and then evaluate them by some criterion
- <u>Hierarchy algorithms</u>: Create a hierarchical decomposition of the set of data (or objects) using some criterion
- <u>Density-based</u>: based on connectivity and density functions
- Grid-based: based on a multiple-level granularity structure
- Model-based: A model is hypothesized for each of the clusters and the idea is to find the best fit of that model to each other

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### Partitioning Algorithms: Basic Concept

- Partitioning method: Construct a partition of a database D
  of n objects into a set of k clusters
- Given a k, find a partition of k clusters that optimizes the chosen partitioning criterion
  - Global optimal: exhaustively enumerate all partitions
  - Heuristic methods: *k-means* and *k-medoids* algorithms
  - <u>k-means</u> (MacQueen'67): Each cluster is represented by the center of the cluster
  - <u>k-medolds</u> or PAM (Partition around medoids) (Kaufman & Rousseeuw'87): Each cluster is represented by one of the objects in the cluster

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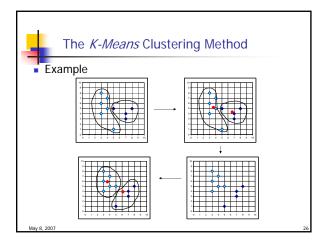
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### The K-Means Clustering Method

- Given k, the k-means algorithm is implemented in 4 steps:
  - Partition objects into k nonempty subsets
  - Compute seed points as the centroids of the clusters of the current partition. The centroid is the center (mean point) of the cluster.
  - Assign each object to the cluster with the nearest seed point.
  - Go back to Step 2, stop when no more new assignment.

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### Comments on the K-Means Method

### Strength |

- Relatively efficient: O(tkn), where n is # objects, k is # clusters, and t is # iterations. Normally, k, t << n.</li>
- Often terminates at a local optimum. The global optimum may be found using techniques such as: deterministic annealing and genetic algorithms

### Weakness

- Applicable only when *mean* is defined, then what about categorical data?
- Need to specify k, the number of clusters, in advance
- Unable to handle noisy data and outliers
- Not suitable to discover clusters with non-convex shapes

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### Variations of the K-Means Method

- A few variants of the k-means which differ in
  - Selection of the initial k means
  - Dissimilarity calculations
  - Strategies to calculate cluster means
- Handling categorical data: k-modes (Huang'98)
  - Replacing means of clusters with modes
  - Using new dissimilarity measures to deal with categorical objects
  - Using a <u>frequency</u>-based method to update modes of clusters
  - A mixture of categorical and numerical data: kprototype method

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### The K-Medoids Clustering Method

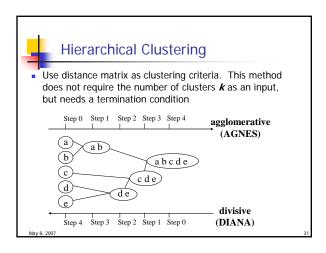
- Find *representative* objects, called <u>medoids</u>, in clusters
- PAM (Partitioning Around Medoids, 1987)
  - starts from an initial set of medoids and iteratively replaces one of the medoids by one of the nonmedoids if it improves the total distance of the resulting clustering
  - PAM works effectively for small data sets, but does not scale well for large data sets
- CLARA (Kaufmann & Rousseeuw, 1990)
- CLARANS (Ng & Han, 1994): Randomized sampling
- Focusing + spatial data structure (Ester et al., 1995)

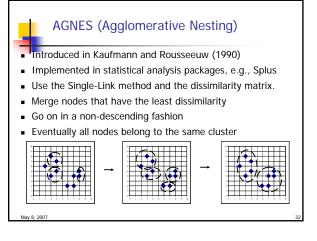
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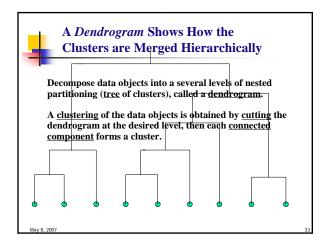


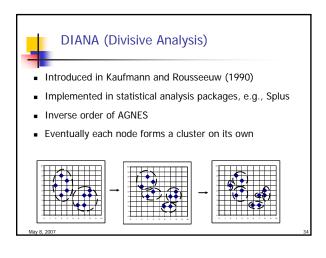
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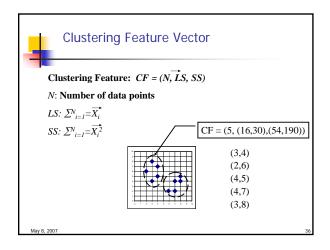


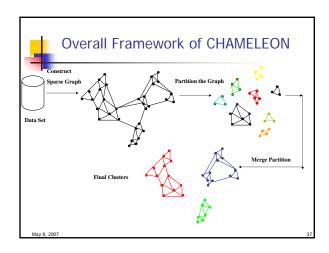




# More on Hierarchical Clustering Methods Major weakness of agglomerative clustering methods do not scale well: time complexity of at least O(n²), where n is the number of total objects can never undo what was done previously Integration of hierarchical with distance-based clustering BIRCH (1996): uses CF-tree and incrementally adjusts the quality of sub-clusters CURE (1998): selects well-scattered points from the cluster and then shrinks them towards the center of the cluster by a specified fraction CHAMELEON (1999): hierarchical clustering using

dynamic modeling







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### **Density-Based Clustering Methods**

- Clustering based on density (local cluster criterion), such as density-connected points
- Major features:
  - Discover clusters of arbitrary shape
  - Handle noise
  - One scan
  - Need density parameters as termination condition
- Several interesting studies:
  - DBSCAN: Ester, et al. (KDD'96)
  - OPTICS: Ankerst, et al (SIGMOD'99).
  - DENCLUE: Hinneburg & D. Keim (KDD'98)
  - CLIQUE: Agrawal, et al. (SIGMOD'98)

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### Density-Based Clustering: Background

- Two parameters:
  - *Eps*: Maximum radius of the neighbourhood
  - MinPts: Minimum number of points in an Epsneighbourhood of that point
- $N_{Eps}(p)$ : {q belongs to D / dist(p,q) <= Eps}
- Directly density-reachable: A point p is directly density-reachable from a point q wrt. Eps, MinPts if
  - 1) p belongs to N<sub>Eps</sub>(q)
  - 2) core point condition:



MinPts = 5

 $Eps=1\ cm$ 

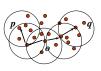
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### ensity-Based Clustering: Background (II)

- Density-reachable:
  - A point  $\rho$  is density-reachable from a point q wrt. *Eps, MinPts* if there is a chain of points  $\rho_1, \ldots, \rho_n, \rho_1 = q, \rho_n = \rho$  such that  $\rho_{i+1}$  is directly density-reachable from  $\rho_i$

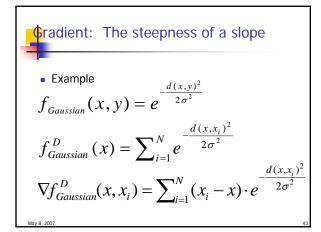


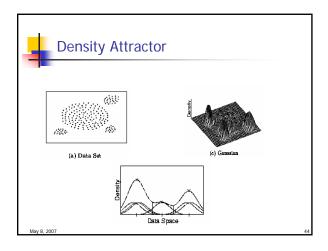
- Density-connected
  - A point p is density-connected to a point q wrt. Eps, MinPts if there is a point o such that both, p and q are density-reachable from o wrt. Eps and MinPts.

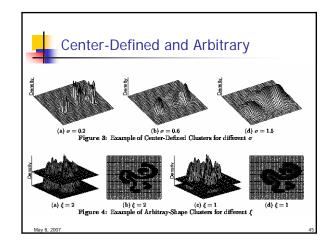


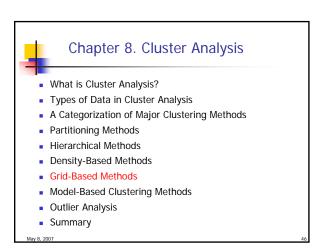
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# DBSCAN: Density Based Spatial Clustering of Applications with Noise Relies on a *density-based* notion of cluster: A *cluster* is defined as a maximal set of density-connected points Discovers clusters of arbitrary shape in spatial databases with noise Discovers clusters of arbitrary shape in spatial databases with noise Eps = 1cm MinPts = 5





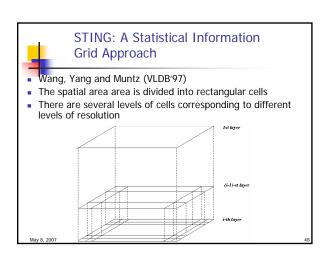






### **Grid-Based Clustering Method**

- Using multi-resolution grid data structure
- Several interesting methods
  - STING (a STatistical Information Grid approach) by Wang, Yang and Muntz (1997)
  - WaveCluster by Sheikholeslami, Chatterjee, and Zhang (VLDB'98)
    - A multi-resolution clustering approach using wavelet method
  - CLIQUE: Agrawal, et al. (SIGMOD'98)





# STING: A Statistical Information Grid Approach (2)

- Each cell at a high level is partitioned into a number of smaller cells in the next lower level
- Statistical info of each cell is calculated and stored beforehand and is used to answer queries
- Parameters of higher level cells can be easily calculated from parameters of lower level cell
  - count, mean, s, min, max
  - type of distribution—normal, uniform, etc.
- Use a top-down approach to answer spatial data queries
- Start from a pre-selected layer—typically with a small number of cells
- For each cell in the current level compute the confidence interval

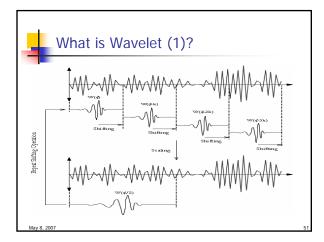
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# STING: A Statistical Information Grid Approach (3)

- Remove the irrelevant cells from further consideration
- When finish examining the current layer, proceed to the next lower level
- Repeat this process until the bottom layer is reached
- Advantages:
  - Query-independent, easy to parallelize, incremental update
  - O(K), where K is the number of grid cells at the lowest level
- Disadvantages:
  - All the cluster boundaries are either horizontal or vertical, and no diagonal boundary is detected

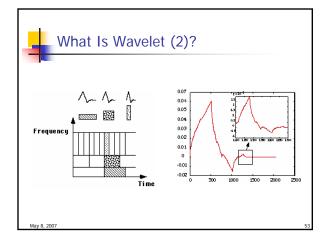
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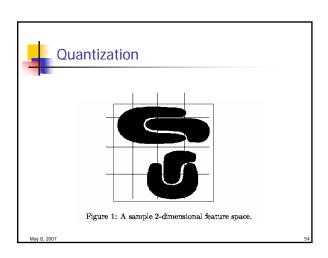


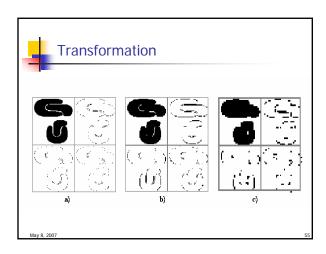


### WaveCluster (1998)

- How to apply wavelet transform to find clusters
  - Summaries the data by imposing a multidimensional grid structure onto data space
  - These multidimensional spatial data objects are represented in a n-dimensional feature space
  - Apply wavelet transform on feature space to find the dense regions in the feature space
  - Apply wavelet transform multiple times which result in clusters at different scales from fine to coarse









### WaveCluster (1998)

- Why is wavelet transformation useful for clustering
  - Unsupervised clustering
     It uses hat-shape filters to emphasize region where points cluster, but simultaneously to suppress weaker information in their boundary
  - Effective removal of outliers
  - Multi-resolution
  - Cost efficiency
- Major features:
  - Complexity O(N)
  - Detect arbitrary shaped clusters at different scales
  - Not sensitive to noise, not sensitive to input order
  - Only applicable to low dimensional data



### CLIQUE (Clustering In QUEst)

- Agrawal, Gehrke, Gunopulos, Raghavan (SIGMOD'98).
- Automatically identifying subspaces of a high dimensional data space that allow better clustering than original space
- CLIQUE can be considered as both density-based and gridbased
  - It partitions each dimension into the same number of equal length interval
  - It partitions an m-dimensional data space into nonoverlapping rectangular units
  - A unit is dense if the fraction of total data points contained in the unit exceeds the input model parameter
  - A cluster is a maximal set of connected dense units within a subspace

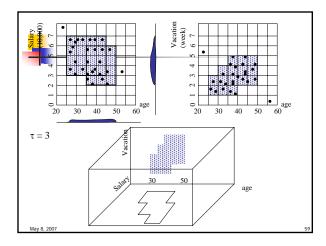
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### **CLIQUE: The Major Steps**

- Partition the data space and find the number of points that lie inside each cell of the partition.
- Identify the subspaces that contain clusters using the Apriori principle
- Identify clusters:
  - Determine dense units in all subspaces of interests
  - Determine connected dense units in all subspaces of interests.
- Generate minimal description for the clusters
  - Determine maximal regions that cover a cluster of connected dense units for each cluster
  - Determination of minimal cover for each cluster

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### Strength and Weakness of CLIQUE

- Strength
  - It <u>automatically</u> finds <u>subspaces of the highest</u> <u>dimensionality</u> such that high density clusters exist in those subspaces
  - It is insensitive to the order of records in input and does not presume some canonical data distribution
  - It scales *linearly* with the size of input and has good scalability as the number of dimensions in the data increases
- Weakness
  - The accuracy of the clustering result may be degraded at the expense of simplicity of the method



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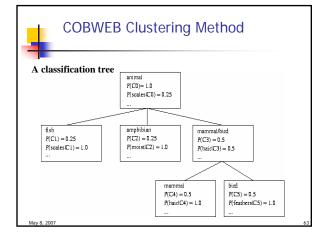
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### Model-Based Clustering Methods

- Attempt to optimize the fit between the data and some mathematical model
- Statistical and AI approach
  - Conceptual clustering
    - . A form of clustering in machine learning
    - Produces a classification scheme for a set of unlabeled objects
    - Finds characteristic description for each concept (class)
  - COBWEB (Fisher'87)
    - A popular a simple method of incremental conceptual learning
    - Creates a hierarchical clustering in the form of a classification
      tree
    - Each node refers to a concept and contains a probabilistic description of that concept

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### More on Statistical-Based Clustering

- Limitations of COBWEB
  - The assumption that the attributes are independent of each other is often too strong because correlation may exist
  - Not suitable for clustering large database data skewed tree and expensive probability distributions
- CLASSIT
  - an extension of COBWEB for incremental clustering of continuous data
  - suffers similar problems as COBWEB
- AutoClass (Cheeseman and Stutz, 1996)
  - Uses Bayesian statistical analysis to estimate the number of clusters
  - Popular in industry

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# Other Model-Based Clustering Methods

- Neural network approaches
  - Represent each cluster as an exemplar, acting as a "prototype" of the cluster
  - New objects are distributed to the cluster whose exemplar is the most similar according to some dostance measure
- Competitive learning
  - Involves a hierarchical architecture of several units (neurons)
  - Neurons compete in a "winner-takes-all" fashion for the object currently being presented

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### What Is Outlier Discovery?

- What are outliers?
  - The set of objects are considerably dissimilar from the remainder of the data
  - Example: Sports: Michael Jordon, Wayne Gretzky,
- Problem
  - Find top n outlier points
- Applications:
  - Credit card fraud detection
  - Telecom fraud detection
  - Customer segmentation
  - Medical analysis

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Outlier Discovery:



Data Values

- Assume a model underlying distribution that generates data set (e.g. normal distribution)
- Use discordancy tests depending on
  - data distribution
  - distribution parameter (e.g., mean, variance)
  - number of expected outliers
- Drawbacks
  - most tests are for single attribute
  - In many cases, data distribution may not be known

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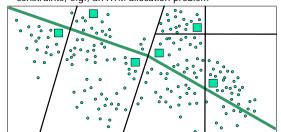
### **Problems and Challenges**

- Considerable progress has been made in scalable clustering methods
  - Partitioning: k-means, k-medoids, CLARANS
  - Hierarchical: BIRCH, CURE
  - Density-based: DBSCAN, CLIQUE, OPTICS
  - Grid-based: STING, WaveCluster
  - Model-based: Autoclass, Denclue, Cobweb
- Current clustering techniques do not <u>address</u> all the requirements adequately
- Constraint-based clustering analysis: Constraints exist in data space (bridges and highways) or in user queries

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## Constraint-Based Clustering Analysis

 Clustering analysis: less parameters but more user-desired constraints, e.g., an ATM allocation problem





### Summary

- Cluster analysis groups objects based on their similarity and has wide applications
- Measure of similarity can be computed for various types of data
- Clustering algorithms can be categorized into partitioning methods, hierarchical methods, density-based methods, grid-based methods, and model-based methods
- Outlier detection and analysis are very useful for fraud detection, etc. and can be performed by statistical, distance-based or deviation-based approaches
- There are still lots of research issues on cluster analysis, such as constraint-based clustering



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