

# Nucleosynthesis in a Simmering Universe

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## Abstract

Primordial nucleosynthesis is a success story of the standard big bang (SBB) cosmology. We explore nucleosynthesis in possible models very different from SBB in which the cosmological scale factor increases linearly with time right through the period during which nucleosynthesis occurs till the present. It turns out that weak interactions remain in thermal equilibrium upto temperatures which are two orders of magnitude lower than the corresponding (weak interaction decoupling) temperatures in SBB. Inverse beta decay of the proton ensures adequate production of helium while producing primordial metallicity much higher than that produced in SBB. Attractive features of such models are the absence of the horizon, flatness and age problems and consistency with classical cosmological tests.

Early universe nucleosynthesis is a major “success story” of the standard big bang (SBB) model. The results look rather good and the observed light element abundances severely constrain cosmological and particle physics parameters.

Surprisingly, a class of models radically different from the standard one has a promise of producing the correct amount of helium as well as the metallicity observed in low metallicity astrophysical objects. This class is defined within the Friedmann - Robertson - Walker framework by a linear variation of the cosmological scale factor with time. How such an evolution can be dynamically realised shall concern us later in this article. For the time being we outline the essential history of a universe that is born and evolves as a Milne universe defined by the metric:

$$ds^2 = dt^2 - R^2(t)[\frac{dr^2}{(1+r^2)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2]$$

with  $R(t) = t$ .

We start by summarizing the early universe nucleosynthesis story in SBB.

A crucial assumption in the standard model is the existence of thermal equilibrium at temperatures around  $10^{12}K$  or  $100MeV$ . At these temperatures, the universe consists of leptons, photons and a contamination of baryons [mainly neutrons and protons] in thermal equilibrium. The ratio of weak reaction rates of leptons to the rate of expansion of the universe (the Hubble parameter) below  $10^{11}K$  (age  $\approx .01$  secs) goes as:

$$r_w \equiv \frac{\sigma n_l}{H} \approx (\frac{T}{10^{10}K})^3 \quad (1)$$

The notations are as described by Weinberg [3]:  $\sigma$  is the weak interaction cross section,  $n_l$  the density of charged leptons and  $H$  the hubble parameter. At these temperatures, the small baryonic contamination begins to shift towards more protons and fewer neutrons because of the neutron - proton mass difference. By  $10^{10}K$  i.e.  $T_9 \equiv 10$ ,  $r_w$  falls below unity, consequently lepton weak interactions fall out of equilibrium and neutrinos decouple. The energy distribution function of the neutrinos, however, maintains a Planckian profile as the universe expands. At  $T \approx 5 \times 10^9 K$  (age of about 4 seconds),  $e^+, e^-$  pairs annihilate. The neutrinos having decoupled, all the entropy of the  $e^+, e^-$  before annihilation, goes to heat up the photons - giving the photons a temperature which is 40% higher than the temperature corresponding to the neutrino Planckian profile. Meanwhile there is a rapid fall in the neutron production by electron and anti - neutrino capture by the proton and this freezes the n/p ratio to a value slightly less than 1/5. This ratio now falls slowly on account of decay of free neutrons. Further, nuclear reactions and photo - disintegration of light nuclei ensure a dynamic buffer of light elements with abundances roughly determined by nuclear statistical equilibrium (NSE). Depending on the baryon-entropy ratio, at a critical temperature around  $T_9 = 1$ , when the n/p ratio has fallen to almost 1/7, the deuterium concentration becomes large enough for efficient evolution of a whole network of reactions leading up to the formation of the most stable light nucleus, viz.  ${}^4He$ . This is the characteristic temperature at which  $D$  conversion into other nuclei becomes a more efficient channel for the destruction of neutrons than neutron decay. At slightly lower

temperatures, the deuterium depletion rate becomes small compared to the expansion rate of the universe [4] - resulting in residual abundances of deuterium and  ${}^3\text{He}$ . Elaborate numerical codes have been developed [5] to describe evolution of the entire history from  $T = 10^{11}\text{K}$  to  $10^7\text{K}$ . Abundances of deuterium, helium - 3, helium - 4 and lithium - 7 are used to constrain the baryon - entropy ratio, the number of light particle species in the model and the neutrino chemical potential.

Taking a cue from the above narrative, we consider nucleosynthesis in a model in which, right through the epoch when  $T \approx 10^{12}\text{K}$  and thereafter, the scale factor  $R(t)$  increases as  $t$  (the age of the universe). With such scaling, the present value of the scale parameter, i.e. the present epoch  $t_o$ , is exactly determined by the present Hubble constant  $H_o = 1/t_o$ . The scale factor and the temperature of radiation are related by  $RT \approx \text{constant}$  with effect from temperatures  $\approx 10^9\text{K}$ . This follows from stress energy conservation and the fact that the baryon - entropy ratio does not significantly change after  $kT \approx m_e$  (rest mass of the electron). From the present age and effective cosmic microwave background temperature ( $2.7\text{K}$ ), one finds the age of universe when  $T \approx 10^{10}\text{K}$  to be around 4.5 years. Such a universe takes some  $10^3$  years to cool to  $10^7\text{K}$ . The rate of expansion of the universe is about  $10^7$  times slower than corresponding rate (for the same temperature) in standard cosmology around  $10^9\text{K}$ . This makes a crucial [big] difference. The first reaction would be that with neutron half life of 888 seconds, there would hardly be any neutrons left at such temperatures. Further, as we shall see later, proton - neutron inter converting weak interaction rates remain in equilibrium till temperatures even below  $T_9 \approx 1$ . This would imply that one would have the n/p ratio falling by the Boltzmann factor [ $\exp(-\Delta m_{n-p}/kT)$ ], again leading to a depletion of the neutron number. “With such a low neutron count - no nucleosynthesis !!” Indeed the standard argument would put the issue at rest here as done recently in [18]. However, and this is the object of this article, such a hasty conclusion, though “obvious”, is incorrect. We shall see that in the conditions as stated above, the mechanism leading to nucleosynthesis is a bit more subtle. The fact of inverse beta decay of the proton not freezing out, saves the day.

To see what would happen in some detail, we start by considering the effect of the slow expansion on the leptonic weak interactions. The process of the neutrinos falling out of thermal equilibrium, for example, is determined by the rate of  $\nu$  production per charged lepton:

$$\sigma_{wk}n_l/c^6 \approx g_{wk}\hbar^{-7}(kT)^5/c^6 \quad (2)$$

and the expansion rate of the universe [ $H = 1/t$ ]. Here  $g_{wk} \approx 1.4 \times 10^{-45} \text{ erg- cm}^3$ . (We again follow the notation and description as given by Weinberg [3]) For  $kT > m_\mu$ ,  $T > 10^{12}\text{K}$

$$\sigma_{wk}n_l/H \approx [\frac{T}{1.6 \times 10^8\text{K}}]^4 \quad (3)$$

Here we have normalised the value of  $RT \approx tT \approx \text{constant}$  from the value  $H_o \approx 65 \text{ km/sec/Mpc}$  for the Hubble constant - corresponding to  $t_o \approx 15 \times 10^9 \text{ years}$ . [ $tT_9 \approx 2.5 \times 10^9$ ]. Increasing  $H_o$  even by a factor of 2 would merely lead to a change in the denominator on the right side of eqn.(3) to  $1.8 \times 10^8\text{K}$ . When  $kT < m_\mu$ , the number density of muons is reduced by a factor [ $\exp(-m_\mu/kT)$ ]. Consequently, the rates of weak

interactions involving muons get suppressed to

$$\sigma_{wk}n_l/H \approx [\frac{T}{1.6 \times 10^8 K}]^4 \exp[-\frac{10^{12} K}{T}] \quad (4)$$

The corresponding rates in the standard big bang model are:

$$\sigma_{wk}n_l/H \approx [\frac{T}{10^{10} K}]^3 \quad (5)$$

for  $kT > m_\mu$ , and

$$\sigma_{wk}n_l/H \approx [\frac{T}{10^{10} K}]^3 \exp[-\frac{10^{12} K}{T}] \quad (6)$$

for  $kT < m_\mu$ . We conclude that leptonic weak interactions involving muons would freeze out at temperatures around  $10^{11} K$  as in the standard model. However, for all leptonic weak interactions mediated by neutral currents and, for weak interactions mediated by charged currents not involving the muons, the suppression factor  $\exp[-10^{12} K/T]$  is absent [3]. It follows that for the following weak interactions mediated by neutral currents

$$e^- + e^+ \longleftrightarrow \bar{\nu} + \nu \quad e^\pm + \nu \longrightarrow e^\pm + \nu \quad e^\pm + \bar{\nu} \longrightarrow e^\pm + \bar{\nu}$$

the ratio of the reaction rates to the expansion rate  $H$ , for temperatures  $kT < m_e$ , would be given by:

$$\sigma_{wk}n_l/H \approx [\frac{T}{1.6 \times 10^8 K}]^4 \exp[-\frac{m_e}{kT}]$$

This would maintain the  $\nu$ 's in thermal equilibrium at all temperatures down to slightly less than  $10^9 K$ . The entropy released from the  $e^+e^-$  annihilation, at  $T_9 \approx 5$ , would heat up all the particles in equilibrium. Both neutrinos and photons would therefore get heated up to the same temperature. The temperature then scales by  $RT = \text{constant}$  as universe expands. Relic neutrinos and photons (CMBR) would therefore have the same Planckian profile ( $T \approx 2.7 K$ ) at present. (The photon number does not significantly change at recombination for a low enough baryon - entropy ratio). This is in contrast to the standard result wherein the relic neutrino temperature is predicted to be 40% lower than the photon temperature.

This has the following effect on hadronic weak decays. With the neutrino and photon temperatures equal, the neutron - proton weak reaction rates are given by the expressions [3]:

$$\begin{aligned} \lambda(n \longrightarrow p) &= A \int (1 - \frac{m_e^2}{(Q+q)^2})^{1/2} (Q+q)^2 q^2 dq \\ &\times (1 + e^{q/kT})^{-1} (1 + e^{-(Q+q)/kT})^{-1} \end{aligned} \quad (7a)$$

$$\begin{aligned} \lambda(p \longrightarrow n) &= A \int (1 - \frac{m_e^2}{(Q+q)^2})^{1/2} (Q+q)^2 q^2 dq \\ &\times (1 + e^{-q/kT})^{-1} (1 + e^{(Q+q)/kT})^{-1} \end{aligned} \quad (7b)$$

These rates have their ratio determined by the neutron - proton mass difference  $\equiv Q \approx 15$  (with temperature measured in units of  $10^9 K$ ):

$$\frac{\lambda(p \longrightarrow n)}{\lambda(n \longrightarrow p)} = \exp(-\frac{Q}{T_9}) \quad (8)$$

The rate of expansion of the universe at a given temperature being much smaller than that in the standard scenario, the nucleons would be in thermal equilibrium till temperatures slightly below  $10^9 K$ .  $X_n \equiv$  the ratio of neutron number to the neutron plus proton number is given by:

$$X_n = \frac{\lambda(p \longrightarrow n)}{\lambda(p \longrightarrow n) + \lambda(n \longrightarrow p)} = [1 + e^{Q/T_9}]^{-1} \quad (9)$$

Let us now re-state the problem at hand. A universe which evolves according to  $R(t) = t$  is some tens of years old at temperatures  $T_9 \approx 1$  and the neutron - proton ratio at such temperatures keeps falling as  $n/p \approx \exp(-15/T_9)$ . One is tempted to naively ask: were nucleosynthesis to commence at temperatures below  $T_9 = 1$ , then as (1) the age of universe is much larger than the neutron life time and (2) with the  $n/p$  ratio reaching very low levels, why would there be any significant nucleosynthesis ?

First of all, as long as n's and p's are held in equilibrium by weak interactions, the age of the universe being large as compared to the neutron lifetime has no effect on the  $n/p$  ratio. As long as neutron - proton inter conversion rates are large as compared to the rate of expansion of the universe,  $X_n$  is given by eqn(9). Secondly, the low level of  $n/p$  at the time when nucleosynthesis commences does not on its own determine the amount of heavier elements produced. If existing neutrons at any given stage are removed to branch off to the nucleosynthesis network, then, as weak interactions are still in equilibrium, inverse beta decay of the protons by electron capture would restore and maintain the  $n/p$  ratio to its equilibrium value. This is similar to an analogous situation in chemical kinetics referred to as the "law of mass action". Given an equilibrium buffer of reactants and products, if any of the reactants or the products are removed, the reaction proceeds to restore the equilibrium concentrations. This operates particularly if the precipitation of the reactants or the products is at a rate that is smaller than the relaxation time of the equilibrium reaction [19]. We shall come to this point after demonstrating the crucial role that inverse beta decay can play in this slow evolution. *Assuming*, for instance, that nucleosynthesis were to commence at a sharply defined temperature around  $T_9 \approx 1$ , one sees from eqn(9) that there is hardly any concentration of neutrons at this temperature. However weak interactions have not frozen off and inverse beta decay can still convert protons into neutrons. If the ratio of number of protons that convert into neutrons after this epoch, to the total baryon number of the universe is roughly 1/8, and all the neutrons so created were to branch into the nucleosynthesis channel with 100% efficiency, we could get the observed  $\approx 25\%$   ${}^4He$ . We can constrain the temperature  $T_{9o}$  at which nucleosynthesis ought to commence in such a case. Eqn(8) implies:

$$\frac{\lambda(p \longrightarrow n)}{\lambda(n \longrightarrow p)} = \exp(-\frac{Q}{kT}) \approx e^{-15/T_9} \quad (10)$$

If  $\tau$  is the neutron life time viz.  $\lambda(n \rightarrow p)^{-1}$  at low temperatures, eqn(10) gives the following equation for the proton ratio:

$$\dot{X}_p \approx -\frac{1}{\tau} e^{-15/T_9} X_p \quad (11)$$

This is exactly integrated, starting from a temperature  $T_{9o}$ , to give:

$$X_p \approx X_{po} \exp\left[-\frac{10^9}{15\tau} e^{-15/T_{9o}}\right]$$

$X_{po} - X_p$  is the number of protons converted to neutrons. If all the neutrons thus produced were to precipitate as  ${}^4\text{He}$  as described above, the amount of helium is just:

$$Y_{\text{He}} \approx 2\left[1 - \exp\left[-\frac{10^9}{15\tau} e^{-15/T_{9o}}\right]\right] \quad (12)$$

This is  $\approx 24\%$  for  $T_{9o} \approx 0.9$ .

In the above analysis, we assumed that nucleosynthesis commenced at a sharply defined temperature and all neutrons formed thereafter branched off into the element production channel. A 100% branching into nucleosynthesis would never be achieved and one has to run a full numerical code as described later. The example suffices to point out that the naive analysis [18] is not correct and one has to proceed with caution. Weak interactions being in equilibrium and slow rate of expansion of the universe, contribute to salvage nucleosynthesis.

Basically, with a judicious choice of the baryon entropy ratio, one can remove neutrons from the equilibrium buffer consisting of neutrons, protons, deuterium and photons at a rate smaller than the relaxation period of the buffer. Inverse beta decay would keep replenishing neutrons into the buffer. The rate of heavier element production would be slow - but steady. One has hundreds of years at one's disposal to have the total helium add up to the right required amount.

We proceed to outline a clearer picture of what actually happens in such a slow evolution. The baryonic content of the universe at temperatures below  $T_9 \approx 10$  consists of protons and neutrons and a buffer of light elements primarily consisting of deuterium [6].

$$X_{2D} \approx X_n X_p \exp[25.82/T_9] 10^{-5} T_9^{1.5} \eta$$

Here  $\eta$  is the baryon entropy ratio. As long as the rate of deuterium depletion into heavier elements is much smaller than that of  $n[p, D]\gamma$  and the reverse reactions,  ${}^2D$  would be maintained near the above equilibrium value obtained by the detailed balancing of the  $n[p, D]\gamma$  reaction. Nucleosynthesis can proceed by the following reversible reactions:

$$n[p, D]\gamma; n[D, {}^3\text{H}]\gamma; n[{}^3\text{He}, {}^4\text{He}]\gamma; n[{}^3\text{He}, {}^3\text{H}]p; p[D, {}^3\text{He}]\gamma; p[{}^3\text{H}, {}^4\text{He}]\gamma;$$

$$D[D, {}^3\text{He}]n; D[D, {}^3\text{H}]p; D[{}^3\text{H}, {}^4\text{He}]n; D[{}^3\text{He}, {}^4\text{He}]p; {}^3\text{He}[{}^3\text{He}, {}^4\text{He}]2p$$

Rates of all these reactions are listed in several review articles eg. [5,6]. At the temperatures of interest, the reverse reactions for all but  $n[p, D]\gamma$  are severely suppressed in

comparison to the forward reaction rates. Thus small amounts of  ${}^4\text{He}$  would keep precipitating out of the network. The most important point that enables sufficient nucleosynthesis to occur is that the suppression of the forward reaction rate on account of low  $D$  abundance is compensated by the large amount of time for which the universe holds at these temperatures. This is in marked contrast to the situation in SBB where the universe holds at these temperatures for just a few seconds. As the equilibrium value of  $D$  is sensitive to the baryon entropy ratio, so would the element production be. Getting the right amount of  ${}^4\text{He}$  translates into an appropriate requirement on the baryon-entropy ratio. In SBB one may recall that at temperatures even higher than the so called  $D$  photodissociation bottleneck, any heavier elements formed would survive. However, and this is the most important point, in SBB the universe holds at these higher temperatures for a very short time and hardly any production has taken place by the time the above bottleneck is arrived at. In the case at hand, nucleosynthesis starts around  $T_9 \approx 7$  at a very small rate. However, the long period for which the universe holds more than compensates for the small rate of production. The rate of production of helium is smaller than the inverse beta decay rate of the proton !! This ensures that all neutrons branching off to the nucleosynthesis channel are compensated by more being formed by the inverse beta decay.

Fortunately one has an extremely user friendly code [5] that we modified to suit the taxing requirements of the much stiffer rate equations that we encounter in our slowly evolving universe. To get convergence of the rate equations for 26 nuclides and a network of 88 reactions (as given in Kawano's code), we were forced to rewrite essential subroutines in quadruple precision. The code incorporates the variation of the baryon entropy ratio during the electron positron annihilation epoch. The results for different values of final baryon entropy ratio  $\eta$  are shown in table I. We find consistency with the  ${}^4\text{He}$  abundances for  $\eta \approx 10^{-8}$ . The metallicity produced is 8 orders of magnitude greater than the corresponding value one gets in the early universe in the Standard model. This is also a consequence of the slow expansion in this model that allows for more time for reactions that build up metallicity. A locally higher  $\eta$  in an inhomogeneous model can further enhance metallicity.

We would like to add here that when we first addressed ourselves to this problem of obtaining the right amount of helium in an  $R(t) = t$  cosmology, we had realized that the inverse beta decay would play a vital role. One has just one parameter, the baryon entropy ratio, to be varied in a hope that one keeps precipitating neutrons in the form of helium at a rate small in comparison to the relaxation time of equilibrium of the buffer. Yet the rate ought not to be so small that even over the long period that the universe takes to cool, significant nucleosynthesis is not achieved. We varied  $\eta$  in our numerical code in search of a value that would yield the right amount of helium. We would have regarded the model as non - viable had our search yielded a very large (say  $10^{-6}$ ) or an extremely small (say  $10^{-11}$ ) value for *eta*. Our search yielded a value  $10^{-8}$  which is the kind of value that is sought for eg. in Weinberg's classic [3]. We find this quite encouraging.

To get the observed abundances of light elements besides  ${}^4\text{He}$ , one would have to fall back upon a host of other mechanisms that were being explored in the SBB in the pre - 1976 days. The most popular processes are: (i) nucleosynthesis by secondary explosions of super massive objects [6], (ii) nucleosynthesis in inhomogeneous models, (iii) effect of inhomogeneous  $n/p$  ratios as the universe comes out of the QGP phase transition, (iv)

spallation of light nuclei at a much later epoch. It is easy to rule out the survival of  $D$  by the processes (ii) and (iii) while the process (i) requires very special initial conditions. It also shares a common difficulty with process (iv), viz.: the production of  $D$  to the required levels is possible but it is accompanied by an overproduction of lithium. Any later destruction of lithium in turn completely destroys  $D$ . Within the framework of the cosmological evolution that we are exploring here, we find the best promise in a model that would combine (ii) and (iv). Table 1 displays the extreme sensitivity of  ${}^4\text{He}$  production to  $\eta$ . In an inhomogeneous model with a spatially varying  $\eta$ , there would hardly be any  ${}^4\text{He}$  production in a region with  $\eta$  lower by (say) a factor of two. Thus we can have proton rich clouds in low density regions and  ${}^4\text{He}$  and metal rich clouds in the higher density regions produced as the universe cools from  $T_9 \approx 5$ . The spallation of the former on the latter, at a subsequent (cooler) epoch, would produce  $D$  without the excess production of lithium [7].

We feel that one should be able to dynamically account for such conditions within the framework of models we outline in the conclusion.

## Conclusion

The purpose of the article is to show that the class of FRW cosmological models where the scale factor grows linearly with time cannot be trivially discarded away on account of SBB nucleosynthesis constraints. In any model in which the rate of expansion of the universe is low enough, inverse beta decay remains in equilibrium and can lead to adequate  ${}^4\text{He}$  and metal production. Further, in principle, it is possible to produce  $D$  by spallation of hydrogen rich clouds over a  ${}^4\text{He}$  - metal rich medium at a later epoch.

One may well ask: (1) Does  $R(t) = t$  coasting lead to a viable cosmology ? and, a related question: (2) How could such an evolution be theoretically realised in a gravity model ?

As regards the first query: An FRW metric with  $R(t) = t$  has interesting features. It has no horizon problem. At any given time  $t > 0$  every observer can see the entire universe. Further, as shown below, there are models in which such a coasting is independent of any “critical density”. Thus the metric does not suffer the flatness problem. Classical cosmological tests, namely: the Hubble diagram (luminosity distance-redshift relation), the angular diameter distance - redshift relation and the galaxy number count-redshift relations do not rule out such a “coasting” cosmology [8,9,18]. In fact the best fit for the latest observations on type IA supernovae [16] is practically indistinguishable from that expected of a  $R(t) = t$  [ $\Omega_M = \Omega_\Lambda = 0$ ] cosmology. The age of universe inferred from a measurement of the Hubble parameter is  $t_o = 1/H_o$  and is comfortably concordant with the age estimates of the oldest objects [clusters, low metallicity clouds etc.]. Finally, the low metallicity that one sees in type II objects poses a problem in SBB. There is no object in the universe that has quite the abundance [metallicity] of heavier elements as is produced in the “first three minutes” (or so) in SBB. One relies on some kind of re - processing, much later in the history of SBB, to get the low observed metallicity in [eg.] old clusters and inter - stellar clouds. This could [for instance] be in the form of a generation of very short - lived type III stars. Large scale production and recycling of metals through



such exploding early generation stars leads to verifiable observational constraints. Such stars would be visible as 27 - 29 magnitude stars appearing any time in every square arc - minute of the sky. Serious doubts have been expressed on the existence and detection of such signals [1]. In the nucleosynthesis model described in this article, the primordial metallicity obtained is quite close to lowest reported metallicity. An  $R(t) = t$  cosmology comes with characteristic predictions. A vanishing deceleration parameter and the equality of effective temperatures of relic microwave background and neutrinos being two of them.

Of late [2], observations have suggested the need for a careful scrutiny and a possible revision of the status of SBB nucleosynthesis from reported high abundance of  $D$  in several  $Ly_\alpha$  systems. Though the status of these observations is still a matter of debate, and (assuming their confirmation), attempts to reconcile the cosmological abundance of deuterium and the number of neutrino generations within the framework of SBB are still on, we feel that alternative scenarios should be explored.

We finally address the second query i.e. the issue of realising linear evolution within the framework of a Friedman cosmology. Within conventional Einstein's theory, such an evolution can be accounted for in a universe dominated by a hypothetical 'K - matter' [8] for which the density scales as  $R^{-2}$ . However, if one requires this matter to dominate even during the nucleosynthesis era, the K - matter would almost close the universe. There would hardly be any baryons in the present epoch. One has to look elsewhere. Fortunately, linear evolution of the scale factor is a generic feature of a large class of non - minimally coupled theories [12, 17]. These are models in which a non - minimal coupling diverging with time is used to dynamically scale the cosmological constant to zero. Such an evolution of the scale factor is also possible in alternative effective gravity and higher order gravity theories. Ellis and Xu [10] for example, consider a higher order gravity theory with action:

$$S = \int d^4x \sqrt{-g} [\alpha R^2 - \beta R] \quad (14)$$

in the weak field approximation, the effective Newtonian potential is:

$$\phi = -\frac{a}{r} + b \frac{\exp(-\mu r)}{r} \quad (15)$$

For  $\mu r \ll 1$  we can have a canonical effective attractive theory. Over large distances, the effective potential is dominated by the first repulsive term alone. A similar possibility occurs in the conformally invariant higher order theory of gravity [11]. Choosing the gravitational action to be the square of the Weyl tensor gives rise to an effective gravity action:

$$S = \int d^4x \sqrt{-g} [\alpha C^2 - \beta R] \quad (16)$$

The dynamics of a conformally flat FRW metric is driven by the anomalous repulsive term  $\beta R$  alone. The FRW - scale factor in such a cosmology approaches linear evolution at large cosmic time. Canonical attractive domains occur in the model as non - conformally flat perturbations in the FRW spacetime.

Linear evolution of the scale factor would also be possible in the following “toy” model [13] that combines the Lee - Wick construction of non - topological soliton [NTS] solutions [14] in a variant of an effective gravity model proposed by Zee [15]. Consider the action:

$$S = \int d^4x \sqrt{-g} [U(\phi)R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + L_m] \quad (17)$$

Here  $\phi$  is a scalar field non - minimally coupled to the scalar curvature through the function  $U(\phi)$ ,  $V(\phi)$  its effective potential and  $L_m$  the matter field action.  $L_m$  includes a Higgs coupling of  $\phi$  to a fermion. Let  $V$  have a minimum at  $\phi_{min}$  and a zero at  $\phi^o$ . We also choose the Higg’s coupling such that the effective fermion mass at  $\phi = \phi_{min}$  is greater than the effective fermion mass at  $\phi = \phi^o$ . Finally we choose the non - minimal function  $U(\phi_{min}) \gg U(\phi^o)$ . These conditions are sufficient for the existence of large NTS’s with the scalar field trapped at  $\phi = \phi^o$  in the interior of a large ball and quickly going to  $\phi = \phi_{min}$  across the surface of the ball. With a judicious choice of the surface tension, these balls could be larger than typical halos of galaxies. The interior and exterior of such a ball would be regions with effective gravitational constant  $[U(\phi^o)]^{-1}$  &  $[U(\phi_{min})]^{-1}$  respectively. With  $[U(\phi_{min})]$  large enough, the universe would evolve as a curvature dominated universe [without any ‘K - matter’].

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Copy of the numerical code is available from the authors. The executable file would need an architecture that would support quadruple precision calculation.

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**TABLE I**

Abundances of Some Light Elements and Metals.

$\eta$	${}^2\text{H}$	${}^3\text{H}$	${}^3\text{He}$	${}^4\text{He}$	${}^7\text{Be}$	${}^8\text{Li}$ & above
( $10^{-9}$ )	( $10^{-18}$ )	( $10^{-25}$ )	( $10^{-14}$ )	( $10^{-1}$ )	( $10^{-11}$ )	( $10^{-8}$ )
9.0	2.007	1.25	8.65	2.03	1.39	8.06
9.1	2.008	1.26	8.63	2.06	1.32	8.63
9.2	2.009	1.26	8.60	2.10	1.23	9.35
9.3	2.010	1.27	8.59	2.11	1.19	9.75
9.4	2.014	1.26	8.56	2.15	1.11	10.66
9.5	2.015	1.27	8.50	2.18	1.05	11.41
9.6	2.016	1.28	8.52	2.19	1.01	11.88
9.7	2.017	1.28	8.49	2.22	0.96	12.69
9.8	2.020	1.29	8.47	2.25	0.91	13.51
9.9	2.020	1.29	8.45	2.28	0.86	14.47
10.0	2.020	1.30	8.43	2.30	0.83	15.19

Initial Temperature  $10^{11} K$ Final Temperature  $10^7 K$