THE DYNAMIC PROPERTIES OF A FOUR-DIMENSIONAL MANIFOLD WITH A UNIFORM ACCELERATION FIELD

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1. ABSTRACT

Friedman's solutions to Einstein's field equations do not make accurate predictions about the universe on the largest scales. Even with multiple free parameters, or perhaps because of them, General Relativity gives conflicting answers when asked for the rate of expansion of the universe. We offer, as an alternative, a model of expansion employing a four-dimensional manifold with one imaginary dimension of time and three real dimensions of space and no other gimmicks. Like all manifolds, this one has an acceleration field. Unlike other manifolds, this field is not assumed to be zero. Here we demonstrate that General Relativity, with a model of uniform acceleration in four dimensions, makes more accurate predictions on all scales than it does with an assumption of symmetrical time. When given the proper metric tensor, General Relativity can even predict the Tully-Fisher Relationship. However, to do this, we must abandon one of our most sacred beliefs: that objects at rest remain at rest.

Key words: cosmology: theory, cosmology: large-scale structure of the universe, cosmology: cosmological parameters, galaxies: kinematics and dynamics

2. INTRODUCTION

With every new revelation from our space telescopes, the *Theory of General Relativity* appears to be more broken. When the field equations are solved using the FLRW metric tensor, they give conflicting answers for the rate of expansion of the universe (H_0) that is beyond the ability of the experimenters to explain (Krishnan et al. 2021). It gives one answer when we look at background radiation, and a different answer when we look at supernovae. In the language of the Scientific Method, the Friedman solutions to the Einstein Field Equations have mutually disproven themselves.

Perhaps there is more new physics to be discovered? General Relativity already depends on multiple ad hoc theories – essentially free parameters in the energy-momentum density term – that have no plausible theoretical foundation (e.g., Dark Matter, Dark Energy, Inflation). It currently holds the record for the worst prediction ever made in physics (Adler et al. 1995). Expecting different results with more new physics would be irrational.

Perhaps an assumption was missed? Yes, perhaps. Shall we start with the first one? All forces cause acceleration. Not all acceleration is caused by forces. Some acceleration is caused by curvature. The curvature of coordinate time with respect to proper time, for example, would be observed as an acceleration without a force. There is no basis for assuming that two particles

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would not accelerate when all the forces are removed. If the curvature is not known, then the acceleration is not known. What law allows us to assume that objects at rest remain at rest?

There is none. Objects in freefall will follow the surface of the manifold. That is the only law that matters. If that surface possesses an acceleration, then free-falling objects on that surface will also accelerate as described by:

$$\vec{A} = \frac{d\vec{u}}{d\tau}$$
$$A^{\alpha}\overrightarrow{e_{\alpha}} = \frac{d(u^{\alpha}\overrightarrow{e_{\alpha}})}{d\tau}$$

Where $\vec{A}$ is the acceleration of a point on the manifold, $\vec{u}$ is the velocity, τ is the evolution parameter (proper time), and $\overrightarrow{e_{\alpha}}$ represents the basis vectors for the four dimensions – coordinate time (t), radius (r), polar angle (θ), azimuthal angle (ϕ) – in this coordinate system where $\alpha = \{t, r, \theta, \phi\}$. This relation can be expanded with the chain rule:

$$A^{\alpha}\overrightarrow{e_{\alpha}} = u^{\alpha}\frac{d\overrightarrow{e_{\alpha}}}{d\tau} + \frac{du^{\alpha}}{d\tau}\overrightarrow{e_{\alpha}}$$
$$\frac{d\overrightarrow{e_{\alpha}}}{d\tau} \equiv \Gamma^{\lambda}_{\alpha\beta}u^{\beta}\overrightarrow{e_{\lambda}}$$
$$A^{\alpha}\overrightarrow{e_{\alpha}} = \Gamma^{\lambda}_{\alpha\beta}u^{\alpha}u^{\beta}\overrightarrow{e_{\lambda}} + \frac{du^{\alpha}}{d\tau}\overrightarrow{e_{\alpha}}$$
$$A^{\alpha} = \Gamma^{\alpha}_{\mu\nu}u^{\mu}u^{\nu} + \frac{du^{\alpha}}{d\tau}$$

Where $du^{\alpha}/d\tau$ is the proper acceleration of an object, and $\Gamma^{\alpha}_{\mu\nu}u^{\mu}u^{\nu}$ is the acceleration from the basis vectors changing with respect to the coordinates. Rearranging, the formula for the proper acceleration of a test particle on the surface of this manifold having a non-zero acceleration field, A, is:

$$\frac{du^{\alpha}}{d\tau} = A^{\alpha} - \Gamma^{\alpha}_{\mu\nu}u^{\mu}u^{\nu} \tag{1}$$

3. GALAXY ROTATION CURVES

In non-relativistic domains, the equation describing the path of this test particle in the presence of a collection of fictitious forces, F, is:

$$\sum_i F_i = \frac{dP}{d\tau}$$
$$\sum_i F_i^{\alpha}\overrightarrow{e_{\alpha}} = \frac{dP^{\alpha}}{d\tau}\overrightarrow{e_{\alpha}}$$
$$\sum_i F_i^{\alpha}\overrightarrow{e_{\alpha}} = m\frac{du^{\alpha}}{d\tau}\overrightarrow{e_{\alpha}}$$
$$\sum_i F_i^{\alpha}\overrightarrow{e_{\alpha}} = m(A^{\alpha} - \Gamma^{\alpha}_{\mu\nu}u^{\mu}u^{\nu})\overrightarrow{e_{\alpha}}$$

Where $\vec{P}$ is the momentum. Replacing the general terms of this equation with specific terms for centripetal motion in a gravitational pseudo-force field yields the formula for motion.

$$-\frac{GMm}{r^2}\vec{r} = m\left(A - \frac{v^2}{r}\right)\vec{r}$$

$$\frac{v^2}{r} = A + \frac{GM}{r^2} \tag{2}$$

$$v = \sqrt{Ar + \frac{GM}{r}} \tag{3}$$

Where G is the gravitational constant, M is the mass within a radius, r, and v is the orbital speed. What, then, would freefall motion look like on the surface of a manifold with a uniform acceleration field?

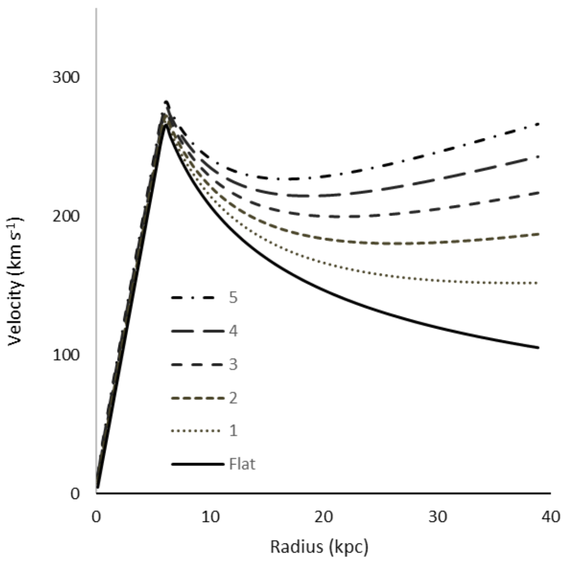

Figure 1 - The rotation curves of orbital motion around a spherical mass of $10^{11}\ M_{\odot}$ having a radius of $6\ kpc$ and constant density, for a variety of freefall accelerations (in units of $10^{-11}\ m\ s^{-2}$).

Figure 1 demonstrates that the greater the acceleration field of the manifold, the faster an object in that field will orbit a given mass. This is an easily identified property that sets quadratic expansion apart from flat spacetime. At higher values of A, we see a flattening of the rotation curves on a galactic scale, and at $A = 0$, we recover Newtonian physics.

4. THE TULLY-FISHER RELATION

Orbital systems in a uniform acceleration field possess a plane of mass that is a function of radius and orbital speed. Eq. (2) can be rearranged to predict the enclosed mass in such a system.

$$M = \frac{(v^2 - Ar)r}{G} \tag{4}$$

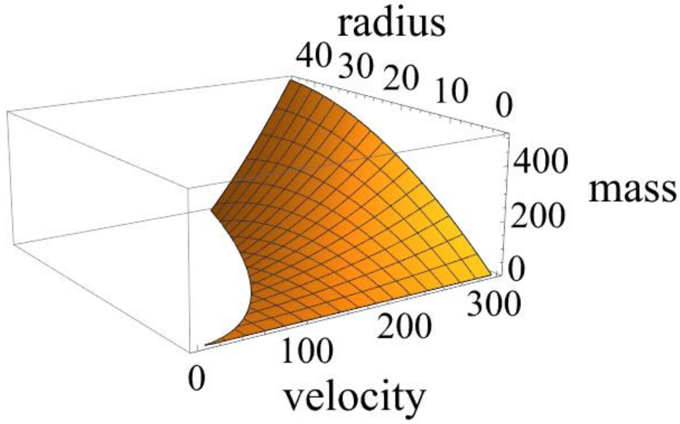

Figure 2 - The fundamental plane of mass, in units of $10^9\ M_\odot$, for $A = 3 \times 10^{-11}\ m\ s^{-2}$, as a function of radius, in kpc, and speed, in $km\ s^{-1}$.

This relationship suggests that there is a radius beyond which the acceleration field outward overcomes the gravitational field inward, as seen in Figure 2, placing an upper bound on the amount of mass that can be enclosed in an orbit. The radius of this inflection point is:

$$M' = \frac{(v^2 - Ar)r}{G}\frac{d}{dr}$$
$$\frac{v^2 - 2Ar}{G} = 0$$
$$r = \frac{v^2}{2A} \tag{5}$$

Substituting Eq. (5) back into Eq. (4) yields the formula for the maximum mass, M_{MAX}, as a function of tangential velocity:

$$M_{MAX} = \frac{\left(v^2 - A\frac{v^2}{2A}\right)\frac{v^2}{2A}}{G}$$
$$M_{MAX} = \frac{v^4}{4AG} \tag{6}$$

A study of the relationship between velocity and baryonic mass was conducted in (McGaugh 2012). Using the data from this study and employing a χ^2 minimization algorithm on Eq. (6), we find a value of $3.65 \times 10^{-11}\ m\ s^{-2}$ for A, thus:

$$M_{MAX} = 51.64v^4 \tag{7}$$

Where velocity has units of $km\ s^{-1}$ and mass has units of $M_\odot$.

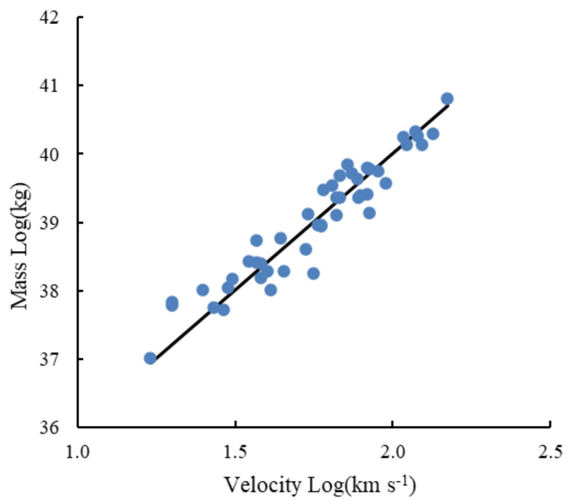

Figure 3 - The relationship between tangential velocity and baryonic mass. The blue circles are the combined gas and stellar mass of gas-rich galaxies and the solid line is the maximum mass possible in quadratically expanding spacetime.

Eq. (7) predicts the Tully-Fisher relation with one free parameter. Quadratic expansion predicts that galaxies will have a brightness that is limited by the fourth power of their orbital speed. This is another easily identified property of a manifold with a uniform acceleration field. The FLRW metric, employing an assumption of Lorentz symmetry, does not predict such a relation.

5. THE METRIC FORMULA

The spacetime interval, Δs, on this manifold is described by:

$$\Delta s^2 = g_{\mu\nu} \Delta x^\mu \Delta x^\nu \tag{8}$$

Where $g_{\mu\nu}$ is the metric tensor, and Δx is the difference between two coordinates in each dimension. If we just consider the coordinate time and radius, and assume the axes are orthogonal, the formula expands to:

$$\Delta s^2 = g_{tt}(\Delta x^t)^2 + g_{rr}(\Delta x^r)^2 \tag{9}$$

Where Δx^t is the change in coordinate time and Δx^r is the change in coordinate radius. The coordinate distances are found by taking the second integral of the acceleration. Note that time is imaginary, so we integrate by substitution (analogous to a Wick Rotation).

$$v^\alpha = \int A \, d\tau'$$
$$\tau' = i\tau$$
$$v^\alpha = i(-V + A\tau) \tag{10}$$
$$x^\alpha = \int \int A \, d\tau' = \int v^\alpha \, d\tau' = V\tau - \frac{A\tau^2}{2} \tag{11}$$

Where v^α is the tangent velocity at proper time, τ. The constant of integration is $-iV$ and is interpreted as the expansion speed of the manifold at $\tau = 0$. Eq. (11) describes uniform acceleration in four dimensions on a complex surface.

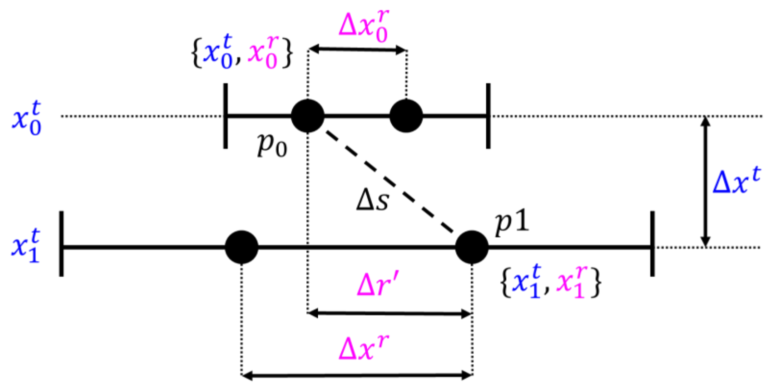

Figure 4 – The spacetime interval, Δs, from p_0 to p_1 showing the line elements proper radius ($\Delta r'$) and coordinate time (Δx^t). Also shown is the the coordinate radius distance, Δx^r.

The imaginary coordinate time distance, Δx^t, over the interval τ_0 to τ_1 can be expressed as:

$$i\Delta x^t = \int_{\tau_0}^{\tau_1} v \, d\tau'$$

$$\Delta x^t = -i\frac{1}{2}(\tau_0 - \tau_1)(-2V + A(\tau_0 + \tau_1)) \tag{12}$$

The real coordinate radius distance, Δx^r, is found by describing the relationship between Δx_0^r and Δx^r as a ratio of the displacement to the size of the manifold, λ, over the same interval.

$$\lambda_0 = V\tau_0 - \frac{A\tau_0^2}{2}, \lambda_1 = V\tau_1 - \frac{A\tau_1^2}{2}$$

$$\frac{\Delta x_0^r}{\lambda_0} = \frac{\Delta x^r}{\lambda_1}$$

$$\Delta x_0^r = -\frac{(2V\tau_0 - A\tau_0^2)}{\tau_1(-2V + A\tau_1)}\Delta x^r \tag{13}$$

Where λ_0 is the manifold size at τ_0 and λ_1 is the manifold size at τ_1. The proper distance, $\Delta r'$, is the coordinate distance, Δx^r, less one half of expansion, expressed as $\Delta x^r - \Delta x_0^r$:

$$\Delta r' = \Delta x^r - \frac{1}{2}(\Delta x^r - \Delta x_0^r)$$

$$\Delta r' = \frac{(-2V(\tau_0 + \tau_1) + A(\tau_0^2 + \tau_1^2))}{2\tau_1(-2V + A\tau_1)}\Delta x^r \tag{14}$$

Given these relations, the metric formula for a two-dimensional distance is:

$$\Delta s^2 = -\frac{1}{4}(\tau_0 - \tau_1)^2(-2V + A(\tau_0 + \tau_1))^2 + \frac{(-2V(\tau_0 + \tau_1) + A(\tau_0^2 + \tau_1^2))^2}{4\tau_1^2(-2V + A\tau_1)^2}(\Delta x^r)^2 \tag{15}$$

6. TYPE IA SUPERNOVAE

The acceleration, A, the initial tangent velocity, V, and the age of the manifold, τ_1, are initial conditions which must be discovered empirically. The acceleration has already been established from the Tully-Fisher relation. The initial tangent velocity, V, can be derived from the measured speed of light if we assume that light (causality) travels at the tangent velocity of the manifold. From Eq. (10), we have:

$$v = -ic = i(-V + A\tau_1)$$
$$V = c + A\tau_1 \tag{16}$$

The age of the universe can be found using a special kind of supernovae, type Ia, that explodes with a predictable luminosity. The radius to this event can be found by solving Eq. (15) for Δx^r and assuming a null geodesic ($\Delta s^2 = 0$).

$$\Delta x^r = \frac{i(\tau_0 - \tau_1)(-2V + A(\tau_0 + \tau_1))}{\sqrt{-\frac{(-2V(\tau_0 + \tau_1) + A(\tau_0^2 + \tau_1^2))^2}{\tau_1^2(-2V + A\tau_1)^2}}} \tag{17}$$

This formula can be simplified by encoding the proper time coordinates as:

$$z = \frac{\lambda_1 - \lambda_0}{\lambda_0}$$
$$\tau_0 = \frac{V + Vz - \sqrt{(1+z)(V^2 + V^2 z - 2AV\tau_1 + A^2\tau_1^2)}}{A + Az}$$
$$D_s(z) = -\frac{z\tau_1(-2V + A\tau_1)}{2 + z} \tag{18}$$

Where D_S is the spatial distance as a function of redshift and z is the redshift. This redshift value is encoded in photons making it possible to use a volume of photons from a known source of luminosity as distance markers. As a function of redshift, the luminous distance, D_L, is:

$$D_L(z) = D_S(z)(1 + z) = -\frac{z\tau_1(-2V + A\tau_1)}{2 + z}(1 + z) \tag{19}$$

The luminous distance is related to the distance modulus, μ, by:

$$\mu = 5Log10\left(\frac{D_L}{pc}\right) - 5 \tag{20}$$

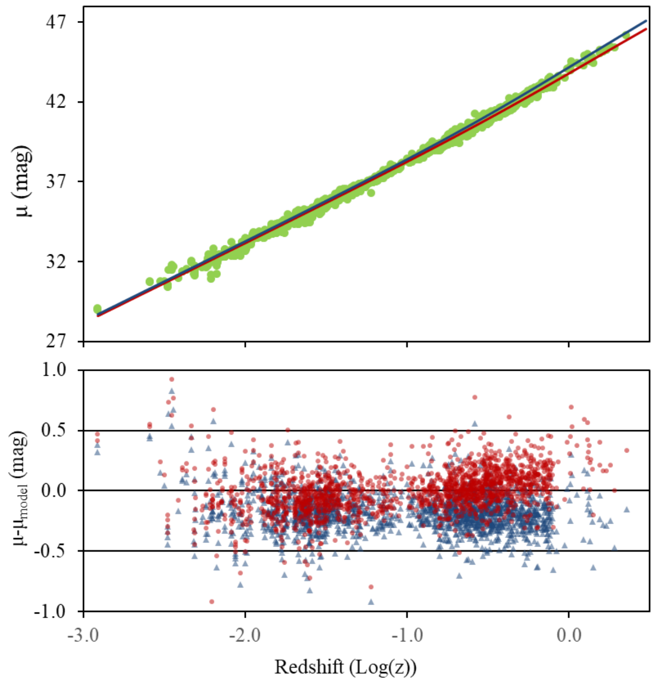

Figure 5 - Top: The distance moduli to a selection of 1071 type Ia supernovae (green) and the magnitude predicted by in Eq. (20) (red) and, for comparison, the magnitutde predicted by the FLRW formula (blue) using the parameters from (Planck Collaboration et al. 2020). Bottom: The difference between the the observed magnitude and the predicted.

The age of the universe can be extracted from the Pantheon data set (Scolnic et al. 2022) using a χ^2 minimization algorithm on Eq. (20). The reduced χ^2 of 0.53 for uniform acceleration with just one free parameter (age) is a better match to the observed data than 1.22 for FLRW with its four free parameters (inflation density, matter density, curvature density and cosmological constant).

A	$3.65 \times 10^{-11}\, m\, s^{-2}$
V	$3.15 \times 10^{8}\, m\, s^{-1}$
τ_1	$4.28 \times 10^{17} s$

Table 1 – The initial conditions: the uniform acceleration, the initial tangent velocity, and the age of the manifold.

The supernovae data describes a manifold that is 13.6 billion years old and currently has a tangent velocity of $3.00 \times 10^{8}\, m\, s^{-1}$. Using this metric, a galaxy with a redshift of 12 would be one billion years old. Using FLRW, the same galaxy would be 360 million years old.

7. THE EINSTEIN FIELD EQUATIONS

To solve the Einstein Field Equations, the displacements of Eq. (15) are converted to infinitesimals by reversing the integration and then taking the limit of Δs as $\tau_0 \to 0$ and $\tau_1 \to \tau$.

$$\Delta s^2 = -\left(\int_{t0}^{t1} (V - A\tau)\, d\tau\right)^2 + \frac{(-2V(\tau_0 + \tau_1) + A(\tau_0^2 + \tau_1^2))^2}{4\tau_1^2(-2V + A\tau_1)^2} (\Delta x^r)^2$$

$$ds^2 = -(V - A\tau)^2 (d\tau)^2 + \left(\frac{1}{2}\right)^2 (\Delta x^r)^2$$

Adding the polar and azimuthal dimensions, we obtain the complete metric formula a four-dimensional manifold using infinitesimals.

$$ds^2 = -(V - A\tau)^2 (d\tau)^2 + \left(\frac{1}{2}\right)^2 (dx^r)^2 + \left(\frac{1}{2}\right)^2 \left(\frac{sin(r\sqrt{\kappa})}{\sqrt{\kappa}}\right)^2 d\theta^2 + \left(\frac{1}{2}\right)^2 \left(\frac{sin(r\sqrt{\kappa})}{\sqrt{\kappa}}\right)^2 Sin(\theta)^2 d\phi^2$$

The metric tensor of this manifold is:

$$g_{\mu\nu} = \begin{pmatrix} -(V - A\tau)^2 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{Sin(r\sqrt{\kappa})^2}{4\kappa} & 0 \\ 0 & 0 & 0 & \frac{Sin(\theta)^2 Sin(r\sqrt{\kappa})^2}{4\kappa} \end{pmatrix} \tag{21}$$

The Christoffel Symbols are enumerated according to the formula:

$$\Gamma^{\lambda}{}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu})$$

$\Gamma^{\tau}_{\tau\tau}$	$-i\frac{A}{-V + A\tau}$	$\Gamma^{r}_{\theta\theta}$	$-\frac{Sin(2r\sqrt{\kappa})}{2\sqrt{\kappa}}$
$\Gamma^{r}_{\phi\phi}$	$-\frac{Sin(\theta)^2 Sin(2r\sqrt{\kappa})}{2\sqrt{\kappa}}$	$\Gamma^{\theta}_{r\theta}$	$\sqrt{\kappa}Cot(r\sqrt{\kappa})$
$\Gamma^{\theta}_{\theta r}$	$\sqrt{\kappa}Cot(r\sqrt{\kappa})$	$\Gamma^{\theta}_{\phi\phi}$	$-Cos(\theta)Sin(\theta)$
$\Gamma^{\phi}_{r\phi}$	$\sqrt{\kappa}Cot(r\sqrt{\kappa})$	$\Gamma^{\phi}_{\theta\phi}$	$Cot(\theta)$
$\Gamma^{\phi}_{\phi r}$	$\sqrt{\kappa}Cot(r\sqrt{\kappa})$	$\Gamma^{\phi}_{\phi\theta}$	$Cot(\theta)$

Table 2 – The non-zero Christoffel Symbols

The Riemann Tensor is expanded with:

$$R^{\lambda}_{\mu\nu\sigma} = \partial_\nu \Gamma^{\lambda}{}_{\mu\sigma} - \partial_\sigma \Gamma^{\lambda}{}_{\mu\nu} + \Gamma^{\eta}{}_{\mu\sigma}\Gamma^{\lambda}{}_{\eta\nu} - \Gamma^{\eta}{}_{\mu\nu}\Gamma^{\lambda}{}_{\eta\sigma}$$

$R^{r}_{\theta r\theta}$	$Sin(r\sqrt{\kappa})^2$	$R^{r}_{\theta\theta r}$	$-Sin(r\sqrt{\kappa})^2$
$R^{r}_{\phi r\phi}$	$Sin(\theta)^2 Sin(r\sqrt{\kappa})^2$	$R^{r}_{\phi\phi r}$	$-Sin(\theta)^2 Sin(r\sqrt{\kappa})^2$
$R^{\theta}_{rr\theta}$	$-\kappa$	$R^{\theta}_{r\theta r}$	κ
$R^{\theta}_{\phi\theta\phi}$	$Sin(\theta)^2 Sin(r\sqrt{\kappa})^2$	$R^{\theta}_{\phi\phi\theta}$	$-Sin(\theta)^2 Sin(r\sqrt{\kappa})^2$
$R^{\phi}_{rr\phi}$	$-\kappa$	$R^{\phi}_{r\phi r}$	κ
$R^{\phi}_{\theta\theta\phi}$	$-Sin(r\sqrt{\kappa})^2$	$R^{\phi}_{\theta\phi\theta}$	$Sin(r\sqrt{\kappa})^2$

Table 3 – The non-zero elements of the Reimann Tensor

Yielding the Ricci Tensor through the application of $R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu}$.

$$R_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2\kappa & 0 & 0 \\ 0 & 0 & 2\sin^2\left(r\sqrt{\kappa}\right) & 0 \\ 0 & 0 & 0 & 2\mathrm{Sin}(\theta)^2\mathrm{Sin}\left(r\sqrt{\kappa}\right)^2 \end{pmatrix}$$

Which, in turn, produces the Ricci Scalar.

$$R = g^{\mu\nu}R_{\mu\nu} = 24\kappa$$

We are working towards finding all the terms for a formula that equates the curvature of spacetime to the contents. The contents of the manifold are modelled as a perfect fluid that moves only through time.

$$T^{\mu\nu} = (\rho_b + \frac{p}{(-V + A\tau)^2})u^{\mu}u^{\mu} + pg^{\mu\nu}$$

$$T^{\mu\nu} = \begin{pmatrix} -\rho_b(V - A\tau)^2 + p\left(-1 - \frac{1}{(V - A\tau)^2}\right) & 0 & 0 & 0 \\ 0 & 4p & 0 & 0 \\ 0 & 0 & 4p\kappa Csc(r\sqrt{\kappa})^2 & 0 \\ 0 & 0 & 0 & 4p\kappa Csc(\theta)^2 Csc(r\sqrt{\kappa})^2 \end{pmatrix}$$

The trace of this tensor is:

$$T^{\mu}{}_{\nu} = g_{\mu\nu}T^{\mu\nu}$$

$$T^{\mu}{}_{\nu} = \begin{pmatrix} p + (V - A\tau)^2(p + \rho_b(V - A\tau)^2) & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

From this, the covariant energy-momentum tensor used in the field equations can finally be constructed.

$$T_{\mu\nu} = g_{\mu\nu}T^{\mu}{}_{\nu}$$

$$T_{\mu\nu} = \begin{pmatrix} -p(V-A\tau)^2-(V-A\tau)^4(p+\rho_b(V-A\tau)^2) & 0 & 0 & 0 \\ 0 & \frac{p}{4} & 0 & 0 \\ 0 & 0 & \frac{pSin(r\sqrt{\kappa})^2}{4\kappa} & 0 \\ 0 & 0 & 0 & \frac{pSin(\theta)^2Sin(r\sqrt{\kappa})^2}{4\kappa} \end{pmatrix}$$

With these terms in hand, the solutions to the Einstein Field Equations can now be derived.

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{(-V+A\tau_1)^4}T_{\mu\nu} \tag{22}$$

EFE(t,t)	$12\kappa(V-A\tau)^2 = -\frac{8G\pi(\rho_b(V-A\tau)^4+p(1+V^2-2AV\tau+A^2\tau^2))}{(V-A\tau)^2}$
EFE(r,r)	$\kappa + \frac{2Gp\pi}{(V-A\tau)^4} = 0$
EFE(θ,θ)	$(1+\frac{2Gp\pi}{\kappa(V-A\tau)^4})Sin(r\sqrt{\kappa})^2 = 0$
EFE(ϕ,ϕ)	$\frac{(2Gp\pi+\kappa(V-A\tau)^4)Sin(\theta)Sin(r\sqrt{\kappa})}{\kappa(V-A\tau)} = 0$

Table 4 – The solutions to the Einstein Field Equations.

These solutions yield formulas for the baryonic density, ρ_b, and the pressure of the manifold, p, as functions of curvature, κ.

$$\rho_b = \frac{\kappa(-2+(V-A\tau)^2)}{2G\pi} \tag{23}$$

$$p = -\frac{\kappa(V-A\tau)^4}{2G\pi} \tag{24}$$

8. CURVATURE

Fluctuations in one or more quantum fields at the dawn of time resulted in tiny density differences which produced standing sound waves as matter alternately collapsed into and expanded back out of these defects. These waves collapsed when the temperature fell enough to allow photons to stream freely. Those photons carried away with them an image of the tiny temperature gradients on the surface of the manifold at the time of the wave collapse. This light is observed today as Cosmic Microwave Background (CMB) radiation, and it contains information about the curvature of spacetime.

Knowing the distance to the surface of last scattering, the wavelength of the fundamental frequency, and the angle that the wave makes the sky, we can use simple geometry to determine the curvature. If we assume that a photon from this surface will only collide with an observer once, we can calculate the distance to the surface.

The first step to an optical depth function is to estimate the present-day baryon density. This value is not yet known, so we will start with a guess. As a rough guess, a spherical curvature ($r = \kappa^{-2}$) is assumed and Eq. (23) provides a seed density for RECFAST++ (Chluba & Thomas 2010). This program generates the free electron fraction, X_e, history using the seed baryon density and our expansion model.

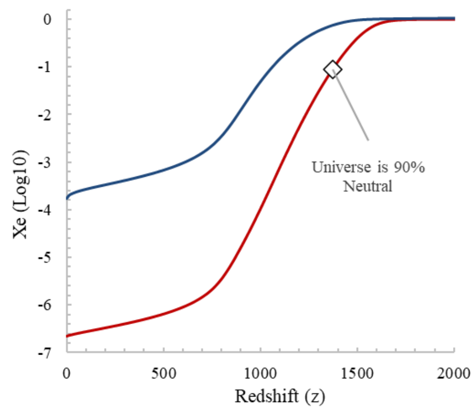

Figure 6 – The free electron fraction, X_e, history for quadratic expansion (red) and FLRW (blue).

The electron number density, n_e, is the product of the interpolated history in Figure 6, as a function of redshift, and the average number of atoms in a volume at that redshift.

$$n_e = X_e \frac{(1+z)^3 \rho_b}{A_H m_H + A_{He} m_{He}} \tag{25}$$

The Thomson Scattering Rate, $\dot{\tau}_T$, is a product of the electron number density, and the scaled Thompson Cross Section, $a\sigma_T$, and the speed of light (that is, the tangent speed of the manifold).

$$\dot{\tau}_T = n_e a \sigma_T i(-V + A\tau) \tag{26}$$

Integrating the scattering rate over the path of the photon yields the Optical Depth, τ_T, which is the effective number of collisions since that photon left the surface of last scattering.

$$\tau_T = \int_{\tau_*}^{\tau_1} \dot{\tau}\, d\tau'$$
$$\tau_T = i \int_{\tau_*}^{\tau_1} \dot{\tau}\, d\tau \tag{27}$$

The age of recombination, τ_*, is defined to be the epoch when $\tau_T = 1$. The history in Figure 7 tells us that the manifold was 9.30 million years old, at a redshift of 1421, when the photons were last scattered by the plasma. Knowing the time of this epoch, Eq. (15) yields the distance to the surface of last scattering, D_{LSS}, of 8.52 Mpc.

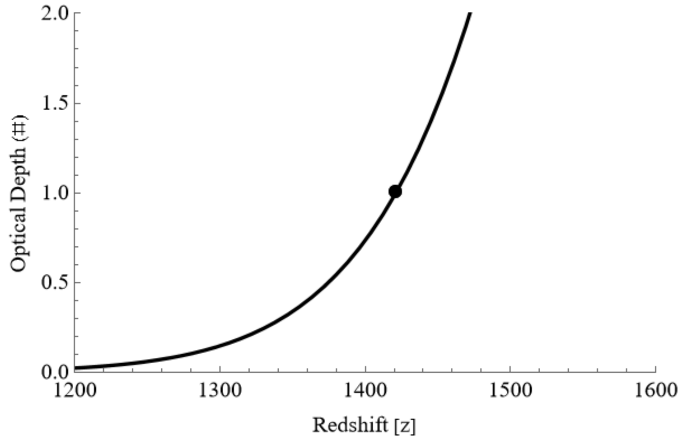

Figure 7 – The Optical Depth to Recombination. The black dot indicates the point at which only one collision of the photon has occurred since leaving the surface of last scattering.

The next leg of the triangle is the sound horizon, how far the wave travelled before collapsing. The velocity of the wave depends on the baryon to photon density ratio.

$$R = \frac{3}{4}\frac{\rho_b(1+\mathrm{z})^3}{\rho_\gamma(1+\mathrm{z})^4} \tag{28}$$

Knowing this, the sound horizon, s_*, is computed as the total distance that this wave travels as space expands.

$$s_* = i\int_0^{\tau_*}\left(i(-V+A\tau)+\frac{i(-V+A\tau)}{\sqrt{3(1+R)}}\right)d\tau \tag{29}$$

The angular acoustic scale, according to (Planck Collaboration et al. 2020), is $100\theta_* = 1.0411$. This is the fraction of the sky occupied by one wavelength of the primordial sound wave. To find the curvature, k,we use a minimization algorithm to solve for:

$$s_* = \int_0^{\theta^*}\frac{\mathrm{Sin}\left(D_{LSS}\sqrt{k}\right)}{\sqrt{k}}d\theta \tag{30}$$

The resulting curvature is then fed back into Eq. (23) and the process is repeated until it converges on a value of $\kappa = 8.75 \times 10^{25}\ km$. Knowing the radius of curvature and the initial conditions, several more properties can be enumerated.

Radius	$4.27\ Gpc$
Radius of Curvature	$2.82\ Gpc$
Baryon Density (ρ_B)	$2.82 \times 10^{-26} kg\ m^{-3}$
Pressure (p)	$-2.54 \times 10^{-9} kg\ m^{-2}$
Total Mass	$6.44 \times 10^{52}\ kg$

Table 5 – Additional properties of a manifold with uniform acceleration.

9. THE PARTICLE HORIZON

CMB radiation is homogenous and isotropic to one part in 500,000. The observed universe has the same temperature and density in every direction. This tells us that all parts of the universe were in causal contact with every other part prior to the last scattering.

The Particle Horizon tells us how far a particle has travelled along a null geodesic. Taking Eq. (17) and setting $\tau_0 = 0$ and $\tau_1 = \tau$, we have a formula for how far a particle has travelled since the beginning of time as a function of proper time.

$$H_p = -\tau(-2V + A\tau) \tag{31}$$

At 8.53 Gpc for the present time, this horizon is considerably smaller than FLRW at 14.14 Gpc. At the time of recombination, the size of the manifold was 3.00 Mpc and the size of the particle horizon was 6.00 Mpc. Quadratic expansion leaves plenty of time, 9.30 Myr, for the universe to thermalize.

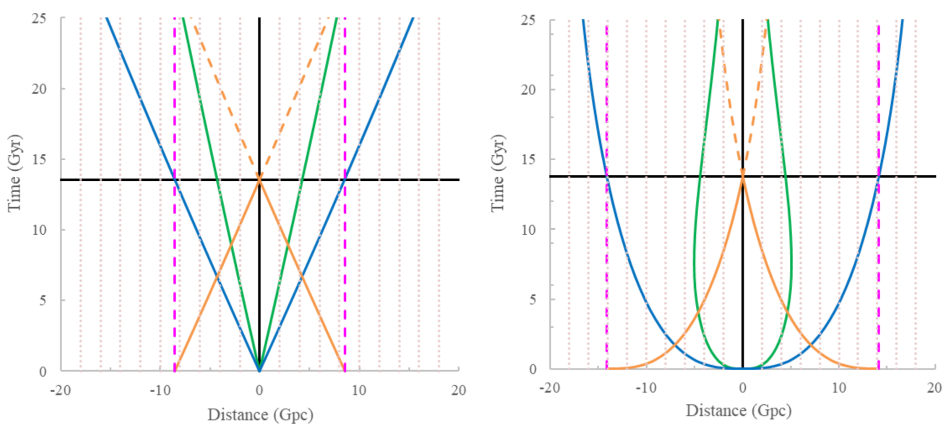

Figure 8 – Cosmological horizons for quadratic expansion (right) and FLRW (left). The solid black lines are the world lines of the observer (here and now), green line is the Hubble radius, blue line is the particle horizon, orange is the light cone. The purple dashed line is the world line for the present-day particle horizon.

Furthermore, a model of uniform acceleration is always in equilibrium. From the formulas we see that the particle horizon is always twice the size of the manifold. Every photon has been in causal contact with every other photon at least twice. In the early universe, the FLRW model expands faster than light and does not have enough time for the photons to catch up before recombination. FLRW predicts a universe that is lumpy and inconsistent.

10. CONCLUSION

The *Theory of General Relativity* has never made a successful prediction on any scale larger than a solar system. As a theory of gravitation, it failed to predict the rotation curves of spiral galaxies. It failed to predict the Tully-Fisher relation. It failed to predict the brightness of distant supernovae and it failed to predict homogenous and isotropic background radiation. Most ironically, it failed to predict a positively curved universe.

However, the fault lies not in the theory, but with the geometry. The FLRW metric tensor, with an assumption of time symmetry and a null acceleration field, has clearly failed. Not only does it disagree with the observed universe, but it does not even agree with itself. The data presented here is incontrovertible evidence that our universe is positively curved and is decelerating, just as the original field equations predicted. When given a model of uniform acceleration on a complex surface, General Relativity can accurately predict galaxy rotation curves, the Tully-Fisher relation, the brightness of supernovae, and the smooth temperature of the night sky without resorting to exotic theories of unproven science.

11. DATA AVAILIBILITY

The Mathematica notebooks supporting this article are available at https://github.com/DonaldAirey/quadratically-expanding-space.

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