**Large Sum KPI Abstract**

We need to compute average unit sales and average sales amounts for a large number of business categories; some of these sums will be computed from lower level averages. For up to 51,200 averages in the range of $0.01 to $7.47, the absolute error is less than 1.0E-6 and loses about one digit of accuracy for each doubling of element size. Sums are relatively insensitive to element order. Sums of squares, used for standard deviation, suffer greater absolute errors but offer better than 1.0E-11 relative errors.

**Large Sum KPI Accuracy Test**

Accuracy assessment double precision summation of 100 batches of 512 integers, each divided by 137, and generated in ordered and randomized sequences. Each input value represents an average amount across a uniform range, and there are 51,200 such averages. The resulting sums (integral and scaled by 137) are down below. Absolute error (delta) and relative percentage error (EPS) for each run is included as well. Observations from these runs are:

1. Sum and Sum SQR are computed to 16 digits for double precision floating point
2. Sums for range 1-512: DELTA-Error RND < DSC < ASC < 1.0E-8
3. Sum Squares for 1-512: DELTA-Error RND < DSC < ASC < 1.0E-7
4. Sums for range 513-1024: DELTA-Error RND < DSC < ASC < 1.0E-7
5. Sum Squares for 513-1024: DELTA-Error ASC < RND < DSC < 1.0E-6
6. The maximum relative error is less than 1.0E-11 *percent*.

The 1-512 integer data range corresponds to average sales amounts of $0.01 to $3.74 while the 513-1024 corresponds to average sales amounts of $3.74 through $7.47. It appears that doubling the size of the average seems to lose one decimal place in absolute accuracy but does not influence the relative error. The absolute errors for sums of squares greatly exceed absolute errors in sums; but the relative errors in sums of squares are much smaller than relative errors of sums.

**Large Sum KPI Accuracy Test Data**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Case** | **Min** | **Max** | **Sum** | **SumSqr** | **SumDP** | **SumSqrDP** |  | **ERR-SUM** |  |  | **ERR-SUMSQR** |  |
| EXP | 1 | 512 | 13132800 | 4487040000 | 95859.8540145985 | 239066.5459001550 |  | **Delta** | **EPS-PCT** |  | **Delta** | **EPS-PCT** |
| RND | 1 | 512 | 13132800 | 4487040000 | 95859.8540145985 | 239066.5459001600 |  | 0 | 0 |  | -5.50062E-09 | -2.30088E-12 |
| ASC | 1 | 512 | 13132800 | 4487040000 | 95859.8540146031 | 239066.5459001650 |  | -4.55475E-09 | -4.75147E-12 |  | -1.04774E-08 | -4.38262E-12 |
| DSC | 1 | 512 | 13132800 | 4487040000 | 95859.8540145945 | 239066.5459001620 |  | 4.04543E-09 | 4.22015E-12 |  | -7.47968E-09 | -3.1287E-12 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| EXP | 513 | 1024 | 39347200 | 31356800000 | 287205.839416050 | 1670669.721349030 |  |  |  |  |  |  |
| RND | 513 | 1024 | 39347200 | 31356800000 | 287205.839416050 | 1670669.721349080 |  | 4.36557E-09 | 1.52002E-12 |  | -4.70318E-08 | -2.81515E-12 |
| ASC | 513 | 1024 | 39347200 | 31356800000 | 287205.839416120 | 1670669.721348990 |  | -6.26314E-08 | -2.18072E-11 |  | 4.30737E-08 | 2.57823E-12 |
| DSC | 513 | 1024 | 39347200 | 31356800000 | 287205.839416030 | 1670669.721349150 |  | 2.63681E-08 | 9.1809E-12 |  | -1.16881E-07 | -6.99606E-12 |