Popoviciu's Inequality in two dimensions

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Abstract

In 1965, T. Popoviciu proved an interesting inequality which deals with the values of a convex function in one variable at three places x_1, x_2, x_3 as well as their means

$$\frac{x_1+x_2}{2}, \frac{x_1+x_3}{2}, \frac{x_2+x_3}{2}, \frac{x_1+x_2+x_3}{3}.$$

In this paper, we investigate Popoviciu's inequality in two dimensions. namely

$$\begin{aligned}&\sum_{i=1}^{3}\sum_{j=1}^{3} f(x_i,y_j)+3\left[\sum_{i=1}^{3} f(x_i,\frac{y_1+y_2+y_3}{3})\right]\\&+3\left[\sum_{i=1}^{3} f(\frac{x_1+x_2+x_3}{3},y_i)\right]+9f(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3})\\\geq&4\sum_{1\leq i<j\leq 3}\sum_{1\leq k<l\leq 3} f(\frac{x_i+x_j}{2},\frac{y_k+y_l}{2})\end{aligned}$$

where f is a coordinated convex function in two variables.

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