System identification of a 3 DOF system

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You are supposed to find the parameters of the 3 DOF systems through a simple system identification (figure 1). Define a model for the system: it depends on a set of parameters (θ). Identify the input (force and i.c.) and the output ($x(t)$) of the system. Give the same input to the model, and compare the measured response $x(t)$ with the predicted one ($\hat{x}(t,\theta)$).

Matlab. Read the help page of the function *lsim*. Alternatively, write a model in Simulink: the simulation of the model can be started by a MATLAB script (file .m) thanks to the function *sim*. For the comparison, look at the function *compare* or *goodnessOfFit*.

Maple. Look for the web page of the function *Simulate*.

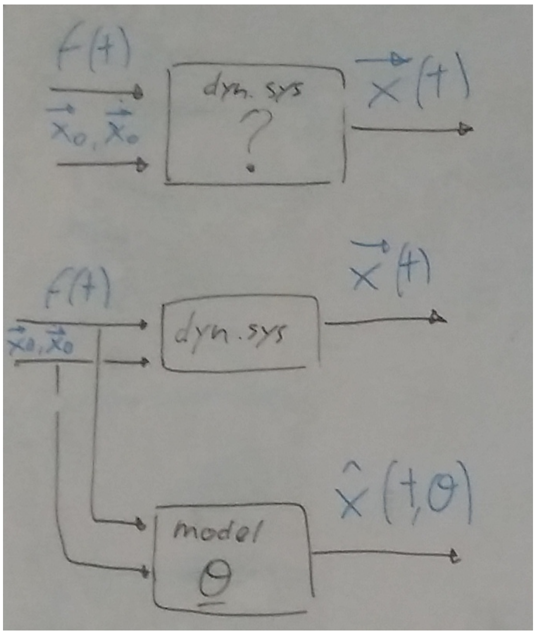

Figure 1: System identification: real system and model.

The residual is the difference between the two signals: $\epsilon(t,\theta) = x(t) - \hat{x}(t,\theta)$ (figure 2). The best choice for the set of parameter θ is the one that minimizes the integral of the square of the residual. In our case (discrete signals with N samples), the optimal value is $\theta_{opt} = \text{argmin}\left(\sum_{i=1}^{N} \epsilon(t_i,\theta)^2\right)$. This corresponds to solving a *least square problem*.

Matlab. You have many possibilities for the minimization: *fminsearch*, *fminunc*, *lsqlin*, *fmincon*, *lsqnonlin*, *lsqcurvefit*.

Maple. Read the web page of the *Optimization Package*.

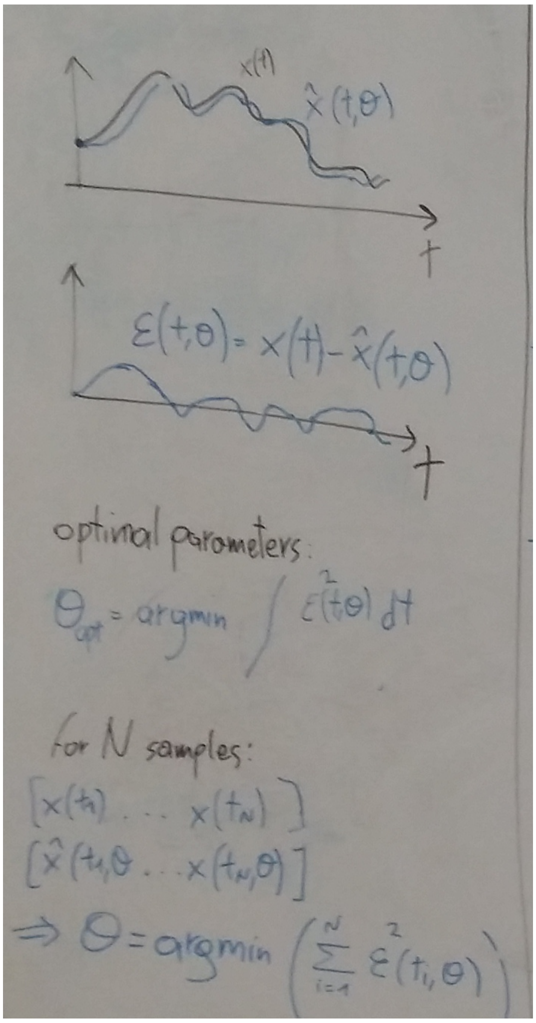

Figure 2: Minimization of the residuals.

Write a linear model for the plant (figure 3). The $[K]$ is given (although with uncertainty): define the parameters of $[M]$ and $[C]$ that you have to identify. Describe the meaning of every identified parameter. Don't forget the constraint between the first mass and the pinion (the motor torque is applied on the pinion). Describe the friction forces applied to the carriages.

Matlab. You can use the functions *ss* (definition of a LTI system) and *tf* (transfer function of a linear system).
Maple. Read the web page of the *SystemObject*.

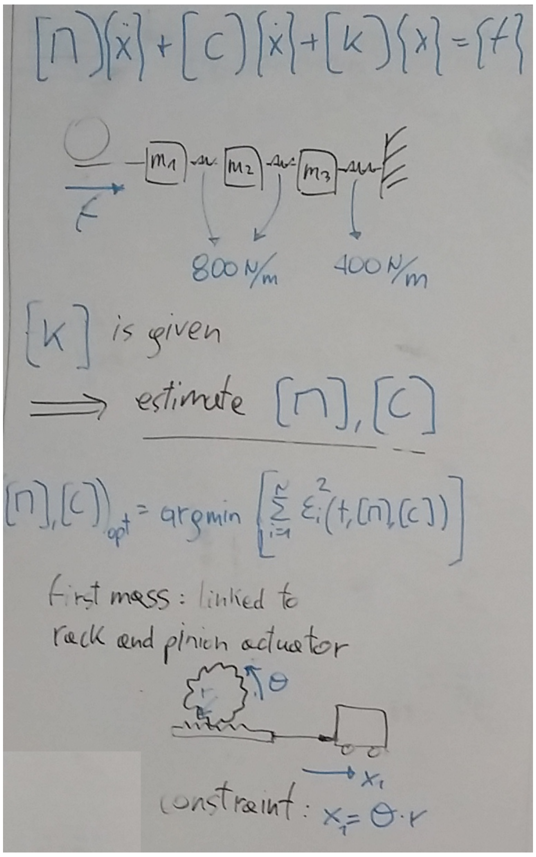

Figure 3: Parameters of the linear system.

The input of voltage required by the software corresponds to a input of force if we assume that the ratio between the two parameters is constant (figure 4). Actually, the relation between the voltage and the current is a dynamical relation (electrical dynamics of the motor).

The system dynamics can be described thanks to the mode superposition. Exciting only one mode could "hide" some parameters (figure 5, figure 6).
Matlab. Eigenvalue problem: use the function *eig*.
Maple You can use the function *Eigenvector*.

The sine sweep (7) has a constant amplitude in the frequency domain (in the range of the excited frequency). The frequency spectrum of the sine sweep response can be analysed through the (discrete) Fourier transform of the time signal. When divided by the spectrum of the input, it gives the (numerical) transfer function.
Matlab. Look for the functions *fft* and *tfestimate*. As a preliminary check for the computation of the spec-

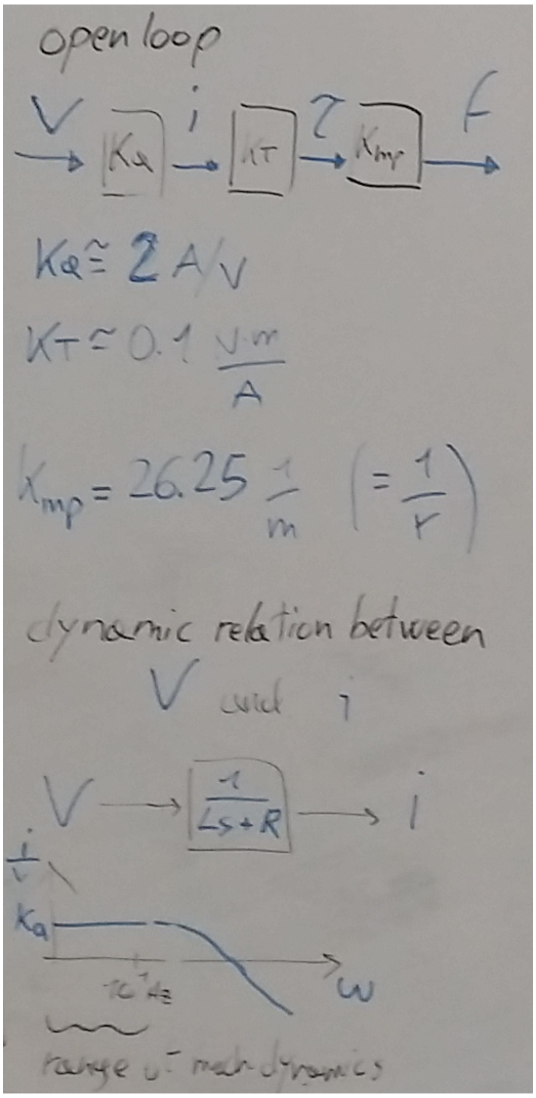

Figure 4: Relation between the commanded voltage and the applied force.

trum, apply your algorithm to an easy signal whose spectrum is well known.
Maple. Look for the web page *Fourier Transforms in Maple*.

Even though they could not correspond to the positions of the degrees of freedom, "nodes" can be identified also for the discrete systems (analogy with continuous systems, figure 8).

Data are affected by noise and disturbances (figure 9) You can apply a filter to the data (explain the choice of the cut frequency). Aliasing should not affect the system identification.
Matlab. For the filter: *filter*, *filtfilt*, *butter*.
Maple. You can use the function *Filter*.

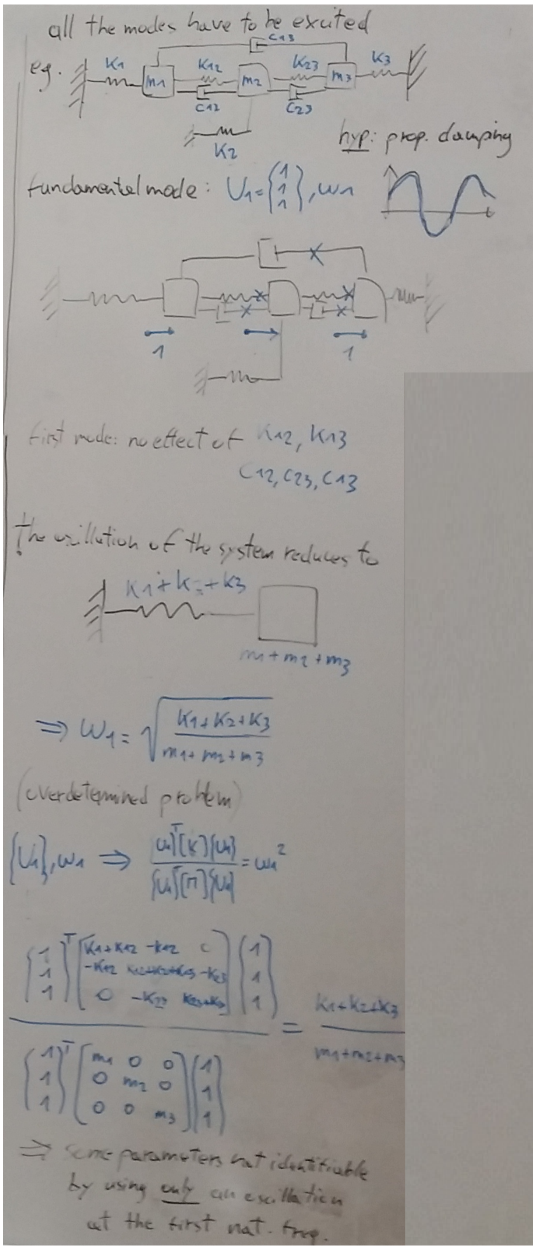

Figure 5: Example: if only one mode is excited, some parameters could not be identified.

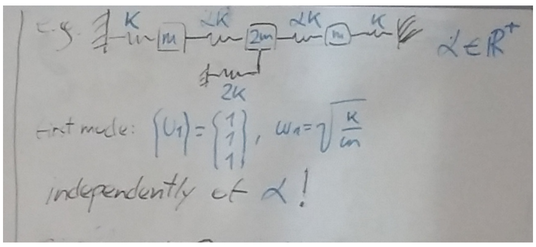

Figure 6: Example: different systems can have the same fundamental mode.

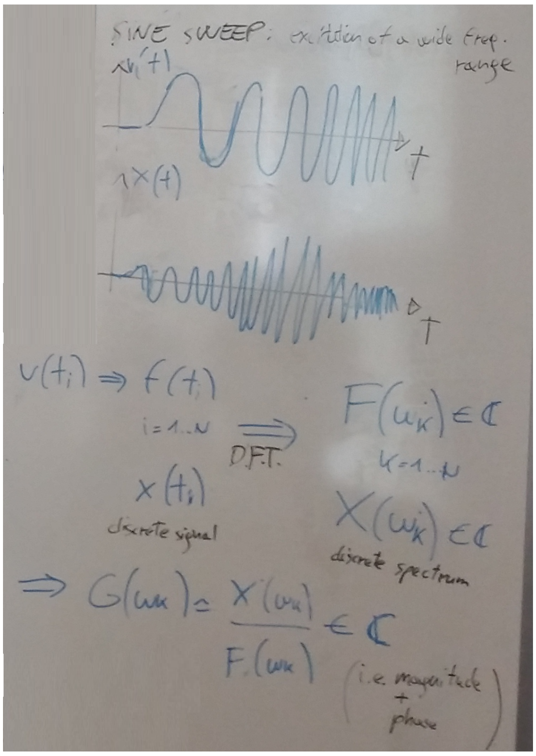

Figure 7: Fourier transform applied to the sine sweep data.

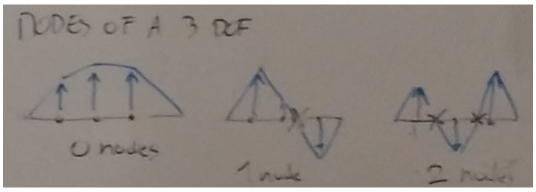

Figure 8: Modal shapes and "nodes" in a 3 DOF system (example).

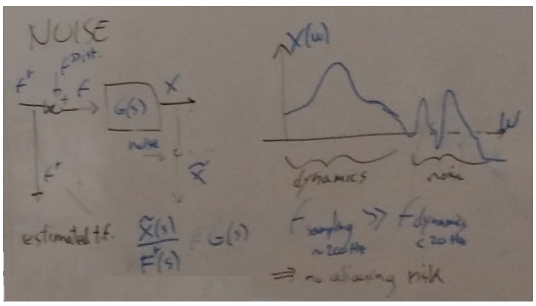

Figure 9: Noise and aliasing.