Mechanical Vibrations - Assignment

Course of *Mechanical Vibrations*

May 10, 2016

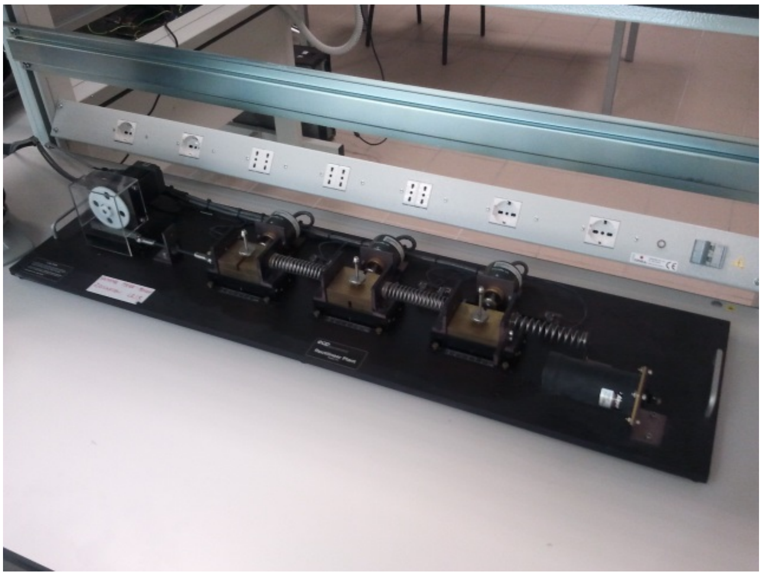

Figure 1: Dynamical system.

You are supposed to identify the parameters of the Rectilinear Control System (Model 210).

1 Dynamical system

Choose a linear model for the system.

- describe the experimental setup and the dynamical system.
- describe the known parameters and the parameters to be identified.
- describe the assumptions that allow a linear model.

2 Parameters Identification

Identify the parameter of your linear model by minimizing the residuals. Comment the residuals of your estimation.

- file *data_steps.mat*: use the step response to verify the ratio between the stiffnesses of the springs. Compute a new estimation for the voltage-to-force coefficient.
- file *data_impulses.mat*: use the impulse response to identify the parameters. Choose between response to impulsive force or response to initial conditions: in the first case, due to the approximation of the force estimation, you can consider the voltage-to-force coefficient as one of the parameters to be estimated.

- repeat the estimation of your parameters by assuming proportional damping. How many parameters have to be identified?
- in order to improve the estimation of the parameters, the springs can be detached and the masses can be blocked at their equilibrium position: describe a possible strategy for the estimation of the parameters by studying the behaviour of three different 1 DOF systems.

3 Modal analysis

Using the parameters of the impulse response (generic damping case), compute the modes.

- use Rayleigh quotient and Matrix Iteration Method to estimate the modes of the undamped system. Compare the results with the ones of the eigenvalue problem.
- use Laplace transform to plot the transfer functions between the applied force and the positions of the degrees of freedom.
- repeat the previous operations for the proportional damping case (and compare the results with the ones of the generic damping case). Use the modes of the proportional damping case to write an analytical expression for the configurations thanks to modal decomposition.

4 Sine sweep data

Use *data_sine_sweep_slow.mat* and *data_sine_sweep_fast.mat* to compute the transfer functions numerically (*data_sine_sweep_fast.mat* has a different sampling period).

- compare the results of the two data sets.
- compare the sine-sweep response with the results of the modal analysis (generic damping case).

Data

Use the nominal values for the stiffness (800 N/m between m_1 and m_2 and m_2 and m_3; $400\frac{N}{m}$ between m_3 and the ground).

The data of the positions x_1, x_2 and x_3 are given in *encoder counts*. The following relation between the measured counts and the displacement holds:

$$\Delta x = 2\pi r_e \cdot \frac{\Delta\text{counts}}{16000}$$

where r_e is the radius of the encoder and $2\pi r_e = 0.0706\text{m}$; 16000 is the number of counts per encoder revolution.

The input data are given by the *voltage* v. The following relation between the applied voltage and the applied force holds:

$$f = (k_a \cdot k_t \cdot k_{mp})\ v$$

Where k_a is the Servo Amp gain, k_t is the Servo Motor Torque constant and k_{mp} is the Motor Pinion pitch radius inverse:

$$\begin{aligned} k_a &\approx 2\ \frac{\text{A}}{\text{V}} \\ k_t &\approx 0.1\ \frac{\text{Nm}}{\text{A}} \\ k_{mp} &= \frac{1}{26.25}\frac{1}{\text{m}} \end{aligned}$$

Hints

The results of the identifications highly depend on the first guess of the optimization. Try different first guesses, and set proper initial values (1-2 kg for the masses, 0-5 N/s for the dampers).

For every data set, analize the response of the system until the motion expires. Split the step response data and the impulse response data into different data sets (you can use them for statistical considerations or for the validation of estimated data).

Comment the residuals. After the identification, use the *normed root mean square output error* for evaluating how well the model fits the measurement of every degree of freedom:

$$errn = \sqrt{\frac{\sum_{k=1}^{N} \left(y(k) - y_m(k, \theta_N)\right)^2}{\sum_{k=1}^{N} y(k)^2}} \cdot 100 \ [\%]$$

The normed root mean square output error is an indicator of the deviation of the estimated motion from the observed one.

Evaluation of the assignment

Write (individually) a short report. It can be written either in English or in Italian. Its contribution on the final grade is up to 5 points.