Lateral vibrations of a beam

Laboratory experience

Course of Mechanical Vibrations

May 8, 2018

Introduction

This document contains the detailed assignments of the laboratory experience of the course of *Mechanical Vibrations*. The requests are subdivided into three main parts: 1) Analytical model 2) Impact hammer excitation and 3) Shaker excitation.

Important notes:

- Every graph of your report must include the units of measurement you used.
- The report is individual, so every student has to write their own.
- You can write the report either in English or in Italian.
- The number of pages does not influence the final grade, what is important is that you answer to all the requests and give your personal interpretation of the results.
- Useful code for MATLAB is available at the end of the document.

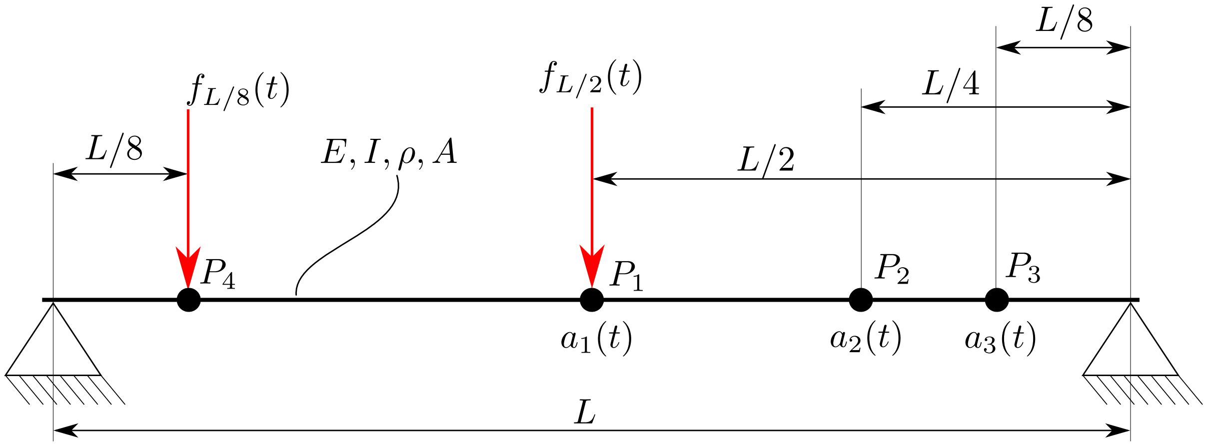

Figure 1: Analytical model and accelerometer positions

Analytical model

The beam data are:

- Young modulus: E = 206 GPa
- Mass density: = 7850 kg/m^3
- Area of the beam cross-section: A = 111 mm^2
- Moment of inertia of the beam cross-section: J = 6370 mm^4
- Length of the beam: L = 0.7 m.
- To build the analytical model and plot the frequency responses, use the following values of the damping for every mode: $\xi_1 = 0.05$ and $\xi_2 = \xi_3 = \xi_4 = 0.01$.

Given the configuration of the experiment perform the following calculations using the beam data above:

1. Referring to figure 1, choose a reference frame and write the analytical model of the beam. Consider the beam as pinned-pinned. Consider only the first four modes to describe the behaviour of the system:
$$w(x,t) = \sum_{n=1}^{4} W_n(x) q_n(t) \tag{1}$$
2. Consider the system as undamped. Calculate the first four *natural frequencies* and plot the first four modal shapes.
3. Consider the system as damped. Start from the generic differential equation of the forced damped modal coordinates. Compute the $q_n(s)/Q_n(s)$ transfer function (TF) between the n-th modal coordinate and the n-th modal force. Highlight the similarity between this TF and that of a generic single dof system.
Consider now the generic transfer function between the acceleration and the force of a single dof damped system. Derive the analytical formula to find the *peak frequency*. Use the formula to compute the first four *peak frequencies* (i.e. the frequencies corresponding to the peaks of the transfer function between acceleration and force) of the beam model.
4. Consider the system as damped. Start from the generic differential equation of the forced damped modal coordinates. Calculate the transfer functions between an impulse force $f_j(t)$ and the accelerations $a_i(t)$. Where j is the point of application of the force, either P_1 or P_4, and i indicates the position of the i-th accelerometer, either P_1, P_2 or P_3 (look at fig 1):
$$A_i(s) = H_{ij}(s)F_j(s) \rightarrow H_{ij}(s) = \frac{A_i(s)}{F_j(s)}, \qquad i = P_1, P_2, P_3 \text{ and } j = P_1, P_4 \tag{2}$$
5. Plot the frequency responses $H_{ij}(\iota\omega)$ (where $\iota = \sqrt{-1}$) in the frequency range $0 - 2200$ Hz. Pay attention to the difference between the ordinary frequency (measured in Hz) and the angular frequency (measured in rad/s). Comment the results (presence or absence of resonant peaks as a function of the position of the accelerometer and the force application point).

Impact hammer excitation

An impact hammer has been used to excite the beam in two different points, P_1 and P_4 (red forces in figure 1). Several tests have been performed, hitting the bar in the two points. The sampling frequency is 25600 Hz, and has been kept constant for all the tests. The results are provided in files with *txt* format. Files columns, from left to right, contains: time (s), force (N), acceleration of accelerometer 1 (m/s^2), acceleration of accelerometer 2 (m/s^2), acceleration of accelerometer 3 (m/s^2).

For the tests with force applied in P_1 and the tests with force applied in P_4, perform the following analysis:

1. Plot, in the time domain, the hammer impact force of the tests. Looking at the graphs some problems can occur, describe them.
   Explain a possible limit of the force applied through the impact hammer in the frequency domain (is there a way to understand which hammer tip type has been used for the experiment?).

2. Plot the accelerations in time domain and their Fourier transform. If there are some problems, describe them and their cause.

3. Considering only the problem-free tests, compute the experimental transfer functions (check MATLAB code at the end of the document) between the force and the recorded accelerations and plot the results.

4. For every accelerometer, compute the mean transfer function through the average of the aforementioned transfer functions. Plot the mean transfer functions along with its standard deviation. Comment the results.

5. Find the experimental *peak frequencies* in the range 0-2000 Hz. Compare the results with the analytical model.

   NB: The shaker has been detached from the beam when performing the tests with the impact hammer.

Shaker excitation

The beam has been excited with an electro dynamical shaker. The exciting force has been generated as a variable-frequency signal. The overall frequency range covered by the excitation is 20-2200 Hz.
Due to the instrumentation limit on the maximum number of points to be collected with a single test, the excitation procedure has been subdivided in the frequency ranges described in table 1. For each frequency range, a data file is provided in *txt* format. The four columns of every *txt* file are: time (s), force (N), acceleration of accelerometer 1 (m/s^2), acceleration of accelerometer 2 (m/s^2), acceleration of accelerometer 3 (m/s^2).

1. Compute the transfer function for every range (you can use the *tfestimate* command in MATLAB) and, by the superposition of the transfer functions, plot the complete experimental transfer function for the three accelerometers in the range 20-2200 Hz.

Table 1: Frequency ranges used to excite the beam with the shaker

Range number	f_{min} (Hz)	f_{max} (Hz)	Test duration (s)	Sampling freq. (kHz)
1	20	100	50	5.12
2	70	330	20	12.8
3	300	600	10	25.6
4	530	780	5	51.2
5	750	1000	5	51.2
6	970	1200	5	51.2
7	1170	1400	5	51.2
8	1370	1600	5	51.2
9	1570	1800	5	51.2
10	1770	2000	5	51.2
11	1970	2200	5	51.2

2. Find the experimental *peak frequencies* in the range 20-2200 Hz. Compare the results with the analytical model and the impact hammer tests.

3. Plot the spectrum, through the Fourier transform, of the force signal. What happens near the resonant frequencies? Give your interpretation (Hint: think of the amount of force you need to push a swing when it is oscillating at its natural frequency).

Useful code and file names

MATLAB code for computing the experimental transfer function:

```
% Tf: transfer function (complex), F: vector of the frequencies
[Tf,F] = tfestimate(f,acc,[],[],[],fs);
% Tm: magnitude of the transfer function
Tm = abs(Tf);
% Tp: phase of the transfer function (in radiants)
Tp = unwrap(angle(Tf));
```

To compute the fast Fourier transform of a signal, check the help page on MATLAB or *Maple* websites.

The data file names composition is described as follow:

- HAMMER TESTS
  File name structure is `hammer_xx_yy_zz.txt`, where:
  - `xx` is either `L2` or `L8`, and indicates the point of application of the force. `L2` stands for P_1 and `L8` stands for P_4.
  - `yy` is an integer indicating the test number.
  - `zz` is an integer indicating the sample frequency used to collect data, expressed in Hz.

Example: the file `hammer_L2_3_25600.txt` contains data collected hitting the beam with the hammer in point P_1. The test number is 3 and the sampling frequency is 25600 Hz.

- SHAKER TESTS
  File name structure is `shaker_xx_yy_zz_ww.txt`, where:
  - `xx` is an integer indicating the range number (referring to the first column of table 1).
  - `yy` is an integer indicating the sample frequency used to collect data, expressed in Hz.
  - `zz` is an four digit integer indicating the minimum frequency of excitation, expressed in Hz.
  - `ww` is an four digit integer indicating the maximum frequency of excitation, expressed in Hz.

  Example: the file `shaker_4_51200_0530_0780.txt` contains data collected using the shaker. The range number is 4 and the sampling frequency is 51200 Hz. The minimum and maximum frequencies of the force signal are 530 Hz and 780 Hz respectively