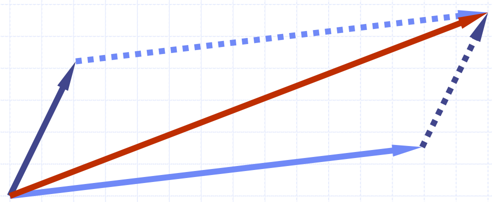
Vectors and Array Lists



The Vector ADT (§5.1)

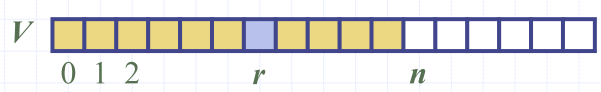
- The Vector ADT extends the notion of array by storing a sequence of arbitrary objects
- An element can be accessed, inserted or removed by specifying its rank (number of elements preceding it)
- An exception is thrown if an incorrect rank is specified (e.g., a negative rank)
- Main vector operations:
  - object elemAtRank(integer r): returns the element at rank r without removing it
  - object replaceAtRank(integer r, object o): replace the element at rank with o and return the old element
  - insertAtRank(integer r, object o): insert a new element o to have rank r
  - object removeAtRank(integer r): removes and returns the element at rank r
- Additional operations size() and isEmpty()

Applications of Vectors

- Direct applications
  - Sorted collection of objects (elementary database)
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

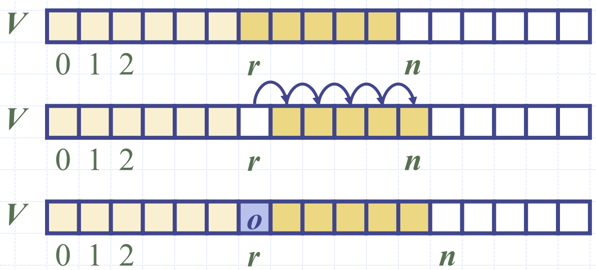
Array-based Vector

- Use an array V of size N
- A variable n keeps track of the size of the vector (number of elements stored)
- Operation ***elemAtRank***(r) is implemented in $O(1)$ time by returning $V[r]$



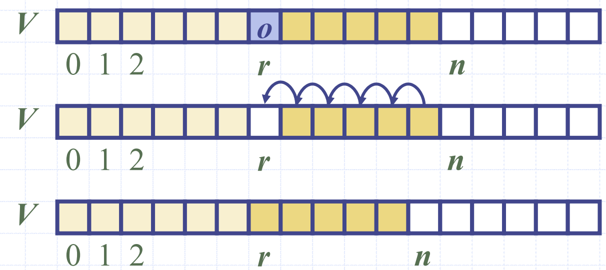
Insertion

- In operation $\textit{insertAtRank}(r, o)$, we need to make room for the new element by shifting forward the $n - r$ elements $V[r], \ldots, V[n - 1]$
- In the worst case ($r = 0$), this takes $O(n)$ time



Deletion

- In operation $\textit{removeAtRank}(r)$, we need to fill the hole left by the removed element by shifting backward the $n - r - 1$ elements $V[r + 1], \ldots, V[n - 1]$
- In the worst case ($r = 0$), this takes $O(n)$ time



Performance

- In the array based implementation of a Vector
  - The space used by the data structure is $O(n)$
  - *size*, *isEmpty*, *elemAtRank* and *replaceAtRank* run in $O(1)$ time
  - *insertAtRank* and *removeAtRank* run in $O(n)$ time
- If we use the array in a circular fashion, $\textit{insertAtRank}(0)$ and $\textit{removeAtRank}(0)$ run in $O(1)$ time
- In an *insertAtRank* operation, when the array is full, instead of throwing an exception, we can replace the array with a larger one

Growable Array-based Vector

- In a push operation, when the array is full, instead of throwing an exception, we can replace the array with a larger one
- How large should the new array be?
  - incremental strategy: increase the size by a constant c
  - doubling strategy: double the size

Algorithm $\textit{push}(o)$
  if $t = S.length - 1$ **then**
    $A \leftarrow$ new array of size …
    for $i \leftarrow 0$ **to** t **do**
      $A[i] \leftarrow S[i]$
    $S \leftarrow A$
  $t \leftarrow t + 1$
  $S[t] \leftarrow o$

Comparison of the Strategies

- We compare the incremental strategy and the doubling strategy by analyzing the total time $T(n)$ needed to perform a series of n push operations
- We assume that we start with an empty stack represented by an array of size 1
- We call amortized time of a push operation the average time taken by a push over the series of operations, i.e., $T(n)/n$

Incremental Strategy Analysis

- We replace the array $k = n/c$ times
- The total time $T(n)$ of a series of n push operations is proportional to

$$n + c + 2c + 3c + 4c + \ldots + kc =$$
$$n + c(1 + 2 + 3 + \ldots + k) =$$
$$n + ck(k + 1)/2$$

- Since c is a constant, $T(n)$ is $O(n + k^2)$, i.e., $O(n^2)$
- The amortized time of a push operation is $O(n)$

Doubling Strategy Analysis

- We replace the array $k = \log_2 n$ times
- The total time $T(n)$ of a series of n push operations is proportional to

$$n + 1 + 2 + 4 + 8 + \ldots + 2^k =$$
$$n + 2^{k+1} - 1 = 2n - 1$$

- $T(n)$ is $O(n)$
- The amortized time of a push operation is $O(1)$

geometric series