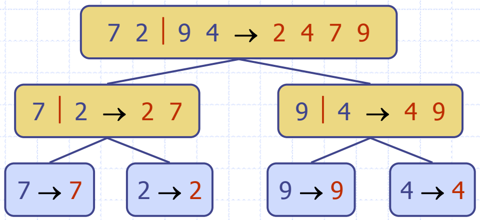
# Merge Sort







# Divide-and-Conquer (§ 10.1.1)

- Divide-and conquer is a general algorithm design paradigm:
  - Divide: divide the input data $S$ in two disjoint subsets $S_1$ and $S_2$
  - Recur: solve the subproblems associated with $S_1$ and $S_2$
  - Conquer: combine the solutions for $S_1$ and $S_2$ into a solution for $S$
- The base case for the recursion are subproblems of size 0 or 1
- Merge-sort is a sorting algorithm based on the divide-and-conquer paradigm
- Like heap-sort
  - It uses a comparator
  - It has $O(n \log n)$ running time
- Unlike heap-sort
  - It does not use an auxiliary priority queue
  - It accesses data in a sequential manner (suitable to sort data on a disk)




# Merge-Sort (§ 10.1)

- Merge-sort on an input sequence $S$ with $n$ elements consists of three steps:
  - Divide: partition $S$ into two sequences $S_1$ and $S_2$ of about $n/2$ elements each
  - Recur: recursively sort $S_1$ and $S_2$
  - Conquer: merge $S_1$ and $S_2$ into a unique sorted sequence

**Algorithm** *mergeSort(S, C)*
　**Input** sequence $S$ with $n$ elements, comparator $C$
　**Output** sequence $S$ sorted according to $C$
　**if** $S.size() > 1$
　　$(S_1, S_2) \leftarrow partition(S, n/2)$
　　$mergeSort(S_1, C)$
　　$mergeSort(S_2, C)$
　　$S \leftarrow merge(S_1, S_2)$




# Merging Two Sorted Sequences

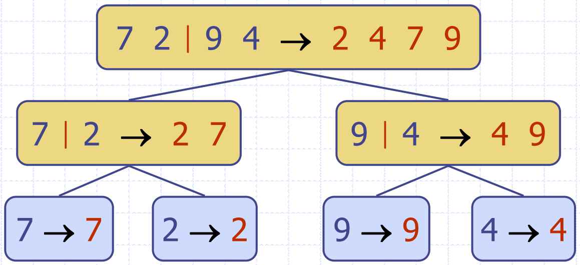
- The conquer step of merge-sort consists of merging two sorted sequences $A$ and $B$ into a sorted sequence $S$ containing the union of the elements of $A$ and $B$
- Merging two sorted sequences, each with $n/2$ elements and implemented by means of a doubly linked list, takes $O(n)$ time

**Algorithm** *merge(A, B)*
　**Input** sequences $A$ and $B$ with $n/2$ elements each
　**Output** sorted sequence of $A \cup B$

　$S \leftarrow$ empty sequence
　**while** $\neg A.isEmpty() \wedge \neg B.isEmpty()$
　　**if** $A.first().element() < B.first().element()$
　　　$S.addLast(A.remove(A.first()))$
　　**else**
　　　$S.addLast(B.remove(B.first()))$
　**while** $\neg A.isEmpty()$
　　$S.addLast(A.remove(A.first()))$
　**while** $\neg B.isEmpty()$
　　$S.addLast(B.remove(B.first()))$
　**return** $S$

# Merge-Sort Tree

- An execution of merge-sort is depicted by a binary tree
  - each node represents a recursive call of merge-sort and stores
    - unsorted sequence before the execution and its partition
    - sorted sequence at the end of the execution
  - the root is the initial call
  - the leaves are calls on subsequences of size 0 or 1

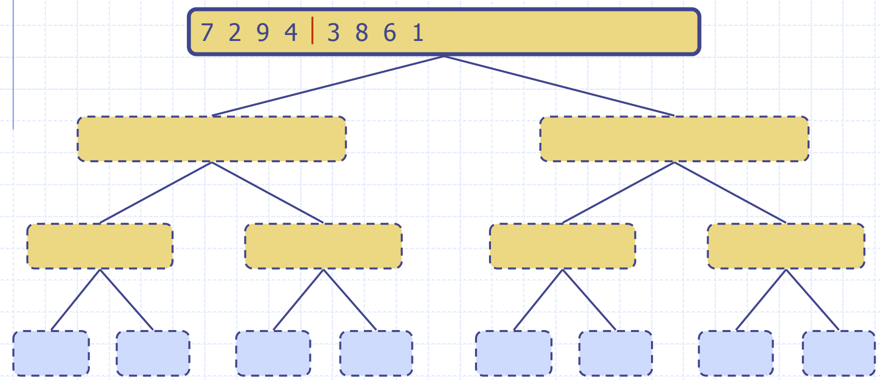






# Execution Example

- Partition

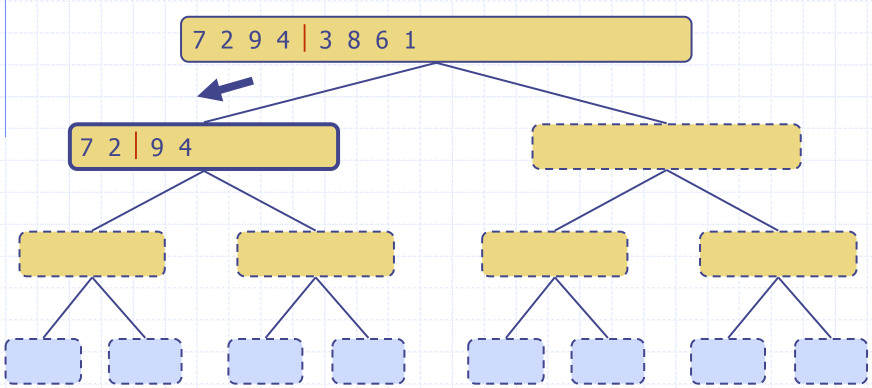






# Execution Example (cont.)

- Recursive call, partition

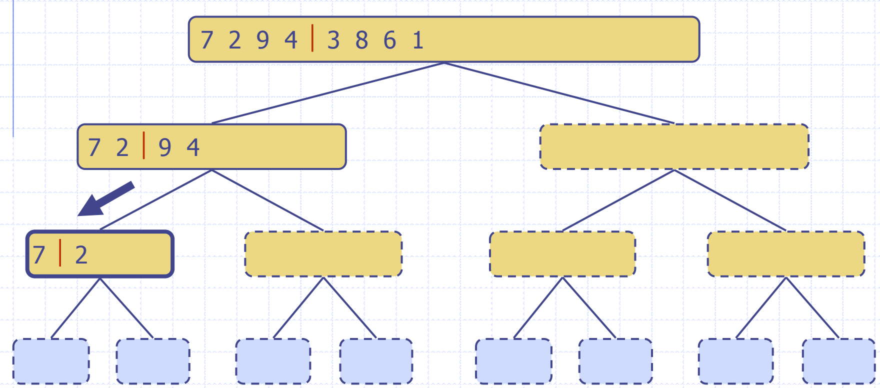






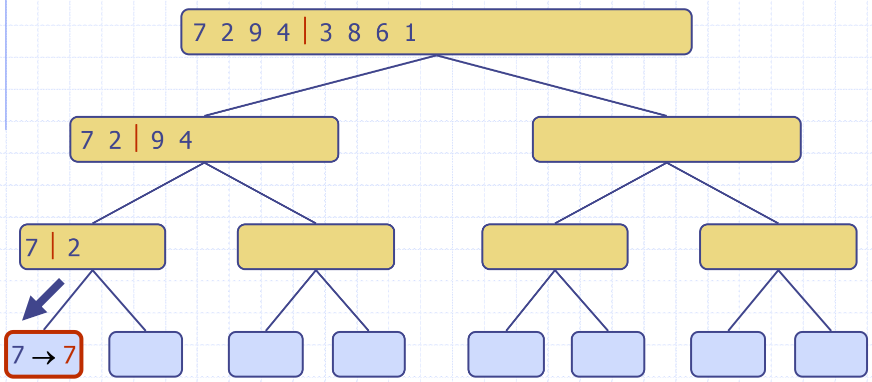
# Execution Example (cont.)

- Recursive call, partition



# Execution Example (cont.)

- Recursive call, base case

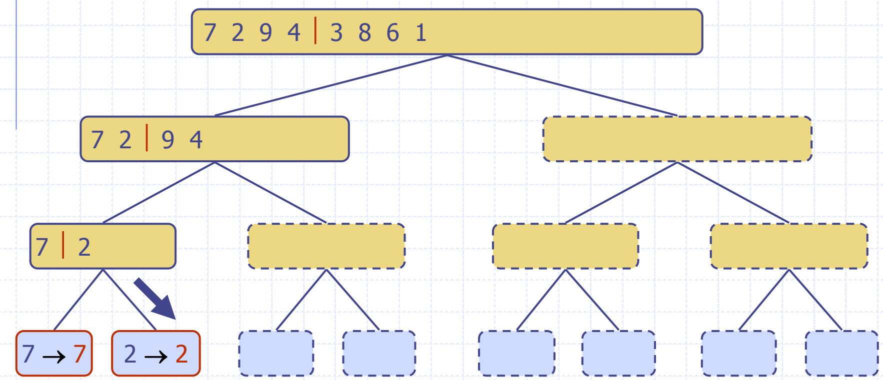






# Execution Example (cont.)

- Recursive call, base case

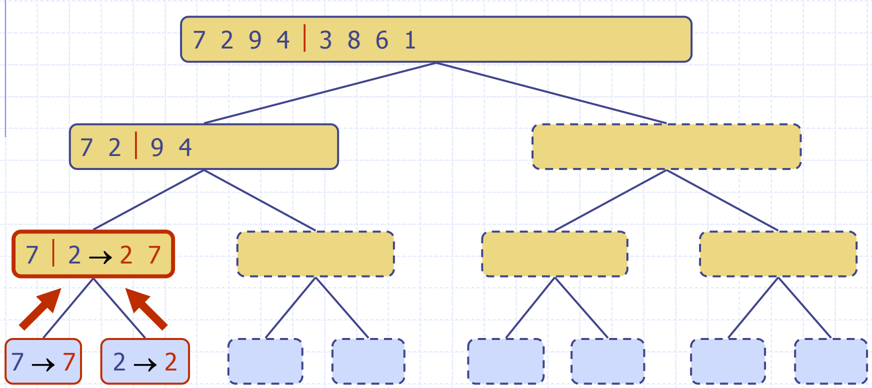






# Execution Example (cont.)

- Merge

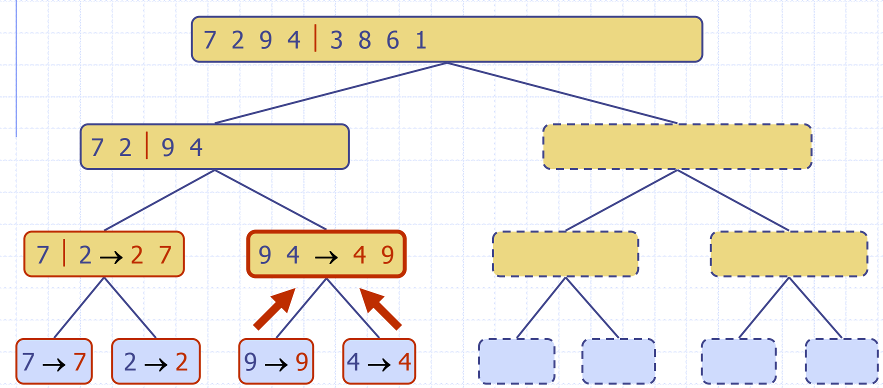






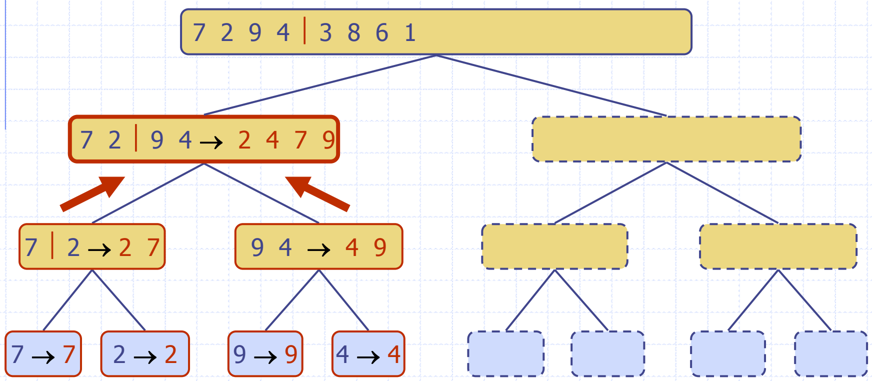
# Execution Example (cont.)

- Recursive call, …, base case, merge



# Execution Example (cont.)

- Merge

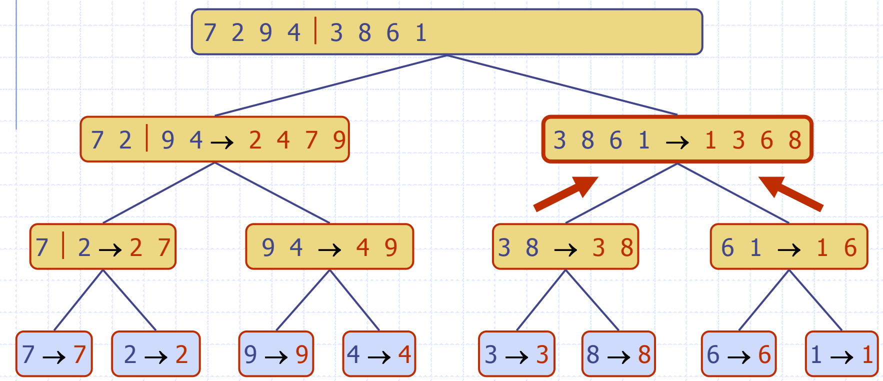






# Execution Example (cont.)

- Recursive call, …, merge, merge

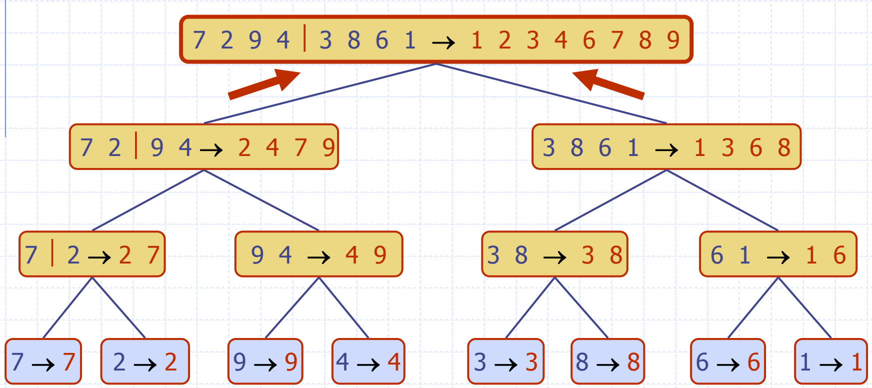






# Execution Example (cont.)

- Merge



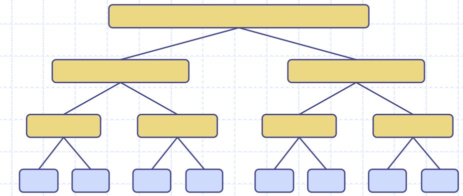




# Analysis of Merge-Sort

- The height $h$ of the merge-sort tree is $O(\log n)$
  - at each recursive call we divide in half the sequence,
- The overall amount or work done at the nodes of depth $i$ is $O(n)$
  - we partition and merge $2^i$ sequences of size $n/2^i$
  - we make $2^{i+1}$ recursive calls
- Thus, the total running time of merge-sort is $O(n \log n)$

| depth | #seqs | size |
|---|---|---|
| $0$ | $1$ | $n$ |
| $1$ | $2$ | $n/2$ |
| $i$ | $2^i$ | $n/2^i$ |
| … | … | … |



# Summary of Sorting Algorithms

| Algorithm | Time | Notes |
|---|---|---|
| selection-sort | $O(n^2)$ | ▪ slow<br>▪ in-place<br>▪ for small data sets (< 1K) |
| insertion-sort | $O(n^2)$ | ▪ slow<br>▪ in-place<br>▪ for small data sets (< 1K) |
| heap-sort | $O(n \log n)$ | ▪ fast<br>▪ in-place<br>▪ for large data sets (1K — 1M) |
| merge-sort | $O(n \log n)$ | ▪ fast<br>▪ sequential data access<br>▪ for huge data sets (> 1M) |