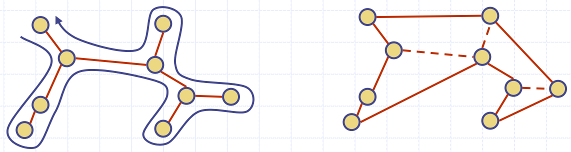
Campus Tour

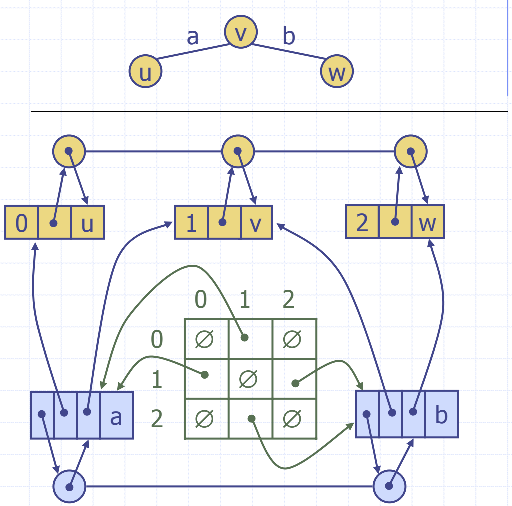


Graph Assignment

- Goals
  - Learn and implement the adjacency matrix structure an Kruskal's minimum spanning tree algorithm
  - Understand and use the decorator pattern and various JDSL classes and interfaces
- Your task
  - Implement the adjacency matrix structure for representing a graph
  - Implement Kruskal's MST algorithm
- Frontend
  - Computation and visualization of an approximate traveling salesperson tour

Adjacency Matrix Structure

- Edge list structure
- Augmented vertex objects
  - Integer key (index) associated with vertex
- 2D-array adjacency array
  - Reference to edge object for adjacent vertices
  - Null for non nonadjacent vertices



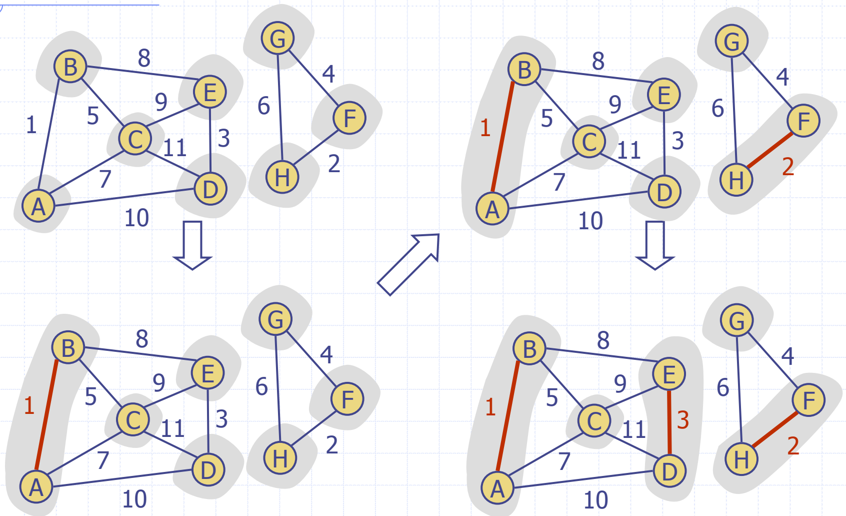
Kruskal's Algorithm

- The vertices are partitioned into clouds
  - We start with one cloud per vertex
  - Clouds are merged during the execution of the algorithm
- Partition ADT:
  - ***makeSet***(***o***): create set {***o***} and return a locator for object ***o***
  - ***find***(***l***): return the set of the object with locator ***l***
  - ***union***(***A***,***B***): merge sets ***A*** and ***B***

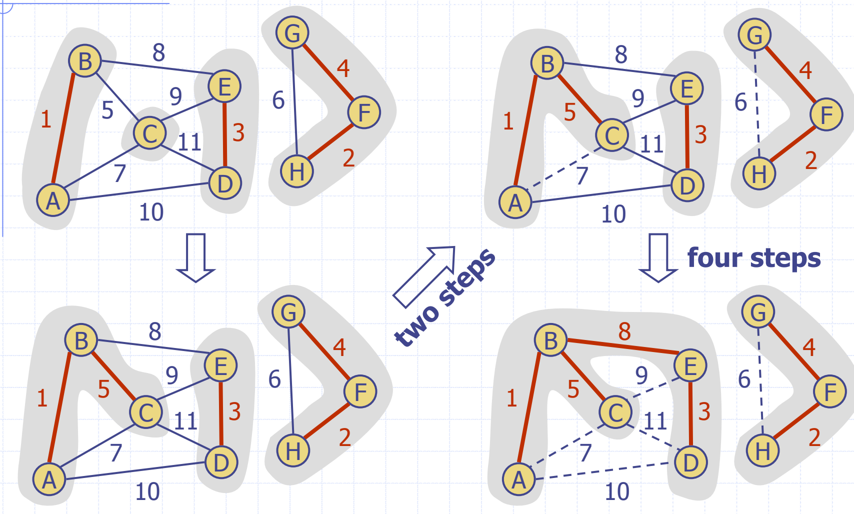
Algorithm ***KruskalMSF***(***G***)
  Input weighted graph ***G***
  Output labeling of the edges of a minimum spanning forest of ***G***

  $Q \leftarrow$ new heap-based priority queue
  for all $v \in G.vertices()$ **do**
    $l \leftarrow makeSet(v)$ { elementary cloud }
    $setLocator(v,l)$
  for all $e \in G.edges()$ **do**
    $Q.insert(weight(e), e)$
  while $\neg Q.isEmpty()$
    $e \leftarrow Q.removeMin()$
    $[u,v] \leftarrow G.endVertices(e)$
    $A \leftarrow find(getLocator(u))$
    $B \leftarrow find(getLocator(v))$
    if $A \neq B$
      $setMSFedge(e)$
      { merge clouds }
      $union(A, B)$

Example



Example (contd.)



Partition Implementation

- Partition implementation
  - A set is represented the sequence of its elements
  - A position stores a reference back to the sequence itself (for operation ***find***)
  - The position of an element in the sequence serves as locator for the element in the set
  - In operation ***union***, we move the elements of the smaller sequence into to the larger sequence
- Worst-case running times
  - ***makeSet, find***: $O(1)$
  - ***union***: $O(\min(n_A, n_B))$
- Amortized analysis
  - Consider a series of k Partiton ADT operations that includes n ***makeSet*** operations
  - Each time we move an element into a new sequence, the size of its set at least doubles
  - An element is moved at most $\log_2 n$ times
  - Moving an element takes $O(1)$ time
  - The total time for the series of operations is $O(k + n \log n)$

Analysis of Kruskal's Algorithm

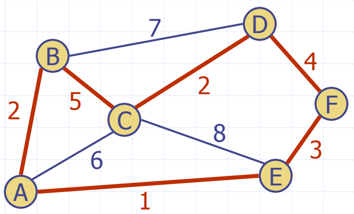
- Graph operations
  - Methods ***vertices*** and edges are called once
  - Method ***endVertices*** is called m times
- Priority queue operations
  - We perform m ***insert*** operations and m ***removeMin*** operations
- Partition operations
  - We perform n ***makeSet*** operations, $2m$ ***find*** operations and no more than $n - 1$ ***union*** operations
- Label operations
  - We set vertex labels n times and get them $2m$ times
- Kruskal's algorithm runs in time $O((n + m) \log n)$ time provided the graph has no parallel edges and is represented by the adjacency list structure

Decorator Pattern

- Labels are commonly used in graph algorithms
  - Auxiliary data
  - Output
- Examples
  - DFS: unexplored/visited label for vertices and unexplored/ forward/back labels for edges
  - Dijkstra and Prim-Jarnik: distance, locator, and parent labels for vertices
  - Kruskal: locator label for vertices and MSF label for edges
- The decorator pattern extends the methods of the Position ADT to support the handling of attributes (labels)
  - ***has***(a): tests whether the position has attribute a
  - ***get***(a): returns the value of attribute a
  - ***set***(a, x): sets to x the value of attribute a
  - ***destroy***(a): removes attribute a and its associated value (for cleanup purposes)
- The decorator pattern can be implemented by storing a dictionary of (attribute, value) items at each position

Traveling Salesperson Problem

- A tour of a graph is a spanning cycle (e.g., a cycle that goes through all the vertices)
- A traveling salesperson tour of a weighted graph is a tour that is simple (i.e., no repeated vertices or edges) and has has minimum weight
- No polynomial-time algorithms are known for computing traveling salesperson tours
- The traveling salesperson problem (TSP) is a major open problem in computer science
  - Find a polynomial-time algorithm computing a traveling salesperson tour or prove that none exists



Example of traveling salesperson tour (with weight 17)

TSP Approximation

- We can approximate a TSP tour with a tour of at most twice the weight for the case of Euclidean graphs
  - Vertices are points in the plane
  - Every pair of vertices is connected by an edge
  - The weight of an edge is the length of the segment joining the points
- Approximation algorithm
  - Compute a minimum spanning tree
  - Form an Eulerian circuit around the MST
  - Transform the circuit into a tour