$y = 2x + 1 \Rightarrow$ linear: grad = 2, y-int = 1

Limits

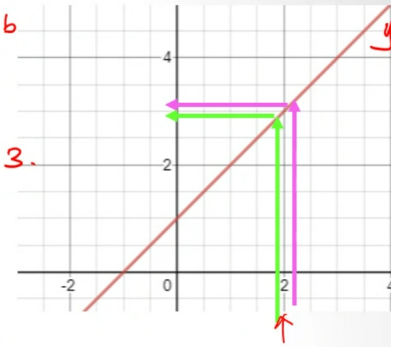
- The limit of $f(x)$ as x approaches a, $\lim_{x \to a} f(x)$ is the value of $f(x)$ approaches as x gets closer and closer to a.
- The actual value of $f(x)$ at a is not considered.
- The important thing is what happens to $f(x)$ as x get nearer and nearer to a.

$$\lim_{x \to a} f(x) = b$$

As $x \to a$, $f(x) \to b$

$$\lim_{x \to 2} (x + 1) = 3$$

As $x \to 2$, $x + 1 \to 3$.



As $x \to 2$, $x^2 - x + 2 \to ?$

Find $\lim_{x \to 2} (x^2 - x + 2)$

$$\lim_{x \to 2^-} x^2 - x + 2 = 4$$

$$\lim_{x \to 2^+} x^2 - x + 2 = 4$$

$\lim_{x \to 2} x^2 - x + 2$ exist $\Rightarrow \lim_{x \to 2} x^2 - x + 2 = 4$

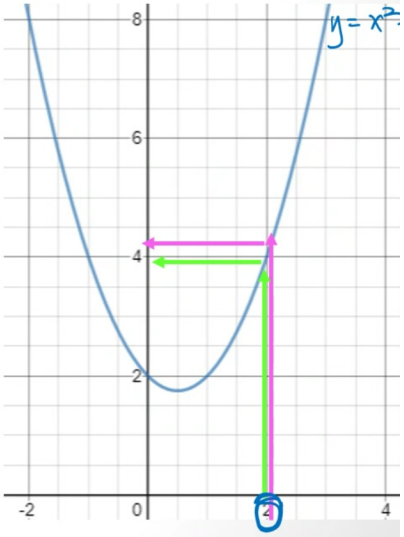
$\therefore$ Limit exists, $\lim_{x \to 2} x^2 - x + 2 = 4$

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) \Rightarrow \lim_{x \to a} f(x) \text{ exist}$$

$\lim_{x \to a} f(x) = b$

As $x \to a$, $f(x) \to b$

x	f(x)	x	f(x)
1.0	2.000000	3.0	8.000000
1.5	2.750000	2.5	5.720000
1.8	3.440000	2.2	4.640000
1.9	3.710000	2.1	4.310000
1.95	3.852500	2.05	4.152500
1.99	3.970100	2.01	4.030100
1.995	3.985025	2.005	4.015025
1.999	3.997001	2.001	4.003001



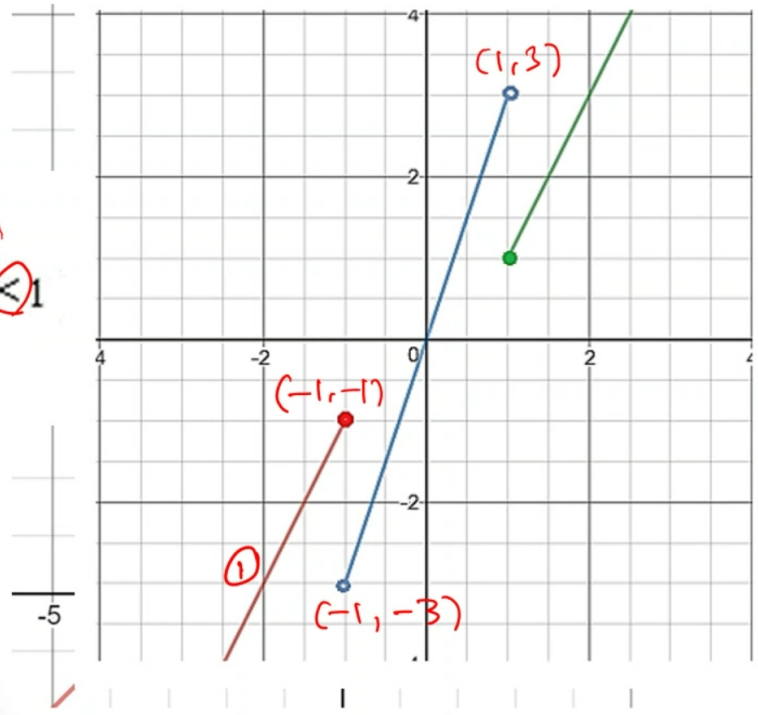
Pre-class

Sketch:

1) $f(x) = \frac{x^2-9}{x-3}$

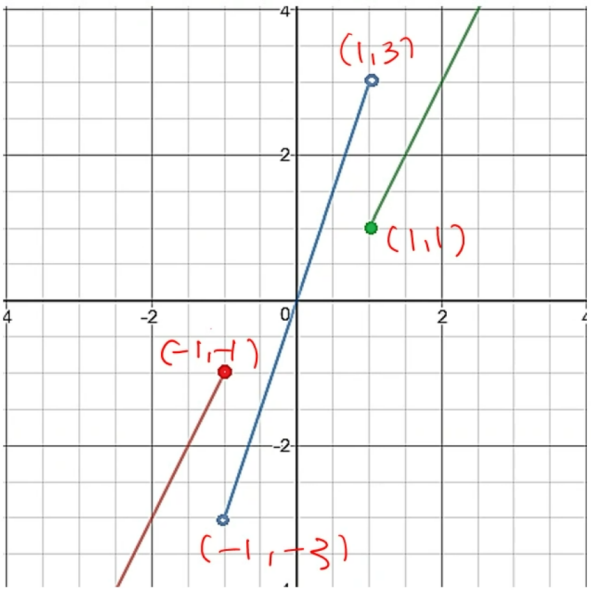
2)

$$f(x) = \begin{cases} 2x+1, & \text{if } x \le -1 \\ 3x, & \text{if } -1 < x < 1 \\ 2x-1 & \text{if } x \ge 1 \end{cases}$$



b)

$$f(x)=\begin{cases}2x+1, & \text{if } x\le -1\\ 3x, & \text{if } -1<x<1\\ 2x-1 & \text{if } x\ge 1\end{cases}$$



For $x=-1$:

S1: $f(-1)=-1$

S2: $\lim_{x\to -1^-}(2x+1)=-1$; $\lim_{x\to -1^+}3x=-3$

$\lim_{x\to -1^-}f(x)\neq\lim_{x\to -1^+}f(x)\Rightarrow$

$\lim_{x\to -1}f(x)$ *does not exist*

$\therefore$ ***There is a point of discontinuity at*** $x=-1$

For $x=1$:

S1: $f(1)=1$

S2: $\lim_{x\to 1^-}3x=3$; $\lim_{x\to 1^+}(2x-1)=1$

$\lim_{x\to 1^-}f(x)\neq\lim_{x\to 1^+}f(x)\Rightarrow\lim_{x\to 1}f(x)$ *does not exist*

$\therefore$ ***There is a point of discontinuity at*** $x=1$

$\therefore f(x)$ is not a continuous function. $f(x)$ is discontinuous at $x=\pm 1$