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Conditional correlation between exchange rates and stock prices

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ABSTRACT

This paper develops a model to explain a time-varying and currency-dependent interaction between stock prices and exchange rates. The linkage is proposed based on portfolio managers' rebalance between foreign and domestic stocks. Stock prices in two industrialized countries are usually correlated but have different sensitivities to a common risk factor. When a common positive shock occurs, the country with higher sensitivity will attract relatively more investment on their stocks, which generates a net order flow toward that country's currency in the foreign exchange market and further causes the country's currency to appreciate. The paper concludes that the correlation between exchange rates and stock prices is determined by the relative sensitivity of two countries' stock prices to the common stock factor. In specific, rising US stock prices are associated with appreciation(depreciation) of the Dollar when US(foreign) stock prices have higher sensitivity.

1. Introduction

Financial press as well as scholarly research have long asserted that stock prices and exchange rates are intertwined. Empirical evidence, however, is mixed. Studies such as Phylaktis and Ravazzola (2005) find a positive correlation between the two variables, while Roll (1992) and Ajayi and Mougou (1996) report a negative relationship. Recent empirical studies suggest that the relationship is time-varying (Kollias, Mylonidis, & Paleologou, 2012), currency-dependent (Granger, Huang, & Yang, 2000; Nieh & Lee, 2001), and its sign might depend on relative interest rates (Katechos, 2011) and direction of international capital flow (Ulklz & Demirci, 2012). Direct observation of historical data of S&P500 Index and Traded Weighted US Dollar Index, as shown in the figure below, can also tell the interaction is more than a simple linear relationship: a booming stock market was accompanied by an appreciation of the Dollar in the 1990s (i.e., positive correlation), while synchronized with the rising stock market after the dot com bubble burst was a depreciation of the Dollar (i.e., negative correlation).

In general, three different types of theories explain the linkage between stock prices and exchange rates. Micro "flow-oriented" models argue that as the US Dollar appreciates, cash flow and profitability of American firms should improve, which leads to rising US stock prices. Macro "flow-oriented" models suggest that a more competitive exchange rate, assuming that the Marshall-Lerner conditions hold, would improve the trade position of an economy and stimulate the real economy through firms' profitability and stock market prices (e.g., Dornbusch & Fisher, 1980). "Stock- oriented" theory claims that a decrease in stock prices would reduce domestic wealth, lower demand of money and interest rate, and eventually causes capital outflow and a depreciation of the country's currency. (e.g., Gavin, 1989). These classical theories propose single but inconsistent signs of the relationship and have difficulty accommodating the time-variation and asymmetry in the relationship.

Understanding the interrelations between exchange rates and stock prices is undoubtedly critical for fund managers who oversee international portfolios. It would also shed light on widely debated spillovers such as international financial contagions. Therefore, the basic goal of this paper is to develop a framework to accommodate the nonlinear interaction between exchange rates and stock prices.

This paper is spirited by foreign exchange market microstructure research. (Evans & Lyons, 2002 and W.Killeen & Lyons, 2001) show that the order flow from electronic brokerage systems has a remarkably high correlation with contemporaneous exchange rate changes. The order flow is dominated by inter-dealer and financial customer order flow (New York Federal Reserve Bank (2009)); and inter-dealer order flow is proportional to the non-public customer order flow (Evans & Lyons, 2002). Hence, exchange rate dynamics

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should be mainly driven by the FX transactions initiated by financial customers, which has been empirically confirmed by (Menkhoff, Sarnoz, Schmeling, and Schrimpf, 2012).

Financial customers' portfolios usually contain domestic and foreign assets, and as market conditions change, portfolio rebalance between domestic and foreign assets produces FX order flows, which further drives exchange rate dynamics. Exchange rate explanation models following this line include (Carlson, Dahl, & Osler, 2008; Dunne, Hau, & Moore, 2010; Ding & Ma, 2013). Meanwhile, such a rebalance behavior also affects the stock market, which provides a linkage between exchange rates and stock prices. Focusing on this linkage, (Hau & Rey, 2004) and (Hau & Rey, 2006) propose simultaneous determination of exchange rates and stock prices. Our model is motivated by these earlier studies and connects exchange rates with the stock market through investors' optimal portfolio rebalancing behavior.

Given correlated stock prices in two countries but with different sensitivity to the common risk factor, when a positive global common stock factor occurs, the country with higher sensitivity would attract relatively more investment on their stocks, therefore generating net order flow toward that country's currency in the foreign exchange market and further causing the appreciation of the currency. We conclude that the exchange rate-stock price correlation is determined by the relative sensitivity of two countries' stock prices to the common stock factor, and it is time-varying and currency-dependent. In specific, rising US stock prices are associated with an appreciation(depreciation) of the Dollar when US(foreign) stock prices have higher sensitivity. Furthermore, the significance of the correlation depends on how closely the two countries' stock prices are correlated.

The mechanism we propose above seems to contradict Hau and Rey (2004) and (2006), which claim that portfolio should be rebalanced away from higher return assets to reduce FX risk, causing the high return assets currency to depreciate, and therefore exchange rate should be negatively correlated with the lag of relative stock returns. The mechanism we propose here says portfolio should be reallocated to higher return assets when higher returns are expected or realizing, causing the high return currency to appreciate. As a consequence, the value of high return currency (low return currency) should be positively (negatively) correlated with contemporaneous stock returns. Hau and Rey (2004) and (2006) focus on the posterior impact of higher return assets on the exchange rate while ours focuses on the contemporaneous impact, and they do not necessarily conflict with each other.

The paper is also in the same spirit of other international portfolio rebalance studies such as (Dunne et al., 2010; Pavlova & Roberto, 2007; Ding & Ma, 2013) as well as Curcuru, Thomas, Warnock, and Wongswan (2014). Compared to the existing literature, the main contributions of our paper are two folds: first, our model proposes an explicit relationship between exchange rates and stock prices, and second, the model has the ability to accommodate the time-variation and asymmetry in the relationship.

The rest of the paper is arranged as follows. Section 2 constructs a theoretical framework and discusses its implications. Section 3 presents extensive empirical evidence. Section 4 concludes.

2. Theoretical model

Our effort to examine the interaction between exchange rates and stock prices starts with modeling exchange rate dynamics mechanisms, which we tie to financial institutions' portfolio reallocation behavior between domestic and foreign assets. To theoretically model this process, we first determine domestic and foreign financial customers' optimal portfolio composition, then we show how FX order flows are generated by the change of this composition as market conditions change, and finally, we derive the relationship between exchange rates and stock prices.

2.1. Model setup

Suppose home and foreign country each has three funds: Bond fund holds domestic and foreign bonds, domestic stocks fund holds domestic bonds and stocks, and foreign stocks fund holds domestic bonds and foreign stocks. They have the following balance sheets:

Assets	Liabilities	Assets	Liabilities
$B_H^1, B_H^*,$	V_H^1	$B_F^1, B_F^*,$	V_F^1
$B_H^2, S_H,$	V_H^2	$B_F^{*2}, S_F^*,$	V_F^2
$B_H^3, S_H^*,$	V_H^3	$B_F^{*3}, S_F^*,$	V_F^3

where V is equity, B^* and B are foreign and domestic bonds respectively. Note that B^* and B can be negative, meaning either domestic or foreign bonds can be shorted to finance other investments. S^* and S are foreign and domestic stocks respectively. Subscript H means assets held by home funds and F means foreign funds. Also, note that all items in the balance sheets are denominated in local currency. We assume there is no addition or withdrawal of equity throughout the trading periods. We further assume each corresponding foreign and domestic funds have symmetric fund size (i.e., $V_H^i = V_F^i$) and risk appetite.[1]

Let $q_t^{B,i}$ and $q_t^{B*,i}$ be quantity of domestic and foreign bonds held in each fund. q_t^S and q_t^{S*} be quantity of domestic and foreign stocks. Again, subscript H means home country and F means a foreign country. Denote price of domestic and foreign bonds by p_t^B and p_t^{B*} and price of domestic and foreign stocks by p_t^S and p_t^{S*}. Also, let e_t be the spot exchange rate quoted as the dollar price of foreign currency (same notation throughout the paper). Thus, each asset held by home and foreign funds in their local currencies are:

$$B_{H,t}^i = q_{H,t}^{B,i} \cdot p_t^B \quad B_{F,t}^{*,i} = q_{F,t}^{B^*,i} \cdot p_t^{B^*} \tag{1}$$

$$B_{H,t}^* = q_{H,t}^{B^*} \cdot p_t^{B^*} \cdot e_t \qquad B_{F,t} = \left(q_{F,t}^B \cdot p_t^B\right)/e_t \tag{2}$$

$$S_{H,t} = q_{H,t}^S \cdot p_t^S \quad S_{F,t}^* = q_{F,t}^{S^*} \cdot p_t^{S^*} \tag{3}$$

$$S_{H,t}^* = q_{H,t}^{S^*} \cdot p_t^{S^*} \cdot e_t \quad S_{F,t} = \left(q_{F,t}^S \cdot p_t^S\right)/e_t \tag{4}$$

We consider bonds risk-free in their local currencies, and their dynamics can be written as:

$$\Delta \log p_t^B = i_t \tag{5}$$

$$\Delta \log p_t^{B^*} = i_t^* \tag{6}$$

where i_t and i_t^* are domestic and foreign interest rates, and Δ denotes the first order difference of the variable.[2] Return variables at period t refers to the return from time t to time $t+1$.[3]

Stock prices often possess long memory and show autoregressive feature. We assume stock prices in foreign and home country have autoregressive coefficient ρ^* and ρ respectively. And also, stock prices in major advanced countries share very similar dynamics, so we assume they are governed by a common stock factor F_t^S with different loading l_s^* and l_s. In addition, denote η_t^* and η_t as the foreign and domestic idiosyncratic risk with zero mean. For simplicity, we assume their variance are constant. Thus, the dynamics of the stock prices in local currencies are:

$$\log p_t^S = \rho \log p_{t-1}^S + l_s F_t^S + \eta_t \tag{7}$$

[1] This assumption is made only to simplify the expression of model solutions. Relaxing it does not fundamentally change the conclusion of the model.

[2] i.e., the percentage change of the variable, the same definition throughout the paper unless specifically noted.

[3] The same definition throughout the paper.

$$\log p_t^{S^*} = \rho^* \log p_{t-1}^{S^*} + l_s^* F_t^s + \eta_t^* \tag{8}$$

Hence the return of stock prices in each country can be written as:

$$r_t = \rho r_{t-1} + l_s \Delta F_t^s + \Delta\eta_t \tag{9}$$

$$r_t^* = \rho r_{t-1}^* + l_s^* \Delta F_t^s + \Delta\eta_t^* \tag{10}$$

For exchange rate dynamics, we follow the major finding in FX microstructure research and assume it is driven by the FX order flow, which we link to fund managers' portfolio rebalance between foreign and domestic assets. At this stage, we certainly do not know details of this rebalance behavior and just denote the exchange rate change as r_t^e:

$$\Delta \log e_t = r_t^e \tag{11}$$

2.2. Optimal portfolio composition

The goal for the managers of each fund is to maximize the expected return of equity:

$$E_t \Delta V_{H,t}^i = \left(1 - W_{H,t}^i\right) \cdot i_t + W_{H,t}^i \cdot E_t \Delta P_t^i \tag{12}$$

where $W_{H,t} = \{B_{H,t}^*/V_{H,t}^1, S_{H,t}/V_{H,t}^2, S_{H,t}^*/V_{H,t}^3\}$, $P_{H,t} = \{p_t^{B*} \cdot e_t, p_t^S, p_t^{S*} \cdot e_t\}$ and i=1, 2, 3. We just use the typical mean-variance framework for the optimization problem, and the optimal portfolio composition for each domestic funds is:

$$\begin{bmatrix} B_{H,t}^* \\ S_{H,t} \\ S_{H,t}^* \end{bmatrix} = \frac{1}{\nu} \begin{bmatrix} \dfrac{i_t^* - i_t}{\sigma_e^2} V_{H,t}^1 \\ \dfrac{r_t - i_t}{\sigma_S^2} V_{H,t}^2 \\ \dfrac{r_t^* - i_t}{\sigma_{S^*}^2 + \sigma_e^2 + 2\sigma_{S^*e}} V_{H,t}^3 \end{bmatrix} \tag{13}$$

where $\sigma_e^2, \sigma_S^2, \sigma_{S*}^2, \sigma_{S*e}$, denote the variance and covariance of the asset prices shown in the subscript, and ν represents fund managers' degree of risk aversion. To simplify our analysis, we assume the investors have constant risk preference over time.

For the foreign funds, similarly, the optimal portfolio composition can be solved as:[4]

$$\begin{bmatrix} B_{F,t} \\ S_{F,t}^* \\ S_{F,t} \end{bmatrix} = \frac{1}{\nu} \begin{bmatrix} \dfrac{i_t - i_t^*}{\sigma_e^2} V_{F,t}^1 \\ \dfrac{r_t^* - i_t^*}{\sigma_{S^*}^2} V_{F,t}^2 \\ \dfrac{r_t - i_t^*}{\sigma_S^2 + \sigma_e^2 + 2\sigma_{Se}} V_{F,t}^3 \end{bmatrix} \tag{14}$$

2.3. Portfolio reallocation

The portfolio reallocations in home country domestic stock fund and foreign country foreign stock fund do not involve FX transactions. Although the bond fund reallocation generates FX transactions, they are not directly related to the stock market. Therefore, we only focus on the domestic foreign stock fund (reallocation between B_H and S_H^*) as well as foreign domestic stock fund (reallocation between B_F^* and S_F) to show FX order flows generation process.

An arrival of market shocks (i.e., ΔF_t^S) changes two countries' stock returns, which triggers portfolio reallocation to obtain the new optimal position. Given our previous assumptions of no addition or withdrawal of the equity, constant risk appetite ν, and constant variances and covariances $\sigma_e^2, \sigma_S^2, \sigma_{S*}^2, \sigma_{S*e}$, a linear reduced-form portfolio reallocation for the home country's foreign stock fund can be written as:

$$\Delta S_{H,t}^* = \Delta(r_t^* - i_t) + E_H r_t^e$$

where $E_{H^{r_t^e}}$ is the fund manager's expectation of exchange rate return. Since stock return is much more volatile than short term interest rate in one-month horizon, percentage change of $(r^* - i_t)$ should be mainly contributed by the change of stock return, hence the reallocation can be simplified as:

$$\Delta S_{H,t}^* = \Delta r_t^* + E_H r_t^e \tag{15}$$

According to dynamics of stock returns defined in equation (10), the equation (15) can be written as:

$$\Delta S_{H,t}^* = (\rho^* - 1) r_{t-1}^* + l_s^* \Delta F_t^s + \Delta\eta_t^* + E_H r_t^e \tag{16}$$

The fund managers would project the impact of their trading behavior on the FX market, which they will take into account for the conditional expected exchange rate return. Suppose $E_{H^{r_t^e}}$ is linearly correlated with the FX order they plan to submit, we have:

$$E_H r_t^e = \delta \Delta S_{H,t}^* \tag{17}$$

Solving equation systems (16,17) gives the reallocation of foreign stocks for the domestic fund as:

$$\Delta S_{H,t}^* = \frac{1}{1-\delta}[(\rho^* - 1) r_{t-1}^* + l_s^* \Delta F_t^s + \Delta\eta_t^*] \tag{18}$$

Following the similar methodology, the reallocation of reallocation between B_F and S_F can be written as:

$$\Delta S_{F,t} = \Delta r_t - E_F r_t^e$$

Once again, we assume the expected exchange rate return is linearly correlated with the FX order they submit, thus:

$$E_F r_t^e = -\delta \Delta S_{F,t} \tag{19}$$

Here foreign funds share the same parameter δ, as we assume the symmetry between domestic and foreign funds. Thus, the reallocation of domestic stocks for the foreign funds is:

$$\Delta S_{F,t} = \frac{1}{1-\delta}[(\rho - 1) r_{t-1} + l_s \Delta F_t^s + \Delta\eta_t] \tag{20}$$

2.4. Order flow and exchange rate dynamics

If positive order flow is defined as net purchase of foreign currency, equation (18) and (20) can give aggregate FX order flow generated by the stocks reallocation as:

$$OF_t^S = \Delta S_{H,t}^* - \Delta S_{F,t} \tag{21}$$

$$OF_t^S = \frac{1}{1-\delta}[(l_s^* - l_s)\Delta F_t^s + (\rho^* - 1) r_{t-1}^* - (\rho - 1) r_{t-1} + (\Delta\eta_t^* - \Delta\eta_t)] \tag{22}$$

As dealers update their quotes according to the order flow, exchange rate dynamics should be proportional to the order flow so that we have:

$$\Delta \ln e_t = \frac{1}{1-\delta}[(l_s^* - l_s)\Delta F_t^s + (\rho^* - 1) r_{t-1}^* - (\rho - 1) r_{t-1} + (\Delta\eta_t^* - \Delta\eta_t)] \tag{23}$$

[4] Note that we assume foreign and domestic funds share the same risk appetite in section.

2.5. Relationship between exchange rates and stock prices

Plugging the stock market shock ΔF_t^S from equation (9) to equation (23) gives the explicit relationship between exchange rate change and stock return:

$$\Delta \ln e_t = \frac{1}{1-\delta}\left[\frac{l_s^* - l_s}{l_s}\Delta \ln p_t^S + (\rho^* - 1)r_{t-1}^* - \left(\frac{l_s^*}{l_s}\rho - 1\right) r_{t-1} + \Delta\eta_t^* - \frac{l_s^*}{l_s}\Delta\eta_t\right] \quad (24)$$

Thus, the contemporaneous correlation between exchange rates and stock prices can be measured by the coefficient of $\Delta \ln p_t^S$ in the equation above.

Given assumption of constant degree of risk aversion (δ), equation (24) implies that the correlation between exchange rates and stock prices is determined by the relative loading to the common stock factor $\left(\frac{l_s^* - l_s}{l_s}\right)$. Specifically, when a positive shock occurs in the stock market, both countries' stock prices will increase, and fund managers would reallocate from low return assets (i.e., bonds in our model) to high return assets (i.e., the other country's stocks in our model). If foreign stock prices are more sensitive to the stock factor (i.e., ($l_s^* > l_s$)), they are expected to have a relatively higher return. Thus, the amount of portfolio rebalancing from domestic assets to foreign assets is larger than that of the opposite direction. This would create net order flow toward the foreign currency in the foreign exchange market, causing the foreign currency to appreciate. Thus increasing stock prices should be correlated with a depreciation of the US dollar (note e_t is quoted as the dollar price of one unit of foreign currency). In the opposite scenario that US stock prices are more sensitive to the common factor, i.e., ($l_s^* < l_s$), positive stock shock would create net order flow toward the US Dollar so that rising US stock price should be correlated with an appreciation of the US Dollar.

In summary, the main conclusion of our model is that the exchange rates and stock price correlation is determined by the relative sensitivity of the foreign and domestic stock prices to the common risk factor (i.e., ($l_s^* - l_s$)). This conclusion has two clear implications: First, as the relative sensitivity can be time-varying and currency dependent, the exchange rates and stock price correlation should be time-varying and currency dependent as well. In specific, if a foreign country has higher (lower) sensitivity in stock prices, the dollar rate of foreign currency is positively (negatively) correlated with US stock prices. Second, the significance of the correlation between exchange rates and stock prices depends on the significance of the co-movement of the two countries' stock prices.

3. Empirical evidence

3.1. Data description

Our empirical tests cover the period between 02/1991 and 09/2020.[5] The starting date was chosen for two primary reasons. First, as highly-leveraged speculation was uncommon before the 1990s, the mechanism proposed in this paper may be insignificant before this time. Second, governments intervened heavily in the FX market before the 1990s. For example, the Plaza Accord of September 1985 was a concerted effort of governments to depreciate the USD. To halt any further depreciation, the Louver Accord of October 1987 coordinated central banks of major industrialized countries to boost the USD. The ending date was the month of the latest data available when this version of the paper was submitted.

[5] DEM ended 12/1998 and EUR started 01/1999.

Table 1
The Augmented Dickey–Fuller test for stock price indices.

	Test statistic Z(t)	10% Critical value	MacKinnon p-value for Z(t)
USD Stock Index	−1.857	−2.570	0.3527
CAD Stock Index	−1.667	−2.570	0.4483
DEM Stock Index	−0.409	−2.579	0.9086
EUR Stock Index	−1.810	−2.570	0.3757
GBP Stock Index	−2.275	−2.570	0.1183
JPY Stock Index	−1.979	−2.570	0.2960

We test the model with five exchange rates: the U.S. Dollar (USD) versus the British Pound (GBP), Canadian Dollar (CAD), Deutsche Mark (DEM), Euro (EUR) and Japanese Yen (JPY). These rates are chosen for the following reasons. First, these currencies are freely traded without strict government regulations and capital mobility restrictions, conditions required by the environment assumed in our model. Second, these currency pairs are the most traded in the world and attract many institutional speculators. Third, they are typical and representative: the CAD represents commodity currencies such as the Australian Dollar and New Zealand Dollar; before the introduction of the Euro, the DEM represented other major European currencies (e.g., the French Franc and Swiss Franc); since its launch, the Euro has shared the dynamics of other major European currencies such as the Swiss France.

We test at the monthly horizon. Shorter horizons (e.g., daily) suffer from noises as well as the unavailability of high-frequency data for variables proposed in the paper. The exchange rate return – is calculated as the log differential of spot exchange rates, which are obtained from the OECD Stats. Despite their quote tradition, all rates are converted into the dollar rates of foreign currencies to be compatible with our theoretical framework. Monthly stock returns are calculated from each country's stock index, which is also extracted from OECD.

3.2. Test specifications

This section tests the three main conclusions of the model: First, the correlation is determined by the relative sensitivity of two countries' stock prices to the common stock factor; Second, the sign of the correlation depends on which country has a relatively higher return in the stock prices; and Third, the significance of the correlation depends on the significance of the correlation of two countries' stock prices.

3.2.1. Variables measurement

We first measure the correlation between exchange rates and stock prices. Since neither exchange rates or stock price indices are stationary, their correlation is measured in returns and can be identified by β_1 in the following regression:

$$\Delta \log e_t = \beta_0 + \beta_1 \Delta \log p_t^S + \varepsilon_t \quad (25)$$

where the significance level of β_1 can also reflect the significance of the correlation.

Next we measure the relative sensitivity of two countries' stock prices (i.e., $l_s^* - l_s$ in equation (24)) and the correlation of the stock prices. If foreign stock prices are more sensitive (i.e., $l_s^* > l_s$), they should increase (decrease) faster than the U.S. counterpart in good (bad) times. Accordingly, if we regress logged values of foreign stock prices (p_t^{S*}) on logged values of American stock prices (p_t^S) with a non-zero constant, the slope should be significantly greater (less) than one. A more rigorous proof is provided in the Appendix to support this method.

Despite the sound intuition, the validity of this regression is not immediately clear because both stock price indexes are non-stationary, which can be confirmed by the results of the Augmented

Table 2
Full sample estimate for the exchange rate-stock price correlation and relative sensitivity of foreign and stock prices.

		β_0	t-statistic	β_1	t-statistic	γ_0	t-statistic	γ_1	t-statistic
CAD	02/91–09/20	−0.6105	−3.20	0.0741	1.83	0.4024	2.72	0.9060	4.70
DEM	02/91–12/98	0.0247	0.35	−0.2418	−0.84	0.1210	0.29	0.6284	3.17
EUR	01/99–09/20	0.0137	0.55	0.0320	1.16	−0.5477	−1.43	1.1710	5.37
GBP	02/91–09/20	0.3313	0.12	0.0103	2.23	0.8266	5.22	0.8720	6.72
JPY	02/91–09/20	4.7756	13.26	0.0557	0.66	1.2797	2.54	0.7063	5.21

β_1 represents the correlation between the exchange rate and stock price from equation (25).
γ_1 represents the sensitivity of foreign stock price index relative to the US stock price index from equation (26).

Table 3
The Stock-Watson tests for the relationship between two countries' stock prices for selected periods[10].

		γ_0	t-statistic	γ_1	t-statistic	Adjusted R-Square
CAD	02/94–01/99	0.1912	0.97	0.7057	2.47	0.17
CAD	01/03–03/13	−0.1555	−0.34	1.2043	4.75	0.24
DEM	02/91–12/98	0.1210	0.29	0.6284	3.17	0.19
EUR	01/99–01/05	−0.6217	−1.93	2.3714	5.42	0.48
GBP	01/95–01/99	0.4236	1.47	0.8910	5.49	0.45
GBP	01/07–01/13	0.0142	0.43	1.2729	6.79	0.53
JPY	01/95–01/99	0.5207	3.95	−0.5384	−1.89	0.22
JPY	01/13–01/15	−0.5776	−1.82	1.4734	6.99	0.57

Table 4
The Johansen and ARDL tests for the relationship between two countries' stock prices[11].

Johansen Test							
		γ_0	z-statistic	γ_1	z-statistic	error correction	z-statistic
CAD	02/94–01/99	0.3457	1.06	0.6105	7.61	−0.2456	−2.17
CAD	01/03–03/13	−0.0348	−0.47	1.1220	8.30	−0.0271	−2.02
DEM	02/91–12/98	0.1874	1.03	0.7267	4.26	−0.1472	−3.14
EUR	01/99–01/05	−0.7414	−1.97	2.2711	6.16	−0.0482	−1.55
GBP	01/95–01/99	0.7415	1.23	0.8212	6.23	−0.5809	−4.16
GBP	01/07–01/13	0.0213	0.57	1.1447	4.97	−0.1761	−2.45
JPY	01/95–01/99	0.3531	2.27	−0.4477	−2.13	−0.1029	−2.03
JPY	01/13–01/15	−1.4433	−1.84	1.9731	6.55	−0.0947	−1.84
ARDL Test							
		γ_0	t-statistic	γ_1	t-statistic	error correction	t-statistic
CAD	02/94–01/99	0.2345	1.23	0.6823	3.19	−0.0481	−2.14
CAD	01/03–03/13	−0.0239	−0.26	1.0723	5.28	−0.0907	−1.98
DEM	02/91–12/98	0.0730	0.59	0.9216	3.37	−0.0715	−2.24
EUR	01/99–01/05	−0.5526	−1.83	2.4682	4.09	−0.0993	−1.59
GBP	01/95–01/99	0.6102	1.61	0.7254	3.63	−0.4045	−3.46
GBP	01/07–01/13	0.0370	0.26	1.0114	6.03	−0.1943	−1.94
JPY	01/95–01/99	0.4536	1.54	−0.3993	−1.96	−0.7861	−1.81
JPY	01/13–01/15	−0.7265	−2.07	1.7438	7.40	−0.2029	−2.58

Dickey–Fuller tests presented in Table 1. This characteristic causes the OLS predicted standard errors unreliable. Therefore we use the Stock- Watson cointegration regression (a.k.a. Dynamic OLS Regression) below to estimate the relative sensitivity and correlation:

$$\log(p_t^{S^*}) = \gamma_0 + \gamma_1 \log(p_t^S) + \sum_{j=-k_1}^{+k_2} \gamma_j \Delta \log(p_{t-j}^S) + \varepsilon_t \qquad (26)$$

where $k1$ and $k2$ denote leads (future) and lags (past) that are equated and selected using the AIC. The advantage of this specification is that its OLS estimator is super consistent, and its confidence interval can be calculated with a normal t-distribution by using heteroscedasticity and serial correlation consistent standard errors.

From equation (26), the relative sensitivity of the stock prices can be measured by $(\gamma_1 - 1)$.[6] The significance of coefficient γ_1 can be an indicator of the significance of the correlation between the two countries' stock prices. We conduct rolling regressions on equation (25) and (26) to display the dynamics of β_1, γ_1 for each currency pair graphically. The window period selected for the regression is 36-month. The results are displayed in Figs. 1 through 5. Table 2 also reports the results of full sample estimate of equation (25) and (26).[7]

As we can see from the figures, both the key variables are not stable and appear to have system switches. In Table 3, we select sub-periods that seem to have relatively stable estimates based on the figures to show the estimates of equation (26). Furthermore, we apply the Johansen as well as ARDL[8] Cointegration tests as a robustness check.

3.2.2. Tests on the dependence of the exchange rate-stock price correlation

A straightforward way to test the dependence of the exchange rate-stock price correlation would be to regress β_1 on $(\gamma_1 - 1)$. Nevertheless, as displayed in the figures and confirmed by the results

[6] A more rigorous proof can be seen from Appendix.

[7] Lead and lag coefficients are insignificant in most cases and lack of economic implications. To simplify the table, we did not report these coefficients.

[8] The optimal number of lags in the ARDL test is automatically determined by the Stata.

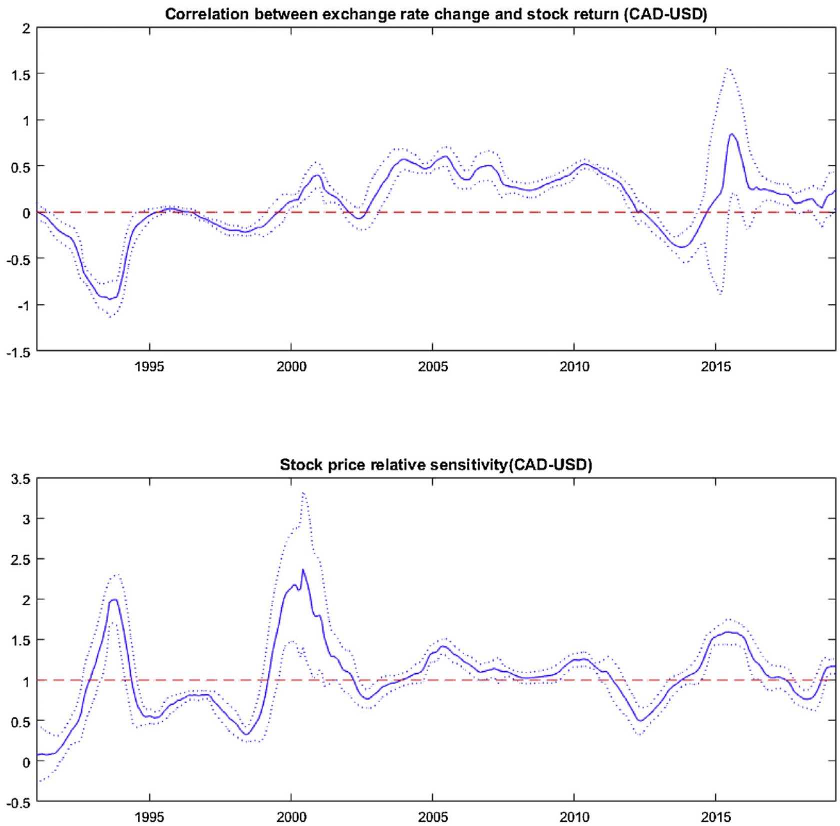


Fig. 1. CAD-USD. The solid lines represent estimated coefficients and dash lines define 95% confidence interval. The same for Figs. 2–5.

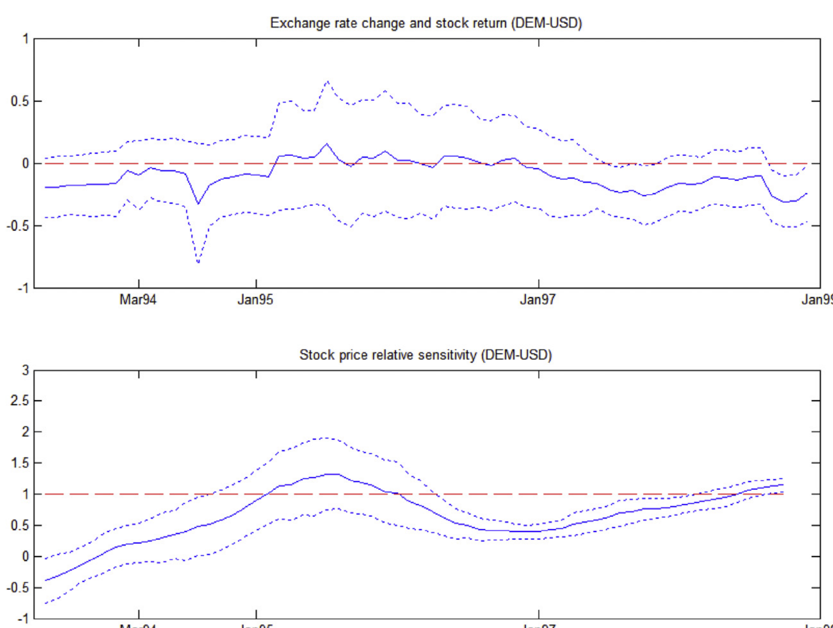


Fig. 2. DEM-USD.

of the Augmented Dickey–Fuller tests illustrated in Table 5, both the variables are non-stationary. To avoid spurious results, we also use the Stock-Watson cointegration test written below to test the relationship:

$$\beta_{1,t} = a_0 + a_1(\gamma_{1,t} - 1) + \sum_{j=-k_1}^{+k_2} a_j \Delta\gamma_{1,t-j} + \varepsilon_t \tag{27}$$

where $k1$ and $k2$ denote leads (future) and lags (past) equated and selected using AIC. Coefficient a_1 should be significantly positive if the implication of our model is correct. The results of this test are reported in Table 6.

Furthermore, we apply the Johansen and ARDL Cointegration tests to test the relationship between β_1 and $(\gamma_1 - 1)$ as a robustness check. Another issue with this test is that the variables on both sides of the equation are generated variables, which may cause gen-

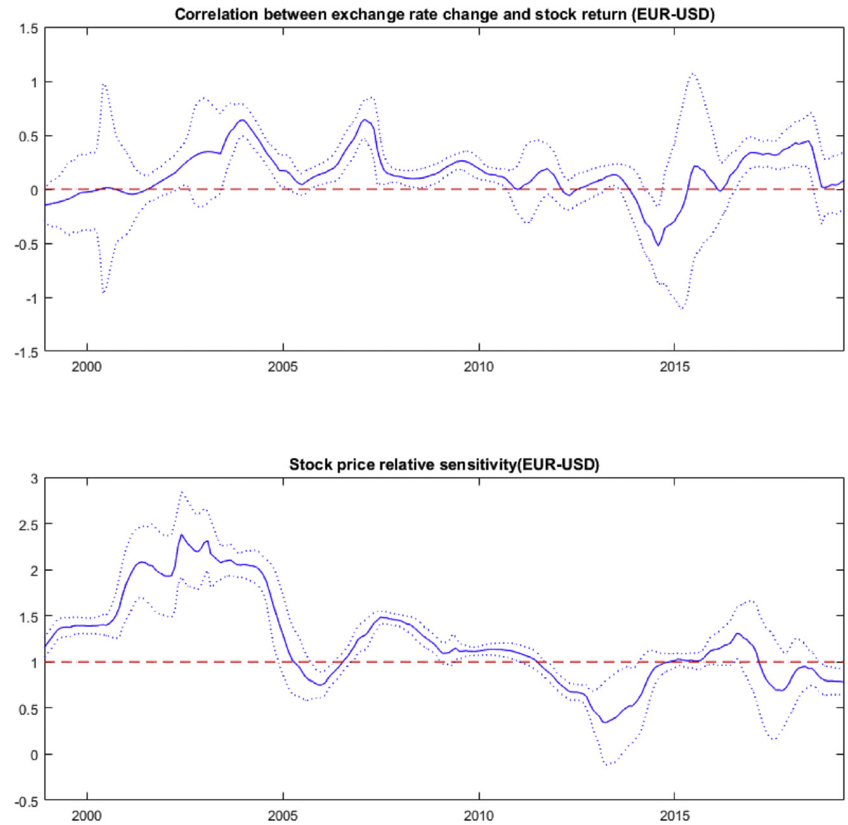


Fig. 3. EUR-USD.

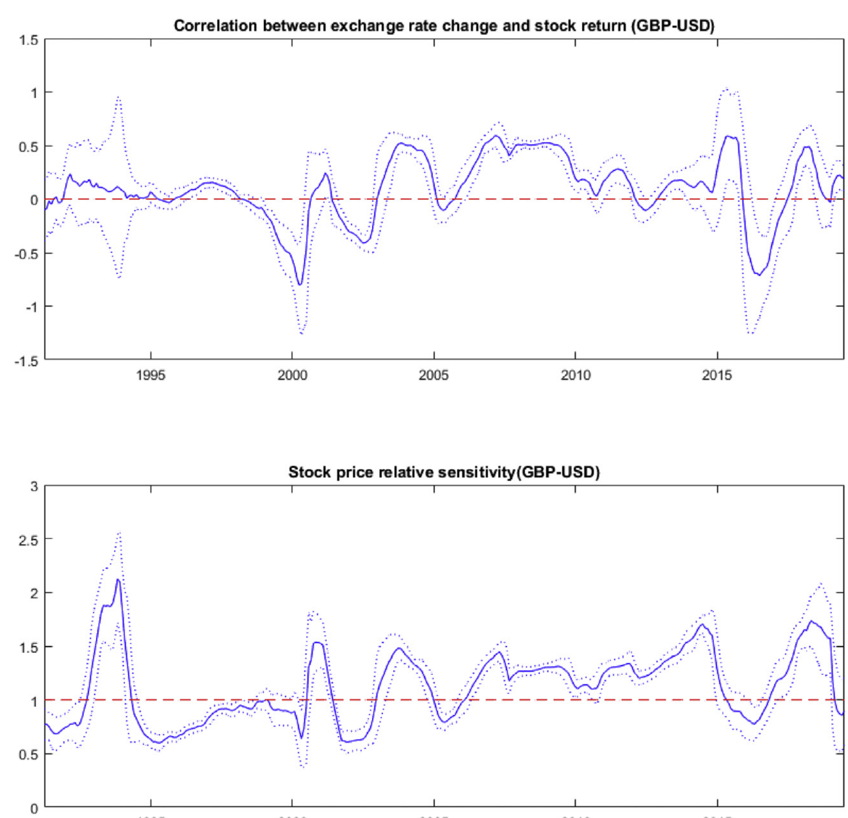
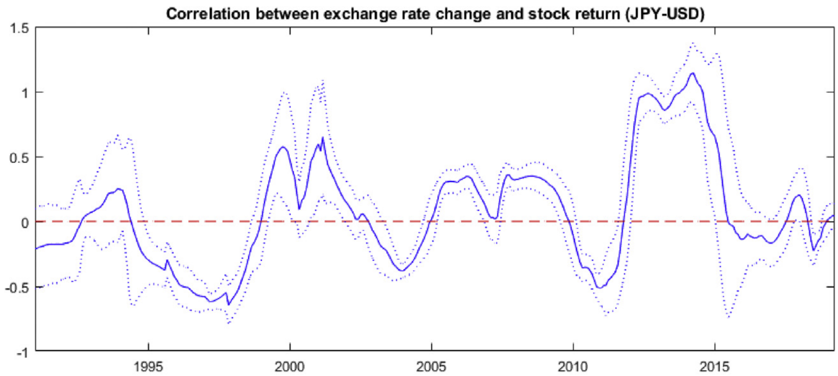


Fig. 4. GBP-USD.



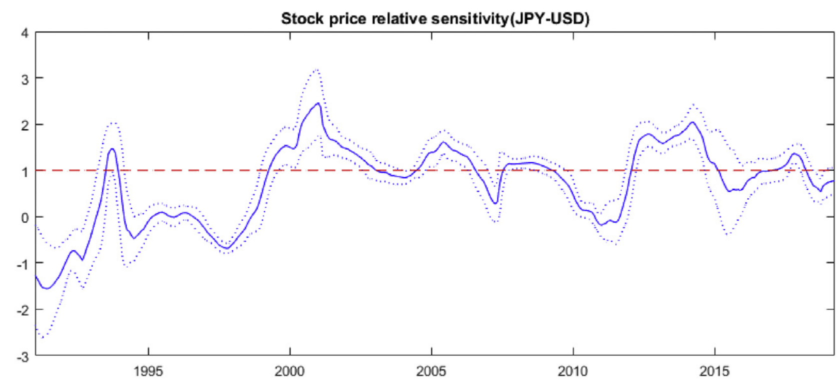


Fig. 5. JPY-USD.

Table 5
The Augmented Dickey–Fuller test for β_1 and γ_1.

	Test statistic Z(t)	10% Critical value	MacKinnon p-value for Z(t)
CAD β_1	−0.958	−2.570	0.7681
DEM β_1	−0.585	−2.579	0.9872
EUR β_1	−1.925	−2.570	0.3206
GBP β_1	−1.721	−2.570	0.4204
JPY β_1	−1.188	−2.570	0.6787
CAD γ_1	−2.234	−2.570	0.1943
DEM γ_1	−1.297	−2.579	0.6307
EUR γ_1	−1.383	−2.570	0.5902
GBP γ_1	−1.845	−2.570	0.3585
JPY γ_1	−2.317	−2.570	0.1666

Table 6
The Stock-Watson cointegration test on the dependence of the exchange rate -stock price correlation.

	CAD	DEM	EUR	GBP	JPY
a_0	0.0916	−0.0557	0.1098	0.0407	0.1754
t-statistic	5.03	−3.36	7.70	2.58	10.35
a_1	0.1974	0.0543	0.1320	0.4024	0.3619
t-statistic	4.29	1.66	4.86	8.27	18.06
Adjusted R^2	0.0952	0.1812	0.09	0.1861	0.5099
Durbin-Watson	0.0306	0.4521	0.0471	0.0451	0.0207

erated regressor bias in the estimation. To address this issue, we apply the bootstrap method to estimate standard errors as an additional robustness check. To simplify the table, only the results of the focused coefficients are reported in Table 7.

3.2.3. *Tests on the sign of the exchange rate-stock price correlation*

In order to test whether the sign of the correlation between exchange rates and stock prices depends on which country has relatively higher return in the stock market, we use the following Probit model:

$$signdummy_t = b_0 + b_1 HRCdummy_t + \varepsilon_t \tag{28}$$

where

$$signdummy_t = \begin{cases} 1 & \text{if} \quad \beta_{1,t} > 0 \\ 0 & \text{if} \quad \beta_{1,t} < 0 \end{cases}$$

$$HRC\,dummy_t = \begin{cases} 1 & \text{if} \quad \gamma_{1,t} > 1 \\ 0 & \text{if} \quad \gamma_{1,t} < 1 \end{cases}$$

Coefficient b_1 should be significantly positive if the conclusion of our model is true.

The variables on both sides of this test are generated variables, which may also cause generated regressor bias in the estimation. To address this issue, we apply the bootstrap method to estimate standard errors as an additional robustness check. The results of the tests are reported in Table 8.

3.2.4. *Tests on the significance of the exchange rate-stock price correlation*

Finally, to test whether the significance of the stock price-exchange rate correlation depends on the significance of the correlation between two countries' stock prices, we run the Stock-Watson cointegration test:

$$\sigma_t^{\beta_1} = c_0 + c_1\sigma_t^{\gamma_1} + \sum_{j=-k_1}^{+k_2} b_j \Delta\sigma_{t-j}^{\gamma_1} + \varepsilon_t \tag{29}$$

where $\sigma_t^{\beta 1}, \sigma_t^{\gamma 1}$ are the standard deviations of the estimates β_1, γ_1 respectively. We run the cointegration test because $\sigma_t^{\beta 1}, \sigma_t^{\gamma 1}$,

Table 7
Johansen and ARDL cointergation tests on the dependence of the exchange rates-stock price correlation.

Johansen Test					
	CAD	DEM	EUR	GBP	JPY
a_0	0.0004	−0.0245	0.1108	0.1464	0.2921
z-statistic without bootstrap	0.43	−2.38	1.45	2.62	6.82
z-statistic with bootstrap	0.56	−1.99	1.32	2.74	5.36
a_1	1.1720	0.0351	0.1471	0.4726	0.3820
z-statistic without bootstrap	4.53	2.13	1.70	2.13	4.87
z-statistic with bootstrap	4.32	2.42	1.86	2.09	5.01
Error Correction Coefficient	−0.0071	−0.0214	−0.0346	−0.0203	−0.0078
z-statistic without bootstrap	−3.67	−3.01	−4.74	−4.05	−5.79
z-statistic with bootstrap	−3.28	−2.79	−5.27	−3.45	−6.05
ARDL Test					
	CAD	DEM	EUR	GBP	JPY
a_0	0.0008	−0.0053	0.0314	0.0014	0.2159
t-statistic	0.80	−1.53	1.76	1.72	2.16
t-statistic with bootstrap	0.42	−1.27	1.88	1.47	2.34
a_1(LR coefficient)	0.6269	0.0187	0.1639	0.3075	0.2722
t-statistic	2.07	1.24	2.51	1.87	2.66
t-statistic with bootstrap	2.37	1.31	2.23	2.26	2.28
Error Correction Coefficient	−0.0094	−0.0194	−0.0275	−0.0362	−0.0187
t-statistic	−3.14	−2.89	−3.73	−5.22	−3.52
t-statistic with bootstrap	−2.78	−3.01	−3.58	−4.35	−3.98

Table 8
The Probit test on the sign of the exchange rate-stock price correlation.

	CAD	DEM	EUR	GBP	JPY
b_0	−0.2892	−1.0410	0.8245	−0.0085	−0.6303
z-statistics without bootstrap	−2.73	−4.65	5.28	−0.08	−6.49
z-statistics with bootstrap	−2.89	−4.72	4.12	−0.15	−6.01
b_1	1.2065	1.2640	−0.0552	0.9386	1.9003
z-statistics without bootstrap	8.10	3.33	−0.28	6.34	11.14
z-statistics with bootstrap	8.81	3.41	−0.37	5.78	9.87

Table 9
Augmented Dickey–Fuller test for ${\sigma_t}^{\beta}{}_1$ and ${\sigma_t}^{\gamma}{}_1$.

	Test statistic Z(t)	10% Critical value	MacKinnon p-value for Z(t)
CAD ${\sigma_t}^{\beta}{}_1$	−1.597	−2.570	0.4851
DEM ${\sigma_t}^{\beta}{}_1$	−1.278	−2.579	0.6392
EUR ${\sigma_t}^{\beta}{}_1$	−1.925	−2.570	0.3206
GBP ${\sigma_t}^{\beta}{}_1$	−2.156	−2.570	0.2225
JPY ${\sigma_t}^{\beta}{}_1$	−2.052	−2.570	0.2641
CAD ${\sigma_t}^{\gamma}{}_1$	−1.452	−2.570	0.5572
DEM ${\sigma_t}^{\gamma}{}_1$	−1.168	−2.579	0.6873
EUR ${\sigma_t}^{\gamma}{}_1$	−1.383	−2.570	0.5902
GBP ${\sigma_t}^{\gamma}{}_1$	−2.364	−2.570	0.1523
JPY ${\sigma_t}^{\gamma}{}_1$	−2.109	−2.570	0.2014

Table 10
Stock-Watson cointegration test on the significance of the exchange rate -stock price correlation.

	CAD	DEM	EUR	GBP	JPY
c_0	0.0692	0.0919	−0.0495	1.1407	0.5171
t-statistic	1.46	9.90	−0.61	2.61	8.45
c_1	0.0060	0.4247	0.1945	1.1239	0.4993
t-statistic	1.71	8.17	2.17	9.73	2.46
Adjusted R^2	0.0748	0.5154	0.0436	0.2537	0.1615
Durbin-Watson	0.0298	0.7462	0.0610	0.1061	0.1247

as confirmed in Table 9, are non-stationary. The results of this test are reported in Table 10.

Furthermore, we apply the Johansen as well as ARDL Cointegration tests as a robustness check. And also, since the variables on both sides of the equation are also generated variables, we use the bootstrap method to estimate standard errors. The results of these robustness tests are reported in Table 11.

3.3. Empirical results

3.3.1. Variables measurement

Figs. 1 through 5 display the dynamics of exchange rate-stock price correlation and relative sensitivity of foreign stock price relative to US stock price for each currency pair. In each figure, the upper sub-figure shows the dynamics of exchange rate change and stock return correlation (β_1). The lower sub-figure shows the dynamics of the relative sensitivity of two countries' stock prices (γ_1). The two dashed lines in each sub-figure indicate the 95% confidence interval for the estimates.

A few common patterns are noticeable: The exchange rate-stock price correlation tends to be negative in the late 1990s while it becomes positive after the 2001 financial crisis. Meanwhile, foreign stock prices appear to be less sensitive relative to the US stock prices in the late 1990s while they become more sensitive in the first decade of the 21st century. Furthermore, the magnitude of the two key variables seem to be correlated with each other to some extent. Moreover, the width of the two variables' confidence band, which reflects the significance level of each estimate, also appears to be somehow synchronized.

Intuitively, US stock prices had a faster growth in the late 1990s due to the booming IT industry, which made US stocks more attractive assets relative to foreign counterparts. A net investment on US stocks generated buying order toward the US dollar in the FX market and caused the Dollar to appreciate. Thus rising US stock prices were correlated with the appreciation of the USD in this period. In contrast, from 2003 through 2007, foreign stocks delivered relatively higher returns, which attracted FX order flow toward foreign currencies and caused the USD to depreciate. Thus rising US stock prices were correlated with a depreciation of the USD in this period.

As we can see from the figures, the correlation between exchange rate and stock price is significant in most periods but not stable and appear to have system switches. And because of this,

Table 11
Johansen and ARDL tests on the significance of exchange rate-stock price correlation.

Johansen Test					
	CAD	DEM	EUR	GBP	JPY
c_0	−0.0871	0.0424	0.1477	0.7311	0.4158
z-statistic without bootstrap	−0.03	0.72	8.71	2.35	6.09
z-statistic with bootstrap	−0.23	0.57	9.03	2.13	4.87
c_1	0.1214	0.5942	0.4202	1.2862	0.5950
z-statistic without bootstrap	2.62	8.92	2.22	7.99	12.74
z-statistic with bootstrap	2.27	9.21	2.16	7.35	13.24
Error Correction Coefficient	−0.0263	−0.0347	−0.0404	−0.0325	−0.0699
z-statistic without bootstrap	−4.60	−2.97	−3.74	−2.36	−5.40
z-statistic with bootstrap	−3.98	−2.14	−3.57	−2.71	−4.95
ARDL Test					
	CAD	DEM	EUR	GBP	JPY
c_0	0.0015	0.0024	0.0531	0.2671	0.2351
t-statistic	0.93	1.03	1.98	2..32	8.19
t-statistic with bootstrap	1.23	0.97	2.24	1.89	7.56
c_1(LR coefficient)	0.0431	0.6086	0.7649	1.0281	0.4720
t-statistic	1.92	6.84	2.43	3.15	7.51
t-statistic with bootstrap	2.31	7.24	2.87	3.41	6.79
Error Correction Coefficient	−0.0210	−0.0415	−0.0375	−0.0370	−0.0791
t-statistic	−3.60	−2.98	−3.07	−2.79	−5.95
t-statistic with bootstrap	−3.12	−2.58	−2.91	−2.98	−5.84

the estimates of β_1 using the full sample, reported in Table 2, does not seem highly significant. The relative sensitivity between foreign and domestic stock prices is also time-varying, but because it is highly significant so that the estimate of γ_1 using the full sample still generates significant results.

Since the estimate of γ_1 is obtained from regressing two non-stationary variables, we select sub-periods that seem to have relatively stable estimates based on the figures to verify the results. Table 3 reports the Stock-Watson test results, and as an additional robustness check, we also conduct the Johansen and ARDL cointegration tests. The results of γ_1 are highly significant in Table 3 and consistent with the results of rolling regressions shown in the figures. The results of the Johansen and ARDL tests, as reported in Table 4 confirm a significant long-run estimate of γ_1. Particularly, the error correction coefficients in these tests are significant in most cases, confirming the existence of a long-run equilibrium relationship.

3.3.2. *Tests on the dependence of the exchange rate-stock price correlation*

Table 6 reports the Stock-Watson test on the relationship between the exchange rates and stock price correlation (β_1) and relative loading of the foreign and domestic stock prices ($\gamma_1 - 1$) as specified in equation (27). According to the model, coefficient a_1 should be significantly positive if the exchange rate-stock price correlation is determined by the relative sensitivity of the foreign and domestic stock prices. As displayed in the table, coefficient a_1 is highly significant (over 1%) across currencies except DEM, which is still 10% significant.

Table 7 reports the results of the Johansen and ARDL cointegration tests. The additional robustness check also shows consistent results as the Stock-Watson test. In particular, the t-statistics generated with the bootstrap method are highly consistent with ones without the bootstrap, which suggests that the generated variables bias is ignorable in these tests. Also note that significant LR coefficient of a_1 and error correction coefficients in these tests suggest the existence of a long-run equilibrium relationship between the two non-stationary variables.

3.3.3. *Tests on the sign of the exchange rate-stock price correlation*

Table 8 reports the results of the Probit test (specified in equation (28)). According to our model, coefficient b_1 should be significantly positive if the sign of the exchange rates and stock price correlation is determined by which country has higher sensitivity in their stock prices. DEM, GBP, and JPY all have over 1% significantly positive b_1 and CAD has 10% significance level. As the correlation is mostly positive for EUR-USD, the coefficient is insignificant for this currency pair due to the singularity of the data. Considering that this test also uses generated variables, we use the bootstrap to estimate standard errors as an additional robustness check. As shown in Table 8, the results with and without bootstrap are highly consistent. These results suggest that when the foreign stock price is more sensitive relative to US stock price, it is significantly more likely the exchange rates and stock prices.[9]

3.3.4. *Tests on the significance of the exchange rate-stock price correlation*

The final test, specified as equation (29), examines whether the significance of the stock prices and exchange rates correlation (measured by the standard deviation of the correlation $\sigma_t^{\beta 1}$) depends on the significance of the correlation between two countries' stock prices (measured by the standard deviation of this correlation $\sigma_t^{\gamma 1}$). The results of the Stock-Watson test are reported in Table 10. The coefficient c_1 should be significantly positive if the conclusion of our model is true. According to the table, the coefficient is highly significant for all currencies except for EUR. Table 11 reports the Johansen and ARDL tests with and without bootstrap as an additional robustness check. Again, the results are consistent with the Stock-Watson test.

3.3.5. *Discussions*

These results are consistent with the existing empirical studies. (Kollias et al., 2012) report that the relationship between exchange rate and stock prices is time-varying. (Granger et al., 2000) and (Nieh & Lee, 2001) suggest the relationship is currency dependent. This paper confirms the features of time-variation and currency-

[9] Exchange rates are quoted as dollar rate of foreign currency correlation is positive.

[10] Lead and lag coefficients are insignificant in most cases and lack of economic implications. We do not report them to simplify the table.

[11] The error correction coefficient in the Johansen Test is the estimated coefficient from _ce1 line reported in Stata, the same for the other Johansen tests. γ_1 reported in the ARDL Test is the LR coefficient reported in Stata. The error correction coefficient in the ARDL Test is the estimated coefficient from ADJ line reported in Stata.

dependency. Furthermore, the paper proposes that these features can be explained by time-varying and currency- dependent relative stock returns through the mechanism of portfolio rebalancing between domestic and foreign stocks. The proposed explanation and mechanism are also consistent with (Katechos, 2011), which finds the sign of the correlation depends on relative interest rates, and (Ulklz & Demirci, 2012) who connect the relationship with the direction of international capital flow.

The mechanism proposed in the paper is also connected with early theories explaining the linkage between stock prices and exchange rates. Micro "flow-oriented" models argue that as the US Dollar appreciates, cash flow and profitability of American firms should improve, which leads to rising US stock prices. "Stock-oriented" theory, such as (Gavin, 1989), claims that a decrease in stock prices would reduce domestic wealth, lower demand of money and interest rate, and eventually causes capital outflow and a depreciation of the country's currency. This paper agrees with these earlier studies to contribute the linkage to capital flows driven by changing returns in financial assets. This paper's contribution, compared to the existing studies, is to argue that the relative return of financial assets between different countries is time-varying and currency-dependent, which further causes the linkage time-varying and currency-dependent.

4. Conclusion

Recognizes the time-variation and currency-dependency in the interaction between the exchange rates and stock prices, this paper develops a model to accommodate such characteristics. The model stems from the major finding of the FX market microstructure research that the exchange rate is driven by customer order flow in which financial customers dominate. The stock market shocks drive the rebalance of institutional investors, i.e., financial customers, between domestic and foreign stocks, which triggers the order flow in the foreign exchange market and further affects the exchange rate.

We conclude that the correlation between exchange rates and stock prices is determined by the relative sensitivity of two countries' stock prices to the common stock factor. Depending on various market conditions, the relative sensitivity is time-varying and currency- dependent, so is the correlation between exchange rates and stock prices. In specific, rising US stock prices are associated with the appreciation (depreciation) of the Dollar when domestic (foreign) stock prices have higher sensitivity. Furthermore, the significance of the correlation depends on how closely two countries' stock prices are correlated.

The paper has great implications for international portfolio management. Fund managers managing foreign stocks inevitably have to bear exchange risks. In the financial world, risk usually refers to volatility, but if the direction of the change serves our interests, volatility is not necessarily a bad thing. Our findings in this paper suggest that if foreign stocks have a higher sensitivity relative to domestic stocks, investing in foreign stocks not only can gain from rising prices in a bullish market but also from the appreciation of the foreign currency. On the contrary, if domestic stocks have a higher sensitivity, foreign currency is more likely to depreciate in a bullish stock market, and such a depreciation will erode capital gain from the investment in foreign stocks.

Future research can develop in three directions. In terms of theory, the model proposed in the paper can accommodate exchange rates between currencies of major developed countries, while how differently the mechanism would work for emerging economies deserves more attention. Empirically, the supporting evidences presented in this paper are based on market data on an aggregate level. It would be helpful for future research to use micro-level data, such as hedge funds portfolio allocation data, to test our proposed mechanisms. In terms of the application of the proposed mechanism in the practice of investment, incorporating the time-varying and currency- dependent interrelations between exchange rates and stock prices is undoubtedly critical for fund managers who oversee international portfolios.

Appendix

In this appendix, we provide our rational of using $(\gamma_1 - 1)$ as the measurement for the relative sensitivity of the stock prices. The dynamics of the stock prices given in the model are:

$$\log p_t^S = \rho \log p_{t-1}^S + l_s F_t^s + \eta_t$$
$$\log p_t^{S^*} = \rho^* \log p_{t-1}^{S^*} + l_s^* F_t^s + \eta_t^*$$

And the regression equation is

$$\log(p_t^{S^*}) = \gamma_0 + \gamma_1 \log(p_t^S) + \sum_{j=-k_1}^{+k_2} \gamma_j \Delta \log(p_{t-j}^S) + \varepsilon_t$$

therefore

$$\gamma_1 = \frac{Cov(\log p_t^{S^*}, \log p_t^S)}{Var(\log p_t^S)}$$
$$\gamma_1 = \frac{Cov(\rho^* \log p_{t-1}^{S^*} + l_s^* F_t^s + \eta_t^*, \rho \log p_{t-1}^S + l_s F_t^s + \eta_t)}{Var(\rho \log p_{t-1}^S + l_s F_t^s + \eta_t)}$$

Denote C as $Cov(\log p_t^{S*}, \log p_t^S)$, V as $Var(\log p_t^S)$, V_F as $Var(F_t^S)$,V_F as $Var(\eta_t)$, then,

$$\gamma_1 = \frac{C}{V} = \frac{\rho^* \rho C + l_s^* l_s V_F}{\rho^2 V + l_s^2 V_F + V_\eta}$$
$$\Leftrightarrow \rho^* \rho CV + l_s^* l_s V_F V = \rho^2 CV + l_s^2 CV_F + CV_\eta$$
$$\Leftrightarrow (\rho^* \rho - \rho^2) CV + l_s^* l_s V_F V - l_s^2 CV_F - CV_\eta = 0$$
$$\Leftrightarrow (\rho^* \rho - \rho^2)\gamma_1 + l_s^* l_s \frac{V_F}{V} - l_s^2 \gamma_1 \frac{V_F}{V} - \gamma_1 \frac{V_\eta}{V} = 0$$
$$\Leftrightarrow \gamma_1 [(\rho^* \rho - \rho^2) - l_s^2 \frac{V_F}{V} - \frac{V_\eta}{V}] = -l_s^* l_s \frac{V_F}{V}$$

Therefore,

$$\gamma_1 = \frac{l_s^* l_s V_F}{l_s^2 V_F + V_\eta - (\rho^* \rho - \rho^2) V}$$

In the equation above, since stock prices are highly consistent and contain unit-root, so the autoregressive coefficient ρ^*, ρ are often very close to 1, hence $\rho^* \approx 1$, $\rho \approx 1$. Also, since the both stock prices are significantly correlated with each other, it implies that the common factor F_t are more dominant than the country's individual factor η_t, so we can assume $V_F \gg V_\eta$. Therefore,

$$\gamma_1 \approx \frac{l_s^*}{l_s}$$
$$\gamma_1 - 1 \approx \frac{l_s^* - l_s}{l_s}$$

Thus, $(\gamma_1 - 1)$ can approximate the relative sensitivity of the stock prices.

Declaration of Competing Interest

The authors whose names are listed immediately below certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript

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