UNIVERSIDAD AUTONOMA DE MADRID

Escuela Politécnica Superior

Single Neuron Modeling:

- Characterize, implement and analyze conductance based models
- Characterize, implement and analyze simplified dynamical models and integrate & fire models
- Characterize, implement and analyze rate models

Neural modeling scales

top-down models

- Abstract models
- Integrate and fire models
- Simplified dynamical models
- Hodgkin-Huxley type coductance based models

bottom-up models

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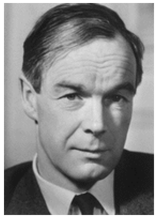
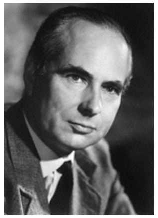
realism

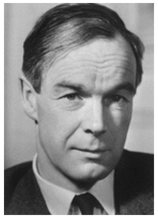
+



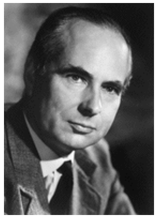
Realistic models of membrane: the HODGKIN-HUXLEY model (H-H):



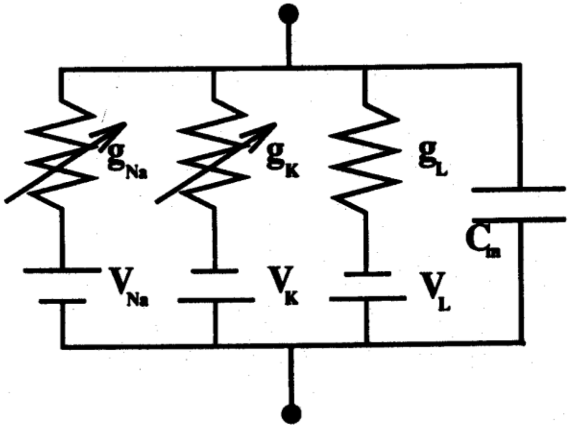
http://es.youtube.com/watch?v=k48jXzFGMc8



Realistic models of membrane: the HODGKIN-HUXLEY model (H-H):



Voltage-gated conductances are described through the following simple equivalente circuit



τ_x = time constant

$\bar{x} \equiv x_\infty$ = steady state value

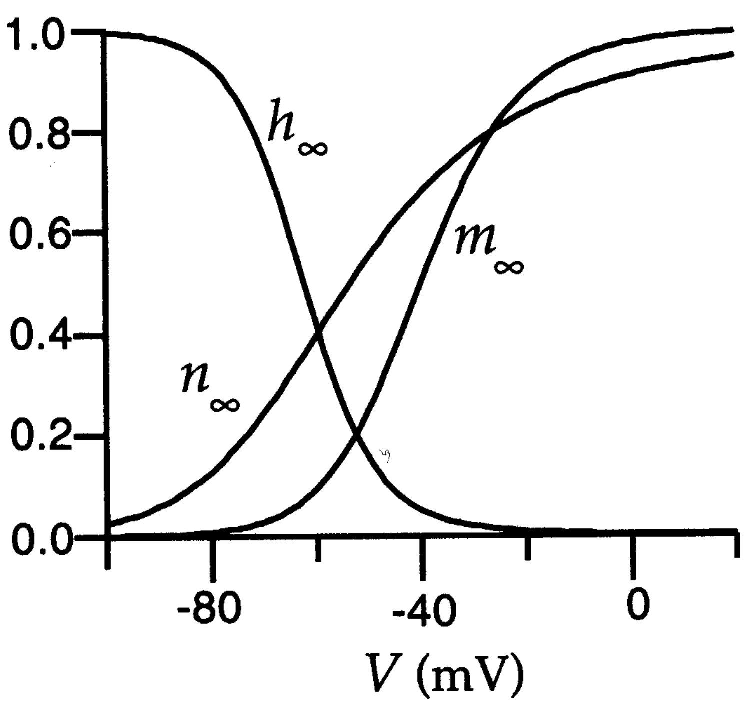
$x = (h,m,n)$

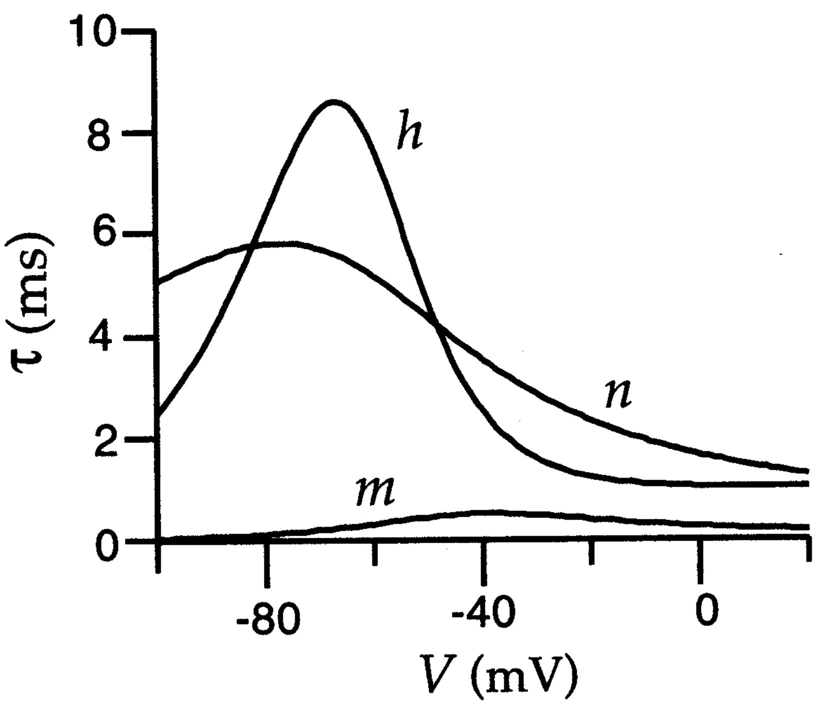
$$
\begin{aligned}
C_m \frac{dV}{dt} &= I_{ext} - g_L(V - V_L) - g_{Na} h m^3 (V - V_{Na}) - g_K n^4 (V - V_k) \\
\tau_h(V) \frac{dh}{dt} &= \bar{h}(V) - h \\
\tau_m(V) \frac{dm}{dt} &= \bar{m}(V) - m \\
\tau_n(V) \frac{dn}{dt} &= \bar{n}(V) - n
\end{aligned}
$$

The original H-H model (squid giant axons) consists of only two major currents, transient Na and persistent K. Most neurons in the nervous system have additional currents with diverse activation inactivation dynamics

HODGKIN-HUXLEY model

Steady state values and time constant depend on the membrane potential





HH equations can be rewritten in terms of α_x, β_x or a_x and b_x (rate constants that depend on V)

$x_\infty = a_x(V) / (a_x(V) + b_x(V))$ y $\tau_x = 1 / (a_x(V) + b_x(V))$

$dn/dt = a_n(V)(1-n) - b_n(V)n$

$dm/dt = a_m(V)(1-m) - b_m(V)m$

$dh/dt = a_h(V)(1-h) - b_h(V)h$

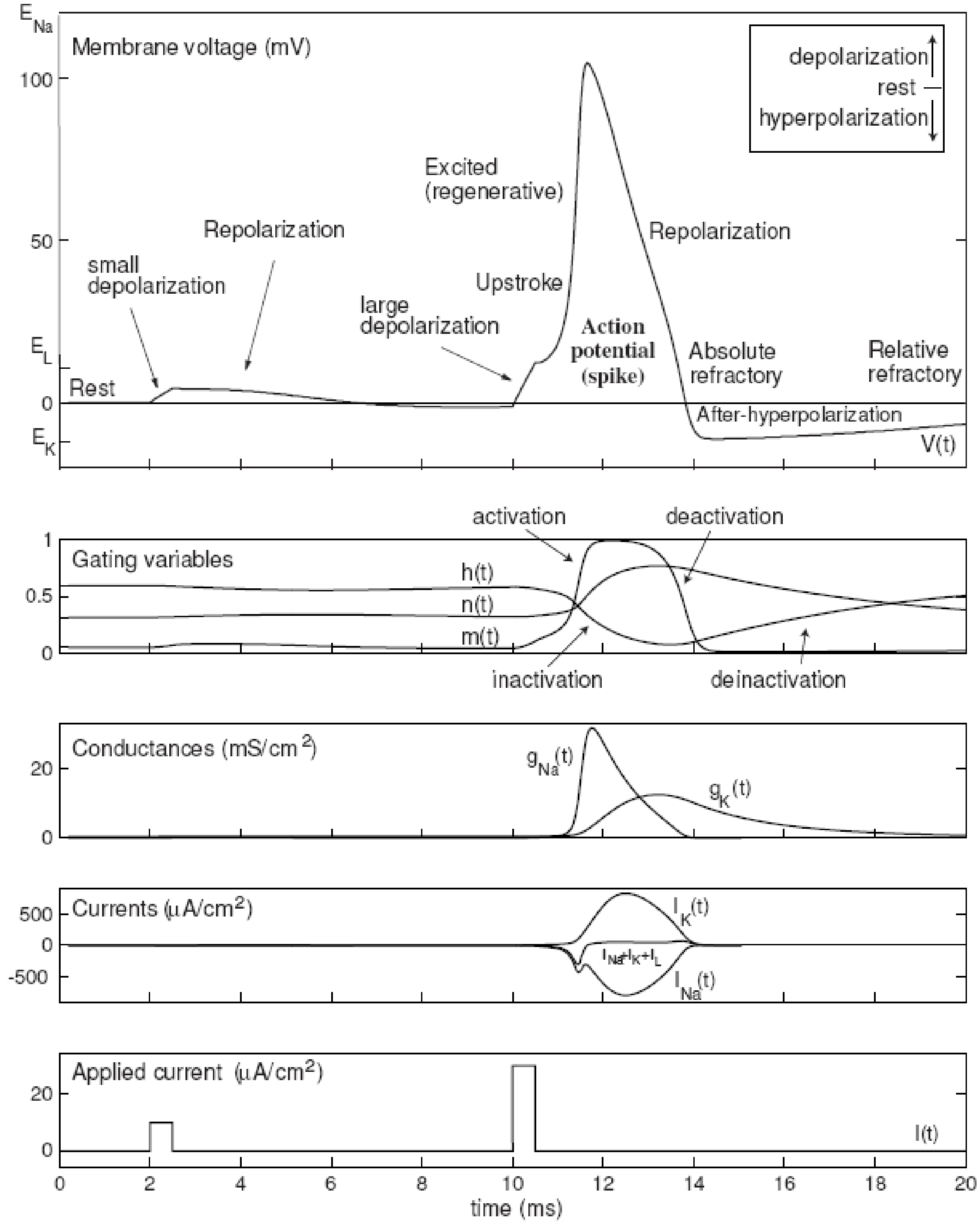
Their dependence on the voltage is obtained experimentally:

$a_n = (0.01(V+55))/(1-\exp(-0.1(V+55)))$ $b_n = 0.125\exp(-0.0125(V+65))$

$a_m = (0.1(V+40))/(1-\exp(-0.1(V+40)))$ $b_m = 4.00\exp(-0.0556(V+65))$

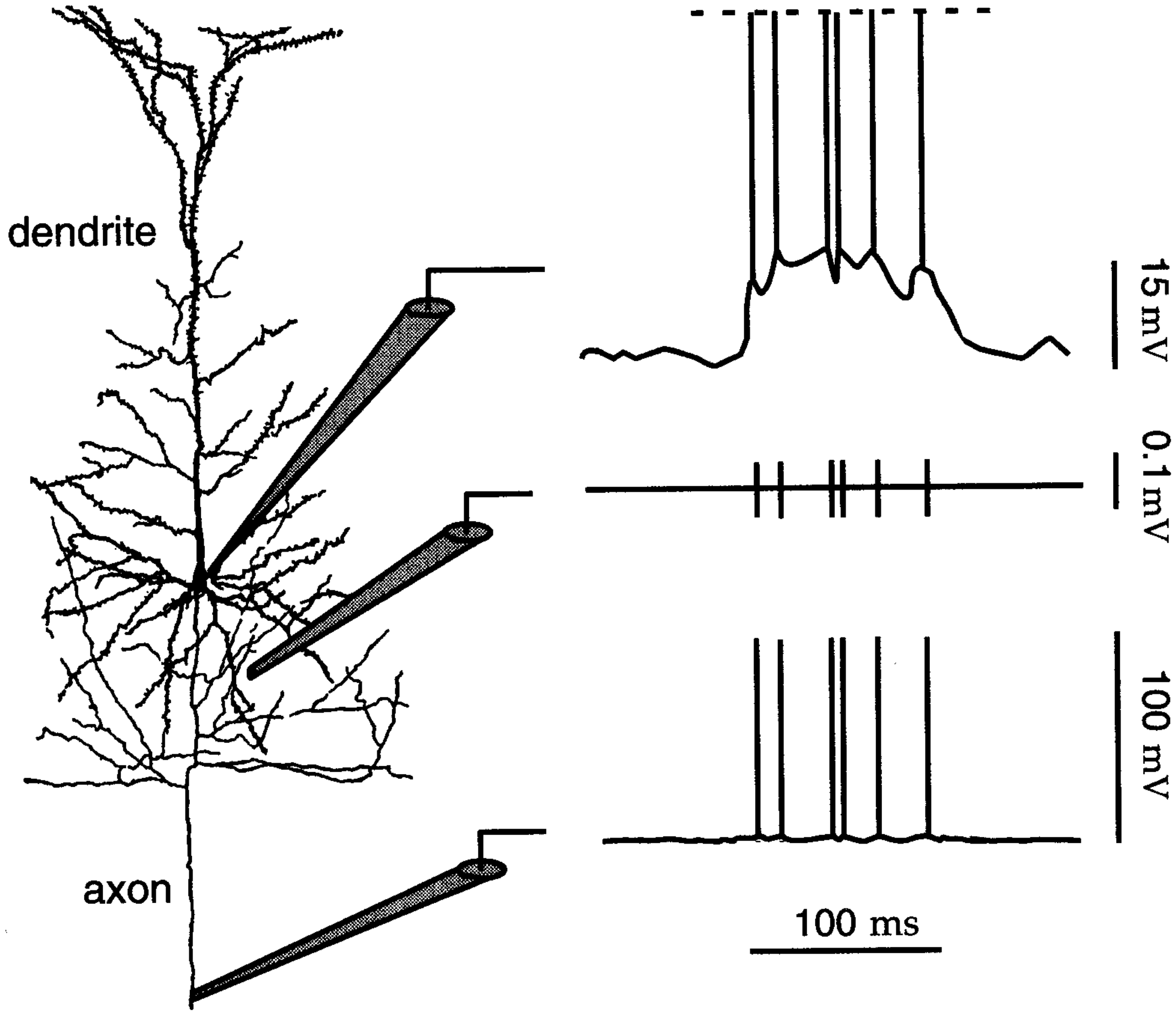
$a_h = 0.07\exp(-0.05(V+65))$ $b_h = 1.0/(1+\exp(-0.1(V+35)))$

H-H MODEL: Generation of a spike



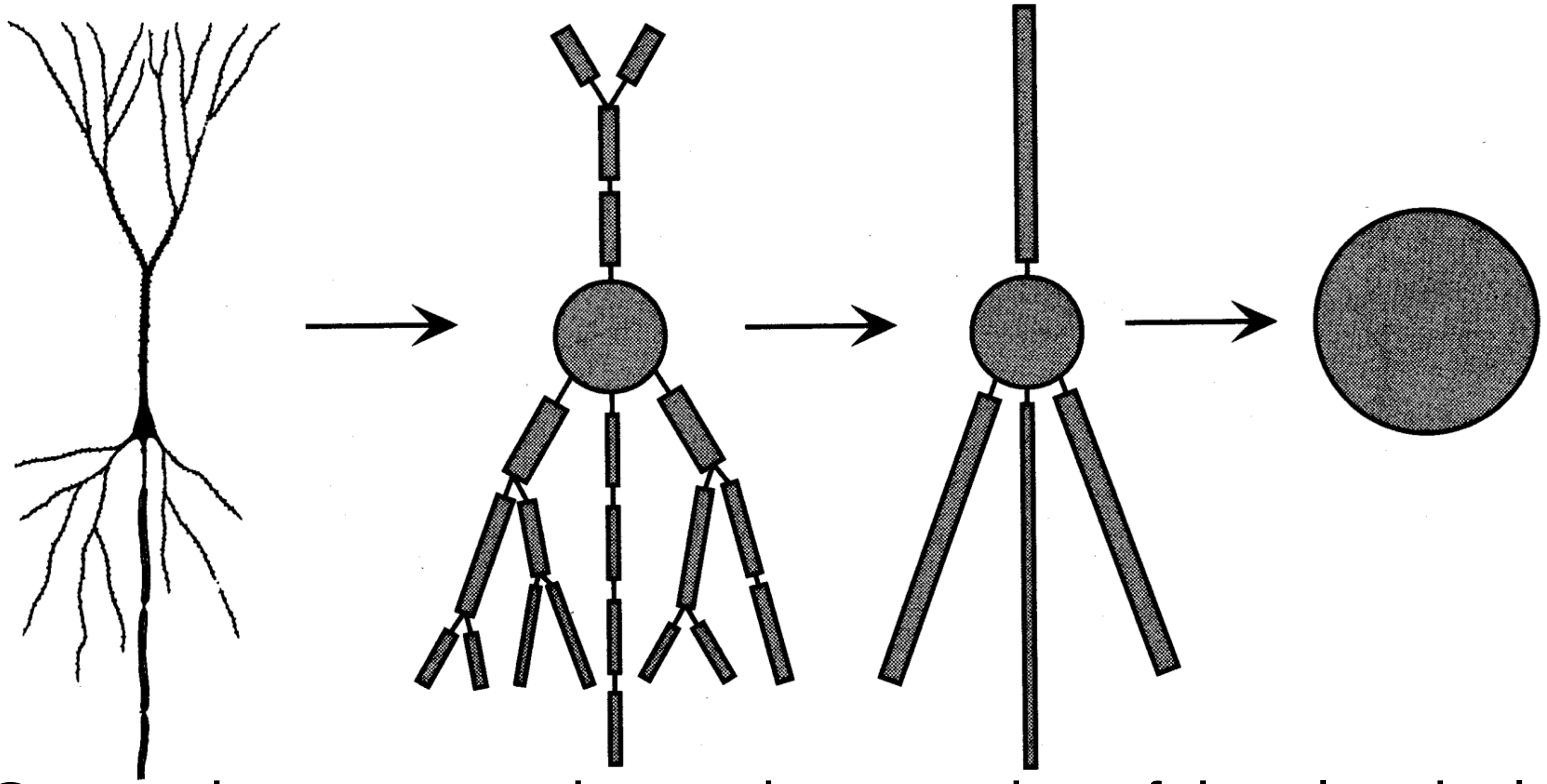
Multi-compartment neuron models

The electrical activity is different at different membrane locations



Multi-compartment neuron models

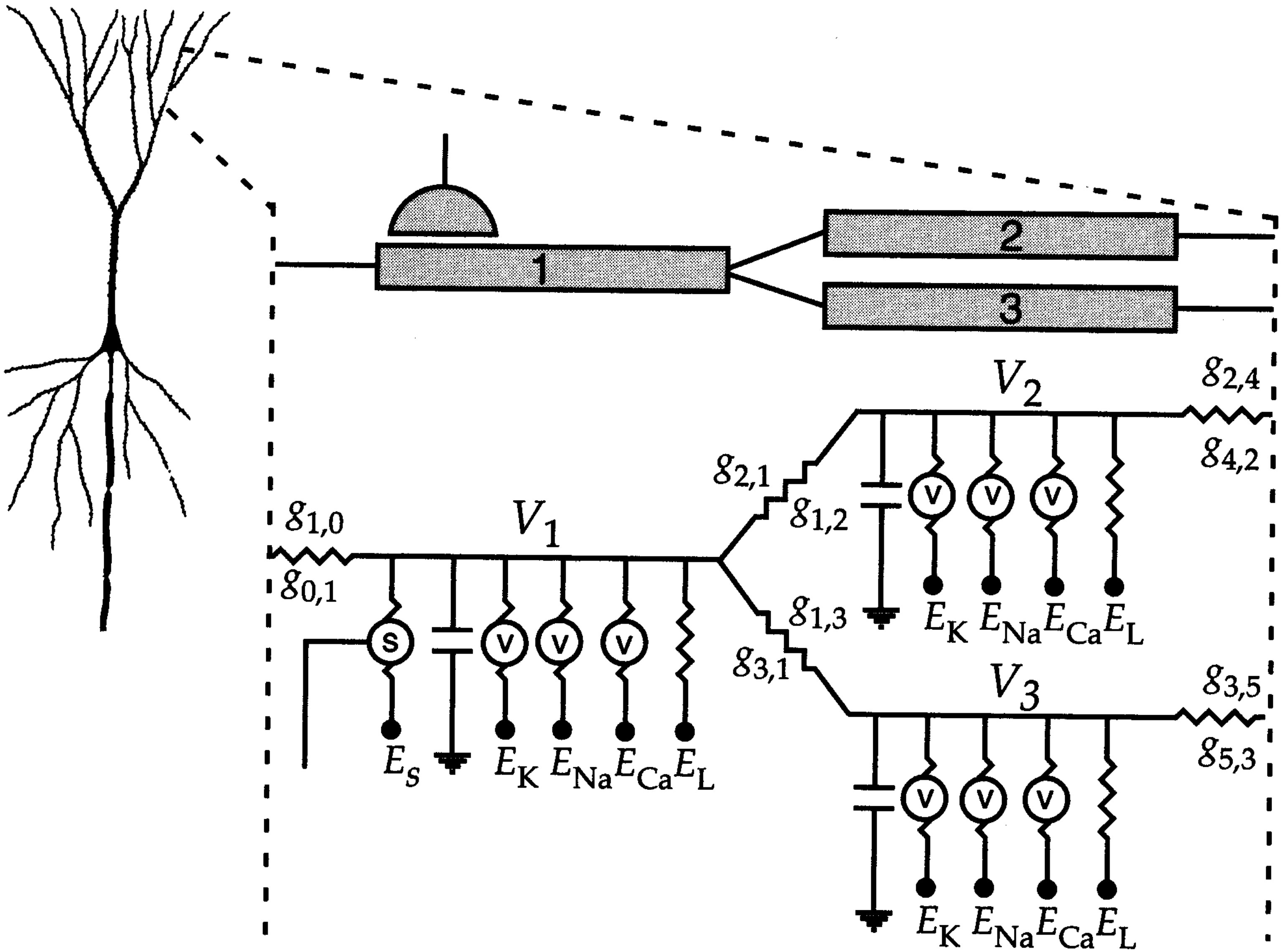
Neurons consist of different structural components: axon, soma, dendrites. For example, many neurons have an extensive dendritic tree



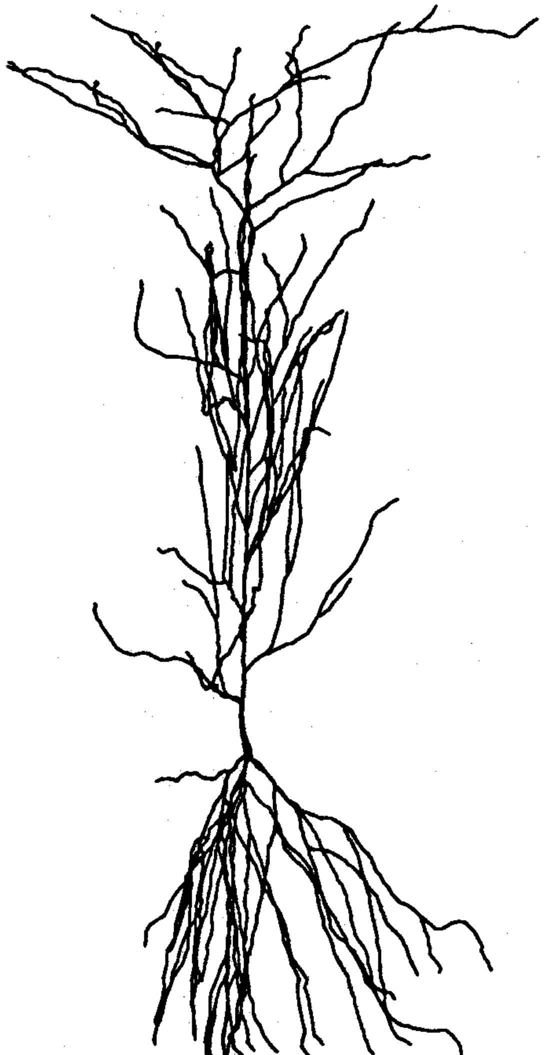
Generation, propagation and processing of the electrical activity can be modeled by using different compartments

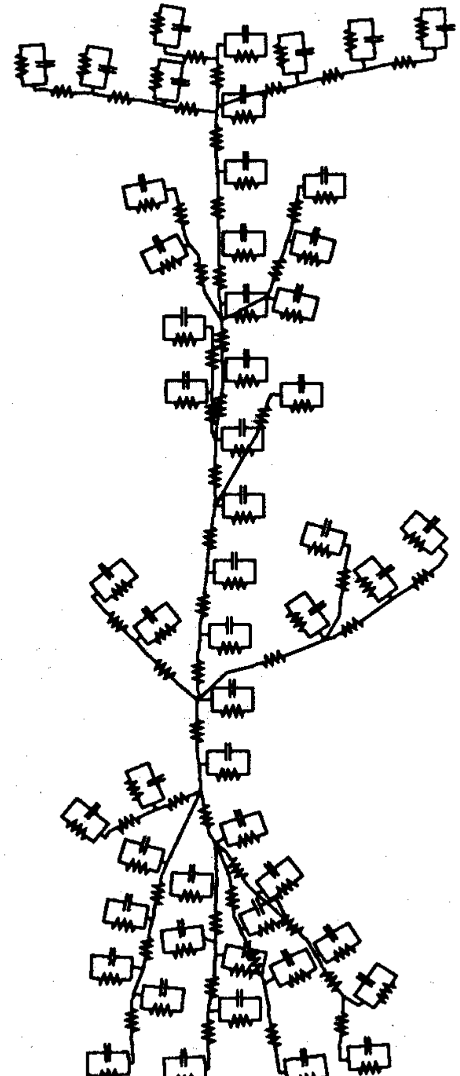
Each compartment can be described by its own equivalent circuit

Each compartment can have active properties for a complex information processing

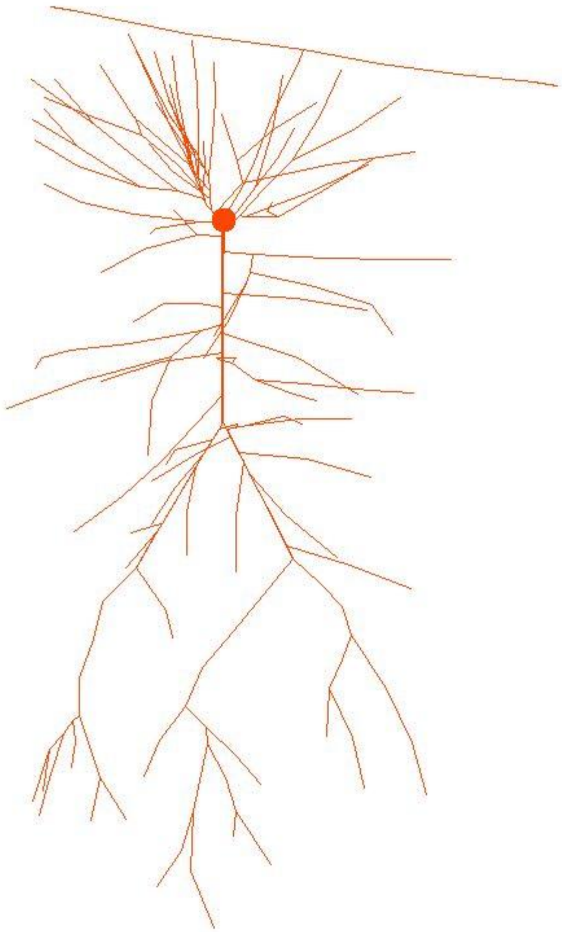


The most realistic models consider this distributed processing within each neuron



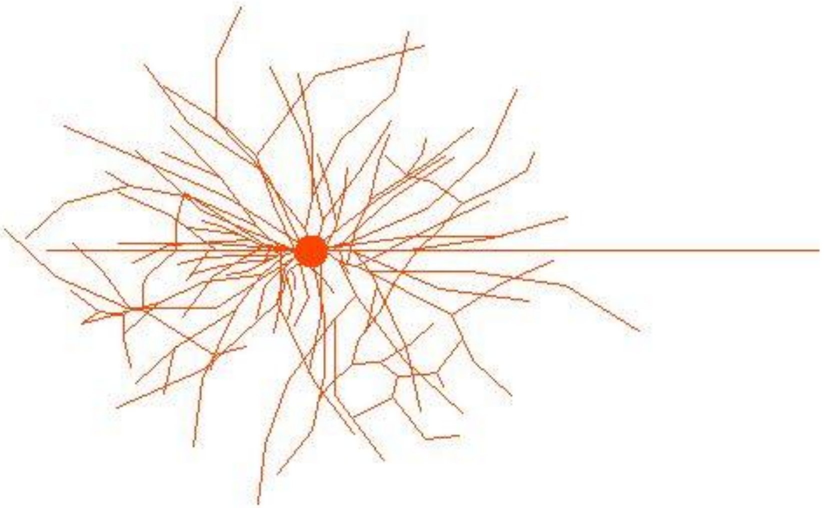


Example: realistic simulation (morphology and physiology) of a CA1 hippocampus neuron



265 compartment

It reproduces the 3D neuron morphology



Example: realistic simulation (morphology and physiology) of a CA1 hippocampus neuron

- Each compartment has an specific distribution of channels
- Note that this H-H type model includes more ionic currents

Ca channel:

$I_{Ca} = g_{Ca}s^2r(V_m-E_{Ca})$

$E_{Ca} = -13.3\mathrm{Ln}([Ca,1]_{in}/1200)$

$alpha_s = -0.16(V_m+26.0)/\{\exp[(V_m+26)/-4.5]-1\}$

$beta_s = 0.04(V_m+12)/\exp[(V_m+12)/10]-1\}$

$alpha_r = 2/\exp[(V_m+94)/10]$

$beta_r = 8/\{\exp[(V_m-68)/-27]+1\}$

K_AHP channel:

$I_{AHP} = g_{AHP}q(V_m-E_K)$

$E_K = -85$ mV

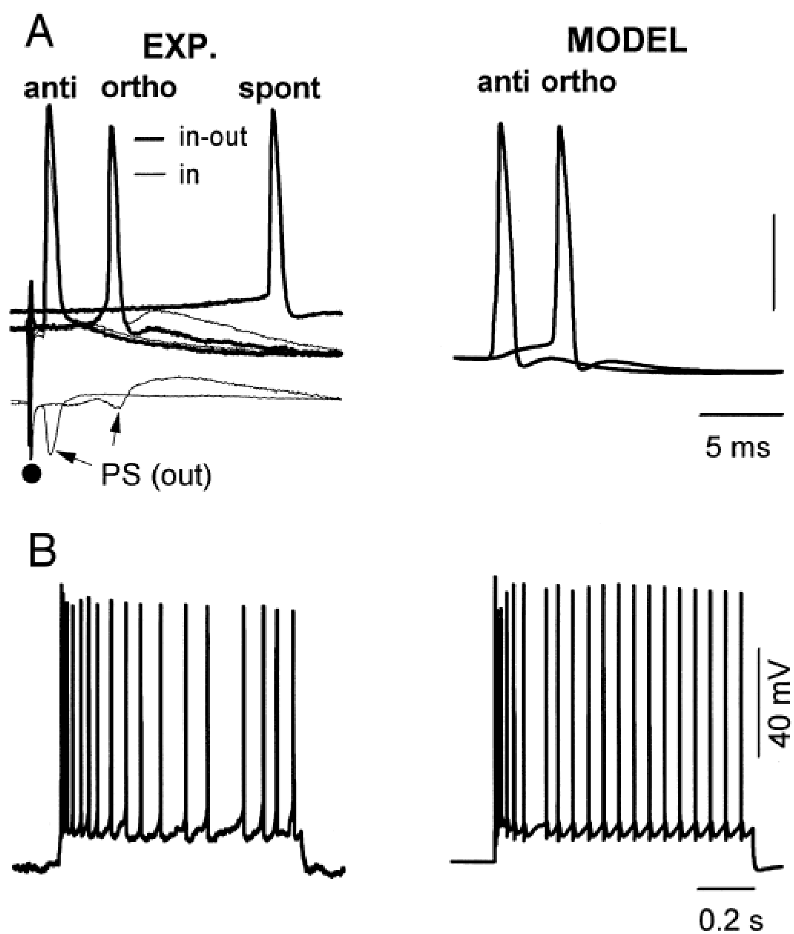
$alpha_q = 0.0048/\exp[(10\mathrm{Log}([Ca,2]_{in}-35)/-2]$

$beta_q = 0.012/\exp[(10\mathrm{Log}([Ca,2]_{in}+100)/5]$

$tau_q = 48$ ms

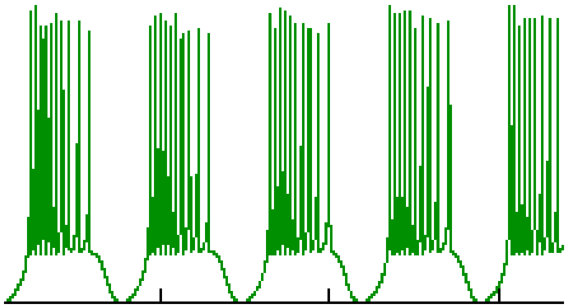
$$\frac{d[Ca,i]_{in}}{dt} = \frac{[Ca,i]_{in}}{\tau_i} - \frac{f_i I_{Ca}}{wzFA}$$

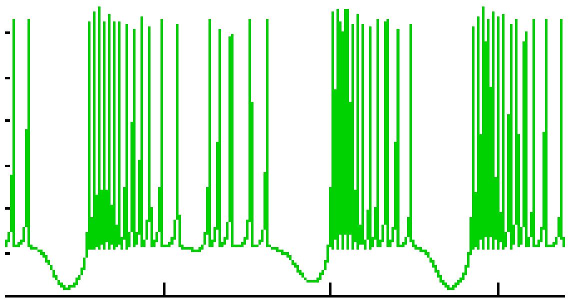
Example: realistic simulation (morphology and physiology) of a CA1 hippocampus neuron



Simplified dynamical models

- It imitates the generation of action potentials by H-H type models but a simpler manner.
- The evolution law is also given by a differential equations system, but with an smoothing nonlinearity degree (exponential vs. polynomial).
- They reduce the number of variables.
- They have a lower computational cost.





Simplified dynamical models

- FitzHugh-Nagumo
- Morris-Lecar
- Hindmarsh-Rose
- Rinzel, etc…

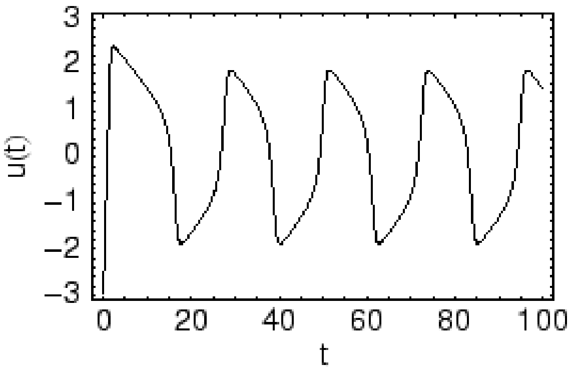
• The integration time is lower and they can reproduce the same bifurcations that the more realistic models.

• They allow to perform theoretical analysis.

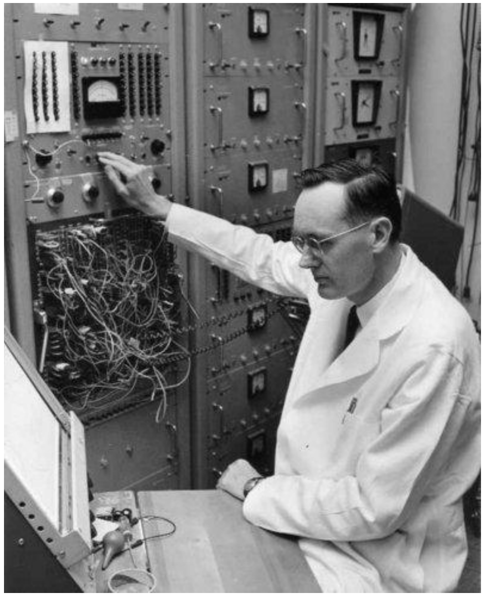
FitzHugh-Nagumo model (FitzHugh 1961, Nagumo 1962)

$$\frac{du}{dt} = u - \tfrac{1}{3}u^3 - w + I \qquad (\tau = 1)$$

$$\frac{dw}{dt} = \varepsilon(b_0 + b_1 u - w) \qquad (\varepsilon = 1/\tau_w)$$



u(t) represents the membrane voltage and *w(t)* mimics activation of an outward current.

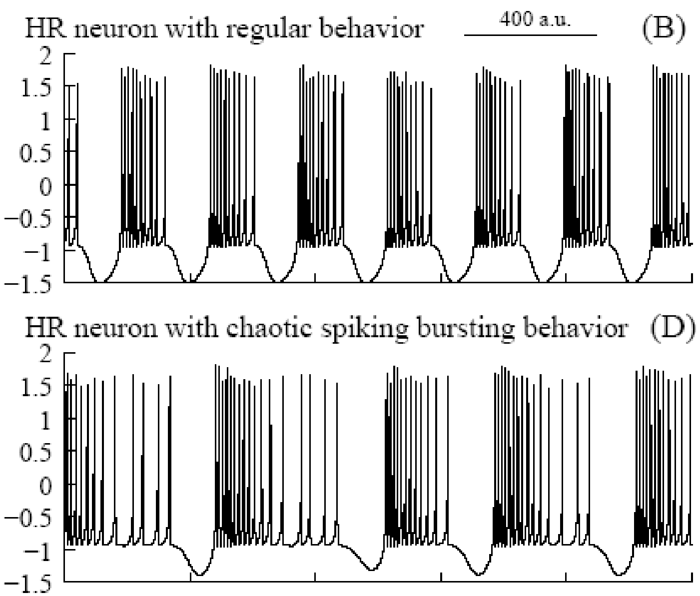


R. Fitzhugh en 1960

Hindmarsh-Rose model (Hindmarsh and Rose 1982)

$$\frac{dx_i(t)}{dt} = y_i(t) + 3x_i^2(t) - x_i^3(t) - z_i(t) + e_i$$

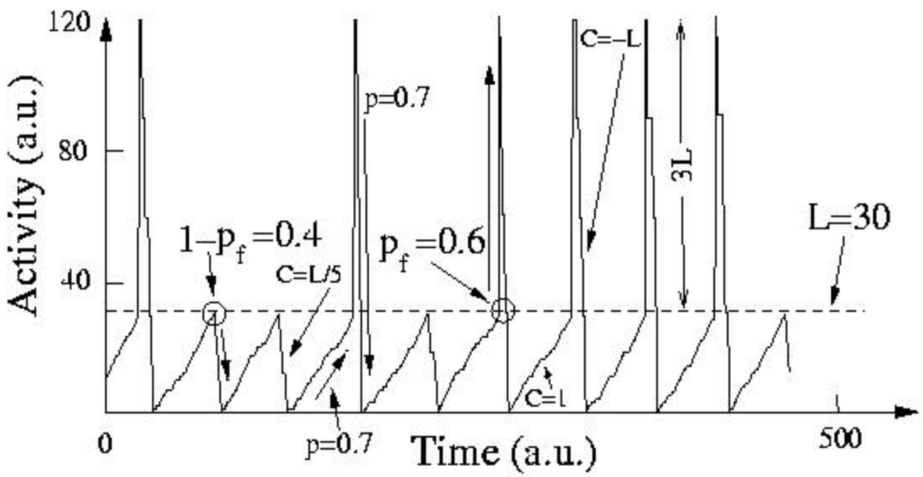
$$\frac{dy_i(t)}{dt} = 1 - 5x_i^2(t) - y_i(t), \quad \frac{1}{\mu}\frac{dz_i(t)}{dt} = -z_i(t) + S\left[x_i(t) + 1.6\right],$$



x(t) denotes the membrane potential, *y(t)* is a fast current and *z(t)* is a slow current

Integrate and fire models

- They describe a variable that simulates the subthreshold membrane potential until a threshold is reached.
- When the threshold is reached the neuron fires an instant action potential in an all-or-none manner.
- After the spike, the variable continues with the subthreshold evolution
- There are many types of integrate and fire models: finite and cellular automatas, stochastic models, realistics with a subthreshold membrane potential description, etc.
- They are optimal options (from the computational cost point of view) to simulate large networks



Abstract models

- They haven't any real parameter. They are based on cualitative observations in biological experiments:
  - Oscillations, activity synchronization, self-organization,
  - Realistic conectivity,
  - Learning rules with biological inspiration.

- They usually allow a theoretical analysis to predict a general behavior that depends on network topology, neuron model or the learning rule.