How do you evaluate the sum of n/(2^n) from n=1 to infinity?

7 Answers

Lai Johnny, M. Phil Mathematics Major, The Chinese University of Hong Kong (1985)
Answered 11 months ago · Author has **3.6K** answers and **1.1M** answer views

Method 1(By Differentiation)

Consider $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$, where $|x| < 1$.

Then differentiating both sides w.r.t. x, we have

$$\sum_{n=1}^{\infty} nx^{n-1} = \frac{1}{(1-x)^2}$$

Putting $x = \frac{1}{2}$ gives

$$\sum_{n=1}^{\infty} \frac{n}{2^{n-1}} = \frac{1}{(1-\frac{1}{2})^2} = 4$$

$$\frac{1}{2}\sum_{n=1}^{\infty} \frac{n}{2^{n-1}} = 2$$

$$\sum_{n=1}^{\infty} \frac{n}{2^n} = 2$$

Method 2(By Shifted difference)

Let $S_n = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots + \frac{n}{2^n}$. Then

$$\frac{1}{2}S_n = \frac{1}{2^2} + \frac{2}{2^3} + \ldots + \frac{n-1}{2^n} + \frac{n}{2^{n+1}}$$

$$\therefore \frac{1}{2}S_n = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots + \frac{1}{2^n} - \frac{n}{2^{n+1}}$$

$$\therefore \quad S_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^{n-1}} - \frac{n}{2^n}$$

$$\sum_{n=1}^{\infty} \frac{n}{2^n}$$

$$= \lim_{n\to\infty}\left(1 + \frac{1}{2} + \frac{1}{2^2} + \cdots \frac{1}{2^{n-1}}\right) - \lim_{n\to\infty}\frac{n}{2^n}$$

$$= \frac{1}{1-\frac{1}{2}} - 0$$

$$= 2$$

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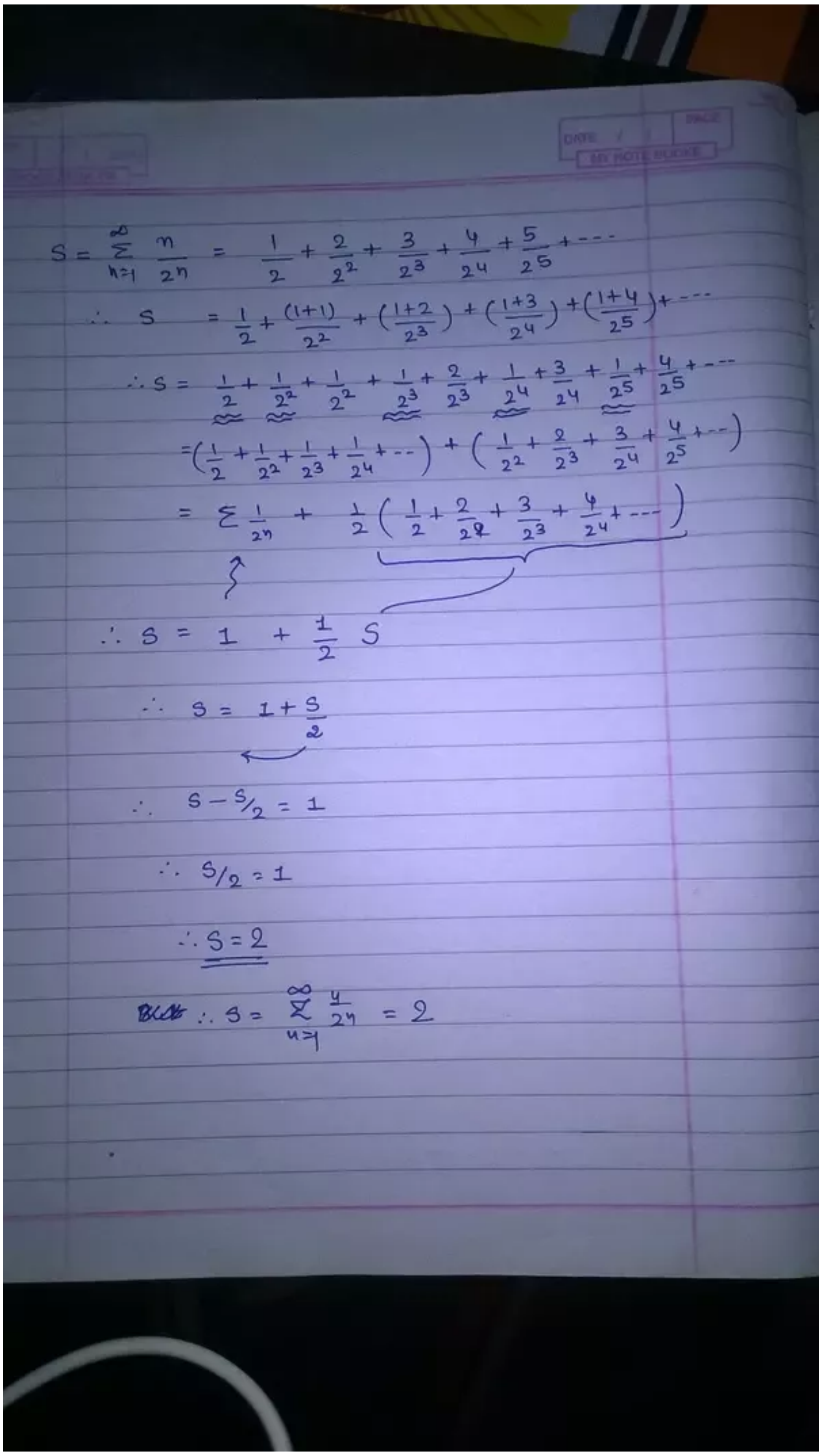
What is the sum, as n goes to infinity, of 1/2^n?

How do I do the sum of Sigma 2^n?

Answered 4 years ago

Sorry Friends, i am little lazy in typing mathematical steps, that's why I'm uploading the solution as calculated here.

If you found any problem, feel free to contact me



$$S = \sum_{n=1}^{\infty} \frac{n}{2^n} = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \frac{5}{2^5} + \cdots$$

$$\therefore S = \frac{1}{2} + \frac{(1+1)}{2^2} + \left(\frac{1+2}{2^3}\right) + \left(\frac{1+3}{2^4}\right) + \left(\frac{1+4}{2^5}\right) + \cdots$$

$$\therefore S = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{2}{2^3} + \frac{1}{2^4} + \frac{3}{2^4} + \frac{1}{2^5} + \frac{4}{2^5} + \cdots$$

$$= \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots\right) + \left(\frac{1}{2^2} + \frac{2}{2^3} + \frac{3}{2^4} + \frac{4}{2^5} + \cdots\right)$$

$$= \sum \frac{1}{2^n} + \frac{1}{2}\underbrace{\left(\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \cdots\right)}$$

$$\therefore S = 1 + \frac{1}{2} S$$

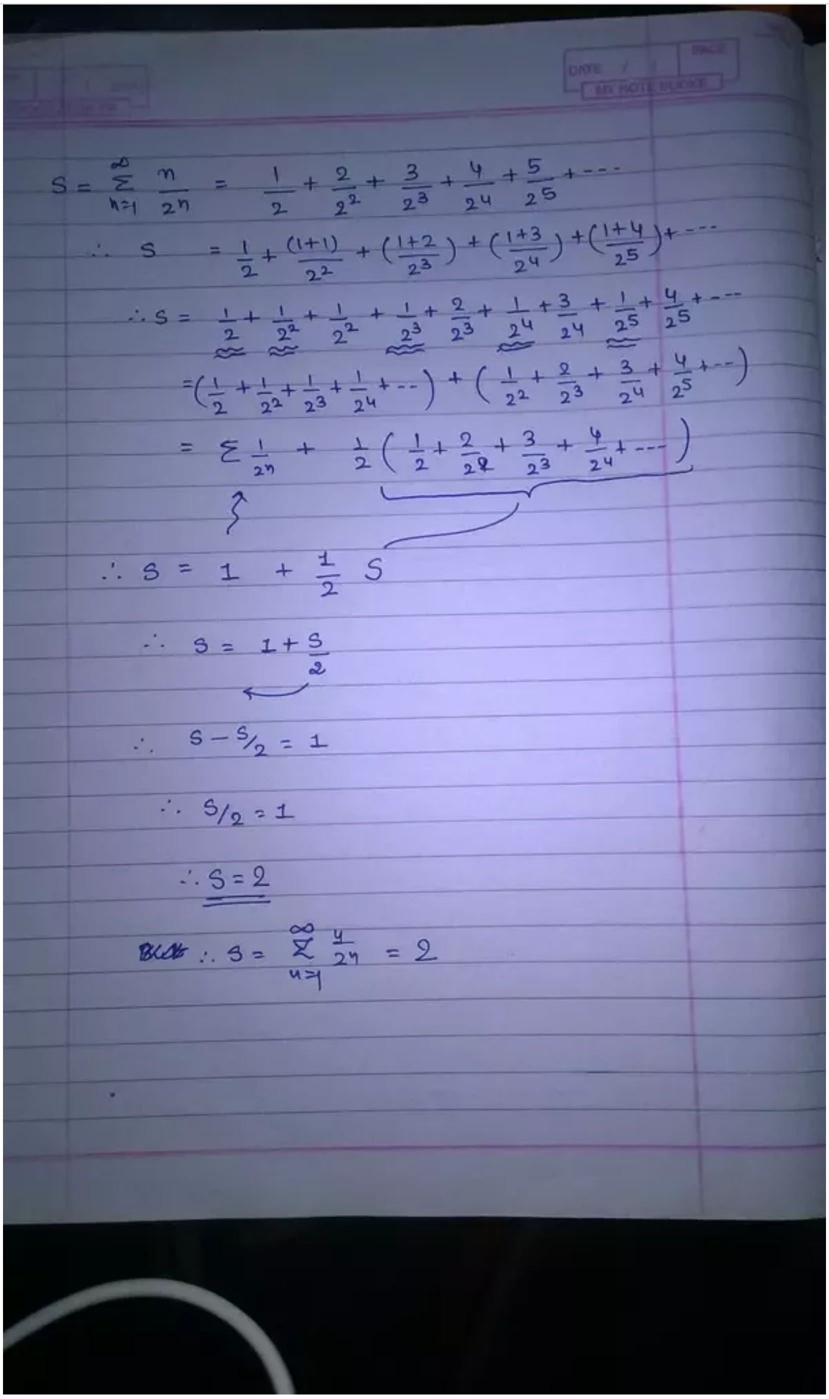
$$\therefore S = 1 + \frac{S}{2}$$

$$\therefore S - S/2 = 1$$

$$\therefore S/2 = 1$$

$$\therefore \underline{\underline{S = 2}}$$

$$\therefore S = \sum_{n=1}^{\infty} \frac{n}{2^n} = 2$$



$$S = \sum_{n=1}^{\infty} \frac{n}{2^n} = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \frac{5}{2^5} + \cdots$$

$$\therefore S = \frac{1}{2} + \frac{(1+1)}{2^2} + \left(\frac{1+2}{2^3}\right) + \left(\frac{1+3}{2^4}\right) + \left(\frac{1+4}{2^5}\right) + \cdots$$

$$\therefore S = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{2}{2^3} + \frac{1}{2^4} + \frac{3}{2^4} + \frac{1}{2^5} + \frac{4}{2^5} + \cdots$$

$$= \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots\right) + \left(\frac{1}{2^2} + \frac{2}{2^3} + \frac{3}{2^4} + \frac{4}{2^5} + \cdots\right)$$

$$= \sum \frac{1}{2^n} + \frac{1}{2}\underbrace{\left(\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \cdots\right)}$$

$$\therefore S = 1 + \frac{1}{2} S$$

$$\therefore S = 1 + \frac{S}{2}$$

$$\therefore S - S/2 = 1$$

$$\therefore S/2 = 1$$

$$\therefore \underline{S = 2}$$

$$\text{But } \therefore S = \sum_{n=1}^{\infty} \frac{n}{2^n} = 2$$

Mohammad Afzaal Butt, B.Sc Mathematics & Physics, Islamia College Gujranwala (1977)

Answered 1 year ago · Author has **19.5K** answers and **9M** answer views

Let $S = \sum_{1}^{n} \frac{n}{2^n}$

$$= \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \frac{5}{2^5} + \frac{6}{2^6} + \cdots$$

$$2S = \frac{2}{2} + \frac{2}{2^1} + \frac{3}{2^2} + \frac{4}{2^3} + \frac{5}{2^4} + \frac{6}{2^5} + \cdots$$

$$2S - S = 1 + \left(\frac{2}{2^1} - \frac{1}{2^1}\right) + \left(\frac{3}{2^2} - \frac{2}{2^2}\right) + \left(\frac{4}{2^3} - \frac{3}{2^3}\right) + \left(\frac{5}{2^4} - \frac{4}{2^4}\right) +$$

It is a geometric sequence of infinite terms with first term 1 and common ra

$$\frac{1}{2} < 1$$

$$\therefore \mathrm{S} = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = 2$$

Ashutosh Nirala

Answered 4 years ago

After looking at solution given by Rajarshi Bandopadhya:

Let S represent the desired sum, so S = 1/2 + 2/(2^2) + 3/(2^3) + ...

From the second term onwards lets take out 1 from numerator, which would give:

S = 1/2 + (1 + 1)/(2^2) + (1 + 2)/(2^3) + ...

Now take the 1 out from the sum, and write it first, followed by the rest of the series

S = 1/2 + 1/(2^2) + 1/(2^3) + ... + 1/(2^2) + 2/(2^2) + ...

First part is a geometric series which sums to 1

From the second part take out 1/2. Thus we'll get:

S = 1 + 1/2 (1/2 + 1/(2^2) + ...)

We see that when we take 1/2 out of the second part, it becomes same as our initial sum, which is S, so we get:

S = 1 + 1/2(S) => S/2 = 1 => S = 2

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How do I do the sum of Sigma 2^n?

What is the sum of (2/3) ^n?

How do you evaluate the sum of sqrt(n) /(2^n) from n=1 to infinity?

How do you evaluate the sum of {[1/ (n^-(1/n))] /[(2n) ^n]} from n=1 to infinity?

How can I do the sum (1 to infinity) of n^2/3^n?

Charlie Li, BSc. Mathematics, The Chinese University of Hong Kong (2020)

Answered 3 years ago · Upvoted by Milap Rajgor, M.Sc Mathematics, Indian Institute of Technology, Mandi

After looking at others answers, I would not say that they are wrong but they are not rigorous. Their result is correct because of the absolute convergence of the series. So, let's check if this series converges absolutely by using ratio test.

The ratio test states that a series $\sum_{n=1}^{\infty} a_n$ converges absolutely if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = r$ and $r < 1$.

$$\lim_{n\to\infty}\frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}}$$

$$= \lim_{n\to\infty}\frac{n+1}{2n}$$

$$= \frac{1}{2} < 1$$

So, the series converges absolutely, and therefore we can rearrange and group the terms like normal real numbers without changing the answer.

First, we can define the partial sum $S_n = \sum_{i=1}^{n} \frac{i}{2^i}$.

We can see that $S_{n+1} = S_n + \frac{n+1}{2^{n+1}}$ so $2^{n+1} S_{n+1} = 2\left(2^n S_n\right) + n + 1$.

Then we can express this using the matrix

$$\begin{pmatrix} 2^n S_n \\ n \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}^n \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Then by some calculation (maybe using wolframalpha), it becomes

$$\begin{pmatrix} 2^n S_n \\ n \\ 1 \end{pmatrix} = \begin{pmatrix} 2^n & -1+2^n & -n+2^{n+1}-2 \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

So $2^n S_n = 2^{n+1} - n - 2$. $S_n = 2 - \frac{n+2}{2^n}$.

One can easily show that $\lim_{n\to\infty} S_n = 2$.
It follows that $\sum_{n=1}^{\infty} \frac{n}{2^n} = 2$.

20.8K views · View upvotes

26 2

Audrey Lizardo, studied at Malayan Colleges Laguna
Answered 3 years ago

Let,
S = 1/2 + 2/4 + 3/8 + 4/16 + 5/32 + ...

Multiply both sides by 2.
2S = 1 + 2/2 + 3/4 + 4/8 + 5/16
-S = 1/2 +2/4 + 3/8 + 4/16 + ...

S = 1 + (1/2 + 1/4 + 1/8 + 1/16 + ...)

S = 1 + (1/2)/(1 – 1/2)

S = 1 + 1

S= 2

11.4K views · View upvotes

18

Zaunty, lives in India
Answered 3 years ago

1/2+1/4+1/8+1/16+1/32+1/64........

Take a square and start dividing it into half.Go on and on .you will find it endless.....

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Anastasiya Romanova, Integrator @Mathematics StackExchange, Brilliant.org, AoPS, and Integral & Series
Answered 7 years ago · Upvoted by Jay Wacker, uses calculus frequently and William Chen, MS in Applied Mathematics from Harvard

How can $\sum_{n=1}^{\infty} \frac{1}{n}$ diverge, but $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converge?

Originally Answered: How can ₩sum_{n=1}^₩infty₩frac{1}{n} diverge but ₩sum_{n=1}^₩infty₩frac{1}{n^2} converge?

In mathematics, the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is called the harmonic series and it is known as a divergent series. The fact that the harmonic series diverges was first proved in the 14th century by Nicole Oresme. Here is how he proved it.

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \underbrace{\left(\frac{1}{3} + \frac{1}{4}\right)}_{> \frac{1}{2}} + \underbrace{\left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right)}_{> \frac{1}{2}} + \cdots$$

$$> 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots$$

Notice that group n contains 2^n terms and the smallest element in group n is larger than $\frac{1}{2^{n+1}}$. For example all elements in group 2 are larger than $\frac{1}{2^3}$. So the sum of the terms in each group is larger than $2^n \cdot \frac{1}{2^{n+1}} = \frac{1}{2}$. Since there are infinitely many groups and the sum in each group is larger than $\frac{1}{2}$, it follows that the harmonic series diverges.

The same idea can be applied to prove $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges. In fact, the series converges to $\frac{\pi^2}{6}$, see Basel problem. The generalization of the harmonic series

$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$

is known as the Riemann zeta function and for $s \in \mathbb{R}\mathrm{e}$ and $s > 1$ the series converges.

Related Answer

Roman Andronov, Solving technical problems is my day job.

Answered 3 years ago · Upvoted by Alon Amit, PhD in Mathematics; Mathcircler. · Author has **901** answers and **8.1M** answer views

How can I calculate this infinite sum $\sum n^2 x^n$?

1. Via the limits notation we show that for any two (real-to-real) differentiable functions $f(x)$ and $g(x)$ it is true that the first derivative of their sum is the sum of their first derivatives (over x):

$$\Big(f(x) + g(x)\Big)'_x = f'_x(x) + g'_x(x) \tag{1}$$

2. By induction on n we generalize (**1**) for an arbitrary (but finite!) number of differentiable functions f_i:

$$\left(\sum_{i=1}^{n} f_i(x)\right)'_x = \sum_{i=1}^{n} f'_i(x) \tag{2}$$

3. However, when we jump fro Continue Reading nber of terms the things stop being so easy and extra care must be taken for our naive wishful thinking (and

Related Answer

Kevin Gomez, Number theorist in training

Updated 1 year ago · Author has **484** answers and **1.1M** answer views

How do you evaluate the sum of (n^2)/(2^n) from n=1 to infinity?

*A2A

I'm going to demonstrate a way to evaluate this sum that doesn't require much more than a year or two of calculus. I think Donald Hartig already showed the basic principles, but let me elaborate just a bit more.

We wish to find

$$S = \sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

$$S^* = \sum_{n=1} \frac{1}{2^n}$$

Most will quickly know the value of this sum as $S^* = 1$. However, we can consider a more general form of this sum

$$S(x) = \sum_{n=1}^{\infty} \frac{x^n}{2^n}$$

We can see that this sum, in general, evaluates to

$$S(x) = \frac{x/2}{1 - x/2} = \frac{x}{2 - x}$$

What happens when we differentiate this? Well...

$$\begin{aligned} S'(x) &= \frac{\mathrm{d}}{\mathrm{d}x} \sum_{n=1}^{\infty} \frac{x^n}{2^n} && = \frac{\mathrm{d}}{\mathrm{d}x} \frac{x}{2 - x} \\ &= \sum_{n=1}^{\infty} \frac{nx^{n-1}}{2^n} && = \frac{2}{(2 - x)^2} \end{aligned}$$

Neat-o! We can first multiple by x to get rid of that pesky x^{n-1}.

$$xS'(x) = \sum_{n=1}^{\infty} \frac{nx^n}{2^n} = \frac{2x}{(2 - x)^2}$$

Call this $T(x)$. Differentiating *this* guy yields

$$\begin{aligned} T'(x) &= \frac{\mathrm{d}}{\mathrm{d}x} \sum_{n=1}^{\infty} \frac{nx^n}{2^n} && = \frac{\mathrm{d}}{\mathrm{d}x} \frac{2x}{(2 - x)^2} \\ &= \sum_{n=1}^{\infty} \frac{n^2x^{n-1}}{2^n} && = \frac{2(2 + x)}{(2 - x)^3} \end{aligned}$$

How does this help? Well, look at $T'(1)$:

$$T'(1) = \sum_{n=1}^{\infty} \frac{n^2 1^{n-1}}{2^n} = \sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

It's our series! And look, we also have a formula for it!

$$T'(1) = \frac{2(2 + 1)}{(2 - 1)^4} = \frac{2 \times 3}{1} = 6$$

And we have arrived at our final answer.

Related Answer

Steve Brown, MSc Applied Mathematics & Computer Science, California State University, Long Beach

What is the sum, as n goes to infinity, of 1/2^n?

> What is the sum, as n goes to infinity, of 1/2^n?

Assuming that n begins at 1:

$$S = \sum_{n=1}^{n=\infty} \frac{1}{2^n} = 1.$$

Proof:

$$\begin{array}{rcll} S & = & & \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots \\ 2S & = & 1+ & \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots \\ \hline S & = & 2S - S & = 1 \end{array}$$

Gujranwala (1977)

Answered 2 years ago · Author has 19.5K answers and 9M answer views

How do you evaluate the sum of (2n) /(2^n) from n=1 to infinity?

Let $S = \sum_{n=0}^{\infty} \frac{2n}{2^n}$

$$= \frac{2}{2} + \frac{4}{2^2} + \frac{6}{2^3} + \frac{8}{2^4} + \frac{10}{2^5} + \cdots$$

$$= 1 + \frac{4}{2^2} + \frac{6}{2^3} + \frac{8}{2^4} + \frac{10}{2^5} + \cdots \qquad (1)$$

$$\implies 2S = 2 + \frac{4}{2} + \frac{6}{2^2} + \frac{8}{2^3} + \frac{10}{2^4} + \cdots \qquad (2)$$

by (2) - (1)

$$S = 2 + 2 - 1 + \left(\frac{6}{2^2} - \frac{4}{2^2}\right) + \left(\frac{8}{2^3} - \frac{6}{2^3}\right) + \left(\frac{10}{2^4} - \frac{8}{2^4}\right) + \cdots$$

$$= 3 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

$$= 3 + \frac{\frac{1}{2}}{1 - \frac{1}{2}} \quad \because S_\infty = \frac{a}{1-r}$$

$$= 3 + 1$$

$$= 4$$

546 views · View upvotes · Answer requested by Alexander James Irons

6 1

Related Answer

Ved Prakash Sharma, former Lecturer at Sbm Inter College, Rishikesh (1971-2007)

Answered 3 years ago · Author has 9.9K answers and 7.8M answer views

How do I do the sum of Sigma 2^n?

Sigma 2^n

=2^1+2^2+2^3+2^4+............+2^n

=a(r^n - 1)/(r-1)

=2(2^n - 1)/(2–1)

=2 . (2^n - 1) , Answer

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6

Related Answer

Arun Iyer, I dabble with mathematics, not really good at it yet

Updated 5 years ago · Author has 803 answers and 3.1M answer views

What is the sum of the series $\sum \frac{1}{n^2}$?

Originally Answered: What is the answer to 1+1/ (2*2) +1/ (3*3) +1/ (4*4) +... = ?

There are many ways to evaluate the given series. One of the simplest one that was originally given by Euler is as follows,

Consider the function $f(x) = \sin x$. This function is zero at $x = \pm n\pi$ and therefore can be written as:

$$f(x) = x\left(1 + \frac{x}{\pi}\right)\left(1 - \frac{x}{\pi}\right)\left(1 + \frac{x}{2\pi}\right)\left(1 - \frac{x}{2\pi}\right)\cdots$$

$$= x\left(1 - \frac{x^2}{\pi^2}\right)\left(1 - \frac{x^2}{2^2\pi^2}\right)\left(1 - \frac{x^2}{3^2\pi^2}\right)\cdots$$

$$3! \quad 5! \quad 7!$$

Comparing the coefficients of x^3 in the two series we get,

$$-\sum_{n=1}^{\infty}\frac{1}{\pi^2 n^2} = -\frac{1}{3!}$$

Therefore,

$$\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$$

This is not the most rigorous proof but it is somewhat of a testament to Euler's ingenuity in observing the pattern (he was the first to solve this problem).

Some interesting links:
1] Different methods to compute $₩sum₩limits_{n=1}^₩infty ₩frac{1}{n^2}$
2] Page on cmu.edu

Robby Goetschalckx, Computer scientist for 11+ years and passionate about math since childhood.

What is the sum to infinity of (1/n^3)?

The result is called Apéry's constant [1]. It does not seem to have any nice closed-form representations.

Footnotes

[1] Apéry's constant - Wikipedia

Imad Zghaib, BA Mathematics & Engineering, Free University of Brussels (1989)

How do you evaluate the sum of (n-1) / (2^n-2) from n=1 to infinity?

(n-1) / (2^n-2) from n=1 to infinity

$$=\sum_{1}^{\infty}\frac{n-1}{2^{n-2}}$$

Let S(x) be the function series =

$$=> S(x) =>$$

$$\sum_{1}^{\infty}\frac{n-1}{x^{n-2}}$$

Let us integrate S(x)

$$=> S1(x) = \int S(x)dx = \int\sum_{1}^{\infty}\frac{n-1}{x^{n-2}}.dx$$

$$=>= \sum_{1}^{\infty}\int\frac{n-1}{x^{n-2}}.dx$$

$$= \sum_{1}^{\infty}\frac{n-1}{(n-1).x^{n-1}}$$

$$=> S1(x) = \sum_{1}^{\infty}x^{n-1}$$

Is a geometric series = $\frac{1}{(1-x)} = S1(x)$

To calculate $\sum_{1}^{\infty}\frac{n-1}{2^{n-2}}$

It suffice to put in S(x) x=2

$$=> \sum_{1}^{\infty}\frac{n-1}{2^{n-2}}$$

$$= S(2)* = \frac{1}{(1-2)^2} = \frac{1}{4}$$

Related Questions

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How do you evaluate the sum of {[1/ (n^-(1/n))] /[(2n) ^n]} from n=1 to infinity?

How can I do the sum (1 to infinity) of n^2/3^n?

What will be series of 2N (sum formula)?

How do I find the sum of n/(3^n) from n=1 to infinity?

How could I find the value of $\sum_{n=1}^{\infty}\frac{n^3}{3^n}$?

How do you evaluate the sum of n=1 to infinity Σ (n^2/6^n)?

How do you evaluate the sum of 3^n from (-infinity) to (n-2)?