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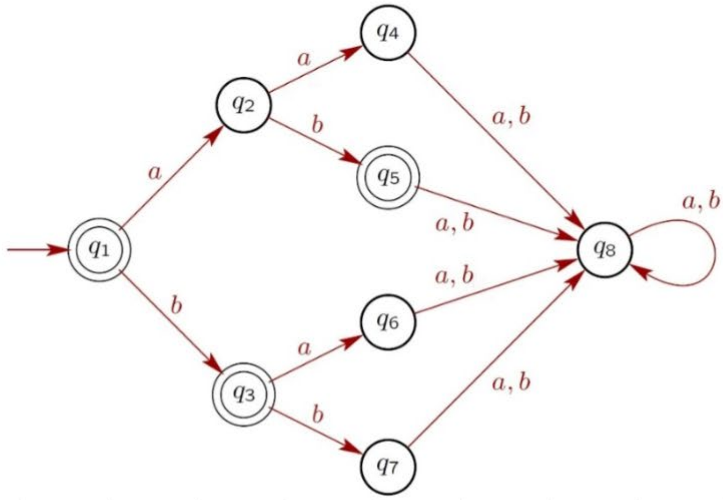
Question 1

Given an DFA that recognizes the language $A = \{\epsilon, b, ab\}$ as follows, please present its symbolic description. (20 marks)



Automation: a DFA $M=\{Q, \Sigma, \delta, q, F\}$

Set of states: $Q=(q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8)$

$\Sigma = \{a, b\}$

$F = \{q_1, q_3, q_5\}$

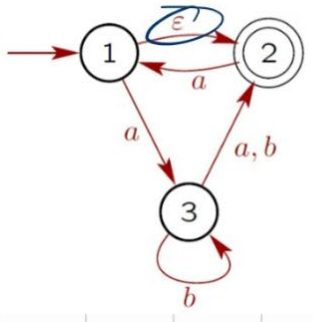
transition function is given:

	a	b
q_1	q_2	q_3
q_2	q_4	q_5
q_3	q_6	q_7
q_4	q_8	q_8
q_5	q_8	q_8
q_6	q_8	q_8
q_7	q_8	q_8
q_8	q_8	q_8

Question 2

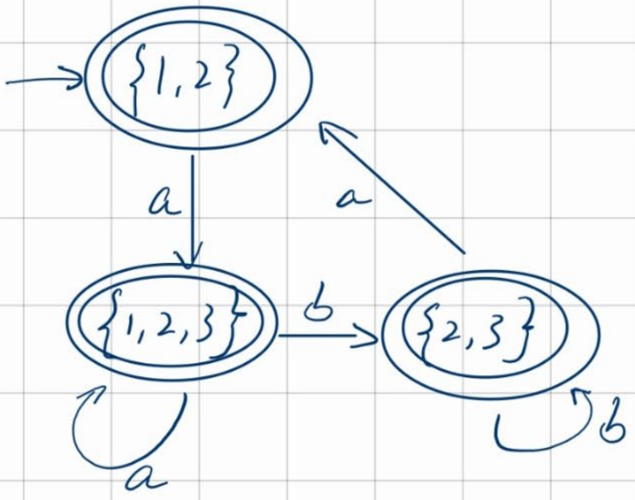
Use the construction given in our lecture to convert the following NFA N into an equivalent DFA. Notably, you only need to draw the corresponding transition graph. (20 marks)

N:



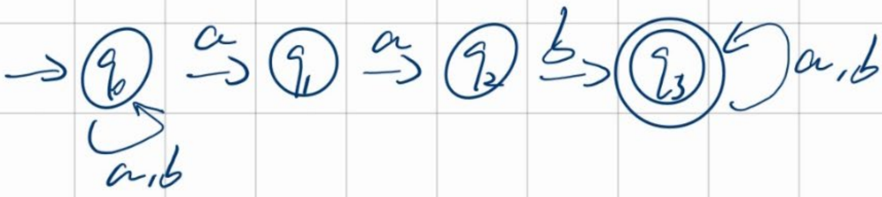
→{1,2}
↓a.

	a	b
{1,2}	{3,1,2}	ϕ
{1,2,3}	{3,1,2}	{3,2}
{2,3}	{1,2}	{2,3}



Question 3

Design an NFA with exactly four states for the language $\{w \in \Sigma^* \mid w \text{ contains the substring } aab\}$, where $\Sigma = \{a, b\}$. You only need to draw the graph. (20 marks)



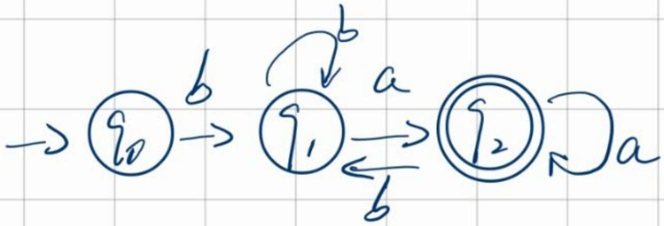
Question 4

For each of the languages that all strings begin with b and end with a, over the alphabet $\Sigma = \{a, b\}$, give a DFA and a regular expression for it. For the DFA, you only need to draw the graph. (20 marks)

(a) Draw the DFA graph. (12 marks)

(b) Give its regular expression. (8 marks)

(a)



(b) $b(a \cup b)^* a$

Question 5

Consider the language $A = \{www | w \in \{a,b\}^*\}$, proving that it is not a regular language through pumping lemma. (20 marks)

(a) Describe pumping lemma for regular languages. (4 marks)

(b) Prove that A is not a regular language. (16 marks)

Proof. (a):

For regular language $L = xyz$ there exists P $(p \geq 0)$ that $s = xy^Pz \in L$ Vice versa.

(b) Define pumping length P

Consider string $s = a^Pb^Pa^Pb^Pa^Pb^P \in A$

length: $|s| = 6P \geq P$, Pumping lemma will be hold,

So split s into $s = xyz$, satisfying:

$$\begin{cases} |y| > 0 \\ |xy| < P \\ xy^iz \in A \text{ for each } i \geq 0. \end{cases}$$

$\Rightarrow$ x and y consist only a(s)

z will hold all the rest a(s) and b(s).

So we have $x = a^i$, for some $i \geq 0$

$y = a^j$, for some $j \geq 0$.

$z = a^kb^Pa^Pb^Pa^Pb^P$, for some $k \geq 0$

$s = xyz = a^{i+j+k}b^Pa^Pb^Pa^Pb^P$

$\Rightarrow i+j+k = P$ ①

From lemma, $xyyz \in A$, but $xyyz = a^{i+2j+k}b^Pa^Pb^Pa^Pb^P$ ②

From ①: $xyyz = a^{P+j}b^Pa^Pb^Pa^Pb^P$

$\because j > 0$,

$\therefore xyyz \neq s \Rightarrow xyyz \notin A$.

Contradiction! A is non-regular.