Xi'an Jiaotong-Liverpool University
西交利物浦大学

Module Code	Examiner	Department	Tel
INT201	Yushi Li	Intelligent Science	5351

1st SEMESTER 23-24 FINAL EXAMINATION

Undergraduate

Decision Computation and Language

TIME ALLOWED: *2 hours*

INSTRUCTIONS TO CANDIDATES

1. **This is a blended close-book exam and the duration is 2 hours.**
2. **Total marks available are 100. This accounts for 80% of the final mark.**
3. **Answer all questions. Relevant and clear steps should be included in the answers.**
4. **Only English solutions are accepted. For online students, answers need to be handwritten and fully and clearly scanned or photographed for submission as one single PDF file via LEARNING MALL.**
5. **Online students should use the format "Module Code-Student ID.filetype" to name their files before submitting to Learning Mall. For example, "INT201-18181881.pdf".**

Question 1

Indicate true or false of the following statements, and briefly justify your answers. (**30 Marks**)

(a) If A is a regular language, then $| A | < \infty$. (**3 Marks**)

(b) If a language A is nonregular, then it has an NFA. (**3 Marks**)

(c) The transition function of an NFA is $\delta : Q \times \Sigma \to Q$. (**3 Marks**)

(d) The regular expression $(01^*0 \cup 1)^*0$ generates the language consisting of all strings over $\Sigma = \{0, 1\}$ having an odd number of 0's. (**3 Marks**)

(e) If a language A is regular, then A has a CFG in Chomsky normal form. (**3 Marks**)

(f) Language A is context-free if and only if there exists a deterministic pushdown automaton D such that A = L(D) (**3 Marks**)

(g) Language A is Turing-decidable if there exists a Turing machine TM such that A = L(TM) **(3 Marks**)

(h) If Language A can be recognized by a multi-tape Turing machine, A is TM-recognizable. (**3 Marks**)

(i) The set of all languages is countable. (**3 Marks**)

(j) If a language A is mapping reducible to a TM-recognizable language B and A is decidable, then B is decidable also. (**3 Marks**)

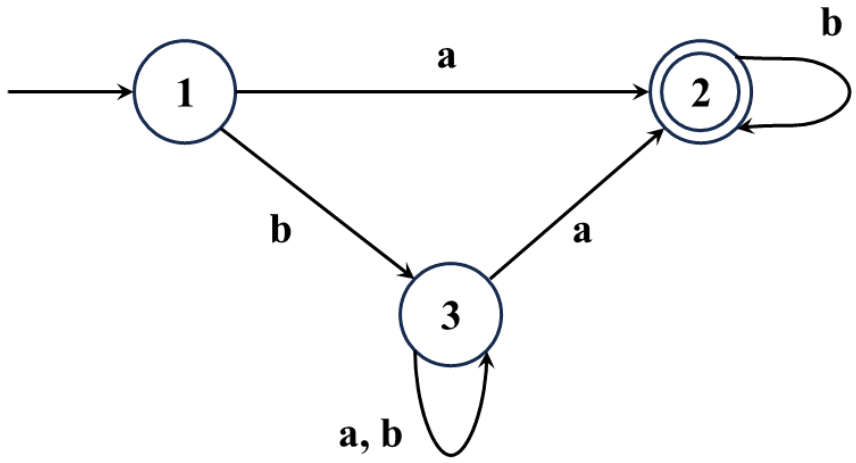
Question 2

Draw an NFA with exactly six states over the alphabet $\Sigma = \{0, 1\}$ that accepts the following language $\{w \in \Sigma^* \mid w$ contains at least two 0s, or exactly two 1s $\}$. (**12 Marks**)

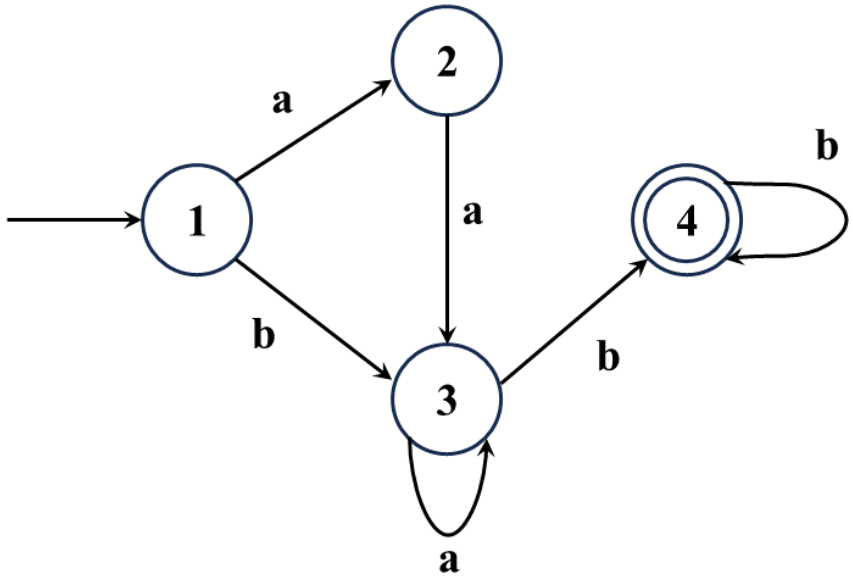
Question 3

Give the regular expressions for the languages recognized by the NFAs below. (**8 Marks**)

(a) (**4 Marks**)



(b) (**4 Marks**)



Question 4

The original CFG is shown as follows, and convert it to Chomsky normal form. (**16 Marks**)

$$S \to QaQ$$
$$Q \to aQb|bQa|QQ|\epsilon$$

Question 5

Consider the following languages $L_1 = \{a^i b^j c^k \mid i = j, i, j, k \geq 0\}$ and $L_2 = \{a^i b^j c^k \mid j = k, i, j, k \geq 0\}$ (**10 Marks**)

(a) Show L_1 is a context-free language by providing a context-free grammar. (**4 Marks**)

(b) What is the language of $L_1 \cap L_2$? (**2 Marks**)

(c) Is context-free language closed under intersection? Justify your answer.

(**4 Marks**)

Question 6

Let $\Sigma = \{a, b\}$, pushdown automata are given by the diagrams below. (**13 Marks**)

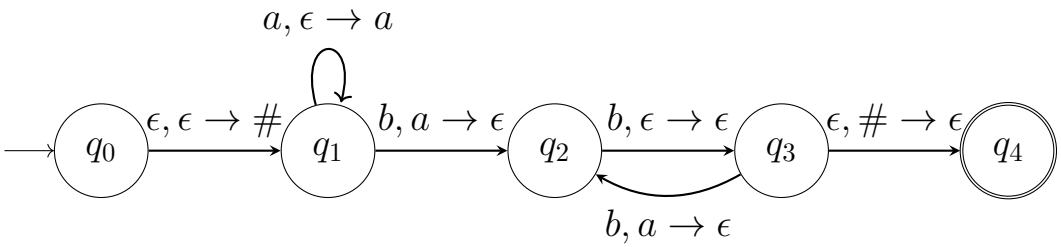


Figure 1: PDA A

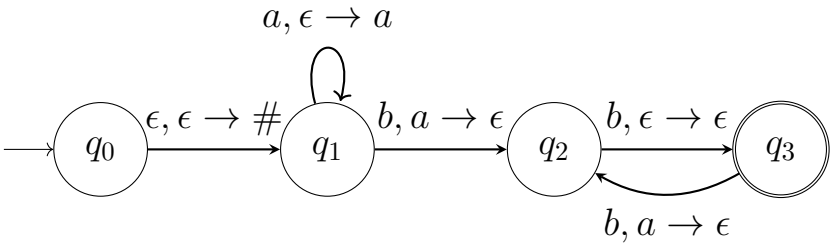


Figure 2: PDA B

(a) What is the language that is being accepted by the PDA A? (**4 Marks**)

(b) Write a context-free grammar that accepts the same language of L(A) (**4 Marks**)

(c) Is $L(A) \subset L(B)$? Justify your answer (**5 Marks**)

Question 7

Let $\Sigma = \{0, 1\}$, and consider the language $A = \{\langle TM \rangle \mid$TM is a Turing machine that accepts string 101$\}$. (**12 Marks**)

(a) Complete the definition of decidable languages. We say that A is decidable, if there exists a Turing machine TM, such that for every string $w \in \Sigma^*$, the following holds: (**4 Marks**)

(b) The Rice's theorem can be used to prove a language of TM descriptions being undecidable. What are the requirements of Rice's theorem? (**4 Marks**)

(c) Prove or disprove A is a decidable language. (**4 Marks**)