

Xi'an Jiaotong-Liverpool University

西交利物浦大學

MODULE CODE	EXAMINER	DEPARTMENT	TEL
INT201	Wenjin Lu	INTELLIGENT SCIENCE	1505

1st SEMESTER 2021-22 EXAMINATIONS (FINAL, OPEN BOOK)

BACHELOR DEGREE – Year 3

DECISION, COMPUTATION AND LANGUAGE

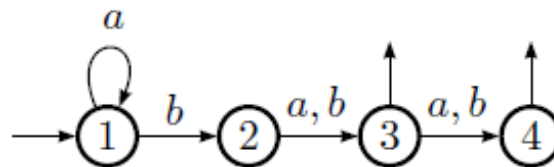
JANUARY 2022

TIME ALLOWED: 2 HOURS

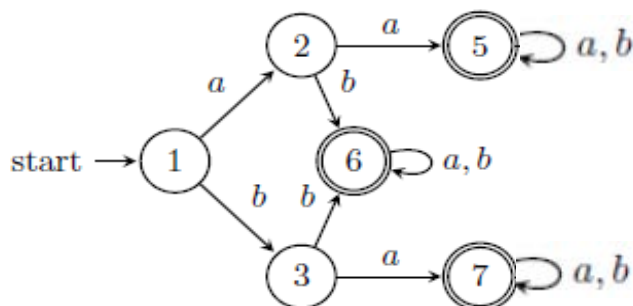
INSTRUCTIONS TO CANDIDATES

- 1、 Total marks available are 100 marks. Marks for this examination account for 80% of the total credit for INT201.
- 2、 The paper consists of 9 questions. Answer all the questions.
- 3、 The number at the right column indicates the mark for each question.
- 4、 Answers should be written in the answer script provided.
- 5、 This is an OPEN BOOK examination. You can reference textbooks but discussions with other students in any way is not allowed.
- 6、 The time of the exam is strictly limited to 2 hours.
- 7、 For students who take the exam online, at the end of the examination, be absolutely sure to submit your answer via Learning Mall.
- 8、 All answers must be in English.

- 1 Given alphabet $A = \{a, b, c\}$ answer the following questions.
- a) Let L be the set of words over A , in which the letters are in alphabetical order and appear at least once, for example abc , $aabbbc$ but not ac , abb , $bbbc$. Write down a regular expression for L . **4**
- b) Give a DFA by diagram that accepts the language. **4**
- 2 Let E, F be regular expressions, and $E = \{0\}$, $F = \{1\}$.
- a) Give a DFA by diagram that accepts $E + F$. **4**
- b) Give a DFA by diagram that accepts EF . **4**
- c) Give a NFA by diagram that accepts $(EF)^*$. **4**
- 3 Given the following DFA, find out its equivalent regular expression. (You can do this either by observation or by applying the 'DFA to regular expression' algorithm.) **6**

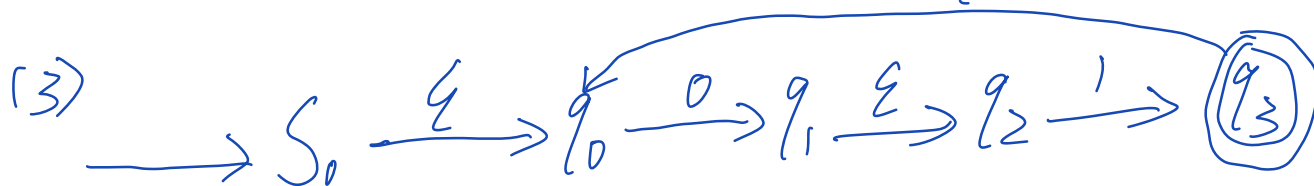
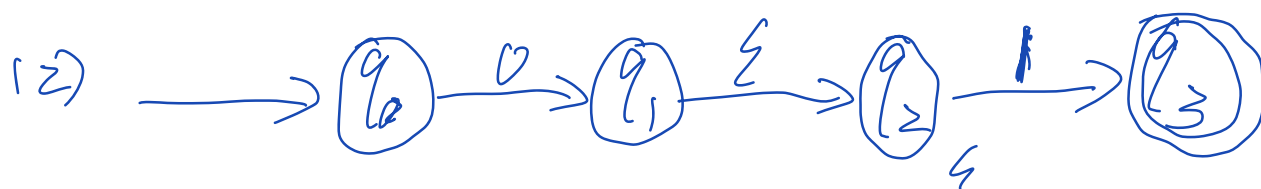
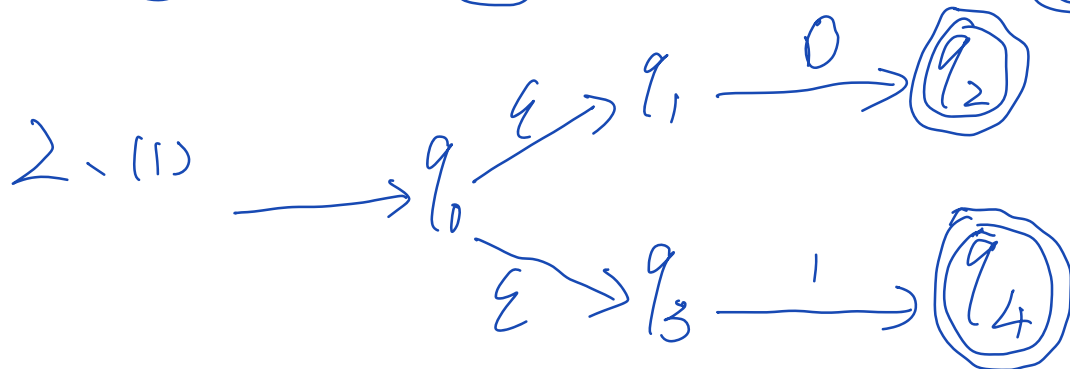
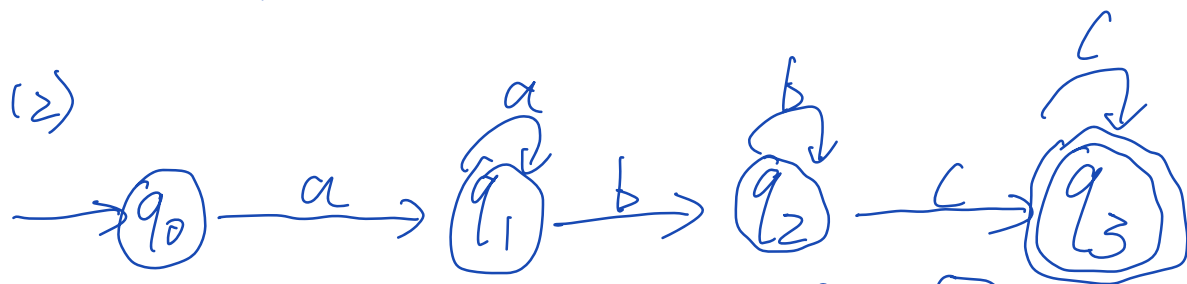


- 4 For the language $L = \{a^k b^l a^m \mid k+l = m\}$, use the Pumping Lemma to prove it is not regular. **8**
- 5 A DFA is given below.



- a) Are there any inaccessible states? If yes, list all inaccessible states. **3**
- b) List all classes of equivalent states in the DFA. **6**
- c) Minimise the DFA by merging the equivalent states. **6**

1. $aa^*bb^*cc^*$



3. $a^*b(ab)^*(ab)^*$

4. Assume L is regular Language

There exists a constant ' p ' for L

Choose $s = a^k b^l c^p$ where $k+l=p$ so $|s| = 2p \geq p$

Divide s into xyz where $|xy| \leq p$ $|y| \geq 1$ $xy^iz \in L$

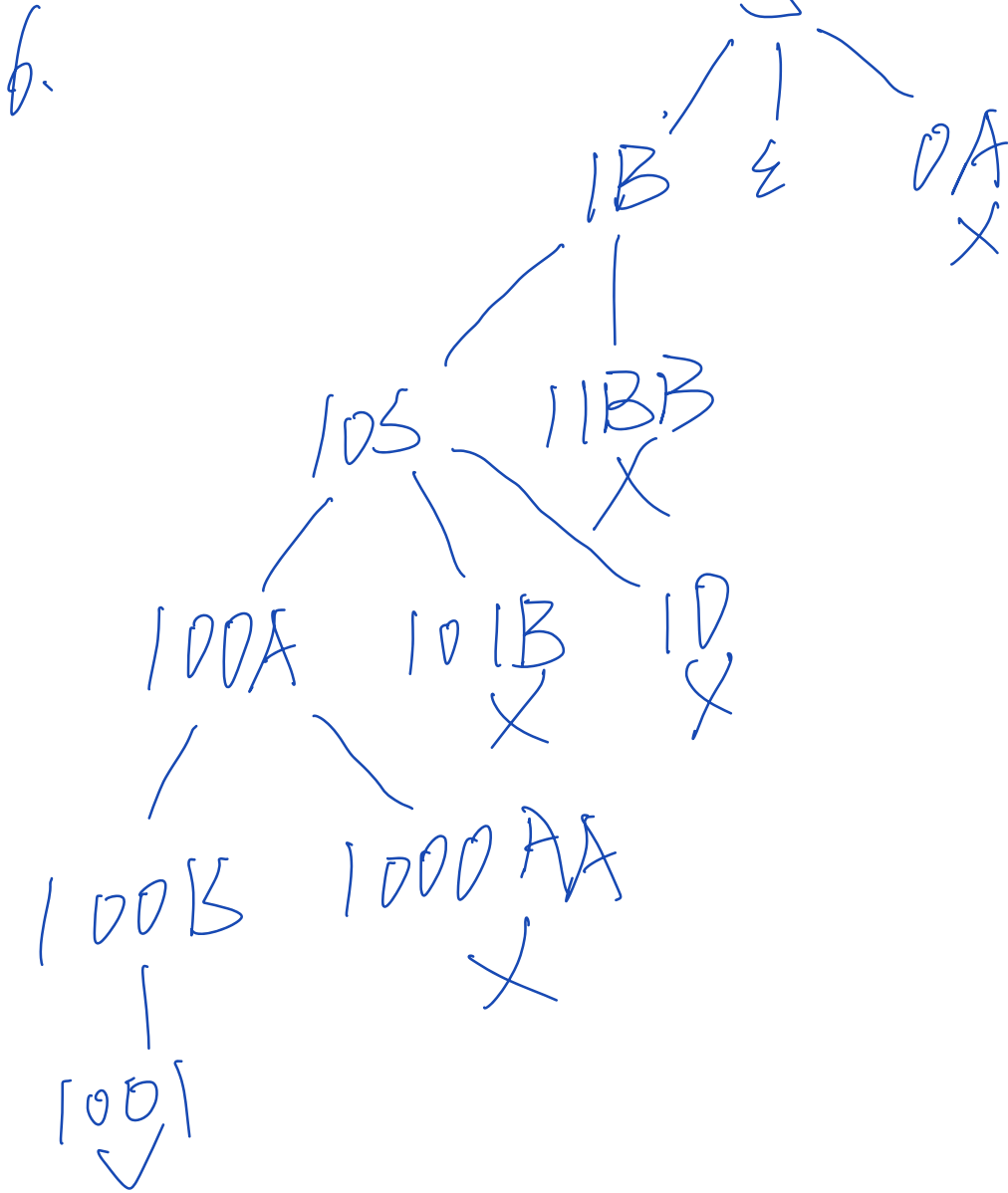
Cause $|xy| \leq p$ so y only can exist b part or $(ab)^p$ part

Case 1: y in b part $x = a^{k+r}$ $y = b^n$ where $r+n=l$

if $i=2$ $s = a^k b^{l+n} c^p$ $|s| > 2p$ and $k+l+n > p$ so cannot

Case 2: y in ab part. $x = a^n$ $y = a^r b^l$ where $r+l = k$
 if $i=2$ $S = a^{k+n} b^l C^p$ $|S| > 2p$ and $k+l+r > p$
 So L is not regular Language.

5. 没学



- 6** Consider a context-free grammar $G = (\{0, 1\}, \{S, A, B\}, S, P)$, where P is the set of productions:

$$\begin{aligned} S &\rightarrow 0A \mid 1B \mid \varepsilon \\ A &\rightarrow 1S \mid 0AA \\ B &\rightarrow 0S \mid 1BB \end{aligned}$$

- a)** Give a leftmost derivation of the string 0011. **4**
b) Give a rightmost derivation of the string 0011. **4**
c) Draw a parse tree for the string 1001. **4**
d) Can the string 1110 be derived from the grammar G ? If yes, write down the associated derivation tree. If not, give a justification. **4**
- 7** Convert the following grammar to Chomsky Normal Form. (You can do this by first substituting variables for the constants and then breaking apart rules the length of whose right hand side is more than 2.) **8**

$$\begin{aligned} S &\rightarrow aAS \mid a \\ A &\rightarrow SbA \mid SS \mid ba \end{aligned}$$

- 8** Consider the Turing machine
 $M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, \{a, b, x, y, B\}, \delta, q_0, B, \{q_4\})$
 where δ is defined as follows:

$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$

Assume the head of the machine initially points to the left-most letter.

- a)** Give the instantaneous descriptions for the input "aabb". Is "aabb" accepted by the machine? **4**
b) Give the instantaneous descriptions for the input "abbb". Is "abbb" accepted by the machine? **4**
c) What is the language accepted by M ? **5**

7. Step 1: $S_0 \rightarrow S$
 $S \rightarrow aAS/a$
 $A \rightarrow \epsilon bA/SS/ba$

Step 2: $S_0 \rightarrow aAS/a$
 $S \rightarrow aA\epsilon/a$
 $A \rightarrow \epsilon bA/SS/ba$

Step 3: $S_0 \rightarrow aN/a$
 $S \rightarrow aN/a$
 $A \rightarrow SM/SS/ba$
 $N \rightarrow AS$
 $M \rightarrow bA$

Step 4: $S_0 \rightarrow RN/a$
 $S \rightarrow RN/a$
 $A \rightarrow SM/SS/XR$
 $N \rightarrow AS$
 $M \rightarrow XA$
 $R \rightarrow a$
 $X \rightarrow b$

8. (a) $aabb$

1. $q_0 a a b b$
2. $x q_1 a b b$
3. $x a q_1 b b$
4. $x q_2 a y b$
5. $q_2 x a y b$
6. $x q_0 a y b$
7. $x x q_1 y b$
8. $x x y q_1 b$
9. $x x q_2 y y$

(b) $a b b b$

11. $x q_2 x y y$
12. $x x q_0 y y$
13. $x x y q_3 y$
14. $x x y y q_3 B$
15. $x x y y B q_4 B$
1. $q_0 a b b b$
2. $x q_1 b b b$
3. $q_2 x y b b$
4. $x q_2 y b b$
5. $x y q_3 b b$

So "aabb" can be
accept

reject, cause
these relevant transition

$$(c) L = \{w = aa^n b^n \mid n \geq 0, w \in \Sigma^+\}$$

- 9 a) What are recursive and recursively enumerable languages? Which one of the two sets stands for decidable problems? 4
- b) It was shown that in lectures that the halting problem is undecidable. Write down a general explanation of what this means, such that it would be useful for someone who knows about Turing Machine but not about undecidability. 10

END OF THE PAPER

9. 没学过, 但 recursive \rightarrow Decidable \hookrightarrow

(b) Halting Problem is to check wheather a TM will halt or not

Like a TM: on input $M, w \in L$

if M halt on w , $H(\langle M, w \rangle) = \text{Yes}$

if M not halt on w , $H(\langle M, w \rangle) = \text{No}$

Now we construct a TM, D , which has reverse

D : on input a $\langle M \rangle$

Use H to decide the $\langle M, \langle M \rangle \rangle$ wheather will halt

if $H: \text{Yes}$, M will halt, D will not halt

if $H: \text{No}$, M will not halt, D will halt

If we use $D(\langle D \rangle)$ to check

1. Assume H : Yes, that means $D(\langle D \rangle)$ will not halt
but D must halt in " D " so it is contradiction

Assume H : No, that means $D(\langle D \rangle)$ will halt

but D can not halt in D so it is contradiction.

Therefore halting problem is undecidable