Module Code	Examiner	Department	Tel
INT201	Yushi Li	Intelligent Science	5351

1st SEMESTER 23-24 RESIT EXAMINATION

Undergraduate

Decision Computation and Language

TIME ALLOWED: *2 hours*

INSTRUCTIONS TO CANDIDATES

1. **This is a blended close-book exam and the duration is 2 hours.**
2. **Total marks available are 100. This accounts for 80% of the final mark.**
3. **Answer all questions. Relevant and clear steps should be included in the answers.**
4. **Only English solutions are accepted. For online students, answers need to be handwritten and fully and clearly scanned or photographed for submission as one single PDF file via LEARNING MALL.**
5. **Online students should use the format "Module Code-Student ID.filetype" to name their files before submitting to Learning Mall. For example, "INT201-18181881.pdf".**

Question 1

Indicate true or false of the following statements, and briefly justify your answers. (**30 Marks**)

(a) If A is regular, then A must be finite. (**3 Marks**)

(b) If A has an NFA, then A is nonregular. (**3 Marks**)

(c) Every non-context-free language is also non-regular. (**3 Marks**)

(d) The language $\{a^n b^n \mid n \geq 0\}$ has regular expression $a^* b^*$. (**3 Marks**)

(e) If B is a context-free language and A $\subseteq$ B, then A is context-free.(**3 Marks**)

(f) All language recognized by a non-deterministic automaton NPDA can be recognized by a deterministic pushdown automaton DPDA (**3 Marks**)

(g) Turing-decidable languages are closed under concatenation (**3 Marks**)

(h) If Language A can be recognized by a Non-deterministic Turing machine, A is TM-recognizable. (**3 Marks**)

(i) All context-free languages are decidable. (**3 Marks**)

(j) If a language A is mapping reducible to a TM-recognizable language B and B is decidable, then A is decidable also. (**3 Marks**)

Question 2
Let $\Sigma = \{a, b\}$, and define $A = \{w \in \Sigma^* \mid w = sba$ for some string $s \in \Sigma^*\}$, i.e., A consists of strings that end in ba. (**8 Marks**)

Question 3
Give regular expressions that generate each of the following languages. In all cases, the alphabet is $\Sigma = \{a, b\}$. (**12 Marks**)

(a) The language $\{w \in \Sigma^* \mid w$ has an odd number of a's $\}$.(**4 Marks**)

(b) The language $\{w \in \Sigma^* \mid w$ ends in a double letter $\}$. (A string contains a double letter if it contains aa or bb as a substring) (**4 Marks**)

(c) The language $\{w \in \Sigma^* \mid w$ does not end in a double letter $\}$. (**4 Marks**)

Question 4

The original CFG is shown as follows, and convert it to Chomsky normal form. (**15 Marks**)

$$
\begin{aligned}
&S \to XSX \mid aY \\
&X \to Y \mid S \\
&Y \to b
\end{aligned}
$$

Question 5
Let $\Sigma = \{a, b, c\}$, and consider the language $A = \{a^i b^j c^k \mid i \geq 0, j \geq 0, k \geq 0,$ and i, j, k all different$\}$. (**10 Marks**)

(a) Complete following the pumping lemma statement for context-free languages.
If A is a context free language, then there is a number p (pumping length) where, if $s \in L$ with $\mid s \mid \geq p$, then there are strings u, v, x, y, z such that

$s = uvxyz$, the following holds: (**4 Marks**)

(b) Use the pumping lemma to show that A is not context-free or show that A satisfies the pumping lemma conditions nonetheless. (**6 Marks**)

Question 6

Let $\Sigma = \{a, b\}$, pushdown automata are given by the diagrams below. (**13 Marks**)

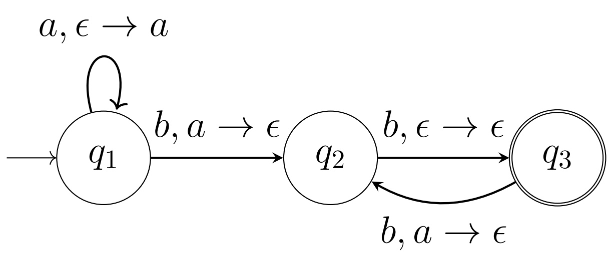

Figure 1: PDA A

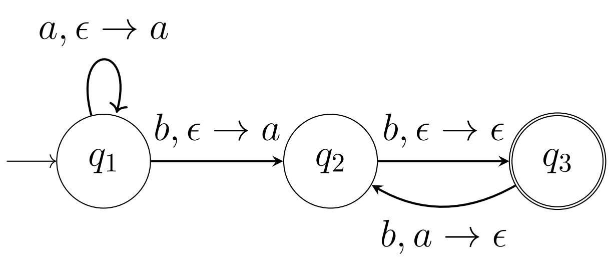

Figure 2: PDA B

(a) What is the language that is being accepted by the PDA A? (**4 Marks**)

(b) Write a context-free grammar that accepts the same language of L(A) (**4 Marks**)

(c) Is $L(A) \subset L(B)$? Justify your answer (**5 Marks**)

Question 7

Let $\Sigma = \{0, 1\}$, and consider the language $A = \{\langle TM \rangle \mid$ TM is a Turing machine that accepts string 101 and $|\langle TM \rangle| \leq 100\}$. In other words, language A consists of Turing machine descriptions with length less than 100 and only those Turing machine that accepts string 101. (**12 Marks**)

(a) State the definition of Turing-recognizable languages. (**2 Marks**)

(b) Is the language A Turing-recognizable? (**4 Marks**)

(c) The Rice's theorem states: let P be a language consisting of TM descriptions where first P is nontrivial. i.e., P contains some TMs, but not all TMs; P does not distinguish computationally equivalent TMs. i.e., for any two TMs M_1, M_2. If $L(M_1) = L(M_2)$, either 1): $\langle M_1 \rangle \in P$ and $\langle M_2 \rangle \in P$ or 2): $\langle M_1 \rangle \notin P$ and $\langle M_2 \rangle \notin P$. Then P is undecidable.

Is language A decidable? Prove it, or disprove it and explain why Rice's theorem does not apply. (**6 Marks**)