Module Code	Examiner	Department	Tel
INT201	Yushi Li	Intelligent Science	5351

1st SEMESTER 23-24 RESIT EXAMINATION

Undergraduate

Decision Computation and Language

TIME ALLOWED: *2 hours*

INSTRUCTIONS TO CANDIDATES

1. **This is a blended close-book exam and the duration is 2 hours.**
2. **Total marks available are 100. This accounts for 80% of the final mark.**
3. **Answer all questions. Relevant and clear steps should be included in the answers.**
4. **Only English solutions are accepted. For online students, answers need to be handwritten and fully and clearly scanned or photographed for submission as one single PDF file via LEARNING MALL.**
5. **Online students should use the format "Module Code-Student ID.filetype" to name their files before submitting to Learning Mall. For example, "INT201-18181881.pdf".**

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Question 1
Indicate true or false of the following statements, and briefly justify your answers. (**30 Marks**)

(a) If A is regular, then A must be finite. (**3 Marks**)

False

(b) If A has an NFA, then A is nonregular. (**3 Marks**)

False

(c) Every non-context-free language is also non-regular. (**3 Marks**)

True

(d) The language $\{a^n b^n \mid n \geq 0\}$ has regular expression a^*b^*. (**3 Marks**)

False non-regular

(e) If B is a context-free language and A $\subseteq$ B, then A is context-free.(**3 Marks**)

True

(f) All language recognized by a non-deterministic automaton NPDA can be recognized by a deterministic pushdown automaton DPDA (**3 Marks**)

False NPDA not always can convert into DPDA

(g) Turing-decidable languages are closed under concatenation (**3 Marks**)

True

(h) If Language A can be recognized by a Non-deterministic Turing machine, A is TM-recognizable. (**3 Marks**)

True

(i) All context-free languages are decidable. (**3 Marks**)

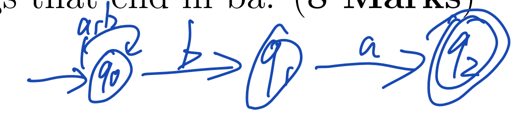
True

(j) If a language A is mapping reducible to a TM-recognizable language B and B is decidable, then A is decidable also. (**3 Marks**)

True

Question 2

Let $\Sigma = \{a, b\}$, and define $A = \{w \in \Sigma^* \mid w = sba$ for some string $s \in \Sigma^*\}$, i.e., A consists of strings that end in ba. (**8 Marks**)



Question 3

Give regular expressions that generate each of the following languages. In all cases, the alphabet is $\Sigma = \{a, b\}$. (**12 Marks**)

(a) The language $\{w \in \Sigma^* \mid w$ has an odd number of a's $\}$.(**4 Marks**)

$(b^*ab^*a)^*b^*ab^*$

(b) The language $\{w \in \Sigma^* \mid w$ ends in a double letter $\}$. (A string contains a double letter if it contains aa or bb as a substring) (**4 Marks**)

$(a\cup b)^*(aa \cup bb)$

(c) The language $\{w \in \Sigma^* \mid w$ does not end in a double letter $\}$. (**4 Marks**)

$(a\cup b)^*(ab \cup ba)$

Question 4

The original CFG is shown as follows, and convert it to Chomsky normal form. (**15 Marks**)

$$S \to XSX \mid aY$$
$$X \to Y \mid S$$
$$Y \to b$$

Question 5

Let $\Sigma = \{a, b, c\}$, and consider the language $A = \{a^i b^j c^k \mid i \geq 0, j \geq 0, k \geq 0,$ and i, j, k all different$\}$. (**10 Marks**)

(a) Complete following the pumping lemma statement for context-free languages.

If A is a context free language, then there is a number p (pumping length) where, if $s \in L$ with $\mid s \mid \geq p$, then there are strings u, v, x, y, z such that

4. Step1: $S_0 \to S$

$S \to XSX \mid aY$

$X \to Y \mid S$

$Y \to b$

Step2: $S_0 \to XSX \mid aY$

$S \to XSX \mid aY$

$X \to b \mid XSX \mid aY$

$Y \to b$

Step3: $S_0 \to XN \mid aY$

$S \to XN \mid aY$

$X \to b \mid XN \mid aY$

$N \to SX$

$Y \to b$

Step4: $S_0 \to XN \mid AY$

$S \to XN \mid AY$

$X \to b \mid XN \mid AY$

$N \to SX$

$Y \to b \quad A \to a$

5. (a) $|vxy| \le p$, $|vy| \ge 1$

for any $i \ge 0$, $uv^i x y^i z \in L$.

(b) Assume A is context-free Language.

there exist constant "p" for A

Choose $S = a^{p+i} b^{p+j} c^{p+k}$ $|S| \ge p$

let S devided into $uvxyz$ five part where $|vxy| \le p$ $|vy| \ge 1$

for any $i \ge 0$ $uv^i x y^i z \in A$

Case 1: the v, y are in same part such that: a, b, c

if v, y are in 'a', $v = a^n$ $y = a^m$

if $i = 2$ $S = a^{p+i+n+m} b^{p+j} c^{p+k}$

there may exists 3 condition:

1. $p+i+n+m = p+j$
2. $p+i+n+m = p+j$
3. $p+i+n+m \ne p+j \ne p+k$

Cause $|mn| \ge 1$ So $\exists (uv^i x y^i z) \notin A$

Case 2: v, y appears in different part such that in 'ab' 'bc'

if v, y are appear in a and b part.

such: $v = a^m$ $y = b^n$

if $i = 2$ $S = a^{p+i+m} b^{p+i+n} c^{p+k}$ there also have three condition.

$\exists (uv^i x y^i z) \notin A$ So A is not context-free language

$s = uvxyz$, the following holds: (**4 Marks**)

(b) Use the pumping lemma to show that A is not context-free or show that A satisfies the pumping lemma conditions nonetheless. (**6 Marks**)

Question 6
Let $\Sigma = \{a, b\}$, pushdown automata are given by the diagrams below. (**13 Marks**)

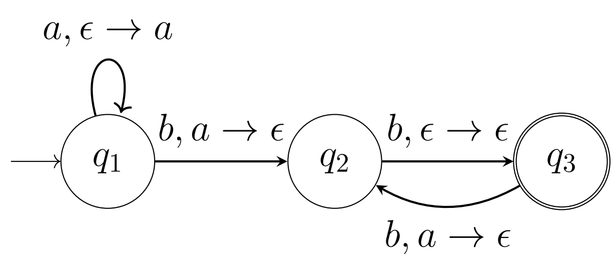


Figure 1: PDA A

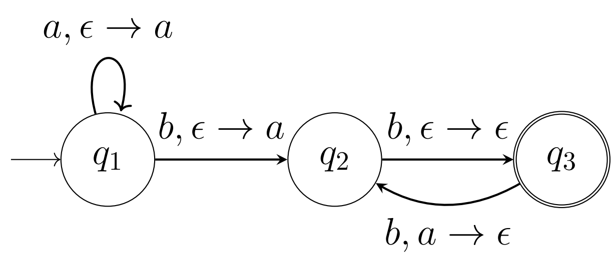


Figure 2: PDA B

(a) What is the language that is being accepted by the PDA A? (**4 Marks**)

(b) Write a context-free grammar that accepts the same language of L(A) (**4 Marks**)

6. (a) $L(A) = \{w = a^n b^{2n} \mid w \in \Sigma^*, n \geq 1\}$

(b) $M = \{V, \Sigma, R, S\}$

$V = \{S, R\}$ $\Sigma = \{a, b\}$ S is start variable

R: $S \to aRbb \mid abb$

$R \to aRbb \mid \varepsilon$

(c) Yes $L(A) \subset L(B)$

Cause $L(B)$ can including all Language of $L(A)$

Such that $a^n b^{2n}$

and there exsits languade $\in L(B)$ and not including

$L(A)$ such that $abbbb \in L(B)$ $\notin L(A)$

(c) Is $L(A) \subset L(B)$? Justify your answer (**5 Marks**)

Question 7

Let $\Sigma = \{0, 1\}$, and consider the language $A = \{\langle TM \rangle \mid$ TM is a Turing machine that accepts string 101 and $|\langle TM \rangle| \leq 100\}$. In other words, language A consists of Turing machine descriptions with length less than 100 and only those Turing machine that accepts string 101. (**12 Marks**)

(a) State the definition of Turing-recognizable languages. (**2 Marks**)

(b) Is the language A Turing-recognizable? (**4 Marks**)

(c) The Rice's theorem states: let P be a language consisting of TM descriptions where first P is nontrivial. i.e., P contains some TMs, but not all TMs; P does not distinguish computationally equivalent TMs. i.e., for any two TMs M_1, M_2. If $L(M_1) = L(M_2)$, either 1): $\langle M_1 \rangle \in P$ and $\langle M_2 \rangle \in P$ or 2): $\langle M_1 \rangle \notin P$ and $\langle M_2 \rangle \notin P$. Then P is undecidable.

Is language A decidable? Prove it, or disprove it and explain why Rice's theorem does not apply. (**6 Marks**)

(a) There exists a $(\langle M \rangle, w)$ where $w \in L$

if $w \in L$ input w on M, M will accept and halt

if $w \notin L$, input w on M, M will reject or loop

(b) Yes, Construct Turing Machine M_A can recognize A

1. on input $\langle TM \rangle$
2. Check $|\langle TM \rangle| \leq 100$

3. Simulate TM on input 101, if TM accept 101, halt and accept

if TM does not accept 101, either halt or loop indefinitely.

So A is Turing-~~re~~recognizable

(c) ① $P \neq \phi$, There exists $L(M_1) = 101$ and $|\langle M_1\rangle| \leq 100$, $L(M_1) \in P$

② $P \neq A$, There exists $L(M_2) = \phi$, $L(M_2) \notin P$

③ for any $L(M_1) = L(M_2)$

if $\langle M_1 \rangle \in P$, but $\langle M_2 \rangle$ may not in P

Cause may $|\langle M_2 \rangle| > 100$

So the Rice's Theorem does not suitable.

Construct a Turing Machine D that can decide A:

1. on input $\langle TM \rangle$ TM is a turing machine
2. check $|\langle TM \rangle| \leq 100$
   a. if $|\langle TM \rangle| \leq 100$ jump to step 3
   b. if $|\langle TM \rangle| > 100$ reject
3. Check wheather String is 101
   a. if TM accept "101" accept $\langle TM \rangle$
   b. if TM reject or loop, reject.

So A is decidable Language.