Module Code	Examiner	Department	Tel
INT201	Yushi Li	Intelligent Science	5351

1st SEMESTER 23-24 FINAL EXAMINATION

Undergraduate

Decision Computation and Language

TIME ALLOWED: *2 hours*

INSTRUCTIONS TO CANDIDATES

1. **This is a blended close-book exam and the duration is 2 hours.**
2. **Total marks available are 100. This accounts for 80% of the final mark.**
3. **Answer all questions. Relevant and clear steps should be included in the answers.**
4. **Only English solutions are accepted. For online students, answers need to be handwritten and fully and clearly scanned or photographed for submission as one single PDF file via LEARNING MALL.**
5. **Online students should use the format "Module Code-Student ID.filetype" to name their files before submitting to Learning Mall. For example, "INT201-18181881.pdf".**

Xi'an Jiaotong-Liverpool University
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Question 1
Indicate true or false of the following statements, and briefly justify your answers. (**30 Marks**)

(a) If A is a regular language, then | A | $< \infty$. (**3 Marks**)

False

(b) If a language A is nonregular, then it has an NFA. (**3 Marks**)

False

(c) The transition function of an NFA is $\delta : Q \times \Sigma \to Q$. (**3 Marks**)

False P(Q)

(d) The regular expression (01*0 ∪ 1)*0 generates the language consisting of all strings over $\Sigma = \{0, 1\}$ having an odd number of 0's. (**3 Marks**)

True

(e) If a language A is regular, then A has a CFG in Chomsky normal form. (**3 Marks**)

True

(f) Language A is context-free if and only if there exists a deterministic pushdown automaton D such that A = L(D) (**3 Marks**)

False

(g) Language A is Turing-decidable if there exists a Turing machine TM such that A = L(TM) (**3 Marks**)

False

(h) If Language A can be recognized by a multi-tape Turing machine, A is TM-recognizable. (**3 Marks**)

True

?

(i) The set of all languages is countable. (**3 Marks**)

False

(j) If a language A is mapping reducible to a TM-recognizable language B and A is decidable, then B is decidable also. (**3 Marks**)

False

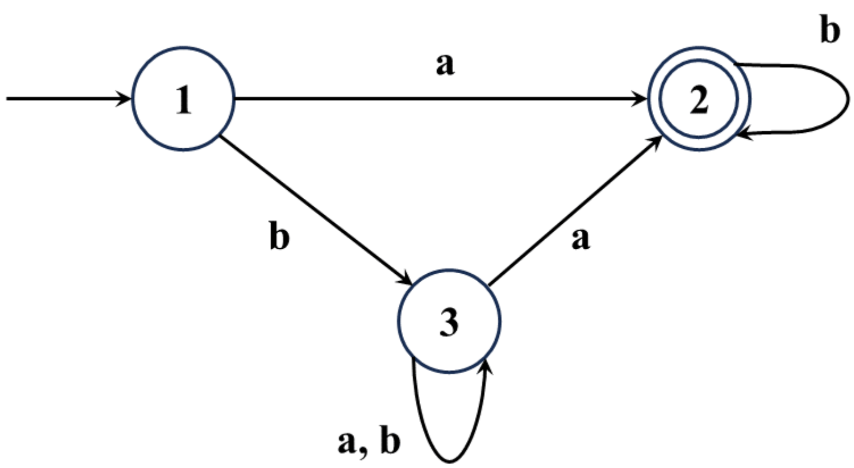
Question 2

Draw an NFA with exactly six states over the alphabet $\Sigma = \{0, 1\}$ that accepts the following language $\{w \in \Sigma^* \mid w$ contains at least two 0s, or exactly two 1s $\}$. (**12 Marks**)

Question 3

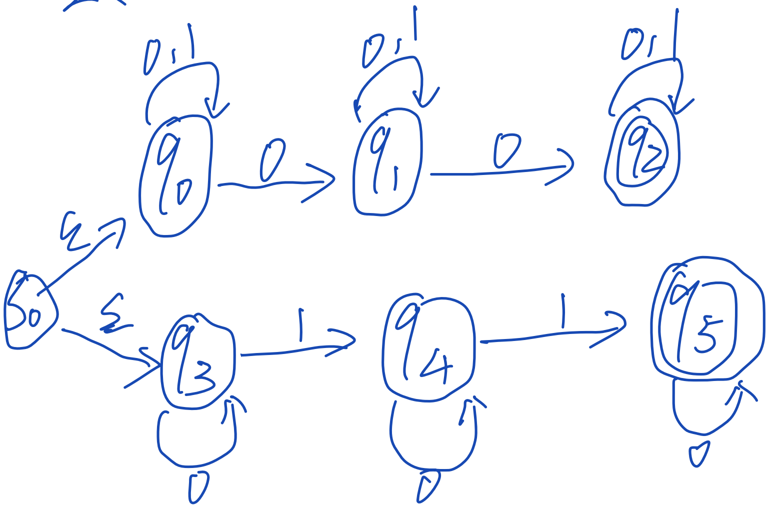
Give the regular expressions for the languages recognized by the NFAs below. (**8 Marks**)

(a) (**4 Marks**)



(b) (**4 Marks**)

2、



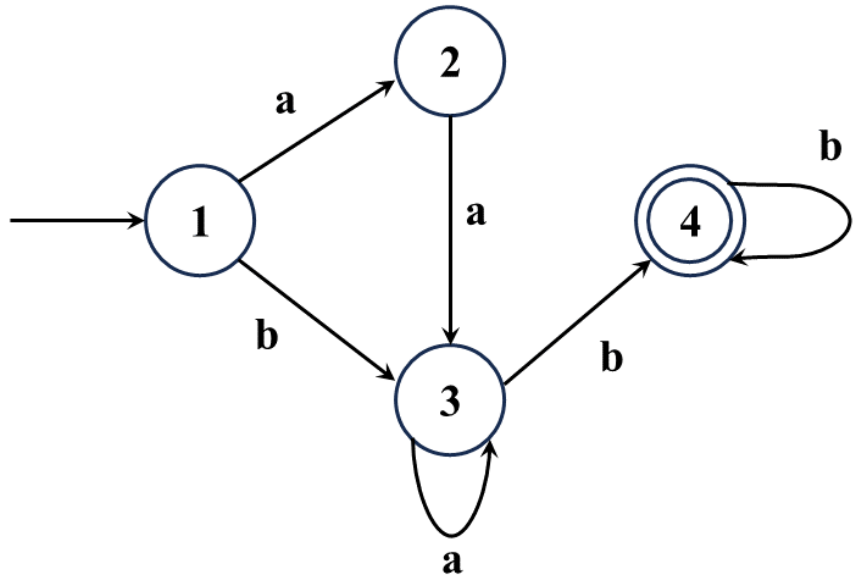
3、(a) $(b(a\cup b)^*a\cup a)b^*$

(b) $(aa\cup b)a^*bb^*$

4、Step 1: $S \rightarrow QaQ \mid aQ \mid Qa \mid a$

$Q \rightarrow aQb \mid ab \mid bQa \mid ba \mid Q \mid QQ$

和 22-23 R 一样



Question 4

The original CFG is shown as follows, and convert it to Chomsky normal form. (**16 Marks**)

$$S \rightarrow QaQ$$
$$Q \rightarrow aQb|bQa|QQ|\epsilon$$

Question 5

Consider the following languages $L_1 = \{a^i b^j c^k \mid i = j, i, j, k \geq 0\}$ and $L_2 = \{a^i b^j c^k \mid j = k, i, j, k \geq 0\}$ (**10 Marks**)

(a) Show L_1 is a context-free language by providing a context-free grammar. (**4 Marks**)

(b) What is the language of $L_1 \cap L_2$? (**2 Marks**)

(c) Is context-free language closed under intersection? Justify your answer.

5. (a) $M = \{V, \Sigma, R, S\}$

$V = \{S, R, C\}$ $\Sigma = \{a, b, c\}$ S is start variable

R: $S \to RC$

$R \to aRb \mid \varepsilon$

$C \to c \mid \varepsilon$

(b) $L = \{a^n b^n c^n \mid n \geq 0\}$

(c) Suppose L is context-free language.

There exsist a constant "p" for L

Choose $S = a^p b^p c^p$ $|S| = 3p \geq p$ devide S into $uvxyz$

where $|vxy| \leq p$ $|vy| \geq 1$ and $i \geq 0$ in $uv^i xy^i z$

Case 1: the v and y are in same part, ~~for~~ Such that in all "a", "b", "c", So if all in "a" $v = a^m$ $y = a^n$ $x = a^r$ where $m+n+r \leq p$

let $i = 2$ $S = a^{p+m+n} b^p c^p$ where $|m+n| \geq 1$ So the three part are not Equal, Same as in "b", "c")

Case 2: the v and y are in different part, Such that in "ab", "b, c"

So if all in "ab" let $v = a^m$ $y = b^n$ e.

if $i = 2$ $S = a^{p+m} y^{p+n} c^p$ where $|m+n| \geq 1$ So the three part are not Equal, Same as in "bc"

(**4 Marks**)

Question 6

Let $\Sigma = \{a, b\}$, pushdown automata are given by the diagrams below. (**13 Marks**)

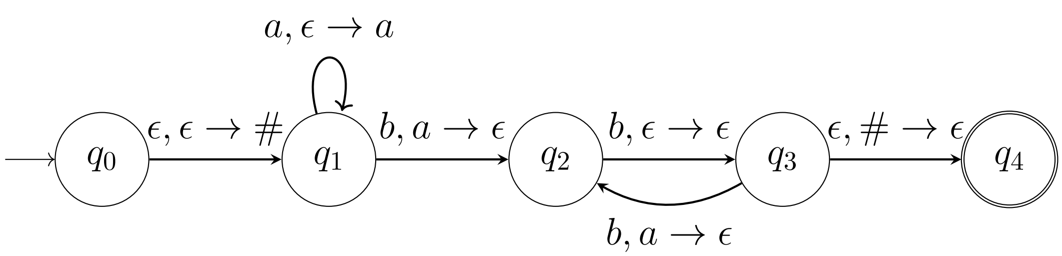


Figure 1: PDA A

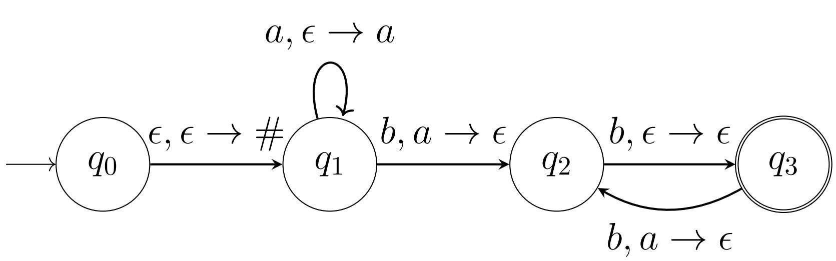


Figure 2: PDA B

(a) What is the language that is being accepted by the PDA A? (**4 Marks**)

(b) Write a context-free grammar that accepts the same language of L(A) (**4 Marks**)

(c) Is $L(A) \subset L(B)$? Justify your answer (**5 Marks**)

Question 7

6. (a) $L = \{w = a^n b^{2n} \mid n \geq 1, w \in \Sigma^*\}$

(b) $M = \{V, \Sigma, R, S\}$

$V = \{S, R\}$ $\Sigma = \{a, b\}$ S is start variable

R: $S \to abb \mid aRbb$ $R \to aRbb \mid \varepsilon$

(c) No, acctrually the $L(A) = L(B)$ cause they can recognize the totally same language.

7. (a) On input $\langle TM, w \rangle$ where $w \in \Sigma^*$.

if TM accept w, accept and halt

if TM reject w, reject and halt.

(b) let P be a subset of TM.

① $P \neq \emptyset$. there exist a Turing machine M such that $\langle M \rangle \in P$

② P is a proper subset of TM, there exists a Turing machine N such that $\langle N \rangle \notin P$

③ for any two Turing machine M_1 and M_2 with $L(M_1) = L(M_2)$

(a) either both $\langle M_1 \rangle$ and $\langle M_2 \rangle$ are in P or

(b) none of $\langle M_1 \rangle$ and $\langle M_2 \rangle$ are not in P.

Let $\Sigma = \{0, 1\}$, and consider the language $A = \{\langle TM \rangle$ |TM is a Turing machine that accepts string 101$\}$. (**12 Marks**)

(a) Complete the definition of decidable languages. We say that A is decidable, if there exists a Turing machine TM, such that for every string $w \in \Sigma^*$, the following holds: (**4 Marks**)

(b) The Rice's theorem can be used to prove a language of TM descriptions being undecidable. What are the requirements of Rice's theorem? (**4 Marks**)

(c) Prove or disprove A is a decidable language. (**4 Marks**)

P is sub set of A

① $P \neq \emptyset$ $L(M_1) = \{101\}$, $\langle M_1 \rangle \in P$

② $P \neq T$ $L(M_2) = \{\emptyset\}$, $\langle M_2 \rangle \notin P$

③ and for any $L(M_1) = L(M_2)$

if $101 \in L(M_1)$, $101 \in L(M_2)$, they are same

if $101 \notin L(M_1)$, $101 \notin L(M_2)$

So $M_1 \in P \Longleftrightarrow M_2 \in P$

According Rice's Theorem, A is nondecidable L.