

Question 1

In each case below, find a CFG generating the given language.

- The set of odd-length strings in $\{a, b\}^*$ with middle symbol a . (10 Marks)
- The set of even-length strings in $\{a, b\}^*$ with the two middle symbols equal. (10 Marks)
- The set of odd-length strings in $\{a, b\}^*$ whose first, middle, and last symbols are all the same. (10 Marks)

(30 Marks Total)

Question 2

Closure properties of Turing-recognizable languages.

Are Turing-recognizable languages closed under Kleene star? Prove it or provide a counter example. (10 Marks)

Are Turing-recognizable languages closed under complement? Prove it or provide a counter example. (10 Marks)

Are Turing-decidable languages closed under complement? Prove it or provide a counter example. (10 Marks)

(30 Marks Total)

Question 3

Show the language $A_{CFG} = \{\langle G \rangle \mid G \text{ is a CFG that generates letter } a\}$ is decidable.

(20 Marks Total)

Question 4

Show the language $T = \{\langle M \rangle \mid M \text{ is a TM that accepts } ww \text{ whenever it accept } w\}$ is undecidable.

(20 Marks Total)