APPENDIX A
PROOF OF THEOREM 1

Theorem 1 (Supermartingale Two-Hop End-to-End Service Reliability with Heterogeneous Services)**.** *The queue's stability condition is* $\mathbb{E}[a(1)] \leq min\left(\mathbb{E}[s_1(1)], \mathbb{E}[s_2(1)]\right)$ *and define*

$$\theta^* = min\{\theta_1^*, \theta_2^*\},$$

where $\theta_i^* = sup\left\{\theta > 0 : \Upsilon_a \leq \Upsilon_{s_i}\right\}, \forall i \in \{1,2\}$*. Then, we can obtain the service reliability of MU* u_k *offloading tasks to the BS at a large slot as*

$$\begin{aligned}&\mathbb{P}(L(n) \geq \sigma)\\&\leq \frac{\mathbb{E}\left[j_a(a(0))\right]\mathbb{E}\left[j_{s_1}(s_1(0))\right]\mathbb{E}\left[j_{s_2}(s_2(0))\right]}{J} e^{-\theta^*\sigma\Upsilon_a},\end{aligned}$$

where

$$J = \min_{a > max\{s_1, s_2\}} \left\{j_a(x) j_{s_1}(s_1) j_{s_2}(s_2)\right\},$$

i.e., the minimum of $j_a(a) j_{s_1}(s_1) j_{s_2}(s_2)$ *when the immediate arrival volume (i.e., a) is greater than the maximum of the two instantaneous services (i.e.,* $max\{s_1, s_2\}$*).*

Proof. Before starting the proof, we give the expression for Doob's inequality, which is used in the last few steps of the proof, *i.e.*, when $X(n)$ is a supermartingale process, we have

$$\mathbb{P}\left(\sup_{0\leq\sigma\leq n} X(n) \geq \zeta\right) \leq \frac{\mathbb{E}\left[X(\sigma)\right]}{\zeta}.$$

According to the Eq. (20)(21), there exists $\varsigma_1 + \varsigma_2 = n$ such that $S_1(n) \otimes S_2(n) = S_1(\varsigma_1) + S_2(\varsigma_2)$. Then, we can derive the delay violation probability defined in Eq. (16) as

$$\begin{aligned}&\mathbb{P}(L(n) \geq \sigma)\\&\quad = \mathbb{P}\left(A(n-\sigma) \geq D_2(n)\right)\\&\quad \leq \mathbb{P}\left(A(n-\sigma) \geq \inf_{0\leq\varsigma\leq n}\{A(0,\varsigma) + \tilde{S}(\varsigma, n)\}\right)\\&\quad \leq \mathbb{P}\left(\sup_{0\leq\sigma\leq n}\left\{A(\sigma,n) - \tilde{S}(n)\right\} \geq 0\right)\\&\quad = \mathbb{P}\left(\sup_{0\leq\sigma\leq n}\left\{A(\sigma,n) - S_1(n)\otimes S_2(n)\right\} \geq 0\right)\\&\quad = \mathbb{P}\left(\sup_{0\leq\sigma\leq n}\left\{A(\sigma,n) - (S_1(\varsigma_1) + S_2(\varsigma_2))\right\} \geq 0\right).\end{aligned}$$

From the definition of θ^*, *i.e.*, Eq. (26), we have that $\Upsilon_a \leq \Upsilon_{s_i}, \forall i \in \{1,2\}$ holds, so we can proceed to derive the following expression:

$$\begin{aligned}&\mathbb{P}\left(\sup_{0\leq\sigma\leq n}\left\{A(\sigma,n) - (S_1(\varsigma_1) + S_2(\varsigma_2))\right\} \geq 0\right)\\&\leq \mathbb{P}\left(\sup_{0\leq\sigma\leq n}\begin{Bmatrix}A(\sigma,n) - (S_1(\varsigma_1) + S_2(\varsigma_2)) - \\ \varsigma_1\Upsilon_a + \varsigma_1\Upsilon_{s_1} - \varsigma_2\Upsilon_a + \varsigma_2\Upsilon_{s_2}\\ -\sigma\Upsilon_a + \sigma\Upsilon_a\end{Bmatrix} \geq 0\right)\\&= \mathbb{P}\left(\sup_{0\leq\sigma\leq n}\begin{Bmatrix}A(\sigma,n) - (n-\sigma)\Upsilon_a + \varsigma_1\Upsilon_{s_1}\\ -S_1(\varsigma_1) + \varsigma_2\Upsilon_{s_2} - S_2(\varsigma_2)\end{Bmatrix} \geq \sigma\Upsilon_a\right).\end{aligned} \tag{1}$$

Then, by the **Definitions** 2 and 3 of the arrival and service martingale, we can build the corresponding supermartingales for both the arrival and service processes, *i.e.*,

$$\begin{aligned}M_A(n) &= j_a(a(n)) e^{\theta^*(A(\sigma,n) - (n-\sigma)\Upsilon_a)},\\M_{S_1}(\varsigma_1) &= j_{s_1}(s_1(\varsigma_1)) e^{\theta^*\left(\varsigma_1\Upsilon_{s_1} - S_1(\varsigma_1)\right)},\\M_{S_2}(\varsigma_2) &= j_{s_2}(s_2(\varsigma_2)) e^{\theta^*\left(\varsigma_2\Upsilon_{s_2} - S_2(\varsigma_2)\right)},\end{aligned}$$

where $\varsigma_1 + \varsigma_2 = n$.

According to the independence assumption of the supermartingale, we use the product of the above independent supermartingales to construct a new supermartingale, *i.e.*,

$$\begin{aligned}M(n) =& j_a(a(n)) j_{s_1}(s_1(\varsigma_1)) j_{s_2}(s_2(\varsigma_2)) \times\\& e^{\theta^*\left(A(\sigma,n) - (n-\sigma)\Upsilon_a + \varsigma_1\Upsilon_{s_1} - S_1(\varsigma_1) + \varsigma_2\Upsilon_{s_2} - S_2(\varsigma_2)\right)}.\end{aligned}$$

Let $g(\sigma) = e^{\theta^*\left(A(\sigma,n) - (n-\sigma)\Upsilon_a + \varsigma_1\Upsilon_{s_1} - S_1(\varsigma_1) + \varsigma_2\Upsilon_{s_2} - S_2(\varsigma_2)\right)}$. Then the delay violation probability, *i.e.*, (1) can be further derived as (2), where (a) is derived from the definition of J, *i.e.*, Eq. (28), step (b) adopts the Doob's inequality, (c) is based on the independence assumption of the supermartingale and the last step is obtained from the non-increasing character of the supermartingale.

$$\begin{aligned}&\mathbb{P}\left(\sup_{0\leq\sigma\leq n}\begin{Bmatrix}A(\sigma,n) - (n-\sigma)\Upsilon_a + \varsigma_1\Upsilon_{s_1}\\ -S_1(\varsigma_1) + \varsigma_2\Upsilon_{s_2} - S_2(\varsigma_2)\end{Bmatrix} \geq \sigma\Upsilon_a\right)\\&= \mathbb{P}\left(\sup_{0\leq\sigma\leq n}\{g(\sigma)\} \geq e^{\theta^*\sigma\Upsilon_a}\right)\\&\overset{(a)}{\leq} \mathbb{P}\left(\sup_{0\leq\sigma\leq n}\{M(n)\} \geq J e^{\theta^*\sigma\Upsilon_a}\right)\\&\overset{(b)}{\leq} \frac{\mathbb{E}\left[M(\sigma)\right]}{J e^{\theta^*\sigma\Upsilon_a}}\\&\overset{(c)}{=} \frac{\mathbb{E}\left[j_a(a(\sigma))\right]\prod_{i=1}^{2}\mathbb{E}\left[j_{s_i}(s_i(\varsigma_i)) e^{\theta^*(\varsigma_i\Upsilon_{s_i} - S_i(\varsigma_i))}\right]}{J e^{\theta^*\sigma\Upsilon_a}}\\&\overset{(d)}{\leq} \frac{\mathbb{E}\left[j_a(a(0))\right]\mathbb{E}\left[j_{s_1}(s_1(0))\right]\mathbb{E}\left[j_{s_2}(s_2(0))\right]}{J} e^{-\theta^*\sigma\Upsilon_a}.\end{aligned} \tag{2}$$

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