Decentralized Search for Shortest Path Approximation in Large-scale Complex Networks

ABSTRACT

Finding approximate shortest paths for extremely large-scale complex networks is a challenging problem, where existing work requires large overhead to achieve accurate results and good path diversity for graphs with millions of vertices. In this paper, we develop an online search method based on preprocessed indexes, to approximate shortest path of arbitrary pairs of vertices. We demonstrate that the algorithm achieves much higher accuracy and path diversity with less index overhead compared to previous work. The algorithm can also achieve differentiated level of accuracies by dynamically controlling the search space to meet various application needs. In order to perform decentralized search more efficiently, we propose a new heuristic index construction algorithm that can greatly increase the approximation accuracy. We further develop support for parallel searches to further reduce the average search time for a large number of queries. We implement our algorithm in distributed way to deal with extreme size graphs. Our algorithm can handle graphs with billions of edges and perform searches for millions of queries in parallel. We evaluate our algorithm on various real world graphs from different disciplines.

CCS Concepts

•**Information systems** → **Data structures;** *Data mining;*

Keywords

Graphs, Shortest Paths, Decentralized Search

1. INTRODUCTION

Various types of graphs are commonly used as models for real-world phenomenon, such as online social networks, biological networks, the world wide web, among others [16]. As their sizes keep increasing, scaling up algorithms to handle extreme size graphs with billions of vertices and edges remains a challenge that has drawn increased attention in recent years. Specifically, straightforward operations in graph theory are usually too slow or costly when they are applied to graphs at this scale. One problem is finding shortest paths in the network, an operation that serves as the building block for many other tasks. For example, a natural application for road network is providing driving directions [1]. In social networks, such applications include social sensitive search [26], analyzing influential people [12], among others. Estimating minimum round trip time between hosts without direct measurement is another application in technology networks [23].

Although previous works have studied the shortest path problem on large road networks extensively and have very effective approaches, a large category of networks known as the complex networks has very different structures. Approaches that worked on road networks do not perform well on complex networks, as the latter follow power law degree distributions and exhibit small diameters. In this paper, we focus on the shortest path problem for complex networks in particular, as their extreme sizes and unique topologies make the problem particularly challenging.

Our design is motivated by recent studies that combine both offline processing and online queries [17, 25, 3, 18, 11]. In these methods, the step of preprocessing aims to construct indexes for the networks, which are later used in the online query phase to dramatically reduce the query time. Among these approaches, landmark based algorithms [24, 8, 17, 10, 25, 18]are widely used for approximate shortest path/distance between vertices. Such algorithms select a small set of landmarks, and construct an index that consists of labels for each vertex, that store distances or shortest paths to landmarks. The approximation accuracy of landmark based algorithms heavily depends on the number of landmarks. To achieve high accuracy, a relatively large set of landmarks is required, which lead to large preprocessing overheads. Indexes that can answer path queries usually have much larger space overhead than indexes that can only answer distance queries. One goal of our design, therefore, is to provide accurate results while still maintain low overhead for indexing.

Previous works only apply online search to explore short cuts between labels of source and target vertices for indexed graphs [10, 18]. The accuracy and diversity of approximated path are rather limited this way, e.g. the approximated path only consist vertices contained in both labels. To overcome this problem, we propose to performs a heuristic search over the indexed graph. The search is guided by locally collected

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information from labels of nearby vertices. The advantage is that the search can expand the search space into edges that have not been indexed to achieve higher accuracy and diversity of approximated path with limited index size. The heuristic search that we use is called decentralized search which was introduced in [13, ?]. Here the "'decentralized"' means that the decision of the search is made based solely on local information which, in our context, is the labels of neighbor vertices at each step of the search.

Decentralized search is very light weighted. The number of visited vertices for decentralized search is bounded by the diameter of the network. Considering that complex networks have relatively a short diameter, decentralized search can finish in limited number of steps. The search can also adjust its search space to balance between different level of performance and required resources for each search. The search is very versatile to meet various application needs without redoing the preprocessing.

The performance of decentralized search relies heavily on the index. To achieve better accuracy without increasing index overhead, we introduce a heuristic index construction algorithm to control shortest paths to be indexed during preprocessing. The propose approach outperforms random shortest path indexing by large margin on real networks.

We implement decentralized search on a distributed platform to support large scale graph with billions of edges. To take advantage of resources available for a cluster of machines, our algorithm can support large amount of queries to run in parallel. There are two properties of decentralized search which makes it very suitable for parallel processing. First, decentralized search has small space complexity and communication complexity. The search does not need to store any information on a per vertex basis like BFS or A* search, very limited space overhead is required for each search. Second, decentralized search only have *RAR* data dependency on the index and underlying graph. Multiple searches can run independently on the same graph and index. These properties makes it possible for a large number of searches running in parallel efficiently without reaching the physical limit of machines, i.e. memory size or network bandwidth. For example, in our experiments, we show that millions of decentralized search can run in parallel on graphs with billions of edges on a cluster of commodity machines, and finish in tens of seconds.

Based on our algorithm design, we further develop a query-processing system based on distributed cloud infrastructure. In this platform, users first submit their graphs for preprocessing needs. The graph processing engine will assign resources according to application's need for accuracy and construct an index for the input graph. Later, users may submit large volumes of queries repeatedly, for which responses will be generated. The light-weighted decentralized search allows a large number of queries to run in parallel so that queries can be answered in a timely manner. Applications that generate queries (on the client side) can provide their desired accuracy levels and the graph processing engine can dynamically adjust search space of decentralized search to meet differentiated levels of accuracies.

1.1 Contributions

Our contributions can be summarized as follows:

- We propose index guided decentralized search for shortest path approximation;
- We design a heuristic index construction algorithm that can improve accuracy of the decentralized search without increasing preprocessing or online search overheads;
- We achieve efficient query processing and good scalability with distributed implementation and parallel processing;
- Experiments on various real world complex networks demonstrate that the proposed algorithm is promising in approximating shortest path compared to existing works.

The rest of this paper is organized as follows. In Section 2 we show previous works on exact and approximate approaches. Section 3 provides notations and definitions used in this paper. We explain index guided decentralized search for shortest path approximation in Section 4. Section 5 discusses index construction algorithm. In Section 6 we show details on our distributed implementation. The evaluations of our algorithm is in Section 7. We conclude our work in Section 8.

2. RELATED WORKS

The majority of the exact approaches are based on either 2-hop cover [5, 2] or tree decomposition [3, 27]. For the former one, finding optimal 2-hop covers is a challenging problem. Reference [2] takes a different approach that solve 2-hop cover problem with graph traversals which has better scalability. Reference [11] borrowed the highway concept from shortest path algorithms on road networks and construct a spanning tree as a "'highway"'. Reference [7] introduced an effective disk-based label indexing method based on independent set.

Since the exact approaches do not scale well, approximate algorithms are also well studied for large-scale complex networks. Landmark based algorithms are extensively studied for approximating shortest path/distance [24, 8, 17, 6, 15]. Although theoretical study of such algorithms does not reveal promising results [24], they work well in practice. Reference [17, 22] studied various landmark selection strategies for constructing better indexes. A common problem for distance-only indexes is that they do not perform well for close pairs of vertices [3]. Algorithms [10, 25, 18] that index shortest paths are proposed to alleviate this problem. The problem has also been formed as a learning problem [4] and mapped to low-dimension Euclidean coordinate spaces [28] to find approximated answers.

Our work falls into the category of applying online searches to indexed graphs. A* search is used for online query based on indexes constructed by landmarks to answer exact shortest path queries [8]. However, the cost of each A* search is still very high for large scale networks. There are also a few work [10, 18] perform online searches on sub-graphs consist of labels of source and target vertex. Although search space is greatly reduced, path accuracy and diversity are compromised with limited number of landmarks due to the constraints of the search space.

3. PRELIMINARIES

In our problem, we consider a graph $G = (V, E)$. For a source node s and a target node t, we are interested in finding a path $p(s,t) = (s, v_1, v_2, ..., v_{l-1}, t)$ with a length of $|p(s,t)|$ close to the exact distance between s and t in the graph. Let $P(s,t)$ be the set of all paths from s to t. The distance between s and t is $d_G(s,t) = min_{p(s,t) \in P(s,t)} |p(s,t)|$. We focus on unweighted, undirected graphs in this paper.

Our method is motivated by the idea of using landmarks as the basis for indexes. Specifically, given a graph G and a small (constant) set of landmarks $(l_1, l_2, ..., l_k)$, an index for the graph contains a label $L(v)$ for each vertex which store the shortest path to each landmark $(l_i, p(v, l_i))$ where $|p(v,l_i)| = d_G(v, l_i)$. The number of landmarks is denoted as k. The label can be calculated by building a shortest path tree SPT from each landmark.

The least common ancestor of two vertex in a tree is the furthest ancestor from the root, we denote it as $LCA(s,t)$. The shortest path distance satisfies the triangle inequality, for an arbitrary pair of vertices s and t, the following bound holds:

$$d_G(s,t) \leq min_l\{d_G(s, LCA_l(s,t)) + d_G(LCA_l(s,t), t)\} \quad (1)$$

This upper bound, which is referred to as LCA distance and denoted by $d_{LCA}(s,t)$, can be used as an approximation of the distance from s and t. And we denote the path indicated by this distance as $p_{LCA}(s,t)$.

4. DECENTRALIZED SEARCH FOR SHORTEST PATH APPROXIMATION

We propose to solve the point-to-point shortest path estimation problem using decentralized search with landmark based index. This section explains how to apply decentralized search on indexed graphs. Several aspects of the search including termination condition, bidirectional search and tie breaking strategy are discussed.

4.1 Index guided decentralized search

To answer a shortest path query, decentralized search iteratively collect local distance information and visit vertex with least approximated distance to the target. More specifically, for a given pair of source s and target vertex t on an indexed graph. The search first set the source vertex as the vertex to visit in first step and appends it to the approximated path p. At each step, suppose the search is visiting u, it traverse all the neighbor vertices of u. For each neighbor vertex v_i, the LCA distance to the target vertex t is calculated. Then the search set the neighbor vertex with least LCA distance to the target as the vertex to visit in next step and appended to p. The search continues until it reach the target vertex t.

However, terminating when the search reaches the target vertex is a valid but not an ideal termination condition. Since the label of each vertex stores the shortest path of each vertex to each landmark, and shortest path follows the optimal substructure, i.e. the path between any two vertices along the shortest path is also the shortest path of them. A search can stop once it reaches any vertex in the label of the target vertex. The remaining path can be extracted from the label of target vertex to form an approximated path. The detailed algorithm of decentralized search is depicted in Algorithm 1, we denote $\tilde{p}$ as the approximated path.

Algorithm 1 Algorithm decentralized search

```
function DecentralizedSearch($s$, $t$)
    $\tilde{p} \leftarrow \emptyset$
    $u \leftarrow s$
    append $u$ to $\tilde{p}$
    while $u \notin L(t)$ do
        $d_{min} \leftarrow \infty$
        $w \leftarrow u$
        for each $v_i$ adjacent to $u$ do
            if $d_{LCA}(v_i, t) < d_{min}$ then
                $d_{min} \leftarrow d_{LCA}(v_i, t)$
                $w \leftarrow v_i$
        $u \leftarrow w$
        append $u$ to $\tilde{p}$
    $\tilde{p} \leftarrow$ path from $u$ to $t$ in $L(t)$ excluding $u$
    append $p_{remain}$ to $\tilde{p}$
    return $\tilde{p}$
```

At each step, by examining neighbor vertices the search is able to explore subset of the graph that is not indexed in the label of source and target vertex, which can potentially increase both accuracy and diversity of the path being found. For example in Fig. 1(a) from vertex 14 to vertex 17, decentralized search find a edge $(6,7)$ as vertex 7 has a LCA distance of 3 which is shorter than LCA distance from vertex 1 to 17.

Theorem 1. *If the target vertex is reachable from the source, the decentralized search terminate in as much as $2\sigma_{max}$ steps, where σ_{max} is the diameter of the graph.*

Proof Sketch. For an arbitrary source vertex s and a reachable target vertex t, the following bound holds:

$$d_{LCA}(s,t) = d_G(s, LCA(s,t)) + d_G(LCA(s,t), t) \leq 2\sigma_{max}$$

And at each step, suppose decentralized search is visiting vertex u and $u \neq t$. Assume the tightest upper bound in equation 1 is achieved on shortest path tree SPT_l rooted at landmark l. Let v be the neighbor vertex of u on path path $p_{LCA_l}(u,t)$. Since SPT_l does not have cycle, $p_{LCA_l}(v,t) \in p_{LCA_l}(u,t)$. Therefore the following equation holds:

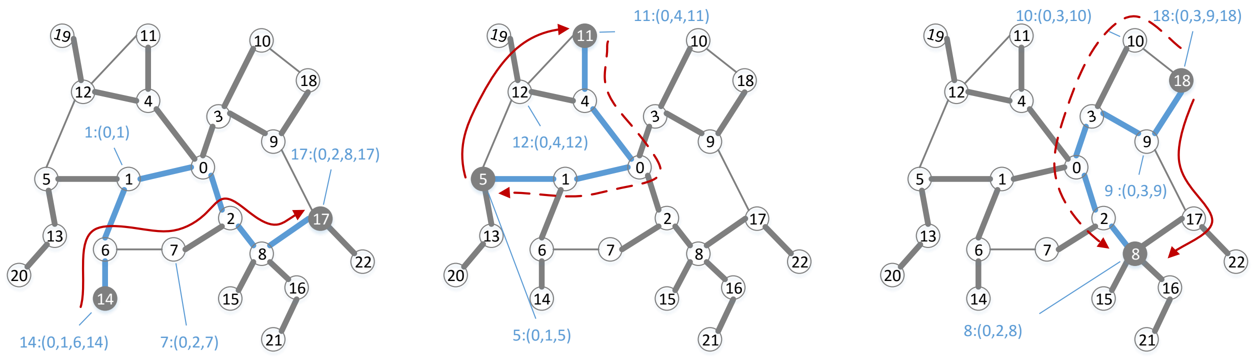
$$d_{LCA}(v,t) = |p_{LCA_l}(v,t)| = |p_{LCA_l}(u,t)| - 1 = d_{LCA}(u,t) - 1$$

Since decentralized search always pick the neighbor with shortest LCA distance to the target, the LCA distance to the target at each step decreases at least by 1. Therefore, the decentralized search terminates in at most $2\sigma_{max}$ steps. □

The time complexity of Decentralized search depends on the max degree and the diameter of the graph. Decentralized search take at most $2\sigma_{max}$ steps to finish. For each step, the search need to check at most δ_{max} neighbor vertices, where δ_{max} is the max vertex degree of the graph. For each neighbor, k LCA computations are required, and time complexity for each LCA computation is $O(h)$[1], where h is the height of the indexed shortest path tree and $h \leq \sigma_{max}$. So the worst case time complexity of decentralized search is $O(k{\sigma_{max}}^2\delta_{max})$.

The space complexity for decentralized search contains two parts, offline index space complexity and online query space complexity. The space required for offline index is $O(k\sigma_{max}n)$, where n is number of vertices. For each query,

[1]$O(h)$ is for simple online algorithm, off-line algorithms can achieve better time complexity $O(1)$.



(a) Decentralized search (b) Bi-directional Decentralized Search (c) Tie breaking strategy

Figure 1: Examples of decentralized search on indexed graph. Bold lines denote the indexed edges. Curved lines denoted paths being found, with arrows showing the direction. Dark vertices denote source and target vertex. Labels of vertices are shown in $vertex : label$ format.

$O(k\sigma_{max})$ space is required to store the labels of target vertex and the vertex that is being examined, $O(2\sigma_{max})$ space is required to store the approximated path. Combining them together, the online search space complexity of decentralized search is $O(k\sigma_{max})$.

4.2 Bi-directional search

In this section, we show how to apply the idea of bidirectional search to decentralized search. In bidirectional decentralized search, the backward search starts at target vertex and is driven by the goal to reach the source vertex. The forward search and backward search may explore different search spaces due to the different start point and search goal. By exploring a different search space the backward search may find a shorter approximated path. This, however, is quite different from the application of bidirectional search in BFS or A* search where the main focus is to reduce search space.

An example of directional decentralized search is shown in Fig. 1(b). Let 11 be source and 5 be target. The backward search can find a shorter path $p_{bwd} = (5, 12, 11)$ than the path $p_{fwd} = (11, 4, 0, 1, 5)$ found by the forward search. The backward search find a shorter path by exploring edge $(5, 12)$. This edge is not discovered by forward search because when the search traversing vertex 4, it prioritize 0 than 12 due to the former one has lower LDA distance to target vertex 5.

It is not guaranteed that the forward search and backward search will eventually meet at any vertex except the source and target vertex due to the different search space. As shown in our previous example, $p_{fwd} \cap p_{bwd} = (11, 5)$. Actually, in decentralized search, the forward search and backward search are mostly two independent search, that the only interaction of them is when combining the search results. Search results can be combined simply by picking the shorter path or finding short cuts connecting resulting paths.

4.3 Tie breaking strategy

Ties happen frequently in decentralized search, especially when the number of landmarks is small. In the decentralized search, a tie means multiple neighbor vertices have same LCA distance to the target in a step. The reason a tie happens is that there is not sufficient information in the index that can separate neighbor vertices for a query. For example in Fig. 1(c), consider a search from 18 to 8. When traversing neighbors of vertex 18, both vertex 10 and 9 have the same LCA distance to target vertex 8, but their actual distances to vertex 8 are different due to that short cut edge $(9, 17)$ is currently invisible to the decentralized search.

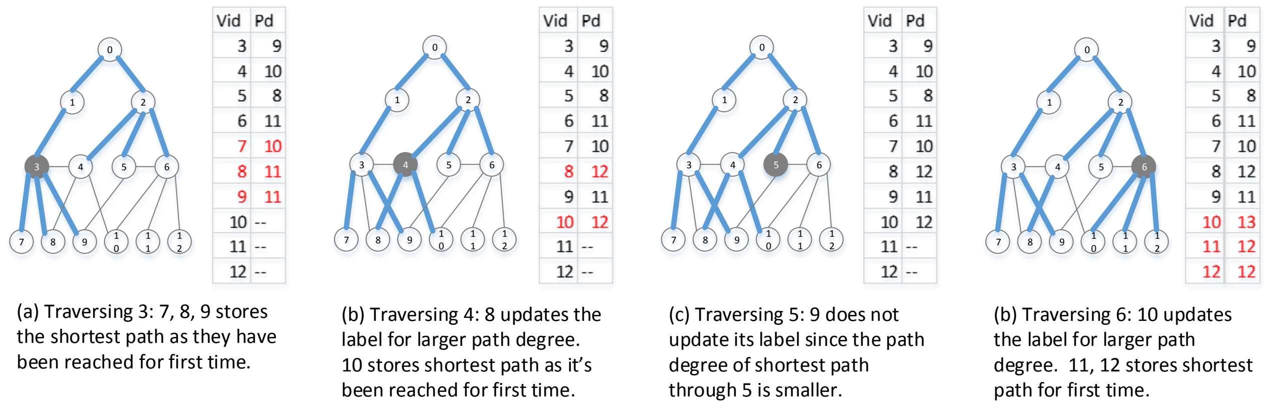
The search space of the decentralized search can be increased by expanding the search onto each tie vertex. This increase both the performance, i.e. chances to find a shorter path and number of paths being found, and the cost of the search. Two extreme ways to deal with ties are either only pick one vertex to visit or visit all vertices in the next step. The former one incurs least search cost and has the least possibility to find a shorter path, we refer to it as single branch decentralized search. The latter one requires most effort and can lead to the shortest path the decentralized search could possibly find, we refer to it as full branch decentralized search.

5. INDEX CONSTRUCTION

This section describes our greedy index construction algorithm. Previous works have studied various landmark selection strategies which have a significant impact on the accuracy of online query. We observe that even with the same landmark set, choosing which shortest path from vertex to landmark to be indexed also plays an important role for the accuracy of online search.

5.1 Greedy index construction algorithm

As the core of decentralized search is to iteratively find vertices that share the least common ancestor with target vertex furthest to the root of the indexed shortest path tree. From the point of view of a vertex, a good shortest path from each landmark to be indexed should be the one that intersects with most of other shortest paths. With this intuition, we design our heuristic greedy index construction algorithm to index the shortest path with highest ”centrality”, i.e. intersects with most other shortest paths. To represent the



Vid	Pd
3	9
4	10
5	8
6	11
7	10
8	11
9	11
10	--
11	--
12	--

(a) Traversing 3: 7, 8, 9 stores the shortest path as they have been reached for first time.

Vid	Pd
3	9
4	10
5	8
6	11
7	10
8	12
9	11
10	12
11	--
12	--

(b) Traversing 4: 8 updates the label for larger path degree. 10 stores shortest path as it's been reached for first time.

Vid	Pd
3	9
4	10
5	8
6	11
7	10
8	12
9	11
10	12
11	--
12	--

(c) Traversing 5: 9 does not update its label since the path degree of shortest path through 5 is smaller.

Vid	Pd
3	9
4	10
5	8
6	11
7	10
8	12
9	11
10	13
11	12
12	12

(b) Traversing 6: 10 updates the label for larger path degree. 11, 12 stores shortest path for first time.

Figure 2: Greedy algorithm index shortest path with highest path degree during breadth first search

"centrality" of a shortest path, we use the sum of vertex centrality along the path. Betweenness centrality fits our needs very well, but the computation complexity to even estimate the value for every vertex in a graph is high [20]. So we use degree as an alternative and refer the sum of degree of vertices along a path as path degree, denoted by Pd.

Our index construction algorithm, which greedily index shortest paths with highest path degree, can be easily modified from the regular index construction procedure. Note that path degree of shortest path follows optimal substructure, i.e. if $(u, .., w, ..., v)$ has the highest path degree among all the shortest path from u to v, then the path degree of $(u, ..., w)$ is also the highest among all the shortest paths from u to w. Normally, the index is constructed with a BFS and a label is assigned to each node the first time the search reach it. The greedy index construction algorithm first add a variable to cache the path degree Pd of the shortest path stored in the label for each vertex. Then during BFS traversal for landmark l, suppose the search is visiting vertex u and reach its neighbor v. If $L_l(v)$ is not empty, and $|L_l(v)| > |L_l(u)|$, then a label update is performed if $Pd(u)+v.degree > Pd(v)$. The detailed algorithm of greedy index construction is depicted in 1. Fig. 2 shows an example of how to greedily select shortest path with the highest path degree during breadth first search. When traversing vertex 4, even though vertex 8 has already been indexed with a shortest path $(0, 1, 3, 8)$ into its label, due to that $(0, 2, 4, 8)$ has a higher path degree, the label of vertex 8 is updated. The same thing happens to vertex 10 while traversing vertex 6.

Note that if the landmark set is relatively large, then following the highest path degree heuristic may lead to redundant labels, i.e. similar index for multiple landmarks. A simple way to solve this is to prioritize shortest paths that have not yet been indexed during index construction.

6. DISTRIBUTED IMPLEMENTATIONS

To handle extremely large graphs and large amount of queries, we implement decentralized search on a distributed general graph processing platform, Powergraph [9]. As decentralized search has low online search space complexity and have very low data dependency upon each other, it is well suited to run multiple searches in a parallel way.

Algorithm 2 Algorithm greedy index construction vertex program running on u

```
function INDEX CONSTRUCTION($root$)
    $PD \leftarrow \emptyset$
    $L \leftarrow \emptyset$
    $Q \leftarrow \emptyset$
    $L[root.id] = root.id$
    $PD[root.id] = root.degree$
    $Q.push(root)$
    while $Q \neq \emptyset$ do
        $u = Q.pop()$
        for $eachvadjecenttou$ do
            if $v.id \notin L$ then
                $L[v.id] = L[u.id] \cup v.id$
                $PD[v.id] = PD[u.id] + u.degree$
                $Q.push(v)$
            else if $L(v).size() < L(u).size() + 1$ then
                $continue$
            else if PD[v.id] < PD[u.id] + u.degree then
                $L[v.id] = L[u.id] \cup v.id$
                $PD[v.id] = PD[u.id] + u.degree$
    return $L$
```

An overview of our shortest path query processing system is shown in Fig. 3. The system first partition the graph with Powergraph onto multiple machines. Then several BFSs are performed to construct the index. After the index has been built, multiple shortest path queries can run in parallel with decentralized search. Large volumes of queries can submit repeatedly, for which responses will be generated.

6.1 Decentralized search vertex-program

Decentralized search can be easily implemented as vertex-programs in Gather-Apply-Scatter model used by Powergraph. Each query instance contains the approximated path and label of target vertex, as index is stored distributively and label of target vertex may not be accessible on each machine locally. Each step of decentralized search is split into gather, apply and scatter phase. In Gather phase, LCA distance to target vertex d_{LCA} is collected from each neighbor

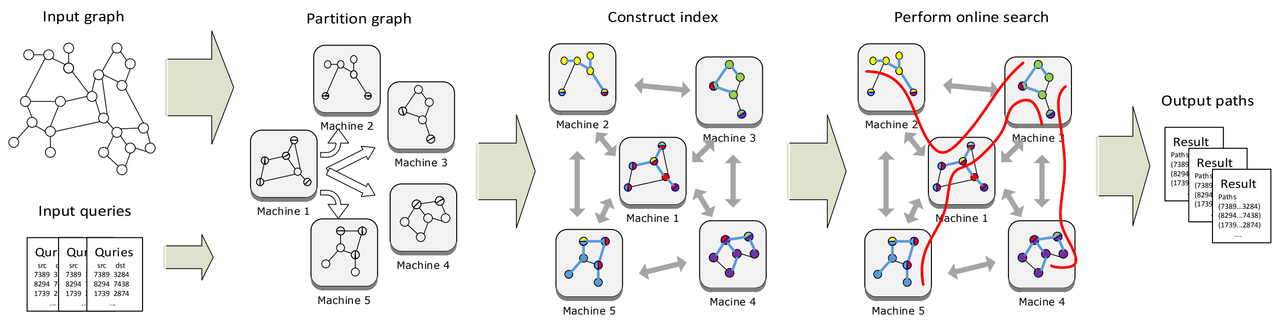


Figure 3: A system overview

and accumulated by finding the neighbor with the smallest LCA distance to target. The accumulated result is returned as a next step candidate. In apply phase, the candidate is appended to the approximated path $p_{appr.}$ and the termination condition is checked. If it is met, the result path will be recorded and the query will be terminated. Otherwise, program will proceed to scatter phase to start a new vertex-program on the candidate vertex and pass on the query instance to them. Algorithm 3 shows the detailed algorithm.

Algorithm 3 Algorithm decentralized search vertex program running on u

```
function GATHER($L(v)$, $L(t)$)
    return $d_LCA(v,t)$, $v.id$
function SUM($d_LCA(v_1,t)$, $vid1$, $d_LCA(v_2,t)$, $vid2$)
    return $min(d_LCA(v_1,t), d_LCA(v_2,t)$, related $vid$
function APPLY($\tilde{p}(s,t)$, $d_LCA(v,t)$)
    $\tilde{p}(s,t) += vid$
    if terminate criteria is met then
        store $\tilde{p}(s,t)$
        $term$ = true
    else
        $term$ = false
function SCATTER($L(t)$, $\tilde{p}(s,t)$, $term$)
    if $\neg term$ then
        Activate(v, $L(t)$, $\tilde{p}(s,t)$)
```

The communication for the decentralized search happen during gather and scatter phase. In gather phase, the label of target vertex need to be passed to multiple machines, and the size is $O(k\sigma_{max})$. And each gather function return a d_{LCA} along side of its id. So for each query, only $O(k\sigma_{max})$ size of data is transferred. In scatter phase, communication happens when a vertex is chosen as the next step candidate and need to be activated, the whole search instance, including approximated path and label of target vertex need be transmitted to the new vertex program. For each query, $O(k\sigma_{max})$ size of data may be transferred. The search will take as much as $2\sigma_{max}$ steps. So the overall communication overhead for each query is $O(k{\sigma_{max}}^2)$.

During decentralized search, only the approximated path $\tilde{p}$ is updated at each step. Therefore, there is only RAR type of data dependency among multiple decentralized searches on the underlying graph. Depends on implementations, there may be output dependency, i.e. WAW, when output $\tilde{p}$ to the same container on each machine.

The low memory and communication cost, along side with RAR only data dependency during the search makes a large number of decentralized search very suitable to run in parallel. To modify the vertex program for parallel processing, each vertex program maintain a list of search instance. During gather phase, label of target vertex for each query is transmitted to other machines, and d_{LCA} is calculated for each query. Each query is updated during apply phase. In scatter phase, each query is examined for whether activating a certain vertex or not.

6.2 Distributed tie breaking strategy

According to tie breaking strategy, multiple neighbors may be returned as candidates at gather phase. In this case, the search instance copies itself into multiple search instances, append each candidate to each search instance at apply phase and activate them at scatter phase. The problem is, as search instances created at apply phase become independent when the search proceeding to next step, during the future steps, even one search instance find a shorter path than others, it can hardly terminate other searches as such synchronizations is too costly in distributed settings. With this limitation, a search may end up with excessive number of child search instances. To overcome this problem, in our implementation, only one candidate will be labeled as a "'main"' candidate at each step. For candidates that are not the "'main"' candidate, an extra termination condition is applied. In the next step, if the search cannot find a shorter path than expected, i.e. $|p_{appr.}| + d_LCA$, the search will stop, and no result will be recorded. This can control the search space effectively without compromise much accuracy.

6.3 Prune LCA computation

A major part of computation load of decentralized search is from large number of LCA computations. It is possible to prune number of LCA computation required at each step for decentralized search to reduce the overall computation load. Suppose the search is visiting vertex u, which means the $d_{LCA}(u,t)$ has already been calculated in previous step. If a neighbor v is a child of u on the indexed shortest path tree SPT_l, then the LCA computation for v and t on SPT_l does not need to be performed as it is clearly that $d_{LCA_l}(v,t) > d_{LCA_l}(u,t)$. In practice, this principle can prune almost half

Table 1: Datasets

Dataset	Type	$\lvert V_{wcc}\rvert$	$\lvert E_{wcc}\rvert$	$\overline{\sigma}$
Wiki	Communication	2.4M	4.7M	3.9
Skitter	Internet	1.7M	11.1M	5.07
Livejournal	Social	4.8M	43.4M	5.6
Hollywood	Collaboration	1.1M	56.3M	3.83
Orkut	Social	3M	117M	4.21
Sinaweibo	Social	58.7M	261.3M	4.15
Webuk	Web	39.3M	796.4M	7.45
Friendster	Social	65M	1.8B	5.03

Datasets with number of vertices and edges in the largest weakly connected components, and average shortest distance $\overline{\sigma}$ of 100,000 vertex pairs.

of the total number of LCA computations of a single search on average.

7. EVALUATIONS

In this section, we show the results of experimental evaluation of decentralized search. We first give an overview of the datasets and introduce the experiment settings of our evaluations. The quality of approximated path generated by decentralized search is evaluated from two aspects, distance accuracy and path diversity, in 7.3 and 7.4 respectively. We then show both overhead of index and online search in 7.5. In the last, we study the scalability of our system in 7.6.

7.1 Datasets

We evaluate our algorithm on 8 graphs from different disciplines as shown in table 1. All graphs are complex networks that have power-law degree distribution and relatively small diameter. To simplify our experiments, we treat each graph as undirected, un-weighted graphs. We only use the largest weakly connected component of each graph which consist of more than 90% of vertices for all graphs. All datasets are collected from Snap [14] and NetworkRepository [21].

7.2 Experiment settings

We evaluate our algorithms in both distributed setting and centralized setting. For the distributed setting, we use 20 Amazon EC2 m4.xlarge virtual machines. Each virtual machine has 4 vCPUs and 16 GB memory. For centralized version, we use a Cloudlab [19] c8220 server with two 10-core 2.2GHz E5-2660 processors and 256GB memory. Powergraph [9] is the platform for distributed version and Snap [14] is the platform for centralized version. All algorithms are implemented in C++.

All the evaluations use the same landmark selection strategy, a variation based on $DEGREE/h$ strategy from reference [17]. When selecting a new landmark, each vertex receive a rank which is the product of its degree and the sum of distance to all the existing landmarks. The vertex with highest rank will be added to the landmark set. We denote number of landmarks as k.

Queries in our experiments are randomly generated. For accuracy evaluations, since performing BFS on large graphs is extremely slow, we randomly choose 1,000 vertices of each graph as source vertices and randomly pick 100 target vertices for each source vertex. All results in 7.3 are averaged among 100,000 queries.

Table 2: Path diversity (k = 2)

Graph	Path cnt(DS)	Path cnt(TS)	$\overline{r_p}$
Wiki	28.9	1.9	0.372
Skitter	24.1	2.4	0.418
Livejournal	30.8	1.9	0.338
Hollywood	9.9	2.6	0.471
Orkut	19.2	3.2	0.465
Sinaweibo	32.0	3.0	0.301
Webuk	704.1	2.0	0.501
Friendster	16.8	2.8	0.39

7.3 Approximation Accuracy

We first evaluate the approximation accuracy of the decentralized search. We use the average approximation error as the measure of accuracy which is defined as follows:

$$E_{\tilde{p}(s,t)} = \frac{|\tilde{p}(s,t)| - d_G(s,t)}{d_G(s,t)}$$

We show the results of 4 variants of decentralized search with mixture of different tie breaking strategy and index construction approach. All the decentralized search are performed with bidirectional search. We also list the performance of an state-of-the-art existing method, TreeSketch [10]. We vary the size of landmark sets and show the averaged results in Fig. 4.

We can see in Fig. 4 that decentralized search achieves better accuracy in most of cases. Especially with small landmark sets, i.e. $k < 5$, decentralized search outperforms TreeSketch on all the graphs. When the full branch decentralized search are carried on the index constructed by our greedy heuristic, the performance gain is most noticeable, with 43.3% to 87.7% lower average error ratio for 1 landmark and 50% to 80% lower average error ratio for 20 landmarks than TreeSketch on all graphs.

Full branch tie strategy always outperforms single branch tie strategy with large margins with same landmark sets. The average error ratio of full branch search with regular index is 17.4% to 45.7% lower than single branch for 1 landmark and 19.5% to 61.8% lower with 20 landmarks on various graphs. As the number of landmark increases, the performance gain also increase.

Decentralized search carried on index constructed by greedy heuristic has lower error ratio than decentralized search with regular index. The average error ratio is 14.5% to 60.2% lower for single branch and 21.1% to 63.3% lower for full branch for 1 landmark. For 20 landmarks, the search is 10.4% to 68.8% lower for single branch and 12.8% to 75% lower for full branch.

7.4 Path Diversity

We show in this section that decentralized search achieves better path diversity by finding more paths and not being constrained by the index. Table 2 shows average number pf paths with shortest approximated distance returned by decentralized search with full branch tie strategies compared to TreeSketch. The average path count of full branch decentralized search is much higher than that of TreeSketch, from 3.73 to 345.13 times for various graphs.

Moreover, decentralized search is not restricted by label of source and target vertices. We define the ratio r_p the number of vertices not in label source and target compared

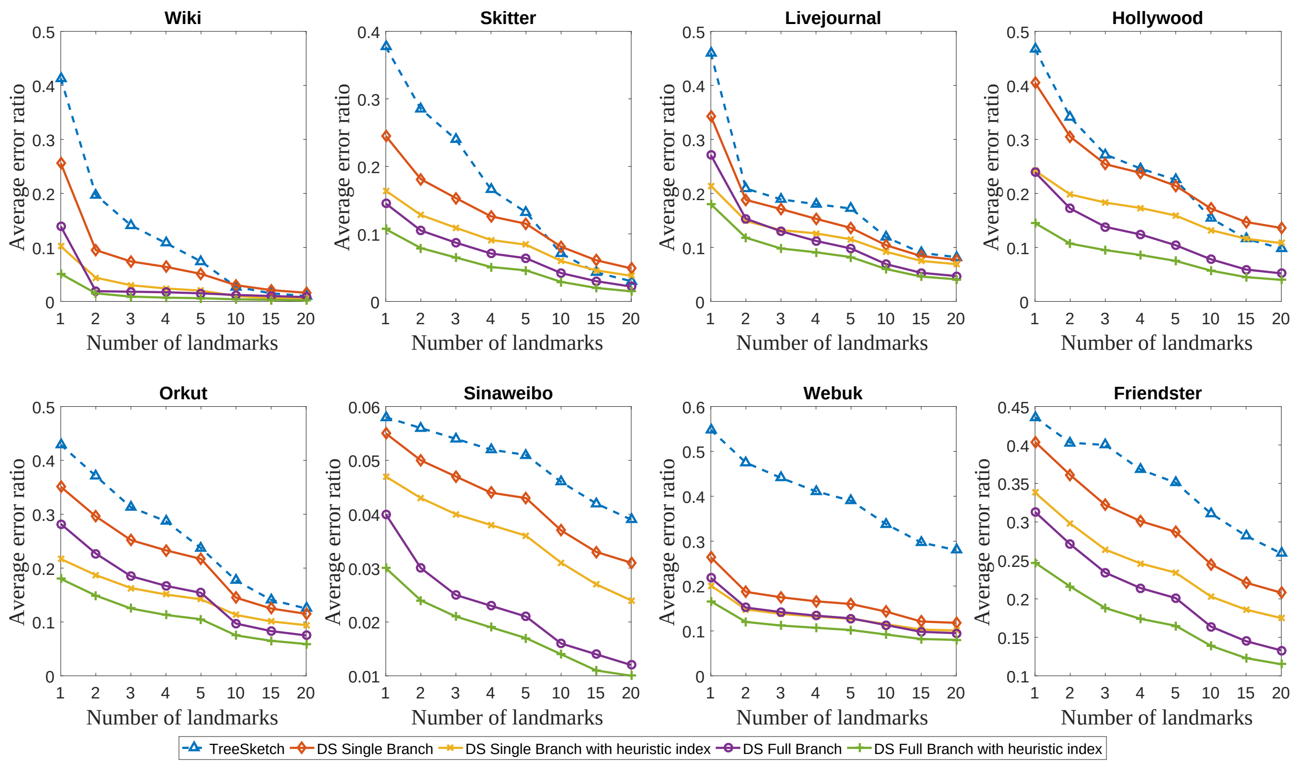


Figure 4: The accuracy of the approximated distance of decentralized search and various optimizations compared to TreeSketch.

Table 3: Index overhead per landmark

Graph	Index size(MB)	Query size(Byte)
Wiki	189.9	369.8
Skitter	143.7	412.9
Livejournal	429.4	419.9
Hollywood	84.1	377.5
Orkut	246.3	390.7
Sinaweibo	4313.8	367.4
Webuk	4068.9	506.5
Friendster	5497.7	437.7

to total vertices except source and target:

$$r_p(s,t) = \frac{|\{u : u \in p(s,t), u \notin L(s) \cup L(t)\}|}{|v : v \in p(s,t), v \neq s, v \neq t|}$$

The higher the r_p's value, the lower the dependence of a path to label of source and target vertices. As shown in Table 2, the average r_p for full branch decentralized search on various graphs ranges from 0.301 to 0.501.

7.5 Overhead

The index and query overhead is shown in table 3. In our implementation, both the label for each vertex and approximated paths are stored as vector of C++ standard library. And each vertex id is represented by 8-byte unsigned long. The size shown in table 3 is the sum of vector size of each vertex.

Fig. 5 shows the online search overhead in log scale for both non-parallel mode and parallel batch mode. By examing all the neighbors of a vertex at each step, decentralized search introduces much higher online search overhead than TreeSketch. Nevertheless, due to that decentralized search has very low foot print, large amount of searches can run independently in parallel efficiently. We can see in Fig. 5, the parallel mode perform 100,000 queries simultaneously, which lead to very low amortized search time for each query. The amortized search time is reduced to from 0.18% to 1.1% for single branch and from 0.15% to 1.3% for full branch compared to non-parallel mode.

For both non-parallel and parallel mode, full branch tie break strategy introduces much higher overhead than single branch strategy. Ranging from 3.3 to 30.8 times higher search time for non-parallel version and from 5.1 to 44.4 times higher average search time for parallel version.

7.6 Scalability

Since our algorithm is designed for large-scale networks, scalability is another major concern of our algorithm. Due to that we implement the algorithm in a distributed setting and execute queries in a parallel way, we study how our algorithm performs when number of machines and queries increase. We only perform single branch decentralized search here as full branch search is equal to multiple independent single branch searches.

We first evaluate the search time when number of machines increasing. Results shown in Fig. 6 are averaged over 1,000,000 queries. We can see the trend is that the average run-time decreases as the number of machines increase. The

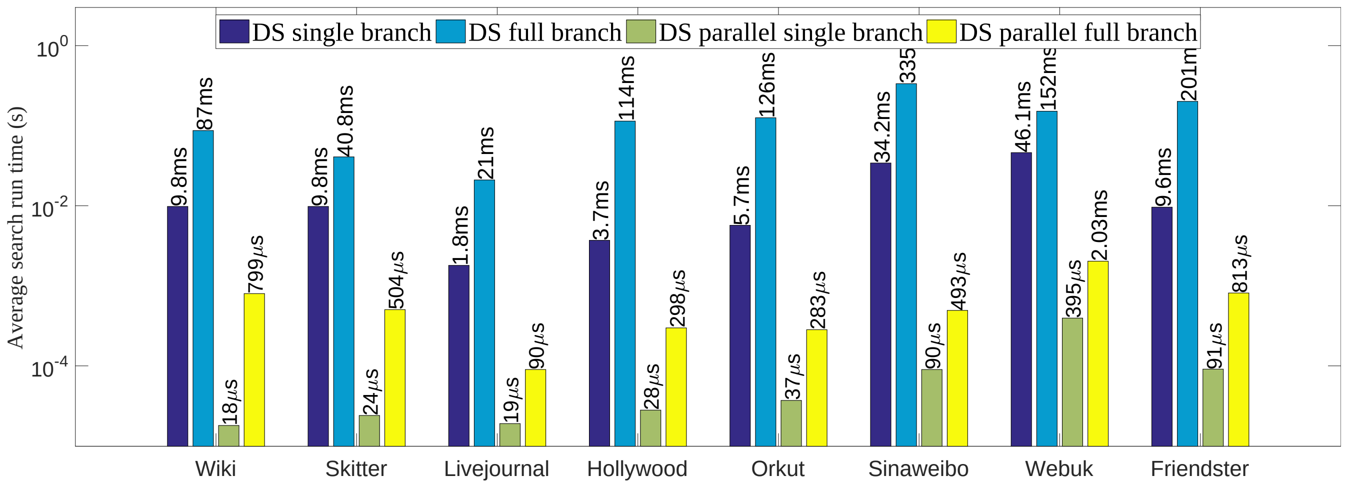


Figure 5: The amortized search time of decentralized search with various optimizations.

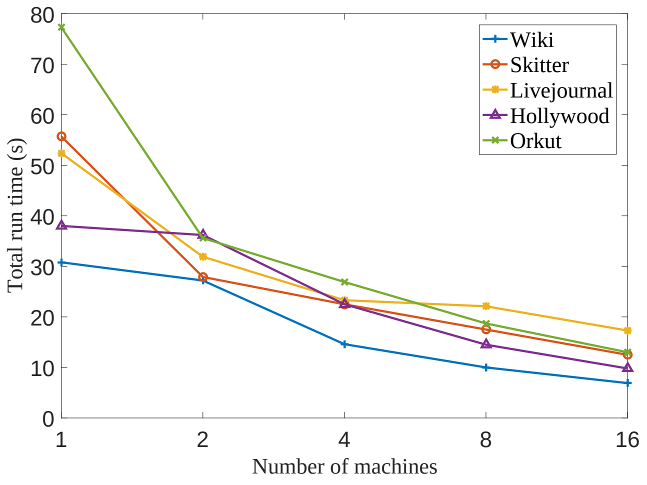


Figure 6: The total run time for 1 million queries as the number of machines increase

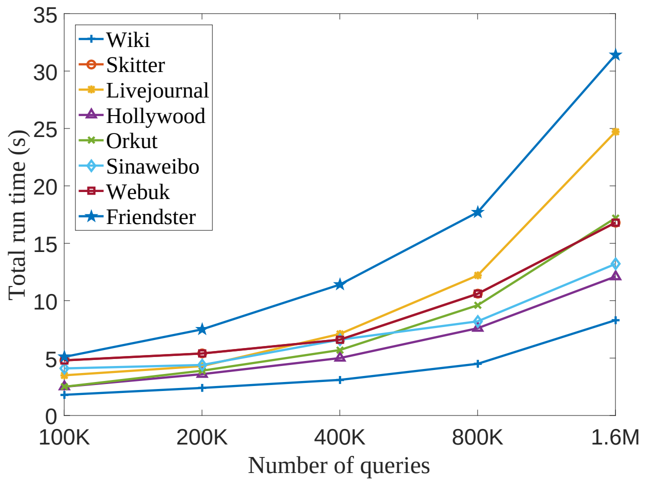


Figure 7: The total run time on 20 machines as the number of queries increase

average search time decreases fast when number of machines is small and slower when large number of machines are used. The total run time on 16 machines range from 17% to 33% of the total run time on a single machine for various graphs.

We observe great scalability of decentralized search as number of queries increase. All experiments are carried on 20 machines in this section. We can see in Fig. 7 that the total search time increases slowly when the number of queries is small. Because for small number of queries, constant overheads such as engine start/stop is the major part of run time. For large number of queries, the total run time growth is still lower than linear growth. The growth of total search time never catch up with the growth of number of queries until it reaches the system limit, i.e. memory limit or network limit.

8. CONCLUSION

In this paper, we describe a novel method to combine online and offline processing to allow approximate shortest path searches for extremely large graphs with high distance accuracy, path diversity and low overhead. We demonstrate that different accuracy and overhead levels can be achieved by controlling the search space of decentralized searches. We also propose a more effective heuristic approach for constructing indexes of the network that can improve the accuracy of the decentralized search without increasing preprocessing and online searching overhead. We implement our algorithm for cloud computing graph processing platforms, and demonstrate that our system can handle extremely large graphs and processing millions of queries in parallel.

9. REFERENCES

[1] I. Abraham, D. Delling, A. V. Goldberg, and R. F. Werneck. A hub-based labeling algorithm for shortest paths in road networks. In *Proceedings of the 10th International Conference on Experimental Algorithms*, SEA'11, pages 230–241, Berlin, Heidelberg, 2011. Springer-Verlag.

[2] T. Akiba, Y. Iwata, and Y. Yoshida. Fast exact shortest-path distance queries on large networks by

pruned landmark labeling. In *Proceedings of the 2013 ACM SIGMOD International Conference on Management of Data*, SIGMOD '13, pages 349–360, New York, NY, USA, 2013. ACM.

[3] T. Akiba, C. Sommer, and K.-i. Kawarabayashi. Shortest-path queries for complex networks: Exploiting low tree-width outside the core. In *Proceedings of the 15th International Conference on Extending Database Technology*, EDBT '12, pages 144–155, New York, NY, USA, 2012. ACM.

[4] M. Christoforaki and T. Suel. Estimating pairwise distances in large graphs. In *Big Data (Big Data), 2014 IEEE International Conference on*, pages 335–344, Oct 2014.

[5] E. Cohen, E. Halperin, H. Kaplan, and U. Zwick. Reachability and distance queries via 2-hop labels. In *Proceedings of the Thirteenth Annual ACM-SIAM Symposium on Discrete Algorithms*, SODA '02, pages 937–946, Philadelphia, PA, USA, 2002. Society for Industrial and Applied Mathematics.

[6] V. Floreskul, K. Tretyakov, and M. Dumas. Memory-efficient fast shortest path estimation in large social networks. In *Eighth International AAAI Conference on Weblogs and Social Media*, 2014.

[7] A. W.-C. Fu, H. Wu, J. Cheng, and R. C.-W. Wong. Is-label: An independent-set based labeling scheme for point-to-point distance querying. *Proc. VLDB Endow.*, 6(6):457–468, Apr. 2013.

[8] A. V. Goldberg and C. Harrelson. Computing the shortest path: A search meets graph theory. In *Proceedings of the Sixteenth Annual ACM-SIAM Symposium on Discrete Algorithms*, SODA '05, pages 156–165, Philadelphia, PA, USA, 2005. Society for Industrial and Applied Mathematics.

[9] J. E. Gonzalez, Y. Low, H. Gu, D. Bickson, and C. Guestrin. Powergraph: Distributed graph-parallel computation on natural graphs. In *Presented as part of the 10th USENIX Symposium on Operating Systems Design and Implementation (OSDI 12)*, pages 17–30, Hollywood, CA, 2012. USENIX.

[10] A. Gubichev, S. Bedathur, S. Seufert, and G. Weikum. Fast and accurate estimation of shortest paths in large graphs. In *Proceedings of the 19th ACM International Conference on Information and Knowledge Management*, CIKM '10, pages 499–508, New York, NY, USA, 2010. ACM.

[11] R. Jin, N. Ruan, Y. Xiang, and V. Lee. A highway-centric labeling approach for answering distance queries on large sparse graphs. In *Proceedings of the 2012 ACM SIGMOD International Conference on Management of Data*, SIGMOD '12, pages 445–456, New York, NY, USA, 2012. ACM.

[12] D. Kempe, J. Kleinberg, and E. Tardos. Maximizing the spread of influence through a social network. In *Proceedings of the Ninth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, KDD '03, pages 137–146, New York, NY, USA, 2003. ACM.

[13] J. Kleinberg. Navigation in a small world. *Nature*, 406:845, 2000.

[14] J. Leskovec and A. Krevl. SNAP Datasets: Stanford large network dataset collection. http://snap.stanford.edu/data, June 2014.

[15] M. Maier, M. Rattigan, and D. Jensen. Indexing network structure with shortest-path trees. *ACM Trans. Knowl. Discov. Data*, 5(3):15:1–15:25, Aug. 2011.

[16] M. Newman. *Networks: an introduction*. OUP Oxford, 2010.

[17] M. Potamias, F. Bonchi, C. Castillo, and A. Gionis. Fast shortest path distance estimation in large networks. In *Proceedings of the 18th ACM Conference on Information and Knowledge Management*, CIKM '09, pages 867–876, New York, NY, USA, 2009. ACM.

[18] M. Qiao, H. Cheng, L. Chang, and J. Yu. Approximate shortest distance computing: A query-dependent local landmark scheme. *Knowledge and Data Engineering, IEEE Transactions on*, 26(1):55–68, Jan 2014.

[19] R. Ricci, E. Eide, and The CloudLab Team. Introducing CloudLab: Scientific infrastructure for advancing cloud architectures and applications. *USENIX ;login:*, 39(6), Dec. 2014.

[20] M. Riondato and E. M. Kornaropoulos. Fast approximation of betweenness centrality through sampling. In *Proceedings of the 7th ACM International Conference on Web Search and Data Mining*, WSDM '14, pages 413–422, New York, NY, USA, 2014. ACM.

[21] R. A. Rossi and N. K. Ahmed. The network data repository with interactive graph analytics and visualization. In *Proceedings of the Twenty-Ninth AAAI Conference on Artificial Intelligence*, 2015.

[22] F. Takes and W. Kosters. Adaptive landmark selection strategies for fast shortest path computation in large real-world graphs. In *Web Intelligence (WI) and Intelligent Agent Technologies (IAT), 2014 IEEE/WIC/ACM International Joint Conferences on*, volume 1, pages 27–34, Aug 2014.

[23] L. Tang and M. Crovella. Virtual landmarks for the internet. In *Proceedings of the 3rd ACM SIGCOMM Conference on Internet Measurement*, IMC '03, pages 143–152, New York, NY, USA, 2003. ACM.

[24] M. Thorup and U. Zwick. Approximate distance oracles. *J. ACM*, 52(1):1–24, Jan. 2005.

[25] K. Tretyakov, A. Armas-Cervantes, L. García-Bañuelos, J. Vilo, and M. Dumas. Fast fully dynamic landmark-based estimation of shortest path distances in very large graphs. In *Proceedings of the 20th ACM international conference on Information and knowledge management*, pages 1785–1794. ACM, 2011.

[26] M. V. Vieira, B. M. Fonseca, R. Damazio, P. B. Golgher, D. d. C. Reis, and B. Ribeiro-Neto. Efficient search ranking in social networks. In *Proceedings of the Sixteenth ACM Conference on Conference on Information and Knowledge Management*, CIKM '07, pages 563–572, New York, NY, USA, 2007. ACM.

[27] F. Wei. Tedi: Efficient shortest path query answering on graphs. In *Proceedings of the 2010 ACM SIGMOD International Conference on Management of Data*, SIGMOD '10, pages 99–110, New York, NY, USA, 2010. ACM.

[28] X. Zhao, A. Sala, C. Wilson, H. Zheng, and B. Y. Zhao. Orion: Shortest path estimation for large social graphs. In *Proceedings of the 3rd Wonference on*

Online Social Networks, WOSN'10, pages 9–9, Berkeley, CA, USA, 2010. USENIX Association.