

Fast Truss Community Query in Large-scale Dynamic Graphs

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ABSTRACT

Recently, there has been significant interest in the study of the community search problem in social and information networks: given one or more query nodes, find densely connected communities containing the query nodes. However, most existing algorithms require linear computational time to the size of the found community for each specific K value. Therefore, state-of-the-art algorithms have limited scalability in large scale graphs, where communities grow to millions of edges.

In this paper, given an undirected graph G and a set of query nodes Q , we study community query using the k -truss based community model. We formulate our problem of finding a connected truss community, as finding a connected k -truss subgraph with all possible k that contains Q . The state-of-art approximation algorithm can achieve this goal with a time complexity of $O(n'm')$ where n' and m' are the size of the result truss community. For queries that only identity and exact size of communities are required, We construct an index structure that can retrieve there information of all connected k -truss communities that contain Q with all possible K values. The algorithm can run in $\sum_{u \in Q} d(u)$, where $d(u)$ is the degree of vertex u . We prove that this is the optimal time complexity for truss community query. Extensive experiments on real-world networks show the effectiveness and efficiency of our algorithms.

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1 INTRODUCTION

Community structures naturally exist in many real-world networks such as social, biological, collaboration, and communication networks. The task of community detection is to identify all communities in a network, which is a fundamental and well-studied problem in the literature. Recently, several papers have studied a related but different problem called community search, which is to find the community containing a given set of query nodes. The need for community search naturally arises in many real application scenarios, where one is motivated by the discovery of the communities in which given query nodes participate. Since the communities defined by different nodes in a network may be quite different, community search with query nodes opens up the

prospects of user-centered and personalized search, with the potential of the answers being more meaningful to a user [5]. As just one example, in a social network, the community formed by a person's high school classmates can be significantly different from the community formed by her family members which in turn can be quite different from the one formed by her colleagues [8].

Various community models have been proposed based on different dense subgraph structures such as k -core [4, 7, 10], k -truss [5], quasi-clique [3], weighted densest subgraph [12], to name a few major examples. Of these, the k -truss as a definition of cohesive subgraph of a graph G , requires that each edge be contained in at least $(k - 2)$ triangles within this subgraph. It is well known that most of real-world social networks are triangle-based, which always have high local clustering coefficient. Triangles are known as the fundamental building blocks of networks [11]. In a social network, a triangle indicates two friends have a common friend, which shows a strong and stable relationship among three friends. Intuitively, the more common friends two people have, the stronger their relationship. In a k -truss, each pair of friends is "endorsed" by at least $(k - 2)$ common friends. Thus, a k -truss with a large value of k signifies strong inner-connections between members of the subgraph. Huang et al. [5] proposed a community model based on the notion of k -truss as follows. Given one query node q and a parameter k , a k -truss community containing q is a maximal k -truss containing q , in which each edge is "triangle connected" with other edges. Triangle connectivity is strictly stronger than connectivity. The k -truss community model works well to find all overlapping communities containing a query node q . We extended this model for the case of multiple query nodes.

AS The time complexity of the state-of-art approximation algorithm can calculate the connected truss community containing all query vertices with the largest k with a time complexity of $O(n'm')$ where n' and m' are the size of the result truss community. Although the linear time complexity for retrieve the whole community is optimal, these algorithms have limited scalability in large scale graphs, where communities grow to millions of edges. In many applications, such as query if a set of users are involved in same community and how cohesive is the community, only the information, such as the identity, the k and the size, of the community are required rather than the detailed community itself. For these queries, We construct an index structure that can retrieve information of all connected k -truss communities that contain Q with all possible K values. The algorithm can run in $\sum_{u \in Q} d_u$, where d_u is the degree of vertex u . We prove that this is the optimal time complexity for truss community query. Note that if further details of found truss communities are required, our index structure can also retrieve the exact community with linear time complexity to the community size which is also the optimal time complexity.

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The rest of this paper is organized as follows. In Section 2 we show previous works on community search and detection as well as dense graph mining. Section 3 provides notations and definitions used in this paper. We explain induced mst graph for truss community search in Section ?? . Section 4 discusses index construction algorithm. The evaluations of our algorithm are in Section 6. We conclude our work in Section 7.

2 RELATED WORKS

The related work to our study include community search and detection, and dense subgraph mining.

Community Search and Detection.

Dense Subgraph Mining.

3 PRELIMINARIES

In our problem, we consider an undirected, unweighted graph $G = (V, E)$. The number of vertices is denoted as $n = |V|$ and number of edges is denoted as $m = |E|$. If the graph is weighted, we use w_u and w_e to denote the weight of vertex u and edge e . We define the set of neighbors of a vertex v in G as $N_v = \{u \in V : (v, u) \in E\}$, and the degree of v as $d_v = |N_v|$. We define a triangle in G as a cycle of length 3. Let $u, v, w \in V$ be the three vertices on the cycle, and we denote this triangle by Δ_{uvw} . Then we define several key concepts in this paper as follows.

DEFINITION 1 (EDGE SUPPORT). *The support of an edge $e_{u,v} \in E$ is defined as $s_{e,G} = |\Delta_{uvw} : w \in V|$. We denote it as s_e when the context is clear.*

DEFINITION 2 (TRUSSNESS). *The trussness of a subgraph $H \in G$ is the minimum support of edges in H plus 2, denoted by $\tau_H = \min\{s_{e,H} + 2 : e \in E_H\}$. The trussness of an edge e is defined as: $\tau_e = \max_{H \in G} \{\tau_H : e \in E_H\}$.*

DEFINITION 3 (K-TRUSS SUBGRAPH). *Given a graph G and $k \geq 2$, $H \subseteq G$ is a k -truss if $\forall e \in E_H, s_{e,H} \geq (k - 2)$.*

DEFINITION 4 (MAXIMAL K-TRUSS SUBGRAPH). *H is a maximal k -truss subgraph if it is not a subgraph of another k -truss subgraph with same trussness k in G .*

We use the same triangle adjacency and triangle connectivity definition as in [5] listed below.

DEFINITION 5 (TRIANGLE ADJACENCY). Δ_1, Δ_2 are adjacent if they share a common edge, i.e., $\Delta_1 \cap \Delta_2 \neq \emptyset$.

DEFINITION 6 (TRIANGLE CONNECTIVITY). Δ_1, Δ_2 are triangle connected if they can reach each other through a series of adjacent triangles, i.e., for $1 \leq i < n$, $\Delta_i \cap \Delta_{i+1} \neq \emptyset$.

DEFINITION 7 (TRIANGLE CONNECTED GRAPH). Two edge e_1, e_2 are triangle connected in a subgraph H if there are two triangle Δ_1, Δ_2 in H and $e_1 \in \Delta_1, e_2 \in \Delta_2$, either $\Delta_1 = \Delta_2$, or Δ_1 is triangle connected with Δ_2 in H . A graph G is triangle connected if all pairs of edges in G are triangle connected.

Finally, we define k -truss community based on the definition of k -truss subgraph and triangle connectivity as follows.

DEFINITION 8 (K-TRUSS COMMUNITY). A k -truss community is a maximal triangle connected k -truss subgraph.

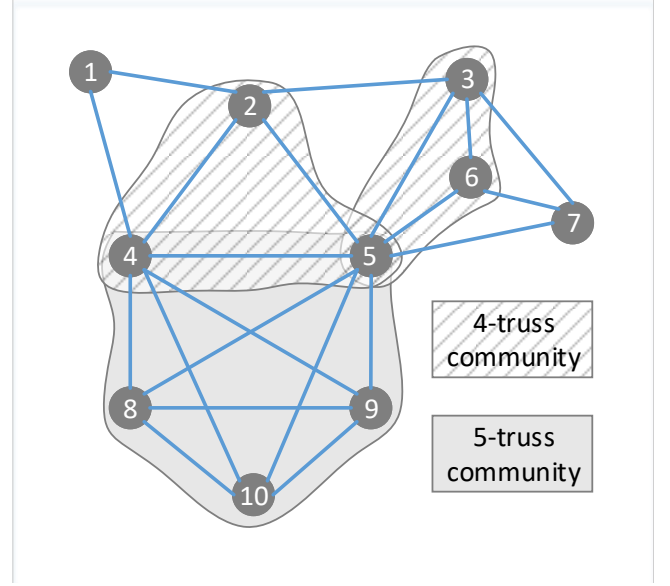


Figure 1: An example graph for k -truss community

Figure 1 shows several examples of k -truss communities. The whole example graph is a 3-truss as every edge has support of at least 1. Note that there are 2 separate 4-truss communities in Figure 1 as they are not triangle connected with each other.

K-truss Community Search The problem of studied in this paper is defined as follows. Given a graph $G(V, E)$, a set of query vertices $Q \subseteq V$, find all truss communities containing Q with maximum k , a specific k or any possible k .

4 INDEXED K-TRUSS COMMUNITY SEARCH

We propose to solve k -truss community search problem using an index based approach. This section describes how to process a k -truss community search query on a static graph, including induced MSF graph construction, creating tree-structured community graph, performing various kind of queries on the preprocessed index. In the next section, we describe index update procedure on dynamic graphs.

4.1 Induced MSF Graph

We first design an induced MSF graph then propose the query algorithm based on it.

Induced MSF Graph Construction. We first compute the edge trussness of graph G_o and then construct a new graph G_m , which we called induced MSF graph, based on the graph G_o and its edges' trussness. We define the induced MSF graph as follows.

DEFINITION 9 (INDUCED MSF GRAPH). *The induced MSF graph is a weighted maximum spanning forest that each edge e in G_o is represented as a vertex x in G_m . An edge y in G_m represents that the two edges, which are represented by the two adjacent vertices of y , are contained in the same triangle in G_o . The weight of the each vertex in G_m is its represented edge's trussness in G_o . The weight of each edge*

in G_m is the lowest edge trussness of its related triangle's edges in G_o .

We have the following theorem for vertex weights and edge weights in induced MSF graph G_m .

THEOREM 1. In induced MSF graph G_m , for each vertex x and each of its adjacent edge y , we have $w_x \geq w_y$.

PROOF. According to Definition 9, w_x is the trussness of the represented edge e in G_o while w_y is the lowest trussness of edges in the represented triangle Δ in G_o . We have $\tau_e \geq \tau_\Delta$, therefore, $w_x \geq w_y$. \square

Algorithm 1: Induced MSF graph construction

Data: $G_o(V_o, E_o)$, edge trussness $\{\tau_e, e \in E_o\}$
Result: inducedMSFgraph $G_m(V_m, E_m)$

```

1  visited  $\leftarrow \emptyset$ ;
2  for  $(u, v) \in E_o$  do
3      suppose  $u$  is the lower degree end of  $(u, v)$ ;
4       $V_m \leftarrow V_m \cup \{(u, v), \tau_{(u, v)}\}$ ;
5      for  $w \in N_u$  do
6          if  $(v, w) \in E_o$  and  $\Delta_{uvw} \notin \text{visited}$  then
7              visited  $\leftarrow \text{visited} \cup \Delta_{uvw}$ ;
8               $\tau_{\Delta_{uvw}} = \min(\tau_{(u, v)}, \tau_{(u, w)}, \tau_{(v, w)})$ ;
9               $V_m \leftarrow V_m \cup \{(u, w), \tau_{(u, w)}\}$ ;
10              $V_m \leftarrow V_m \cup \{(v, w), \tau_{(v, w)}\}$ ;
11              $E_m \leftarrow E_m \cup \{((u, v), (u, w)), \tau_{\Delta_{uvw}}\}$ ;
12              $E_m \leftarrow E_m \cup \{((u, v), (v, w)), \tau_{\Delta_{uvw}}\}$ ;
13              $E_m \leftarrow E_m \cup \{((u, w), (v, w)), \tau_{\Delta_{uvw}}\}$ ;
14         end
15     end
16 end
17 run Kruskal's algorithm on  $G_m$ ;
18 return  $G_m$ 

```

The truss decomposition algorithm [11] is used to compute trussness of all edges $\{\tau_e, e \in E_o\}$ in G_o . Although it is possible to directly compute k -truss communities based on edge trussness with BFS traversals, such an algorithm suffers from high time complexity for redundant edge access [5]. algorithm 1 uses both G_o and edge trussness as inputs to construct the induced MSF graph G_m for optimal query time. The algorithm iterates through all edges of G_o and create a vertex in G_m for each edge (u, v) in G_o with weight $\tau_{(u, v)}$. Then for each unvisited neighbor triangle Δ_{uvw} of edge (u, v) , the algorithm creates three edges $((u, v), (u, w))$, $((u, v), (v, w))$ and $((u, w), (v, w))$ in G_m with same weight $\tau_{\Delta_{uvw}} = \min(\tau_{(u, v)}, \tau_{(u, w)}, \tau_{(v, w)})$. After that, one can simply run Kruskal's algorithm to get the maximum spanning forest.

We show an example of induced MSF graph in Figure 2. We outline the induced MSF graph of the example graph in Figure 1 with bold lines. The rest lines are edges that are generated by algorithm 1 but discarded by Kruskal's algorithm.

[***** some possible error of time complexity in [5] *****]

The time and space complexity for computation of edge trussness of

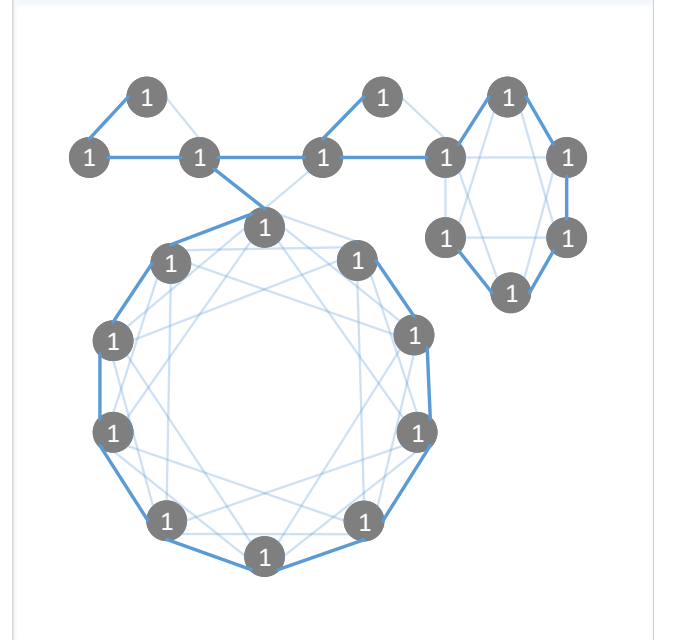


Figure 2: An example induced MSF graph of the example graph in Figure 1

G_o are $O(\sum_{(u, v) \in E_o} \min\{d_u, d_v\})$ and $O(m)$ respectively [5]. Listing all the triangles in G_o takes $O(\sum_{(u, v) \in E_o} \min\{d_u, d_v\})$ time and $O(\sum_{(u, v) \in E_o} \min\{d_u, d_v\})$ space. Finally, running Kruskal's algorithm takes $O(\sum_{(u, v) \in E_o} \min\{d_u, d_v\} \log m)$ time. As G_m is a maximum spanning forest, so the induced MSF graph index takes $O(|V_m|) = O(m)$ space.

Query on Induced MSF Graph. To query the k -truss communities of a query vertex q in G_o , the algorithm iterate through adjacent edges of the vertex q . For each neighbor edge (u, q) that is unvisited by the algorithm, it is marked as a seed edge for a new community C_i . Suppose the edge (u, q) in G_o is represented as a vertex x in G_m , the algorithm starts a BFS/DFS from vertex x in G_m and only expands through edges with weight $\geq k$ to find the connected component CC . Then it finds the represented edge e of each vertex $v \in CC$ and adds e to the community C_i . The union of all communities $A = \bigcup C_i$ is all the k -truss communities the vertex q belongs to.

THEOREM 2. The union of all communities $\bigcup C_i$ found by algorithm 2 is the union of all the k -truss communities containing query vertex q .

PROOF. According to Definition 9, a vertex x in induced MSF graph G_m with weight $w_x \leq k$ means the represented edge e in G_o has trussness $\tau_e \leq k$ and thus can be included in a k -truss community. An edge (x, y) in induced MSF graph G_m with weight $w_{(x, y)} \leq k$ means the represented triangle Δ in G_o has all three edges with trussness higher or equal to k and thus the triangle is included in a k -truss community containing all three edges of it. Adjacent edges in G_m means adjacent triangles in G_o and connected components in G_m means triangle connected components

in G_o . So, BFS/DFS search starts with a seed vertex x with weight constraint will find the maximal connected component including x which representing the k -truss community that e belongs to in G_o (x represents e in G_m). Therefore, performing such BFS/DFS searches on each edge of the query vertex will find all the k -truss communities that the query vertex belongs to. \square

Algorithm 2: Query on induced MSF graph

Data: $G_o(V_o, E_o)$, $G_m(V_m, E_m)$, an integer k , a query vertex q
Result: a union of all k -truss communities $\cup C_i$ containing q

```

1  $i \leftarrow 0, visited \leftarrow \emptyset;$ 
2 for  $u \in N_q$  do
3   if  $(u, q) \notin visited$  then
4     find representing vertex  $x$  of  $(u, q)$  in  $G_m$ ;
5      $CC \leftarrow$  connected component containing  $x$  with edges
      of weight  $\geq k$ ;
6      $C_i \leftarrow \emptyset;$ 
7     for  $v \in CC$  do
8       find represented  $e$  of  $v$  in  $G_o$ ;
9        $visited \leftarrow visited \cup e;$ 
10       $C_i \leftarrow C_i \cup e;$ 
11   end
12    $i \leftarrow i + 1;$ 
13 end
14 end
15 return  $\cup C_i$ 

```

Since the query process is performing a BFS on a maximum spanning forest, each query takes $O(|A|)$ time and $O(|A|)$ space, where $|A|$ is the number of edges in A . Although such time complexity is already optimal if the detailed communities are required. We propose a new index structure that can be constructed upon the induced MSF graph to further reduce the time complexity if details of k -truss communities are not required.

4.2 Tree-structured Community Graph

We first show how to construct the tree-structured community graph based on induced MSF graph. Then we design an algorithm to efficiently query the tree-structured community graph.

Tree-structured Community Graph Construction. A key observation in [2] is that, for $k \geq 2$, each k -truss of G_o is the subgraph of a $(k-1)$ -truss of G_o . With this observation, for k -truss communities, we have the following theorem.

THEOREM 3. *A k -truss community C_k is the subgraph of a l -truss community C_l , if C_k and C_l are triangle connected and $l < k$. If k -truss community C_k is the subgraph of both l_1 -truss community C_{l_1} and l_2 -truss community C_{l_2} , then $l_1 \neq l_2$.*

PROOF. For the first part, since $l < k$, if edges in C_k are triangle connected through triangles with trussness of k , then they are also triangle connected through triangles with trussness of l .

[***** do we call it k -truss or l_1 -truss. *****]

For the second part, suppose $l_1 = l_2$, then edges in C_{l_1} and C_{l_2} are

triangle connected through C_k . So $C_{l_1} \cup C_{l_2}$ meets the definition of k -truss community (Definition 8) and becomes a larger k -truss community. This contradicts with C_{l_1} and C_{l_2} are k -truss communities themselves, i.e., they are maximal k -truss. \square

Algorithm 3: Tree-structured community graph Construction

Data: $G_m(V_m, E_m)$
Result: $G_t(V_t, E_t)$, h

```

1  $Q \leftarrow \emptyset, parent \leftarrow \emptyset;$ 
2 while  $V_m \neq \emptyset$  do
3    $seed \leftarrow$  an unvisited vertex in  $V_m, Q \leftarrow Q \cup seed;$ 
4   while  $Q \neq \emptyset$  do
5      $x = Q.pop();$ 
6     for  $z \in N_x$  do
7        $Q \leftarrow Q \cup z, parent[z] \leftarrow x;$ 
8     end
9     if  $x \in parent$  then
10       $y \leftarrow parent[x], C_a \leftarrow C_y^{max};$ 
11      while  $\tau_{C_a} > w_{(x,y)}$  do
12         $C_c \leftarrow C_a, C_a \leftarrow$  parent of  $C_a$  in  $G_t$ ;
13        if  $C_a = \emptyset$  then
14           $\tau_{C_a} \leftarrow -1$   $\triangleright$  Reach the top of the tree;
15        end
16      end
17      if  $\tau_{C_a} < w_{(x,y)}$  then
18        if  $w_{(x,y)} = w_x$  then
19          create  $C_x^{max}, h[x] \leftarrow C_x^{max};$ 
20           $C_x^{max}.parent \leftarrow C_a, C_c.parent \leftarrow C_x^{max};$ 
21        else
22          create  $C_x^{max}, h[x] \leftarrow C_x^{max};$ 
23          create  $C_{(x,y)}, C_{(x,y)}.parent \leftarrow C_a;$ 
24           $C_c.parent \leftarrow C_{(x,y)};$ 
25           $C_x^{max}.parent \leftarrow C_{(x,y)};$ 
26        end
27      else
28        if  $w_{(x,y)} = w_x$  then
29           $h[x] \leftarrow C_a;$ 
30        else
31          create  $C_x^{max}, h[x] \leftarrow C_x^{max};$ 
32           $C_x^{max}.parent \leftarrow C_a;$ 
33        end
34      end
35    else
36      create  $C_x^{max}, h[x] \leftarrow C_x^{max};$ 
37       $V_t \leftarrow V_t \cup C_x^{max};$ 
38    end
39    remove  $x$  from  $V_m$ ;
40  end
41 end
42 return  $G_t(V_t, E_t), h$ 

```

According to Theorem 3, we can build another tree-structured index upon our existing induced MSF graph to further facilitate KTruss computation. In this new tree-structured index, we use vertices to represent k-truss communities, i.e., we assign each k-truss community a unique ID and a representing vertex in the new index. If one k-truss community is the subgraph of another k-truss community, we assign an edge to connect the representing vertices. Each vertex can have a list associated with it including the status of the related k-truss community, such as the trussness of the community, the size of the community, etc. We call this new index the tree-structured community graph and denote it as G_t . For each vertex of G_t , we also have meta data of the represented k-truss communities, e.g., the trussness, the size, etc., stored with it. These meta-data can be gathered very easily through the index construction process. For the ease of query, we build a hash table h that for each edge e in G_o (vertex x in G_m), we record the ID of the k-truss community that includes it with highest order k . We denote such a k-truss community as C^{max} . We have the following theorem for G_t .

THEOREM 4. *The tree-structured community graph G_t is a forest.*

PROOF. First, according to Theorem 3, there is only one ancestor for each k-truss community. Also, there is no inter level edges according to the definition of maximal KTruss. So, if the graph contains a loop, then a KTruss may contain more than 1 ancestors.

Second, G_t can be disconnected as not all k-truss communities are triangle connected with each other. \square

algorithm 3 shows the procedure to build the tree-structured community graph G_t . The algorithm uses BFS to traverse the induced MSF graph G_m . For each vertex x , if it does not have a parent vertex in the BFS traversal, then the algorithm uses it as a seed vertex to create a new index tree. Otherwise it is combined to the same index tree $T \in G_t$ as its parent vertex y . According to Theorem 1, we have the following equation.

$$w_x \geq w_{(x,y)}, w_y \geq w_{(x,y)} \quad (1)$$

THEOREM 5. *For a vertex x and its neighbor vertex y in induced MSF graph G_m , if their representing edges in G_o are contained in the same k -truss community with trussness of k , then $k \leq w_{(x,y)}$.*

PROOF. We denote G'_m as the graph that is constructed the same way as G_m but with all triangles in G_o as edges, i.e., G_m is the maximum spanning forest of G'_m . Since G_m is the maximum spanning forest, it has the cycle property, i.e., for any cycle in G'_m , if the weight of an edge in the cycle is smaller than the individual weights of all the other edges in the cycle, then this edge cannot belong to a maximum spanning forest. So there is no path in G'_m between x and y that has all edges with weight $> w_{(x,y)}$. Suppose x and y representing e_x and e_y in G_o , this means that e_x is not triangle connected to e_y through edges in G_o with trussness $> w_{(x,y)}$. Therefore, it is not possible for e_x and e_y to exist in the same k-truss community with $k > w_{(x,y)}$. \square

Having a parent y in the BFS search only means that the vertex x can be combined to the current index tree T . We still have a problem to solve: On which part of T should the algorithm add the

vertex x ? According to Theorem 5 and Equation 1, the algorithm needs to backtrack T from C_y^{max} to find an ancestor vertex C_a that meets $\tau_{C_a} \leq w_{(x,y)}$ and use it as the merge point of x . We refer to the index vertices $C_y^{max}, \dots, C_i, \dots, C_a$ as the backtrack branch for vertex x in T and denote it as B .

Once the algorithm has found C_a , it needs to check the relations of $\tau_{C_a}, w_{(x,y)}$ and w_x to decide how to merge vertex x to T . Note that they follow $\tau_{C_a} \leq w_{(x,y)} \leq w_x$, so we have 4 cases shown in algorithm 3. As long as $\tau_{C_a} \neq w_x$, we create a new index vertex C_x^{max} with trussness $\tau_{C_x^{max}} = w_x$. If $\tau_{C_a} < w_{(x,y)} < w_x$, we also create a new index vertex $C_{(x,y)}$ with trussness $\tau_{C_{(x,y)}} = w_{(x,y)}$. Then we adjust the tree structure of T with new index vertices. Finally, we update the hash table to record in which index vertex x is.

For each vertex of G_m , the backtrack procedure takes $O(k_{max})$ time, where k_{max} is the highest trussness of any k-truss community in G_o . Since the index construction process is a BFS on a maximum spanning tree, the tree-structured community graph construction algorithm takes $O(k_{max}m)$ time. As each vertex in G_t represents a k-truss community in G_o , and G_t is a forest. The algorithm takes $O(m)$ space and the index size is also $O(m)$ space. Although in practice, the size of G_t is much smaller than $O(m)$.

Query on Tree-structured Community Graph. Tree-structured community graph supports three basic types of k-truss community queries of a single query vertex q as listed below.

- K-truss query: Given a vertex q and an integer k , find the k-truss community that contains q .
- Max-k-truss query: Given a vertex q , find the k-truss community with highest possible trussness that contains q .
- Any-k-truss query: Given a vertex q , find all the k-truss communities that contains q .

Max-k-truss query is naturally supported by simply looking up the hash table h and comparing trussness of $h[x_e]$ for each neighbor edge. We show the queries process algorithms for k-truss query and any-k-truss query in algorithm 4. A common operation used in both query algorithms is what we called backtrack branch search, which is defined in Definition 10 below. We can see that if a specific k is provided, the backtrack branch search will stop once the trussness falls below k . On the other hand, if no k is provided, a value of 0 is used and the search will reach the root of the tree.

DEFINITION 10 (BACKTRACK BRANCH SEARCH). *Given a vertex $C_0 \in G_t$ and an integer k , the backtrack branch search returns a list of vertices C_0, \dots, C_i, \dots that C_{i+1} is the parent vertex of C_i in G_t and any vertex C_i meets $\tau_{C_i} \geq k$. We refer to the searching results C_0, \dots, C_i, \dots as backtrack branch and denote it as B .*

Tree-structured community graph also supports all three types of queries when the input is a set of query vertices Q . The query process algorithms simply takes intersections of the query results of each individual query vertex for k-truss queries and any-k-truss queries. For max-k-truss queries, the query process algorithm needs to calculate the least common ancestors in G_t of the results of each individual query vertex.

For single vertex queries, the time complexity is $O(d_q)$ for max-k-truss queries and $O(\sum_{e \in N_q} \tau_{h[x_e]})$ for k-truss and any-k-truss queries. The space complexity is $O(1)$ for max-k-truss queries,

Algorithm 4: Query on Tree-structured community graph

Data: $G_o(V_o, E_o)$, $G_t(V_t, E_t)$, the hash table h , a query vertex q or a set of query vertices Q , [an integer k]
Result: a set of k -truss community IDs R

```

1 function branch_search ( $C \in G_t, G_t, [k = 0]$ )
2    $B \leftarrow \emptyset$ ;
3   while  $C \neq \emptyset$  and  $\tau_C \geq k$  do
4      $B \leftarrow B \cup C$ ;
5      $C \leftarrow \text{parent of } C \text{ in } G_t$ ;
6   end
7   return  $B$ 
8 end

9 function query_k ( $q, G_o, G_t, k$ )
10   $R \leftarrow \emptyset$ ;
11  for  $e \in N_q$  do
12     $B = \text{branch\_search}(h[x_e], G_t, k)$ ;
13    if  $\tau_{B[-1]} = k$  then
14       $R \leftarrow R \cup B[-1]$ ;  $\triangleright B[-1]$  is the last element in  $B$ 
15    end
16  end
17  return  $R$ 
18 end

19 function query_anyk ( $q, G_o, G_t$ )
20   $R \leftarrow \emptyset$ ;
21  for  $e \in N_q$  do
22     $B = \text{branch\_search}(h[x_e], G_t)$ ;
23     $R \leftarrow R \cup B$ ;
24  end
25  return  $R$ 
26 end

```

$O(d_q)$ for k -truss queries and $\sum_{e \in N_q} \tau_{h[x_e]}$ for any- k -truss queries. For multiple vertices max- k -truss queries, since the least common ancestor computation takes $O(H)^1$ time, where H is the height of the tree. The query time is $O(\sum_{q \in Q} (\max_{e \in N_q} \tau_{h[x_e]} + d_q))$ and the space is $O(\max_{q \in Q} \max_{e \in N_q} \tau_{h[x_e]})$. Multiple vertices k -truss queries take $O(\sum_{q \in Q} \sum_{e \in N_q} \tau_{h[x_e]})$ time and $O(\max_{q \in Q} d_q)$ space. For multiple vertices any- k -truss queries, the time and space complexity is $O(\sum_{q \in Q} \sum_{e \in N_q} \tau_{h[x_e]})$ and $O(\max_{q \in Q} \sum_{e \in N_q} \tau_{h[x_e]})$ respectively.

5 DYNAMIC UPDATES

This section describes update procedures of both induced MSF graph and tree-structured community graph in dynamic graphs. We focus on edge insertion/deletion as vertex insert/deletion can be represented by inserting/deleting an isolated vertex and several following edge insertions/deletions.

¹ $O(H)$ is for simple online algorithm, off-line algorithms can achieve time complexity of $O(1)$ [1].

Table 1: Datasets

Dataset	Type	$ V_{wcc} $	$ E_{wcc} $	$\bar{\sigma}$
Wiki	Communication	2.4M	4.7M	3.9
Skitter	Internet	1.7M	11.1M	5.07
Livejournal	Social	4.8M	43.4M	5.6
Hollywood	Collaboration	1.1M	56.3M	3.83
Orkut	Social	3M	117M	4.21
Sinaweibo	Social	58.7M	261.3M	4.15
Webuk	Web	39.3M	796.4M	7.45
Friendster	Social	65M	1.8B	5.03

Datasets with the number of vertices and edges in the largest weakly connected components, and the average shortest distance $\bar{\sigma}$ of 100,000 vertex pairs.

5.1 scope of changes

The scope of affected edges when a new edge is inserted/deleted has been well studied in [5]. Suppose an edge e_0 has been added/removed from G_o , we have the following rules.

- **Edge insertion.** For $e \in E_o \cup e_0$ with $\tau_e < \overline{\tau_{e_0}}$, e may have $\tau'_e = \tau_e + 1$ if e and e_0 are triangle connected in G_o with triangles have lowest edge trussness of τ_e .
- **Edge deletion.** For $e \in E_o \setminus e_0$ with $\tau_e < \overline{\tau_{e_0}}$, e may have $\tau'_e = \tau_e + 1$ if e and e_0 are triangle connected in G_o with triangles have lowest edge trussness of τ_e .

6 EVALUATIONS

In this section, we show the results of experimental evaluation of truss community query on our index structure.

6.1 Datasets

We evaluate our algorithm on 8 graphs from different disciplines as shown in table 1. All graphs are complex networks that have power-law degree distributions and relatively small diameters. To simplify our experiments, we treat them as undirected, un-weighted graphs and only use the largest weakly connected component of each graph. All datasets are collected from Snap [6].

6.2 Experiment settings

We evaluate our algorithms, we use a Cloudlab [9] c8220 server with two 10-core 2.2GHz E5-2660 processors and 256GB memory. We base our algorithm on Snap [6]. All algorithms are implemented in C++.

6.3 Query time

We first compare the query time of truss community search based on our index structure with the state-of-art related work.

6.4 Index construction time and size

We show in this section that our index achieves the similar construction time and size as previous work.

6.5 Index update time

The index update time upon graph changes, i.e. edge deletion, is shown in ...

7 CONCLUSION

In this paper, we describe ...

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