GANs (Generative Adversarial Nets)

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Contents

Introduction

Alchemist
(Generator)

Jeweler
(Discriminator)

The coolest idea in ML in the last twenty years - Yann Lecun

Introduction

Alchemist

(Generator)

She have to make gold bars that others can not distinguish.

Introduction

He must distinguish the Fake.

Jeweler
(Discriminator)

Introduction

both ability going up

Vs

Alchemist
(Generator)

Jeweler
(Discriminator)

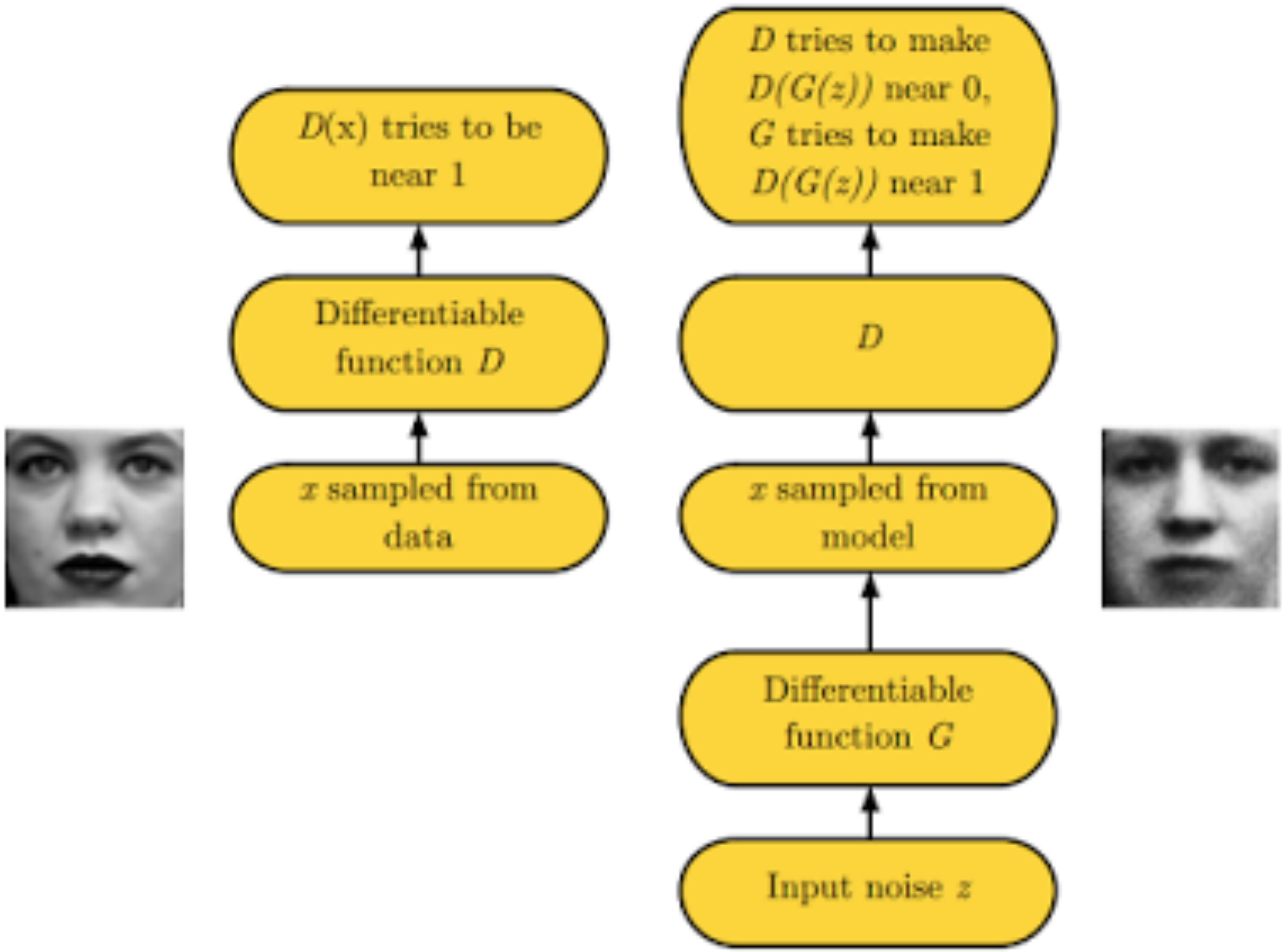
Goal : It can not really tell whether this is real gold or not

Adversarial Nets

Alchemist

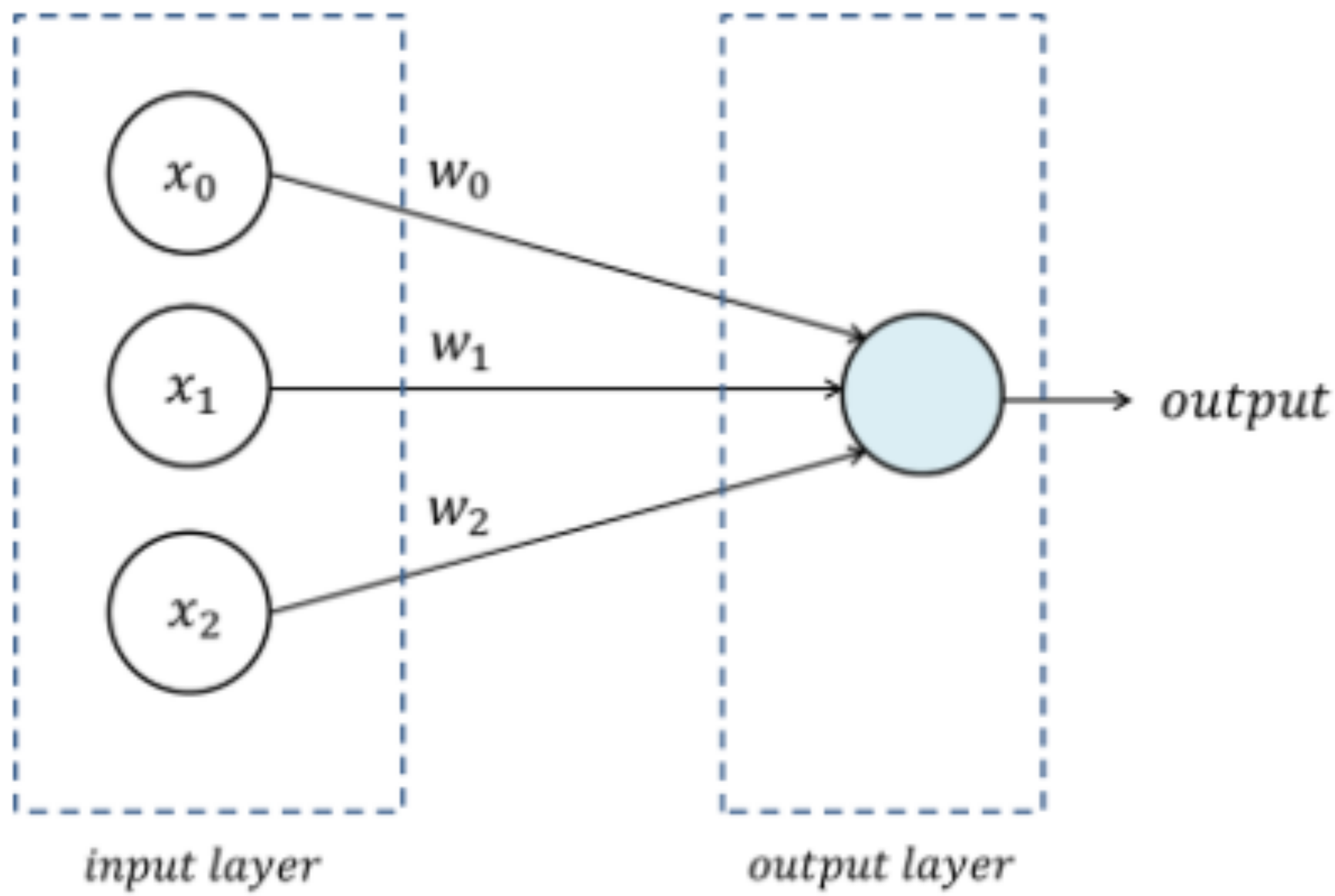
Jeweler

Adversarial Nets



출처 : NIPS 2016 Tutorial: Generative Adversarial Networks

Single-layer Perceptron



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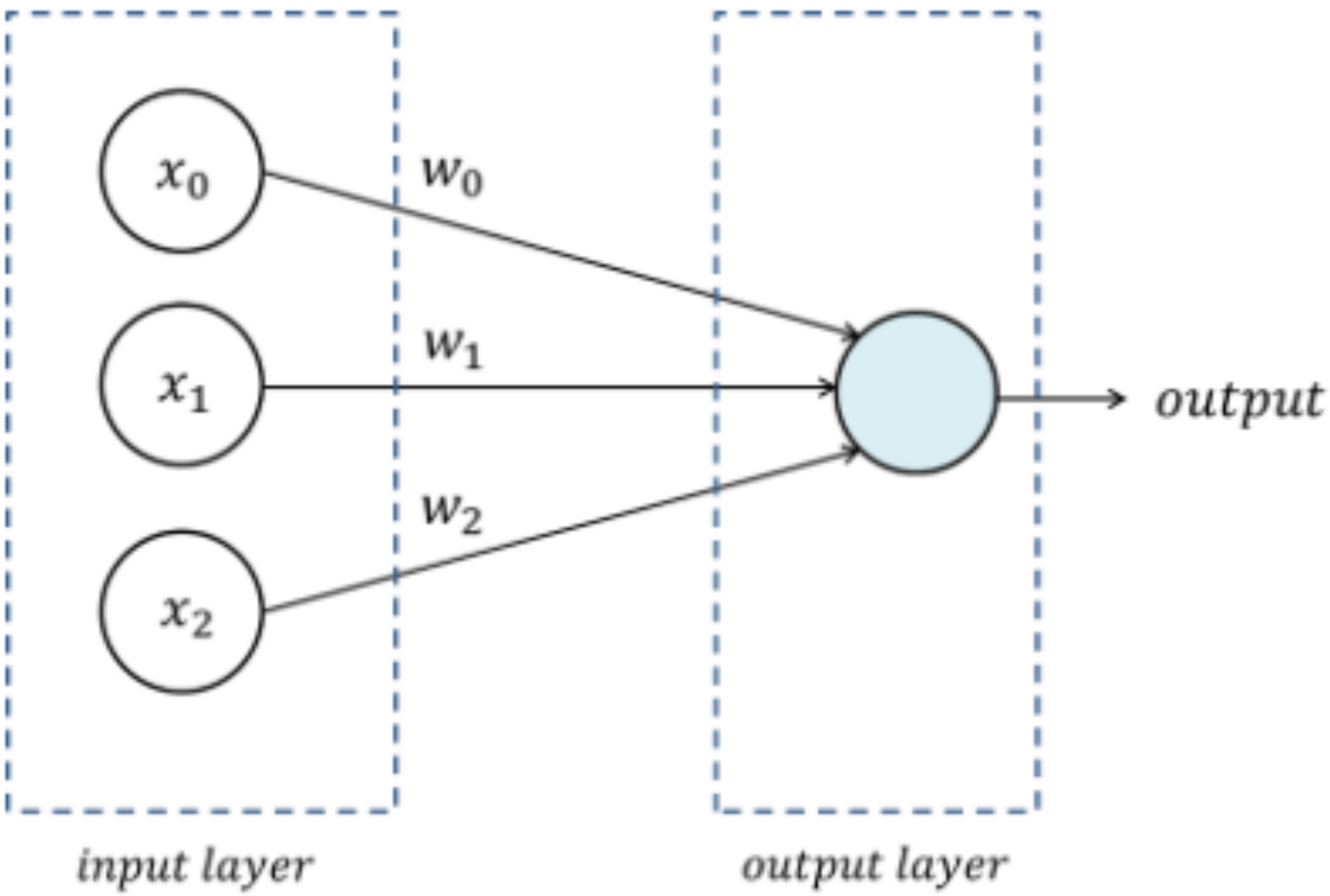
Multi-layer Perceptron



+ Hidden Layer + backpropagation

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Adversarial Nets

Define multilayer perceptron G (Generator)

Generator distribution : P_g

generator input : noise variables $P_z(z)$

Data space : $G(z; \theta_g)$

Adversarial Nets

Define multilayer perceptron D(Discriminator)

Data space : $D(x; \theta_d)$

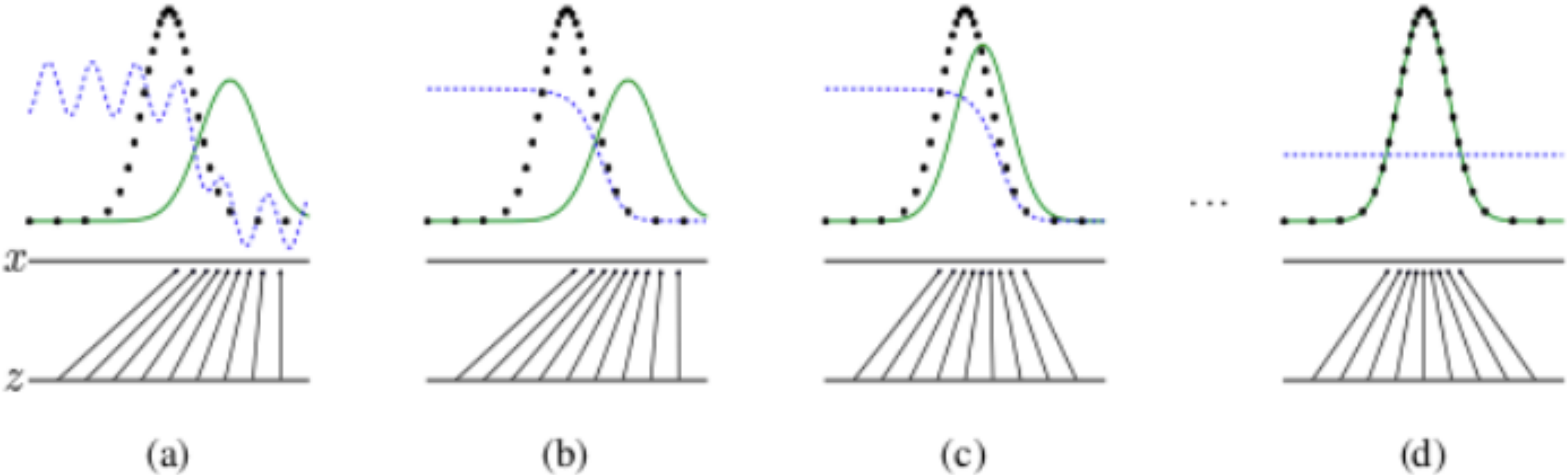
Output : Probability (Fake or not)

Adversarial Nets

It could use another method,
but this paper used multilayer perceptron model for D and G.

$$\min_G \max_D V(D, G) = \mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}(\boldsymbol{x})}[\log D(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z} \sim p_{\boldsymbol{z}}(\boldsymbol{z})}[\log(1 - D(G(\boldsymbol{z})))].$$

Adversarial Nets



········ : data generating distribution

x , z : domain

: generative distribution

: x = G(z) mapping

········ : discriminator distribution

Theoretical Results

Two result

1. Introduced method will get global optimum when $P_g = P_{data}$
2. They prove that the algorithm presented in the paper finds the global optimum

Theoretical Results

Proposition

For G fixed, the optimal discriminator D is

$$D_G^*(x) = \frac{p_{data}(x)}{p_{data}(x) + p_g(x)}.$$

Proof. The training criterion for the discriminator D, given any generator G, is to maximize the quantity $V(G, D)$

$$\begin{aligned} V(G, D) &= \int_x p_{data}(x) log(D(x))dx + \int_z p_z(z) log(1 - D(G(z)))dz \\ &= \int_x p_{data}(x) log(D(x)) + p_g(x) log(1 - D(x))dx \end{aligned}$$

For any $(a, b) \in \mathbb{R}^2 \setminus \{0, 0\}$, the function $y \rightarrow \mathrm{a}log(y) + \mathrm{b}log(1 - y)$ achieves its maximum in $[0, 1]$ at $\frac{a}{a+b}$. The discriminator does not need to be defined outside of $Supp(p_{data}) \cup Supp(p_g)$, concluding the proof. ■

Theoretical Results

Main Theorem

The global minimum of the virtual training criterion C(G) is Achieved if and only if $P_g = P_{data}$. At that point, C(G) achieves the value $-\log 4$

$$
\begin{aligned}
C(G) &= \max_D V(G, D) \\
&= \mathbb{E}_{x \sim p_{data}}\left[\log D_G^*(x)\right] + \mathbb{E}_{z \sim p_z}\left[\log(1 - D_G^*(G(z)))\right] \\
&= \mathbb{E}_{x \sim p_{data}}\left[\log D_G^*(x)\right] + \mathbb{E}_{x \sim p_g}\left[\log(1 - D_G^*(x))\right] \\
&= \mathbb{E}_{x \sim p_{data}}\left[\log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)}\right] + \mathbb{E}_{x \sim p_g}\left[\log \frac{p_g(x)}{p_{data}(x) + p_g(x)}\right]
\end{aligned}
$$

Theoretical Results

Kullback-Leibler divergence

$$C(G) = -\log(4) + KL\left(p_{\text{data}} \left\| \frac{p_{\text{data}} + p_g}{2}\right.\right) + KL\left(p_g \left\| \frac{p_{\text{data}} + p_g}{2}\right.\right)$$

Jensen-Shannon divergence

$$C(G) = -\log(4) + 2 \cdot JSD\left(p_{\text{data}} \| p_g\right)$$

Theoretical Results

Algorithm 1 Minibatch stochastic gradient descent training of generative adversarial nets. The number of steps to apply to the discriminator, k, is a hyperparameter. We used $k = 1$, the least expensive option, in our experiments.

for number of training iterations **do**

  for k steps **do**

  - Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \dots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.
  - Sample minibatch of m examples $\{\boldsymbol{x}^{(1)}, \dots, \boldsymbol{x}^{(m)}\}$ from data generating distribution $p_{\text{data}}(\boldsymbol{x})$.
  - Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} \left[\log D\left(\boldsymbol{x}^{(i)}\right) + \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right) \right].$$

  end for

  - Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \dots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.
  - Update the generator by descending its stochastic gradient:

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right).$$

end for

The gradient-based updates can use any standard gradient-based learning rule. We used momentum in our experiments.

Theoretical Results

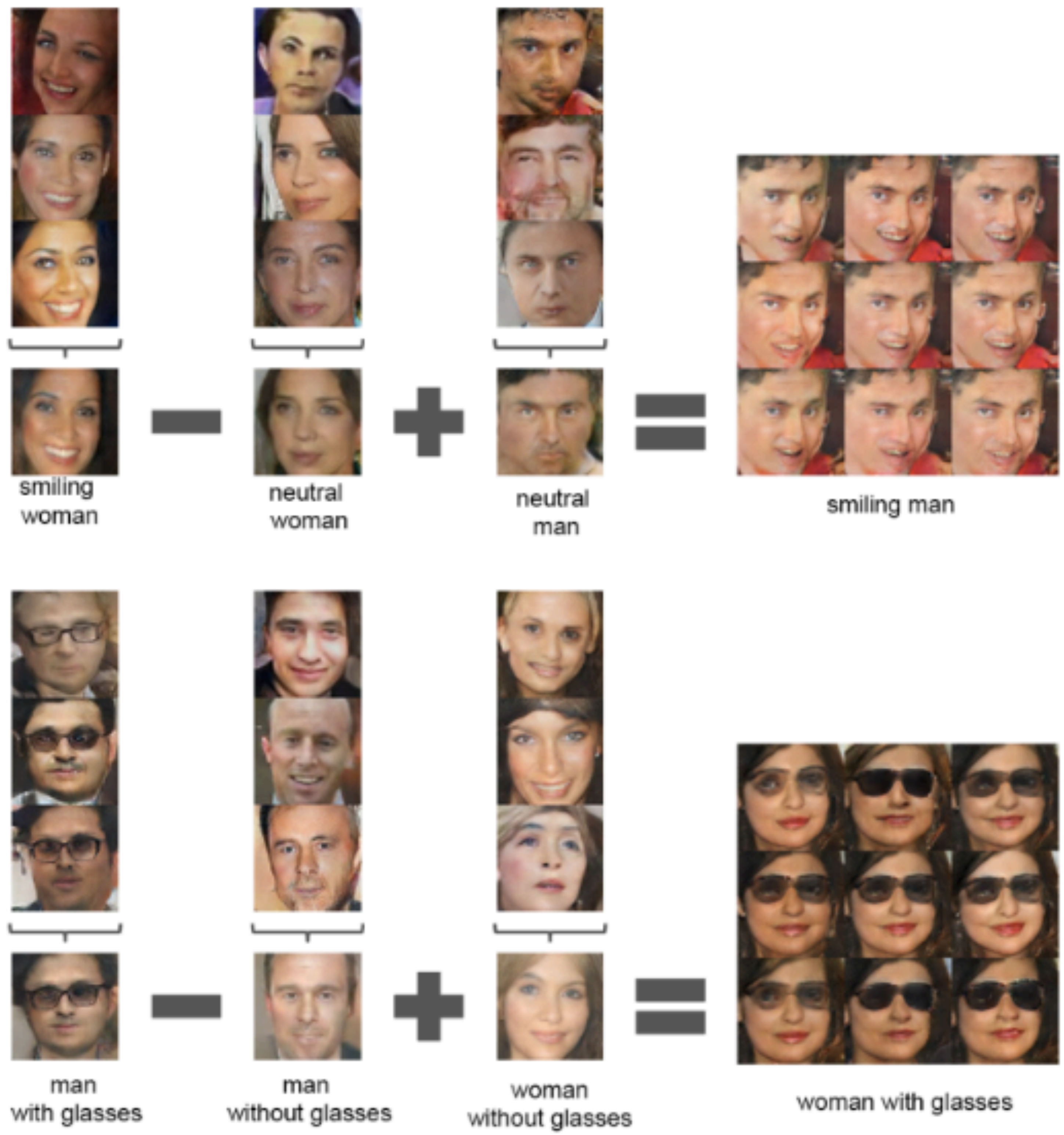
Convergence of Algorithm

Proposition 2. *If G and D have enough capacity, and at each step of Algorithm 1, the discriminator is allowed to reach its optimum given G, and p_g is updated so as to improve the criterion*

$$\mathbb{E}_{\boldsymbol{x}\sim p_{data}}[\log D_G^*(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{x}\sim p_g}[\log(1 - D_G^*(\boldsymbol{x}))]$$

Proof. Consider $V(G, D) = U(p_g, D)$ as a function of p_g as done in the above criterion. Note that $U(p_g, D)$ is convex in p_g. The subderivatives of a supremum of convex functions include the derivative of the function at the point where the maximum is attained. In other words, if $f(x) = \sup_{\alpha\in\mathcal{A}} f_\alpha(x)$ and $f_\alpha(x)$ is convex in x for every α, then $\partial f_\beta(x) \in \partial f$ if $\beta = \arg\sup_{\alpha\in\mathcal{A}} f_\alpha(x)$. This is equivalent to computing a gradient descent update for p_g at the optimal D given the corresponding G. $\sup_D U(p_g, D)$ is convex in p_g with a unique global optima as proven in Thm 1, therefore with sufficiently small updates of p_g, p_g converges to p_x, concluding the proof. $\square$

Experiments & Conclusions



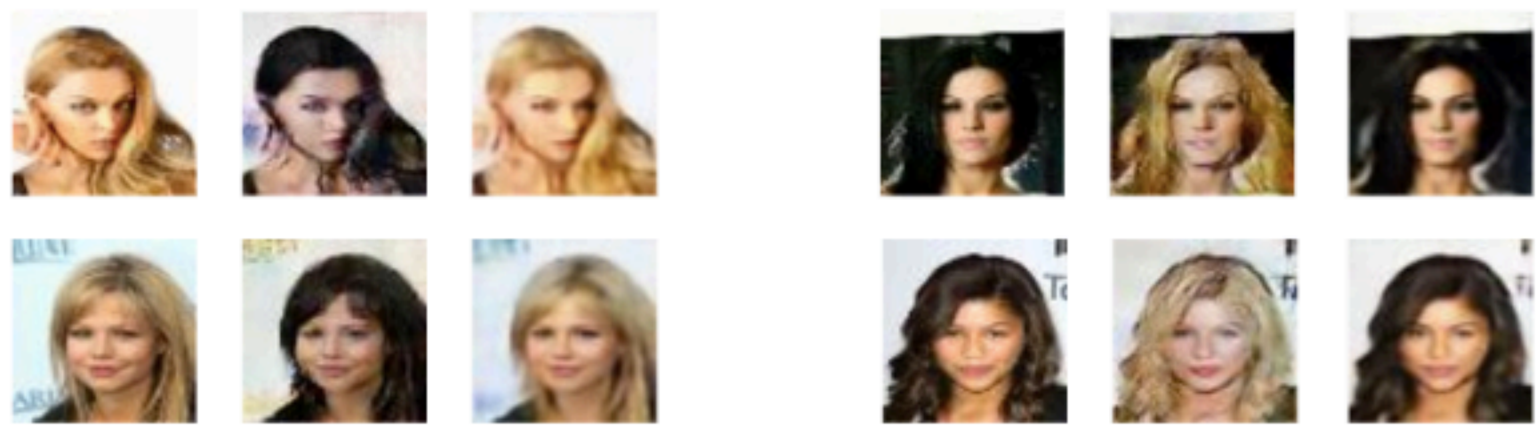
DCGAN , Alec Radford et al.2016

Experiments & Conclusions

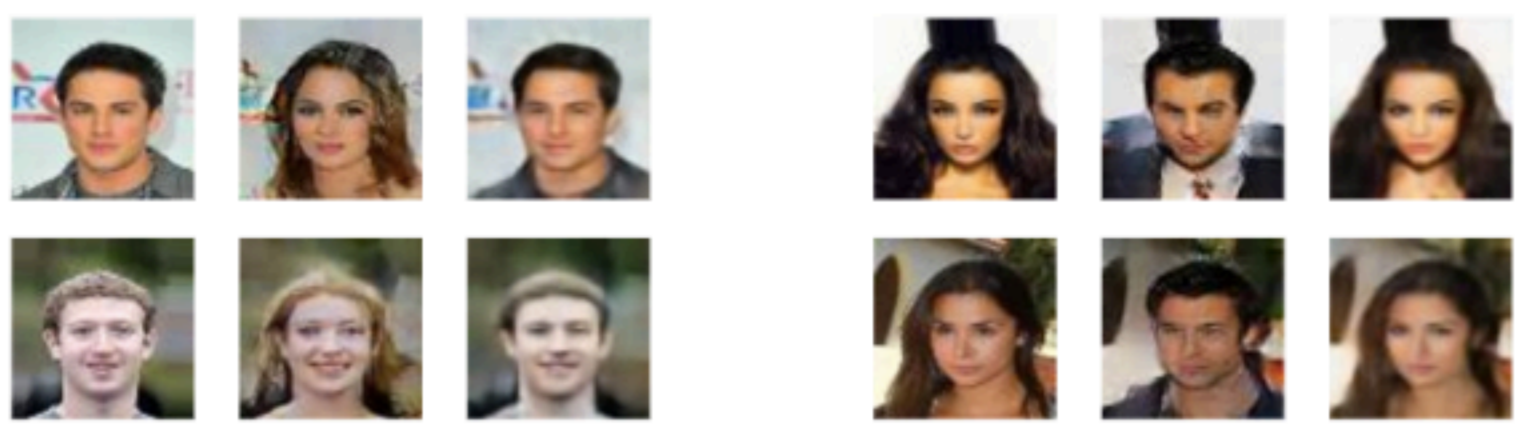
BEGAN , David Berthelot et al.2017

Experiments & Conclusions

Example results of hair color conversion



Example results of gender conversion (CelebA)



DiscoGAN , Kim taeksoo et al.2017

QnA